

$$\frac{\frac{\frac{\rho_1(f) = \forall \alpha, \alpha \rightarrow \text{int}}{\rho_1 \vdash f : \alpha? \rightarrow \text{int}} (p3)}{A} \quad \frac{\frac{\rho_1 \vdash 20 : \text{int}}{\rho_1[f \mapsto \forall \alpha, t_x \rightarrow \text{int}] \vdash f \ 20 : \text{int}} (p9)}{\boxed{\vdash} \vdash \text{let } f \ x = \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) \text{ in } f \ 20 \text{ end} :} (p8)$$

A:

$$\frac{\frac{\frac{???}{\rho \vdash x : \text{int}} (p???) \quad \frac{\rho \vdash 10 : \text{int}}{\rho \vdash x < 10 : \text{bool}} (p5)}{\rho \vdash x < 10 : \text{bool}} (p1) \quad \frac{\rho \vdash 42 : \text{int}}{B} (p1)}{\frac{[x \mapsto t_x, f \mapsto t_x \rightarrow \text{int}] \vdash \text{if } x < 10 \text{ then } 42 \text{ else } f(x+1) :} (p7)$$

B:

$$\frac{\frac{\rho(x) = \forall \alpha, \alpha \rightarrow \text{int}}{\rho \vdash f :} (p3) \quad \frac{\frac{\frac{\rho(x) = \forall \text{int}}{\rho \vdash x : \text{int}} (p3)}{\rho \vdash x+1 : \text{int}} (p4)}{\rho \vdash f(x+1) : \text{int}} (p9)$$