University of New South Wales

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Assignment 1

Complex Analysis

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Since f_1 is complex differentiable on $\mathbb{C}$, f_1 cannot have any singularities in the unit ball and so all the of singularities of f_1/f_2 in the unit ball come from $1/f_2$. We therefore turn our attention to $1/f_2$. It is simple to see that if $1/f_2$ has infinitely many poles in the unit ball then f_2 has infinitely many zeros in the unit ball (at the same locations). Say these zeros are at $\{z_n\}_n$ then by the complex analogue of the Bolzano-Wierstrass theorem there exists a subsequence $\{x_{n_k}\}_k \subseteq \{x_n\}_n$ such that $\lim_{k \longrightarrow \infty} x_{n_k}$ exists.
Suppose x^* is the limit point of this sequence then by theorem 1 of lecture notes 4, we have that $f_2 \equiv 0$ for all z in the unit ball.
This means that if f_1/f_2 has infinitely many poles in the unit ball then $f_2 \equiv 0$ in the unit ball. So if we disregard this degenerate case then f_1/f_2 cannot have infinitely many poles in the unit ball.

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