UNIVERSITY OF NEW SOUTH WALES

SCHOOL OF MATHEMATICS AND STATISTICS

Assignment

Number Theory

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Question 1

Part a

Use the character table given in lectures for $\mathbb{Z}_5$, extended to a Dirichlet character,to evaluate

$$\sum_{i=1}^{4} \chi_i(n)\overline{\chi_i(b)}, \qquad \text{for each } b \in \mathbb{U}_5.$$

Part b

Use the results of (a) to prove, in detail, that there are infnitely many primes congruent to $1 \mod 5$, $2 \mod 5$, and $3 \mod 5$ and $4 \mod 5$.

Solution

Part a

$$\sum_{i=1}^{4} \chi_i(n)\overline{\chi_i(b)} = \begin{cases} 0 & \text{if } n \not\equiv b \mod 5 \\ 4 & \text{if } n \equiv b \mod 5 \end{cases}$$

This follows immediatly from the orthogonality relation proved in lectures.

Part b