# OOPA

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# A quick note on code layout

After much thought, I decided to layout my code as follows. I have a class called Options.cs which is used to price European options using the Heston model. This class is then used within the Calibrator.cs class which is used to calibrate the Heston model to real world data. On top of these classes, I have the class MCPaths.cs, which is used to generate the paths (either as standard or with anithetic sampling) for the Monte Carlo pricing class, which is called OptionsMC.cs. This class is used to price European, Asian and lookback calls, both with standard Monte Carlo and with parallel Monte Carlo using anithetic sampling. Finally, I wrote several classes which implement some of the interfaces which we were given. The first such class, InterfaceFill.cs, implements the base level interfaces. OtherInterfaceFill.cs and AnotherInterfaceFill.cs are used to implement interfaces which are derived from these base level interfaces. FinalInterfaceFill.cs was used to implement other interfaces, but was only neccesary for my own testing of the Heston.cs class.

Having written code to price European put and call options in the Heston model I was able to fill out the below table as requested, where the parameters used were

$$r = 2.5\%$$
  
 $\theta^* = 3.98\%, \ \kappa^* = 157.68\%, \ \sigma = 57.51\%, \ \rho = -57.11\%, \ v = 1.75\%$  (1)  
 $S = 100.$ 

Strike $K$	Option exercise $T$	Price $C(0, S, v)$
100	1	7.27
100	2	11.74
100	3	15.48
100	4	18.77
100	15	43.17

Table 1: Prices of "at the money" call options in the Heston model using Heston formula.

## Task 2.3

Having written code to price European put and call options in the Heston model using a Monte Carlo Algorithm I was able to fill out the below table as requested, where the parameters used were

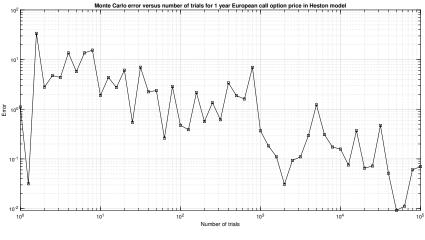
$$r = 10\%$$
  
 $\theta^* = 6\%, \ \kappa^* = 200\%, \ \sigma = 40\%, \ \rho = 50\%, \ v = 4\%$   
 $S = 100.$  (2)

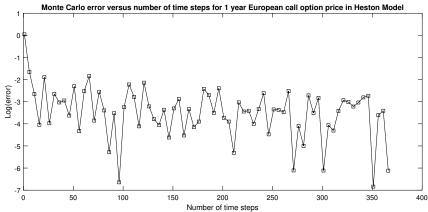
Strike K	Option exercise $T$	Price $C(0, S, v)$
100	1	13.6
100	2	22.6
100	3	30.1
100	4	37.2
100	15	77.9

Table 2: Prices of "at the money" call options in the Heston model using a Monte Carlo method.

## **Task 2.4**

The above graph shows the convergence of the Monte Carlo pricing and the Heston formula pricing of a 1 year European Call option as the number of paths used increases. The number of time steps used was fixed at 365 and the Heston parameters were as seen in (2). The graph below shows the convergence of the pricing when the number of trials was fixed at 100000 and the time steps increase. It is clear that the number of trials used is more important for the accuracy of our Monte Carlo, once we are working with a reasonable number of time steps per year





Having used the BFGS algorithm to create a calibrator for the Heston model using the market data given in the question I was able to achieve the following results.

Parameters	Calibrated value
$\kappa^*$	1.60287
$\theta^*$	0.10323
$\sigma$	0.45889
$\rho$	-0.47647
v	0.09975

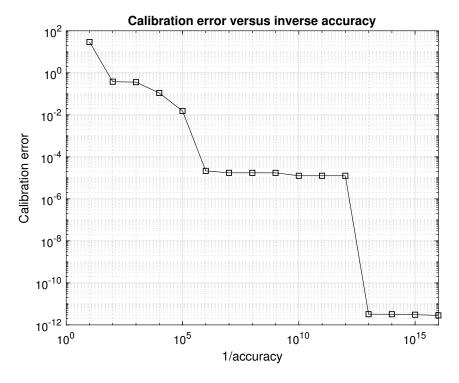
Table 3: Calibrated parameters using accuracy  $1 \times 10^{-3}$ , maxIts = 1000, S = 100 and parameters (1) as initial guess.

What I found was that with accuracy  $1\times 10^{-3}$  the calibrated parameter values would vary dependent on the initial guess parameters, while with accuracy  $1\times 10^{-15}$  they would converge to the below parameters from most reasonable starting points which I tried. The calibration returned FinishedOk in the cases of both of these tables, with mean square error to the order of 0.01 in the first case and  $1\times 10^{-12}$  in the second, as I will explore in the next task.

