Let $\mathcal{S}$ be the set of stars of interest and $\mathcal{E}_i, i \in \mathcal{S}$ be the set of elements with measured abundances for each star. The abundances are measured relative to that of Hydrogen, and given as $X_{ij}, j \in \mathcal{E}_i, i \in \mathcal{S}$. These are taken to be normally distributed in log-space, so that $\log X_{ij}$ has variance σ_{ij}. To model accretion processes, we assume that the abundance of a given element for a given star has been modified by accretion, such that if $X_{ij,0}$ is the bulk distribution,

$$X_{ij} = X_{j,0}\left[(1-f_i) + f_i \exp\left(\delta_i \Theta(T_j - T_{c,i})\right)\right], \tag{1}$$

where $X_{j,0}$ is the bulk abundance of element j, taken to be uniform across stars, f_i is the accreted mass fraction in the photosphere of star i, δ_i is an enhancement or depletion factor for the material around star i, Θ is the Heaviside step function, equal to zero for negative arguments and one for positive ones, T_j is the (known) condensation temperature of element j, and $T_{c,i}$ is the condensation temperature associated with star i. As a result,

$$\log X_{ij} = \log X_{j,0} + \ln\left[(1-f_i) + f_i \exp\left(\delta_i \Theta(T_j - T_{c,i})\right)\right], \tag{2}$$

and $\log X_{ij}$ has the same variance as $\log X_{ij,0}$. For notational convenience we let

$$Q_{ij} \equiv \ln\left[(1-f_i) + f_i \exp\left(\delta_i \Theta(T_j - T_{c,i})\right)\right]. \tag{3}$$

We would like to infer $X_{ij,0}$ as well as f_i, δ_i and $T_{c,i}$. The likelihood function $\mathcal{L}$ for this problem is

$$\ln \mathcal{L} = \sum_{i\in\mathcal{S}} \sum_{j\in\mathcal{E}_i} \left[-\frac{1}{2}\ln(2\pi\sigma_{ij}) - \frac{(\log X_{ij} - Q_{ij} - \log X_{j,0})^2}{2\sigma_{ij}^2}\right]. \tag{4}$$

Now let

$$A_j \equiv \sum_{i\in\mathcal{S}\,\text{s.t.}\, j\in\mathcal{E}_i} \frac{1}{\sigma_{ij}^2}, \tag{5}$$

$$B_j \equiv \sum_{i\in\mathcal{S}\,\text{s.t.}\, j\in\mathcal{E}_i} \frac{2(Q_{ij} - \log X_{ij})}{\sigma_{ij}^2}, \tag{6}$$

and

$$C_j \equiv \sum_{i\in\mathcal{S}\,\text{s.t.}\, j\in\mathcal{E}_i} (Q_{ij} - \log X_{ij})^2 \sigma_{ij}^{-2}. \tag{7}$$

Then

$$\ln \mathcal{L} = -\sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{E}_i} -\frac{1}{2}\ln(2\pi\sigma_{ij}) - \sum_{j\in\cup_i\mathcal{E}_i} \frac{1}{2}\left(A_j \log X_{j,0}^2 + B_j \log X_{j,0} + C_j\right). \tag{8}$$

Completing the square for each j yields

$$\ln \mathcal{L} = -\sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{E}_i} -\frac{1}{2}\ln(2\pi\sigma_{ij}) - \sum_{j\in\cup_i\mathcal{E}_i} \frac{A_j}{2}\left(\log X_{j,0} + \frac{B_j}{2A_j}\right)^2 + \frac{1}{2}\left(C_j - \frac{B_j^2}{4A_j}\right). \tag{9}$$

Integrating the likelihood over $\log X_{j,0}$ yields the marginalised likelihood function

$$\ln \bar{\mathcal{L}} = -\sum_{i\in\mathcal{S}}\sum_{j\in\mathcal{E}_i} -\frac{1}{2}\ln(2\pi\sigma_{ij}) - \sum_{j\in\cup_i\mathcal{E}_i} -\frac{1}{2}\ln\frac{2\pi}{A_j} + \frac{1}{2}\left(C_j - \frac{B_j^2}{4A_j}\right). \tag{10}$$

Dropping the constant offsets, we find

$$\ln \bar{\mathcal{L}} = -\sum_{j\in\cup_i\mathcal{E}_i} \frac{1}{2}\left(C_j - \frac{B_j^2}{4A_j}\right). \tag{11}$$

This may be used to find the joint distribution of δ_i, f_i, and $T_{c,i}$, at which point sampling may be used to determine the distributions of $X_{j,0}$.