



UNSW
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UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

Assignment

Functional Analysis

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Declaration

I declare that this assessment item is my own work, except where acknowledged, and has not been submitted for academic credit elsewhere.

I certify that I have read and understood the University Rules in respect of Student Academic Misconduct.

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Date

Question 1

See that

$$\begin{aligned}\langle Tx, x \rangle &= \overline{\langle x, Tx \rangle} \\ &\geq 0\end{aligned}$$

which means

$$\begin{aligned}\overline{\langle x, Tx \rangle} &= \langle x, Tx \rangle \\ &= \langle T^*x, x \rangle\end{aligned}$$

so

$$\langle (T - T^*)x, x \rangle = 0$$

for all $x \in \mathcal{H}$.

We just want to show that $(T - T^*) = 0$. Say $\langle Jx, x \rangle = 0$ for all $x \in \mathcal{H}$ then for $x, y \in \mathcal{H}$ see that

$$\begin{aligned}\langle J(x + y), x + y \rangle &= \langle Jx, y \rangle + \langle Jy, x \rangle + \underbrace{\langle Jx, x \rangle}_{=0} + \underbrace{\langle Jy, y \rangle}_{=0} \\ &= \langle Jx, y \rangle + \langle Jy, x \rangle\end{aligned}\tag{1}$$

and see that

$$\begin{aligned}\langle J(x + iy), x + iy \rangle &= \langle Jx, iy \rangle + \langle Jiy, x \rangle + \underbrace{\langle Jx, x \rangle}_{=0} + \underbrace{\langle Jiy, iy \rangle}_{=0} \\ &= -i\langle Jx, y \rangle + i\langle Jy, x \rangle\end{aligned}\tag{2}$$

Now multiplying (1) by i and subtracting it from (2) yields

$$\langle Jx, y \rangle = 0$$

for all $x, y \in \mathcal{H}$ which implies $J \equiv 0$. Letting $J = (T - T^*)$, yields $(T - T^*) = 0$ as required.

So $T \equiv T^*$. □