Parameters	Calibrated value
$\kappa^*$	1.94372
$\theta^*$	0.09980
$\sigma$	0.35828
$\rho$	-0.56084
v	0.09891

Table 4: Calibrated parameters using accuracy  $1 \times 10^{-15}$ , maxIts = 1000, S = 100 and parameters (1) as initial guess.

As I have previously noted, I found that when I used a smaller accuracy parameter my parameters would calibrate to the same values, while with larger accuracy it would depend on the initial guess. The below graph shows the mean squared error of the calibrated model when different accuracies were used, with initial guess and market data as in Task 2.5.



After calibrating the model with the two accuracy levels in Task 2.5, I used the parameters in Tables 3 and 4 to test the calibrated models against the market data we were given. As expected given the tiny error of the model with parameters from Table 4, it's prices agreed to several decimal places with the market data. This wasn't the case with the previous model, as can be seen below.

Strike $K$	Option exercise $T$	Market Call Price
80	1	25.72
90	1	18.93
80	2	30.49
100	2	19.36
100	1.5	16.58

Table 5: Market data from the question

Strike $K$	Option exercise $T$	Market Call Price
80	1	25.80
90	1	18.95
80	2	30.59
100	2	19.31
100	1.5	16.52

Table 6: Calibrated model prices with parameters from Table 3

I was also able to find situations where the calibration did not work as it should. For example, using parameters (1), I created market data using the Heston formula using the same strike prices and maturities as in the market data in the question. I then, from several initial guesses, with accuracy  $1 \times 10^{-15}$ , observed whether the calibrated model's parameters matched the actual parameters I had used. In many cases they did, but in certain cases, whilst the calibrator converged, it was not to the right parameters. For example, with initial guess  $\kappa^* = \theta^* = \sigma = \rho = v = 10\%$ , the calibrator converged to the following parameters, with a mean squared error of 0.01:

$$\theta^* = -0.31\%, \ \kappa^* = -491.12\%, \ \sigma = 50.25\%, \ \rho = -0.83\%, \ v = 2.92.\%$$

And while we can see that with these parameters the prices will roughly match the market data, it is clear that with any other strike prices and maturities this calibrated model will be way off. This is a case of a "local minimum" for the Heston calibration function, whereby it converges,

Strike $K$	Option exercise $T$	Model price	Market Price
80	1	23.00	23.03
90	1	14.47	14.51
80	2	26.24	26.19
100	2	11.69	11.74
100	1.5	9.68	9.62

Table 7: Calibrated model prices with parameters from Table 3

but not to the right parameters! Using very large parameters in the original guess also lead to situations like this, with calibration being successful but with huge mean squared error between the calibrated model prices and the market data.

Having written code to price Asian arithmetic options in the Heston model using a Monte Carlo algorithm I was able to fill out the below table as requested, using parameters (2).

Strike $K$	Option exercise $T$	$T_1,, T_m$	MC Price
100	1	0.75, 1.00	12.0
100	2	0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75	11.3
100	3	1.0, 2.0, 3.0	19.9

Table 8: Prices of "at the money" Asian call options in the Heston model using a Monte Carlo method.

## **Task 2.8**

Having written code to price lookback options in the Heston model using a Monte Carlo algorithm I was able to fill out the below table as requested, using parameters (2).

Option Exercise $T$	MC Price
1	19.1
3	36.7
5	49.5
7	58.9
9	66.9

Table 9: Prices of lookback options in the Heston model using a Monte Carlo method.

# Additional investigation

One means of additional investigation I undertook was to write code for parallel implementation of some of our Monte Carlo methods. I used Parallel.For loops in my code to allow parallel implementation and then used the Interlocked.Exchange command to update variables so as to ensure that everything happened in the right order. Interestingly, I found that this consistently reduced computation time for European and Asian options by 15-30%, but, while on average it reduced computation time for lookback options there was a lot more variance in the times taken here. I priced the same option 5 times in a row with both methods using the Stopwatch class in C# to get computation times.

Normal MC	Parallel MC
7.4789	6.1622
7.1997	6.0032
7.3187	6.0654
7.7478	6.3696
6.9976	5.2767

Table 10: 5 computation time in seconds for price of 1 year European call with 100000 trials and 365 time steps.

The average time for normal MC was 7.3485 seconds compared to 5.9754 seconds for parallel MC, an improvement of 18.69%.

