Logic practical

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1 Logic exercises

1. Consider a vocabulary (sometimes called a *propositional language*) with 4 propositional symbols: A, B, C and D.
   (a) How many models are there for this language?
   (b) How many models for the formula $A \vee \neg C$?
   (c) How many models for the formula $A \wedge \neg C$?
   (d) How many models for the formula $A \vee \neg A$?
   (e) How many models for the formula $A \wedge \neg A$?
   (f) How many *literals* are there for this language?
2. Which of the following are correct?
   (a) $\models A \vee \neg A$ (This statement is equivalent to True $\models A \vee \neg A$, i.e. it asserts that $A \vee \neg A$ is valid).
   (b) $\models A$
   (c) $A \models B$
   (d) $A \wedge B \models B$
   (e) $A \wedge \neg A \models B$
   (f) $A \Leftrightarrow B \models A \vee B$
   (g) $A \Leftrightarrow B \models \neg A \vee B$
3. (*) (The Deduction theorem). Prove that for any two propositional formulae α and β:
   - $\alpha \models \beta$ if and only if $\models \alpha \Rightarrow \beta$
4. Convert each of the following formulae into CNF:
   (a) $A \wedge B$
   (b) $A \Leftrightarrow B$
   (c) $A \wedge \neg B \Rightarrow C$

5. For each of the following inference procedures for propositional logic, decide (i) whether they are sound and (ii) whether they are complete.

   (a) For any KB and any α: $KB \vdash \alpha$ (i.e. any formula can be derived from any knowledge base.)

   (b) For any KB and any α: $KB \wedge (\beta \wedge \neg\beta) \vdash \alpha$

   (c) For any KB and any α construct all possible models of KB. Derive α from KB if and only if α is true in at least one of these models.

   (d) For any KB and any α construct all possible models of KB. Derive α from KB if and only if α is true in all of these models.

2 Graph colouring

The graph colouring problem is as follows. You are given an undirected graph and some colours. You have to work out whether the graph can be coloured: i.e. whether it is possible to assign a colour to each vertex of the graph so that directly connected vertices are always of different colours.

Suppose your input is the graph $A-B-C$ and you have two colours. Encode this problem as a CNF in such a way the CNF is satisfied if and only if the graph can be coloured.

Write a Python program which takes as input an arbitrary graph colouring problem and solves it using the PicoSAT SAT solver. This SAT solver can be conveniently accessed using the pycosat Python package (`https://pypi.python.org/pypi/pycosat`). You can choose how the graph colouring problem is represented, just be sure that your code can deal with *any* graph colouring problem. Of course, big graph colouring problems might take a while to solve ...