Lab 3

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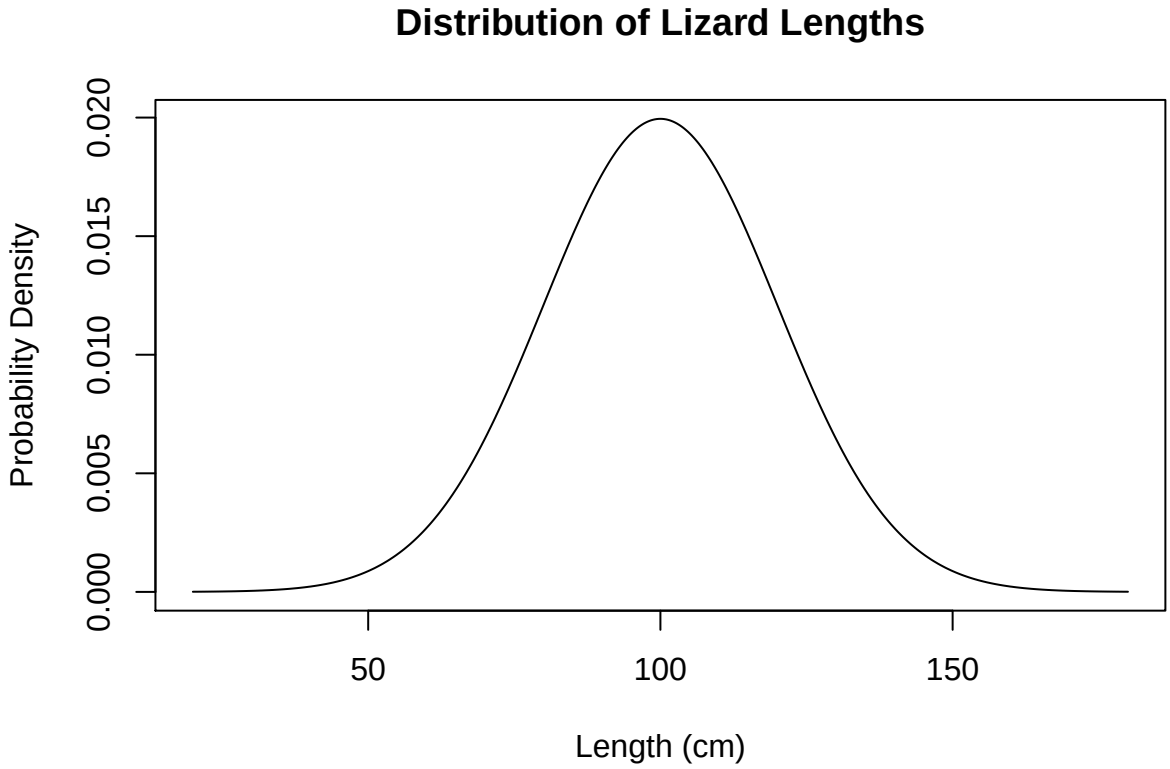
Section: 91973 Friday 9am