Normal MC	Parallel MC
6.1713	4.3416
5.8967	3.7910
5.9571	3.9080
5.9075	4.0570
5.9759	4.3758

Table 11: Computation time in seconds for price of 1 year Asian call with 100000 trials, 365 time steps, and observation times 0.25, 0.5, 0.75, 1.

The average time for normal MC was 5.9817 seconds compared to 4.0947 seconds for parallel MC, an improvement of 31.55%.

Normal MC	Parallel MC
27.70	27.71
29.41	22.44
27.76	24.36
28.02	21.78
24.80	27.25

Table 12: Computation time in seconds for price of 1 year lookback option with 100000 trials and 365 time steps.

The average time for normal MC was 27.5395 seconds compared to 24.7094 seconds for parallel MC, an improvement of 10.28%.

The other additional investigation I undertook was to implement variance reduction methods for Monte Carlo. I used anithetic sampling whereby you generate two paths at once. You first sample from the normal distribution and generate a path as usual. The difference is that you then change the sign of your normal sample and use this to generate a second path. In theory this method has two benefits. Firstly you need only sample from the normal distribution half as many times to generate N paths as you would with normal sampling. Secondly it is often the case that this kind of sampling reduces the variance of Monte Carlo prices. Intuitively we ensure that the normal samples for each trial are stratified, with exactly as many coming from the negative side of the normal distribution as from the positive. Mathematically if we have two samples  $Y_1$  and  $Y_2$  and some function f it follows that

$$Var\left(\frac{f(Y_1) + f(Y_2)}{2}\right) = \frac{Var(f(Y_1)) + Var(f(Y_2)) + 2Cov(f(Y_1), f(Y_2))}{4}$$

and so if  $Cov(f(Y_1), f(Y_2))$  is negative we get a reduction in variance, which if often the case for anthethic samples  $(Y_1 = -Y_2)$ . This was the case in my implementation of this method in the Heston model for pricing European options. Anithetic sampling reduced the variance of the Monte Carlo prices by quite a large factor. I priced the same 1 year European call option 25 times with both methods, computed the variance of their prices and repeated 5 times.

Normal MC	Anithetic MC
0.020	0.005
0.013	0.009
0.024	0.006
0.015	0.013
0.110	0.009

Table 13: Variance of 25 pricings of a 1 year European call option with 100000 trials and 365 price steps.

This is useful as it makes our method more accurate. So to summarise, in my additional investigation, I used parallelisation and anithetic sampling to successfully reduce both the computational time and variance of my Monte Carlo options pricing.

## Code

## Options.cs class

```
2 using System;
3 using MathNet.Numerics.Integration;
4 using System. Numerics;
5 using System.Collections.Generic;
6 using System.Linq;
7 using System.Text;
8 using System.Threading.Tasks;
10 namespace HestonCalibrationAndPricing
11 {
       /// <summary>
12
       /// This class prices options within the Heston model
13
      /// </summary>
14
      public class Options
15
16
17
           public const int numberParams = 5;
           public const int kappaIndex = 0;
18
           public const int thetaIndex = 1;
19
           public const int sigmaIndex = 2;
           public const int rhoIndex = 3;
21
           public const int vIndex = 4;
22
23
           private double r;
24
           private double kappaStar;
           private double thetaStar;
26
           private double sigma;
27
28
           private double rho;
           private double v;
29
30
           private double S;
31
           public Options(double r, double S, double kappaStar, double thetaStar,
33
               double sigma, double rho, double v)
34
               if (r \le 0 \mid | S \le 0 \mid | sigma \le 0 \mid | v \le 0)
35
```

```
37
                                                        throw new System.ArgumentException("r, S, sigma, v must be
                                                                    positive");
38
                                            this.r = r; this.S = S; this.kappaStar = kappaStar; this.thetaStar
39
                                                        = thetaStar;
                                           this.sigma = sigma; this.rho = rho; this.v = v;
40
                               }
41
                               public Options(double r, double S, double[] parameters)
43
44
                                            this.r = r; this.S = S;
45
                                           kappaStar = parameters[kappaIndex];
46
47
                                            thetaStar = parameters[thetaIndex];
                                           sigma = parameters[sigmaIndex];
48
49
                                           rho = parameters[rhoIndex];
50
                                           v = parameters[vIndex];
51
                                           if (r <= 0 || S <= 0 || sigma <= 0 || v <= 0)
52
                                           {
53
                                                        throw new System.ArgumentException("r, S, sigma, v must be
                                                                    positive");
                                           }
55
56
                               }
57
                                   /// <summary>
59
                                  /// Prices a European call option within the Heston model.
61
                                   /// </summary>
                                  /// <param name = "T">The maturity date of the option in years.</param
62
                                   /// <param name = "K">The options' strike price.</param>
63
                                  /// <returns>Option price.</returns>
64
                               public double EuropeanCallPrice(double T, double K)
65
66
                                           if (T < 0 | | K < 0)
67
                                           {
68
                                                        throw new System.ArgumentException("T, K must be non-negative")
69
                                           }
70
71
                                           // set up paramters
72
                                            double[] b = { kappaStar - rho * sigma, kappaStar };
                                           double[] u = \{ 0.5, -0.5 \};
74
                                            double a = kappaStar * thetaStar;
75
76
                                           Complex i = new Complex(0, 1);
77
                                            // function implementing part of Heston formula
78
                                           Func<int, double, double> RealP = (j, phi) =>
79
80
81
                                                        Complex temp1 = new Complex(-b[j], rho * sigma * phi);
82
83
                                                        Complex tempp1 = new Complex(-phi * phi, 2 * u[j] * phi);
                                                        Complex d = Complex.Pow(temp1 * temp1 - sigma * sigma * tempp1,
84
                                                                      0.5);
                                                        85
                                                                    j] - rho * sigma * phi * i + d, -1);
86
                                                        Complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma)) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * T * i + (a / (sigma * sigma))) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * ((b[j] - a)) + (complex c = r * phi * sigma)) * (
87
                                                                    rho * sigma * phi * i - d) * T - 2 * Complex.Log((1 - g *
                                                                    Complex.Exp(-T * d)) / (1 - g)));
                                                        Complex bigD = ((b[j] - rho * sigma * phi * i - d) / (sigma * phi * phi * i - d) / (sigma * phi * i 
                                                                    sigma)) * ((1 - Complex.Exp(-T * d)) / (1 - g * Complex.Exp
                                                                     (-T * d));
                                                        Complex littlePhi = Complex.Exp(c + bigD * v + phi * i * Math.
89
                                                                    Log(S));
                                                        Complex value = Complex.Exp(-i * phi * Math.Log(K)) * littlePhi
```

