University of New South Wales

School of Mathematics and Statistics

Assignment 1

Complex Analysis

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We can identify any Mobius transformation

$$f(z) = \frac{az+b}{cz+d} \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{1}$$

with a matrix in $GL_2(\mathbb{C})$.
Also see that if

$$f \sim \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{2}$$

$$g \sim \begin{bmatrix} e & f \\ g & h \end{bmatrix} \tag{3}$$

then

$$(g \circ f) \sim \begin{bmatrix} e & f \\ g & h \end{bmatrix} \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{4}$$

$$\sim \begin{bmatrix} ea+fc & eb+fd \\ ag+ch & gb+dh \end{bmatrix}. \tag{5}$$

Noice that this identificaiton is not unqiue as

$$\frac{az+b}{cz+d} \sim \lambda \begin{bmatrix} a & b \\ c & d \end{bmatrix} \tag{6}$$

for $\lambda \in \mathbb{C}$. It turns out that the group of automorphisms on $\hat{\mathbb{C}}$ (the projective line) is isomorphic the the projective linear group, that is

$$\mathrm{Aut}(\hat{\mathbb{C}}) \cong \mathrm{PGL}(2, \mathbb{C}). \tag{7}$$

We are interested finite subgroups of $\mathrm{Aut}(\hat{\mathbb{C}})$ and hence we are interested in finite subgroups of $\mathrm{PGL}(2, \mathbb{C})$.