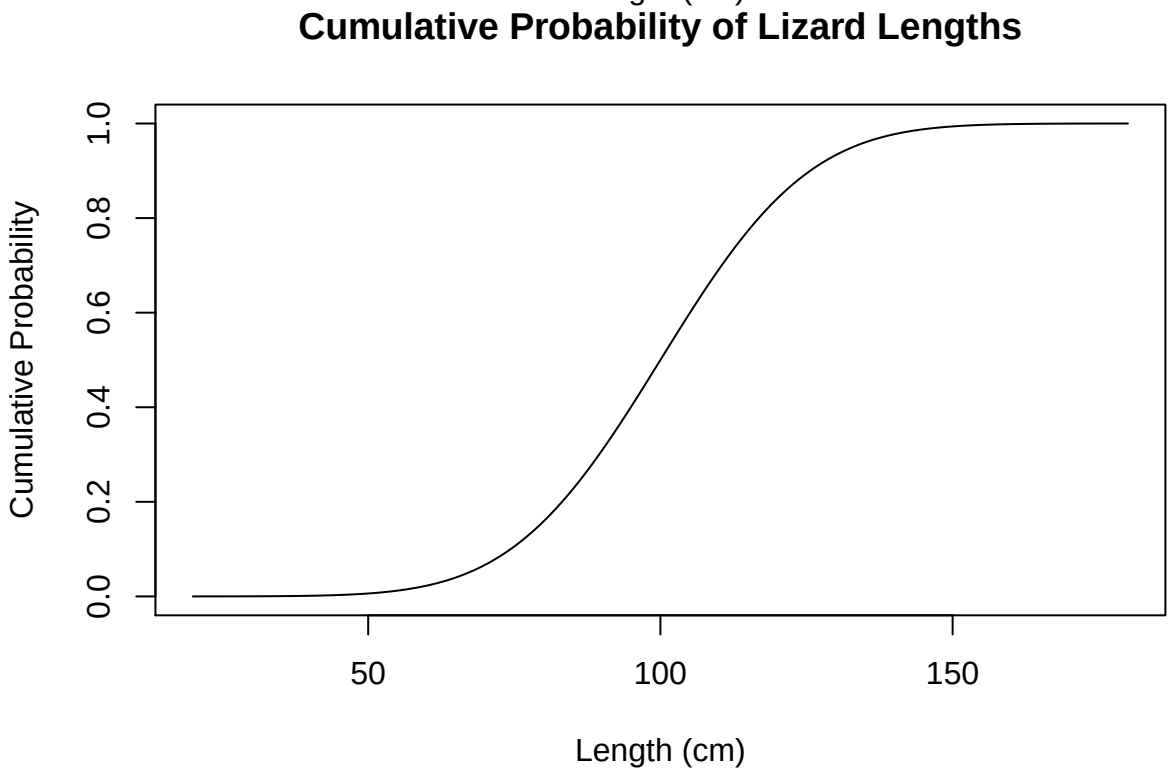
Question 1

Script

```
#set parameters
mu <- 100
sigma_squared <- 400
sigma <- sqrt(sigma_squared)

#plot the probability density function of lizard lengths
q <- seq(mu - 4 * sigma, mu + 4 * sigma, by = .1)
pdf <- dnorm(q,mu,sigma)
plot(pdf~q, type = 'l',
     xlab = "Length (cm)",
     ylab = "Probability Density",
     main = "Distribution of Lizard Lengths")

#plot the cumulative distribution function of lizard lengths
cdf <- pnorm(q, mu, sigma)
plot(cdf~q, type = 'l',
     xlab = "Length (cm)",
     ylab = "Cumulative Probability",
     main = "Cumulative Probability of Lizard Lengths")
```

Output


Distribution of Lizard Lengths
Probability Density
0.000
0.005
0.010
0.015
0.020
50
100
150
Length (cm)


Cumulative Probability of Lizard Lengths
Cumulative Probability
0.0
0.2
0.4
0.6
0.8
1.0
50
100
150
Length (cm)

Answers

Question 2

Script

```
#set parameters
mu <- 100
sigma_squared <- 400
sigma <- sqrt(sigma_squared)
#what is the probability density for a length of 75 cm?
dnorm(75, mu, sigma)

#what is the probability that a lizard will be less than
# or equal to 75 cm?
pnorm(75, mu, sigma)

#greater than 120 cm?
1 - pnorm(120, mu, sigma)

#between 95 and 115 cm?
pnorm(115, mu, sigma) - pnorm(95, mu, sigma)

#at least 40 cm different from the mean?
2 * pnorm(mu - 40, mu, sigma)

#closer than 1.3 sigma to the mean?
pnorm(mu + 1.3 * sigma, mu, sigma) - pnorm(mu - 1.3 * sigma, mu, sigma)

#further than 1.5 sigma from the mean?
2 * pnorm(mu - 1.5 * sigma, mu, sigma)

#further than 0.7 sigma from the mean?
2 * pnorm(mu - .7 * sigma, mu, sigma)

#what are the quartiles of the distribution?
#1st quartile
qnorm(.25, mu, sigma)

#2nd quartile
qnorm(.5, mu, sigma)

#3rd quartile
qnorm(.75, mu, sigma)

#4th quartile
qnorm(1, mu, sigma)

#2/3 of observations are expected to lie below what value?
qnorm(2/3, mu, sigma)

#80% of observations are expected to lie above what value?
qnorm(1 - .8, mu, sigma)
```