{

```
/ (i * phi);
 91
                                                 return value.Real;
                                      };
 92
 93
                                      double[] P = new double[2];
 94
 95
                                      // integrate with appropriate number of steps and length to
 96
                                                 approximate infinite integral
                                      P[0] = 0.5 + (1.0 / Math.PI) * SimpsonRule.IntegrateComposite(x => 0.5 + (1.0 / Math.PI)) * SimpsonRule.IntegrateComposite(x => 0.5 + (1.0 / Math.PI))) * SimpsonRule.IntegrateComposite(x => 0.5 + (1.0 / Math.PI)))) * SimpsonRule.Integrate(x => 0.5 + (1.0 / Math.PI)))) * 
                                                RealP(0, x), 0.000001, 50, 100);
                                      P[1] = 0.5 + (1.0 / Math.PI) * SimpsonRule.IntegrateComposite(x =>
 98
                                                RealP(1, x), 0.000001, 50, 100);
 99
                                      return S * P[0] - K * Math.Exp(-r * T) * P[1];
                            }
101
102
                               /// <summary>
103
                               /// Prices a European put option within the Heston model.
104
105
                               /// </summary>
                               /// <param name = "T">The maturity date of the option in years.</param
106
                               /// <param name = "K">The options' strike price.</param>
107
                              /// <returns>Option price.</returns>
108
109
                            public double EuropeanPutPrice(double T, double K)
                            {
110
                                      if (T < 0 | | K < 0)
                                      {
112
                                                 throw new System.ArgumentException("T, K must be non-negative")
114
115
                                      // put call parity
116
                                      return EuropeanCallPrice(T, K) - S + K * Math.Exp(-r * T);
117
118
119
                               /// <summary>
120
                               /// Forms an array of the model parameters.
121
                               /// </summary>
                              /// <returns>Model parameters.</returns>
123
                            public double[] ParamsAsArray()
124
125
                                       double[] paramsArray = new double[Options.numberParams];
126
                                      paramsArray[kappaIndex] = kappaStar;
127
                                      paramsArray[thetaIndex] = thetaStar;
128
                                      paramsArray[sigmaIndex] = sigma;
                                      paramsArray[rhoIndex] = rho;
130
                                      paramsArray[vIndex] = v;
131
132
                                      return paramsArray;
133
                 }
135
136 }
```

