Lecture 4: First-Order Logic

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April 10, 2019

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1 Pros & Cons of Propositional Logic

1.1 Pros

- Propositional logic is **declarative**: pieces of syntax correspond to facts.
- Propositional logic allows partial/disjunctive/negated information (unlike most data structures and databases).
- Propositional logic is **compositional** (i.e. meaning of $B_{1.1} \wedge P_{1.2}$ is derived from meaning of $B_{1.1}$ and of $P_{1.2}$).
- Meaning in propositional logic is **context-independent**.

1.2 Cons

Propositional logic has very limited expressive power (e.g. cannot say "pits cause breezes in adjacent squares" except by writing one sentence for each square).

2 First-Order Logic

Whereas propositional logic assumes world contains facts, first-order logic assumes the world contains:

Objects People, houses, numbers, theories etc.

Relations Red, round, bogus, prime, etc.

Functions Father of, best friend, one more than etc.

3 Atomic Sentences

The simplest form of first-order logic is in **atomic sentences**.

Atomic Sentence $predicate(term_1, ..., term_n)$ or $term_1 = term_2$.

Term \$function(term_1, ..., term_n) or *constant* or *variable*.

4 Complex Sentences

Complex sentences are made from atomic sentences using connectives.

5 Truth in First-Order Logic

Sentences are true with respect to a model and an interpretation. Models contain ≥ 1 objects (**domain elements**) and relations among them.

Interpretation specifies referents for:

- constant symbols $\implies$ objects
- predicate symbols $\implies$ relations
- function symbols $\implies$ functional relations