Lecture 4: First-Order Logic

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April 14, 2019

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1 Pros & Cons of Propositional Logic

1.1 Pros

- Propositional logic is **declarative**: pieces of syntax correspond to facts.
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases).
- Propositional logic is **compositional** (i.e. meaning of $B_{1.1} \wedge P_{1.2}$ is derived from meaning of $B_{1.1}$ and of $P_{1.2}$).

- Meaning in propositional logic is **context-independent**.

1.2 Cons

Propositional logic has very limited expressive power (e.g. cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square).

2 First-Order Logic

Whereas propositional logic assumes world contains facts, first-order logic assumes the world contains:

Objects People, houses, numbers, theories etc.

Relations Red, round, bogus, prime, etc.

Functions Father of, best friend, one more than etc.

3 Atomic Sentences

The simplest form of first-order logic is in **atomic sentences**.

Atomic Sentence $predicate(term_1, ..., term_n)$ or $term_1 = term_2$.

Term \$function(term_1, ..., term_n) or *constant* or *variable*.

4 Complex Sentences

Complex sentences are made from atomic sentences using connectives.

5 Truth in First-Order Logic

Sentences are true with respect to a model and an interpretation. Models contain ≥ 1 objects (**domain elements**) and relations among them.

Interpretation specifies referents for:

- constant symbols $\implies$ objects
- predicate symbols $\implies$ relations
- function symbols $\implies$ functional relations

6 Problems with FOL

6.1 Frame Problem

Finding an elegant way to handle non-change rather than repeated frame axioms.

6.2 Qualification Problem

True descriptions of real actionss require endless caveats— what if gold is slippery or nailed down or...

6.3 Ramification Problem

Real actions have many secondary consequences— what about dust on the gold, war and tear on gloves, etc.