### OptionsMC.cs class

```
1 using System;
2 using MathNet.Numerics.Distributions;
3 using System.Collections.Generic;
4 using System.Linq;
5 using System.Text;
6 using System.Threading.Tasks;
7 using System.Threading;
8
9 namespace HestonCalibrationAndPricing
10 {
11 /// <summary>
```

```
methods.
       /// </summary>
13
      public class OptionsMC
14
15
           public const int kappaIndex = 0;
16
           public const int thetaIndex = 1;
17
           public const int sigmaIndex = 2;
18
           public const int rhoIndex = 3;
19
           public const int vIndex = 4;
20
21
           private double r;
22
           private double K;
23
           private double kappaStar;
           private double thetaStar;
25
26
           private double sigma;
27
           private double rho;
           private double v;
28
29
           private double S;
30
           public OptionsMC(double r, double K, double kappaStar, double thetaStar
               , double sigma, double rho, double v, double S)
32
               if (2 * kappaStar * thetaStar <= sigma * sigma)</pre>
33
               {
34
                   throw new System.ArgumentException("Feller condition violated."
                       );
               }
37
               if (r <= 0 || K <= 0 || sigma <= 0 || S <= 0 || v <= 0)
38
39
                   throw new System.ArgumentException("r, K, sigma, S must be
40
                        positive");
               }
41
42
43
               this.r = r;
               this.K = K; this.kappaStar = kappaStar; this.thetaStar = thetaStar;
44
               this.sigma = sigma; this.rho = rho; this.v = v; this.S = S;
46
47
           \verb"public OptionsMC" (double r, double kappaStar, double thetaStar, double
48
               sigma, double rho, double v, double S)
           {
49
               if (2 * kappaStar * thetaStar <= sigma * sigma)</pre>
50
51
                   throw new System.ArgumentException("Feller condition violated."
52
                       );
               }
53
               if (r \le 0 \mid | K \le 0 \mid | sigma \le 0 \mid | S \le 0 \mid | v \le 0)
54
55
                    throw new System.ArgumentException("r, K, sigma, S must be
56
                        positive");
               }
57
58
               this.r = r;
               this.kappaStar = kappaStar; this.thetaStar = thetaStar;
               this.sigma = sigma; this.rho = rho; this.v = v; this.S = S;
60
62
           public OptionsMC(double r, double K, double[] paramss, double S)
63
64
65
               this.r = r;
               this.K = K; kappaStar = paramss[kappaIndex]; thetaStar = paramss[
                   thetaIndex];
               sigma = paramss[sigmaIndex]; rho = paramss[rhoIndex]; v = paramss[
67
                   vIndex]; this.S = S;
               if (2 * kappaStar * thetaStar <= sigma * sigma)</pre>
68
               {
```