Output

```
#what is the probability density for a length of 75 cm?
```

```
## [1] 0.009132454
```

```
#what is the probability that a lizard will be less than
# or equal to 75 cm?
```

```
## [1] 0.1056498
```

```
#greater than 120 cm?
```

```
## [1] 0.1586553
```

```
#between 95 and 115 cm?
```

```
## [1] 0.372079
```

```
#at least 40 cm different from the mean?
```

```
## [1] 0.04550026
```

```
#closer than 1.3 sigma to the mean?
```

```
## [1] 0.806399
```

```
#further than 1.5 sigma from the mean?
```

```
## [1] 0.1336144
```

```
#further than 0.7 sigma from the mean?
```

```
## [1] 0.4839273
```

```
#what are the quartiles of the distribution?
#1st quartile
```

```
## [1] 86.5102
```

```
#2nd quartile
```

```
## [1] 100
```

```
#3rd quartile
```

```
## [1] 113.4898
```

```
#4th quartile
```

```
## [1] Inf
```

```
#2/3 of observations are expected to lie below what value?
```

```
## [1] 108.6145
```

```
#80% of observations are expected to lie above what value?
```

```
## [1] 83.16758
```

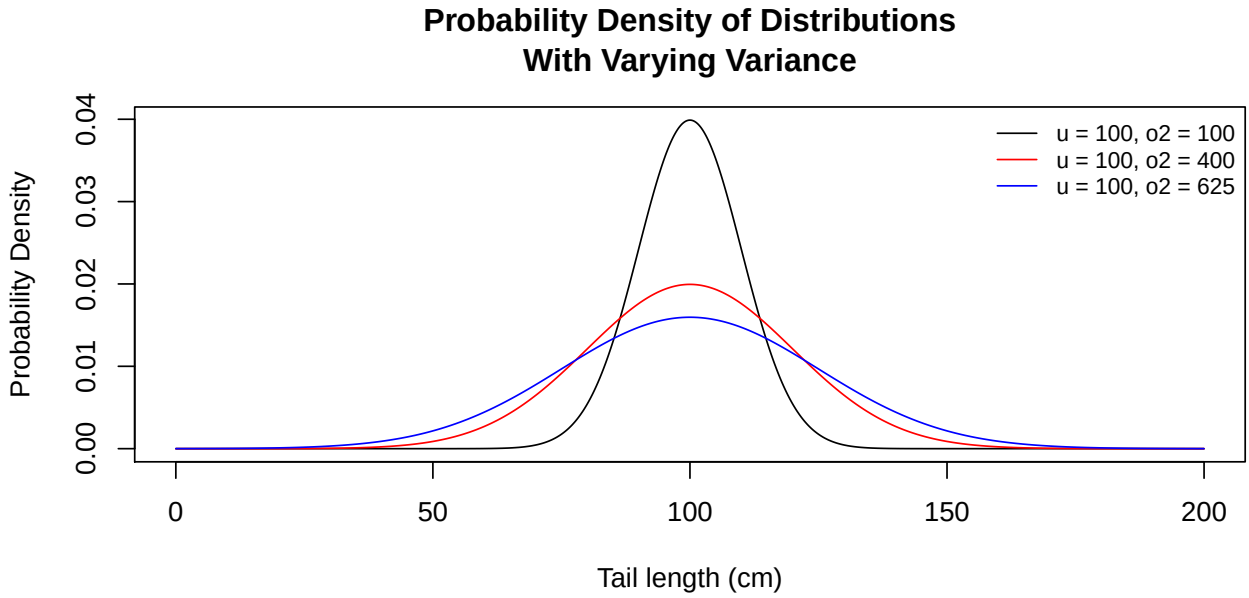
Answers

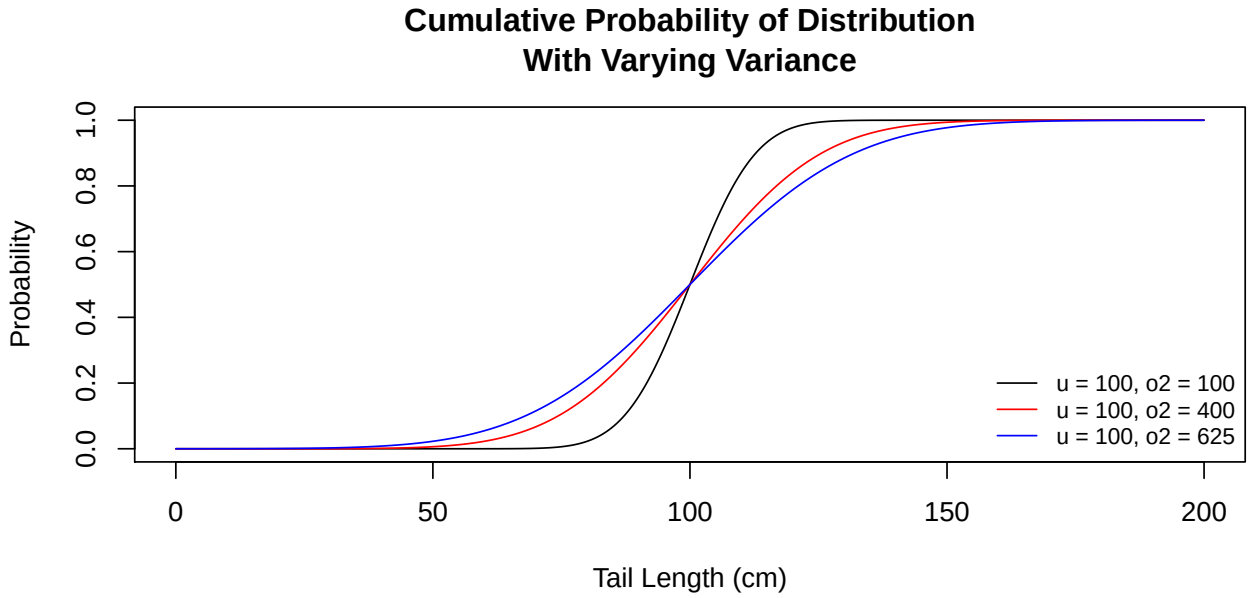
Question 3

Script

```
#set parameters
mu <- rep(100,3)
sigma_squared <- c(100,400,625)
sigma <- sqrt(sigma_squared)
q <- seq(min(mu) - 4 * max(sigma), max(mu) + 4 * max(sigma), by = .1)

