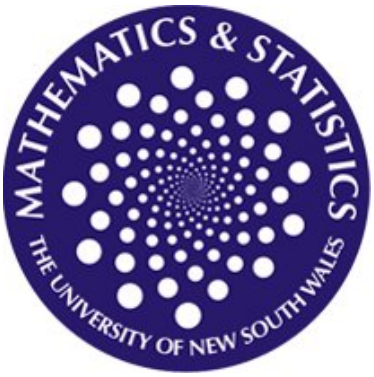
UNSW
AUSTRALIA



University of New South Wales

School of Mathematics and Statistics

Assignment 1

Complex Analysis

Author:
Adam J. Gray

Student Number:
3329798

Question 5

We proprose the function

$$f(z) := \frac{z}{\log(|z|)}$$

and prove that it satisfies the desired attributes. If we write $z = re^{i\theta}$ and $f(z) = u(r,\theta) + iv(r,\theta)$ then we can use the polar form of the Cauchy Riemann equations.
It is not hard to show that that Cauchy Riemann equations have an equivelent polar form

$$\frac{\partial u}{\partial r} = \frac{1}{r}\frac{\partial v}{\partial \theta} \qquad \frac{\partial v}{\partial r} = -\frac{1}{r}\frac{\partial u}{\partial \theta}$$

then see that

$$\begin{aligned}\frac{\partial u}{\partial r} &= \frac{\partial}{\partial r}\left[\frac{\cos(\theta)}{\log(r)}\right] \\ &= \frac{-\cos(\theta)}{r\log^2(r)}\end{aligned}$$

and

$$\begin{aligned}\frac{\partial v}{\partial \theta} &= \frac{\partial}{\partial \theta}\left[\frac{\sin(\theta)}{\log(r)}\right] \\ &= \frac{\cos(\theta)}{\log(r)}\end{aligned}$$