 $/\!/\!/$  This class prices options within the Heston model using Monte Carlo

```
throw new System.ArgumentException("Feller condition violated."
70
               }
71
               if (r <= 0 || K <= 0 || sigma <= 0 || S <= 0 || v <= 0)
72
73
                    throw new System.ArgumentException("r, K, sigma, S must be
74
                        positive"):
               }
75
           }
76
77
            /// <summary>
78
            /// Prices a European call option within the Heston model using Monte
79
                Carlo methods.
            /// </summary>
            /// <param name = "T">The maturity date of the option in years.</param
81
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
82
                 our path generator to take to reach time T.</param>
            /// <param name = "numberPaths">The number of simulations we wish to
                run.</param>
            /// <returns>Option price.</returns>
84
           public double EuropeanCallOptionPriceMC(double T, int
85
               numberTimeStepsPerPath, int numberPaths)
86
               if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
87
                    throw new System.ArgumentException("Parameters must be positive
89
                        ");
               }
90
91
               // set up counter and path generator
92
               double count = 0;
93
               MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
94
95
               // generate paths, evaluate payoff on each, then average and
96
                    discount
               for (int i = 0; i < numberPaths; i++)</pre>
97
98
                    count += Math.Max(path.PathGenerator(T, S,
99
                        numberTimeStepsPerPath) - K, 0);
               }
100
               return Math.Exp(-r * T) * count / numberPaths;
101
           }
103
            /// <summary>
104
            /// Prices a European put option within the Heston model using Monte
105
                Carlo methods.
            /// </summary>
106
            /// <param name = "T">The maturity date of the option in years.</param
107
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
108
                 our path generator to take to reach time T.</param>
            /// <param name = "numberPaths">The number of simulations we wish to
109
                run.</param>
            /// <returns>Option price.</returns>
           public double EuropeanPutOptionPriceMC(double T, int
111
               numberTimeStepsPerPath, int numberPaths)
           {
112
               if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
113
114
                    throw new System.ArgumentException("Parameters must be positive
115
                        ");
               }
116
117
               // set up counter and path generator
118
               double count = 0;
119
               MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
```

```
// generate paths, evaluate payoff on each, then average and
122
                    discount
                for (int i = 0; i < numberPaths; i++)
123
124
                    count += Math.Max(K - path.PathGenerator(T, S,
125
                        numberTimeStepsPerPath), 0);
126
                return Math.Exp(-r * T) * count / numberPaths;
128
129
            /// <summary>
130
            /// Prices a European call option within the Heston model using Monte
131
                Carlo methods with parallelisation.
            /// </summary>
132
            /// <param name = "T">The maturity date of the option in years.</param
133
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
134
                 our path generator to take to reach time T.</param>
            /// <param name = "numberPaths">The number of simulations we wish to
135
                 run.</param>
            /// <returns>Option price.</returns>
136
           public double EuropeanCallOptionPriceMCParallel(double T, int
137
               numberTimeStepsPerPath, int numberPaths)
           {
138
               if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
               {
140
                    throw new System.ArgumentException("Parameters must be positive
                        ");
142
143
               // set up counter and path generator
144
                double count = 0;
145
               MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
146
147
               // generate paths in parallel, evaluate payoff on each
148
               Parallel.For(0, numberPaths, (i) =>
149
               {
                    double pathAdd = Math.Max(path.PathGenerator(T, S,
151
                        numberTimeStepsPerPath) - K, 0);
                    // update count in such a way as to protect thread safety
152
                    Interlocked.Exchange(ref count, count + pathAdd);
153
               });
155
                // average and discount
157
               return Math.Exp(-r * T) * count / numberPaths;
           }
158
159
            /// <summary>
160
            /// Prices a European call option within the Heston model using Monte
                Carlo methods using anithetic sampling .
            /// </summary>
162
            /// <param name = "T">The maturity date of the option in years.</param
163
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
                 our path generator to take to reach time T.</param>
             /// <param name = "numberPaths">The number of simulations we wish to
                run.</param>
            /// <returns>Option price.</returns>
166
           public double EuropeanCallOptionPriceMCAnithetic(double T, int
167
               numberTimeStepsPerPath, int numberPaths)
               if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
169
170
                    throw new System.ArgumentException("Parameters must be positive
171
                        ");
               }
```

```
// set up counter and path generator
174
                double count = 0;
175
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
176
177
178
                // generate two paths half as many times, evaluate payoff on each
                int halfNumPaths = (int)Math.Ceiling(numberPaths / 2.0);
179
                for (int i = 0; i < halfNumPaths; i++)</pre>
180
181
                    count += Math.Max(path.PathGeneratorAnithetic(T, S,
182
                        numberTimeStepsPerPath)[0] - K, 0) + Math.Max(path.
PathGeneratorAnithetic(T, S, numberTimeStepsPerPath)[1] - K
                         , 0);
                }
183
184
                // average and discount
                return Math.Exp(-r * T) * count / (2 * halfNumPaths);
186
            }
187
188
            /// <summary>
189
            /// Prices a European call option within the Heston model using Monte
                Carlo methods using both parallelisation and anithetic sampling .
191
            /// </summary>
            /// <param name = "T">The maturity date of the option in years.</param>
192
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
193
                our path generator to take to reach time T.</param>
            /// <param name = "numberPaths">The number of simulations we wish to
194
                run.</param>
            /// <returns>Option price.</returns>
195
            public double EuropeanCallOptionPriceMCAnitheticParallel(double T, int
196
                numberTimeStepsPerPath, int numberPaths)
197
                if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
198
199
                {