#calculate pdfs
pdfs <- sapply(sigma, FUN = function(x){dnorm(q,mu,x)})

#plot 3 PDFs on the same graph, each with a mean of 100,
#but with different variances. plot each line in a different
#color
par(mfrow=c(2,1)) #put 2 plots on one figure
plot(pdfs[,1]~q, type = 'l',
     xlab = "Tail length (cm)",
     ylab = "Probability Density",
     main = "Probability Density of Distributions\nWith Varying Variance")
lines(pdfs[,2]~q, col = 'red')
lines(pdfs[,3]~q, col = 'blue')

#add legend
legend("topright",
       legend = c("u = 100, o2 = 100", "u = 100, o2 = 400", "u = 100, o2 = 625"),
       col = c('black','red','blue'),
       lty = 1,
       bty = "n",
       cex = .8)

#the second plot should show three corresponding CDFs
#calculate the CDFs
cdfs <- sapply(sigma, FUN = function(x){pnorm(q,mu,x)})
plot(cdfs[,1]~q, type = 'l',
     xlab = 'Tail Length (cm)',
     ylab = "Probability",
     main = "Cumulative Probability of Distribution\nWith Varying Variance")
lines(cdfs[,2]~q, col = 'red')
lines(cdfs[,3]~q, col = 'blue')

#add legend
legend("bottomright",
       legend = c("u = 100, o2 = 100", "u = 100, o2 = 400", "u = 100, o2 = 625"),
       col = c('black','red','blue'),
       lty = 1,
       bty = "n",
       cex = .8)
```