                    throw new System.ArgumentException("Parameters must be positive
200
                        ");
                }
201
                // set up counter and path generator
203
204
                double count = 0;
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
205
206
                // generate two paths half as many times, in parallel, evaluate
                    payoff on each
                int halfNumPaths = (int)Math.Ceiling(numberPaths / 2.0);
209
                Parallel.For(0, halfNumPaths, (i) =>
210
                    double[] paths = path.PathGeneratorAnithetic(T, S,
                        numberTimeStepsPerPath);
                    double pathAdd = Math.Max(paths[0] - K, 0) + Math.Max(paths[1]
                        - K. O):
213
                    Interlocked.Exchange(ref count, count + pathAdd);
                });
214
215
                // average and discount
216
                return Math.Exp(-r * T) * count / (2.0 * halfNumPaths);
217
219
            /// <summary>
220
            /// Prices a European put option within the Heston model using Monte
221
                Carlo methods using both parallelisation and anithetic sampling .
            /// </summary>
222
            /// <param name = "T">The maturity date of the option in years.</param>  
223
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
224
                our path generator to take to reach time T.</param>
            /// <param name = "numberPaths">The number of simulations we wish to
225
                run.</param>
```

```
/// <returns>Option price.</returns>
226
           public double EuropeanPutOptionPriceMCAnitheticParallel(double T, int
227
                numberTimeStepsPerPath, int numberPaths)
228
                if (T <= 0 || numberTimeStepsPerPath <= 0 || numberPaths <= 0)
229
230
                    throw new System.ArgumentException("Parameters must be positive
231
                        ");
                }
233
                // set up counter and path generator
234
                double count = 0:
235
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
236
237
                // generate two paths half as many times, in parallel, evaluate
238
                    payoff on each
                int halfNumPaths = (int)Math.Ceiling(numberPaths / 2.0);
239
                Parallel.For(0, halfNumPaths, (i) =>
240
241
                    double[] paths = path.PathGeneratorAnithetic(T, S,
242
                        numberTimeStepsPerPath);
                    double pathAdd = Math.Max(K - paths[0], 0) + Math.Max(K - paths
243
                        [1], 0);
                    Interlocked.Exchange(ref count, count + pathAdd);
244
                });
245
                // average and discount
247
                return Math.Exp(-r * T) * count / (2.0 * halfNumPaths);
           }
249
250
251
           /// <summary>
252
           /// Checks that the times used for pricing Asian options make sense.
253
           /// </summary>
254
           /// <param name = "T">An array containing the onservation times of the
255
               Asian option.</param>
           /// <param name = "exerciseT">The Asian option's exercise time.</param>
256
           private void CheckAsianOptionInputs(double[] T, double exerciseT)
258
259
                if (T.Length == 0)
                    throw new System.ArgumentException("Need at least one
260
                        monitoring date for Asian option.");
261
                if (T[0] <= 0)
262
                    throw new System.ArgumentException("The first monitoring date
                        must be positive.");
264
                for (int i = 1; i < T.Length; ++i)
265
266
                    if (T[i - 1] >= T[i])
                        throw new System.ArgumentException("Monitoring dates must
268
                            be increasing");
                }
269
270
                if (T[T.Length - 1] > exerciseT)
271
                    throw new System.ArgumentException("Last monitoring time must
272
                        not be greater than the exercise time");
           }
273
274
             /// <summary>
275
            /// Prices an Asian call option within the Heston model using Monte
276
                 Carlo methods.
             /// </summary>
277
             /// <param name = "T">An array containing the onservation times of the
278
                 Asian option.</param>
             /// <param name = "exerciseT">The Asian option's exercise time.</param
279
                >
```

```
run.</param>
             /// <param name = "numberTimeStepsPerPath">The number of steps we wish
281
                  our path generator to take to reach time exerciseT.</param>
             /// <returns>Option price.</returns>
282
           public double PriceAsianCallMC(double[] T, double exerciseT, int
283
                numberPaths, int numberTimeStepsPerPath)
284
                if (numberTimeStepsPerPath <= 0 || numberPaths <= 0)</pre>
286
                {
                    throw new System.ArgumentException("Monte Carlo settings must
287
                        be positive");
                }
288
                // check parameters make sense
290
                CheckAsianOptionInputs(T, exerciseT);
291
292
                // set up parameter, path generator and counter
293
294
                int M = T.Length;
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
295
                double pathCounter = 0;
297
                // generate paths between observation times, evaluate payoff
298
                    function
                for (int i = 0; i < numberPaths; i++)</pre>
299
                    double priceCount = 0;
301
                    double holder = S;
                    double deltaT = T[0];
303
304
                    for (int j = 0; j < M; j++)
305
306
                        if (j > 0)
307
                            deltaT = T[j] - T[j - 1];
308
309
310
                        // generate path from previous time point to next with
                            appropriate number time steps
                        int stepNumber = (int)Math.Ceiling(deltaT *
                            numberTimeStepsPerPath / exerciseT);
                        holder = path.PathGenerator(deltaT, holder, stepNumber);
312
                        priceCount += holder;
313
314
                    double pathPayoff = Math.Max(priceCount / M - K, 0);
                    pathCounter += pathPayoff;
316
                7
317
318
                // average and discount
319
                return Math.Exp(-r * exerciseT) * (pathCounter / numberPaths);
320
           }
321
           /// <summary>
323
            /// Prices an Asian put option within the Heston model using Monte
324
                 Carlo methods.
             /// </summary>
325
             /// <param name = "T">An array containing the onservation times of the
                  Asian option.</param>
             /// <param name = "exerciseT">The Asian option's exercise time.</param
             /// <param name = "numberPaths">The number of simulations we wish to
328
                 run.</param>
             /// <param name = "numberTimeStepsPerPath">The number of steps we wish
329
                  our path generator to take to reach time exerciseT.</param>
330
             /// <returns>Option price.</returns>
           public double PriceAsianPutMC(double[] T, double exerciseT, int
331
                numberPaths, int numberTimeStepsPerPath)
332
                if (numberTimeStepsPerPath <= 0 || numberPaths <= 0)</pre>
```