Output





Answers

Question 4

Script

```
#if the mean height of British men is 177 cm with a standard
#deviation of 7.1 cm, what proportion of British men are
#excluded from being spies by this height restriction?
#assume height follows a normal distribution
```

```r
#set parameters for men
men_mu <- 177
men_sigma <- 7.1

#proportion excluded
(p_men_excluded <- 1 - pnorm(180, men_mu, men_sigma))

#the mean height of british women is 163.3 cm,
#with a standard deviation of 6.4 cm. assuming a normal distribution
#of female height, what fraction of women meet MI5's  height standard?

#set parameters for women
women_mu <- 163.3
women_sigma <- 6.4

#fraction of women that meet MI5's height standard
pnorm(173, women_mu, women_sigma)

#imagine that mi5 wants to change its maximum height for
#female spies. its goal is to exclude the same proportion
#of women as men.
#What should the new maximum height for women be?
#(round your answer to the nearest centimeter)
(women_q <- qnorm(1 - p_men_excluded, women_mu, women_sigma))
round(women_q, 0)

#Sean Connery, the original James Bond, is 183 cm tall.
#By how many standard deviations does he exceed the height
# limit for spies?
(183 - men_mu) / (men_sigma)
```

Output

```r
#if the mean height of British men is 177 cm with a standard
#deviation of 7.1 cm, what proportion of British men are
#excluded from being spies by this height restriction?
#assume height follows a normal distribution
```

```
## [1] 0.3363172
```

```r
#the mean height of british women is 163.3 cm,
#with a standard deviation of 6.4 cm. assuming a normal distribution
#of female height, what fraction of women meet MI5's  height standard?
```

```
## [1] 0.9351929
```

```r
#imagine that mi5 wants to change its maximum height for
#female spies. its goal is to exclude the same proportion
#of women as men.
#What should the new maximum height for women be?
#(round your answer to the nearest centimeter)
```

```
## [1] 166.0042
```

```
## [1] 166
```

```
#Sean Connery, the original James Bond, is 183 cm tall.
#By how many standard deviations does he exceed the height
# limit for spies?
```

```
## [1] 0.8450704
```

Answers

Question 5

Script

```
#What is the probability that a normal random variable will
#have a value within 1 standard deviation of the mean?
1 - 2 * pnorm(-1)

#What is the probability that it will be within 5 standard
#deviations of the mean?
1 - 2 * pnorm(-5)

#Fill in the blank: A normal random variable has a 50%
#probability of lying within ___ standard deviations of
#the mean
abs(qnorm(.25))

#Fill in the blank: A normal random variable has a 95%
#probability of lying within ___ standard deviations of
#the mean
abs(qnorm(.025))

#Fill in the blank: A normal random variable has a 99%
#probability of lying within ___ standard deviations of
#the mean
abs(qnorm(.005))
```

Output

```
#What is the probability that a normal random variable will
#have a value within 1 standard deviation of the mean?
```

```
## [1] 0.6826895
```

```
#What is the probability that it will be within 5 standard
#deviations of the mean?
```

```
## [1] 0.9999994
```

```
#Fill in the blank: A normal random variable has a 50%
#probability of lying within ___ standard deviations of
#the mean
```

```
## [1] 0.6744898
```

```r
#Fill in the blank: A normal random variable has a 95%
#probability of lying within ___ standard deviations of
#the mean
```

```
## [1] 1.959964
```

```r
#Fill in the blank: A normal random variable has a 99%
#probability of lying within ___ standard deviations of
#the mean
```

```
## [1] 2.575829
```

Answers