/// <param name = "numberPaths">The number of simulations we wish to

```
{
334
                    throw new System.ArgumentException("Monte Carlo settings must
335
                        be positive");
336
337
                // check parameters make sense
338
                CheckAsianOptionInputs(T, exerciseT);
339
340
                // set up parameter, path generator and counter
                int M = T.Length;
342
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
343
                double pathCounter = 0;
344
345
                // generate paths between observation times, evaluate payoff
                    function
                for (int i = 0; i < numberPaths; i++)</pre>
347
348
                    double priceCount = 0;
349
350
                    double holder = S;
                    double deltaT = T[0];
351
                    for (int j = 0; j < M; j++)
353
354
                        if (j > 0)
355
                             deltaT = T[j] - T[j - 1];
356
357
                        \ensuremath{//} generate path from previous time point to next with
358
                             appropriate number time steps
359
                        int stepNumber = (int)Math.Ceiling(deltaT *
                            numberTimeStepsPerPath / exerciseT);
                        holder = path.PathGenerator(deltaT, holder, stepNumber);
360
                        priceCount += holder;
361
362
                    double pathPayoff = Math.Max(K - priceCount / M, 0);
363
                    pathCounter += pathPayoff;
364
                }
365
366
                // average and discount
                return Math.Exp(-r * exerciseT) * (pathCounter / numberPaths);
368
369
370
             /// <summary>
371
             /// Prices an Asian call option within the Heston model using Monte
                 Carlo methods with parallelisation.
             /// </summary>
             /// <param name = "T">An array containing the onservation times of the
374
                  Asian option.</param>
             /// <param name = "exerciseT">The Asian option's exercise time.</param
             /// <param name = "numberPaths">The number of simulations we wish to
                 run.</param>
             /// <param name = "numberTimeStepsPerPath">The number of steps we wish
                  our path generator to take to reach time exerciseT.</param>
             /// <returns>Option price.</returns>
378
            public double PriceAsianCallMCParallel(double[] T, double exerciseT,
                int numberPaths, int numberTimeStepsPerPath)
                if (numberTimeStepsPerPath <= 0 || numberPaths <= 0)</pre>
381
                {
382
                    throw new System.ArgumentException("Monte Carlo settings must
383
                        be positive");
                }
385
                // check parameters make sense
386
                CheckAsianOptionInputs(T, exerciseT);
387
388
                // set up parameter, path generator and counter
```

```
int M = T.Length;
390
                double pathCounter = 0;
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
392
393
                // generate paths between observation times in parallel, evaluate
394
                    payoff function
                Parallel.For(0, numberPaths, (i) =>
395
                {
396
                    double priceCount = 0;
                    double holder = S;
398
                    double deltaT = T[0];
399
                    Parallel.For(0, M, (j) =>
400
                    {
401
                        if (j > 0)
                            deltaT = T[j] - T[j - 1];
403
404
                        // generate path from previous time point to next with
405
                            appropriate number time steps
                        int stepNumber = (int)Math.Ceiling(deltaT *
                            numberTimeStepsPerPath / exerciseT);
407
                        holder = path.PathGenerator(deltaT, holder, stepNumber);
                        Interlocked.Exchange(ref priceCount, priceCount + holder);
408
                    });
409
                    double pathPayoff = Math.Max((priceCount / M) - K, 0);
410
                    Interlocked.Exchange(ref pathCounter, pathCounter + pathPayoff)
411
                });
412
                // average and discount
414
                return Math.Exp(-r * exerciseT) * (pathCounter / numberPaths);
415
           }
416
417
             /// <summary>
418
             /// Prices a Lookback option within the Heston model using Monte Carlo
419
                  methods.
             /// </summary>
420
             /// <param name = "exerciseT">The Lookback option's exercise time.</
421
                 param >
             /// <param name = "numberPaths">The number of simulations we wish to
422
                 run.</param>
             /// <param name = "numberTimeStepsPerPath">The number of steps we wish
423
                  our path generator to take to reach time exerciseT.</param>
             /// <returns>Option price.</returns>
424
           public double PriceLookbackCallMC(double exerciseT, int numberPaths,
425
                int numberTimeStepsPerPath)
426
                if(exerciseT <= 0 || numberPaths <=0 || numberTimeStepsPerPath <=
427
                    0)
                {
428
                    throw new System.ArgumentException("Parameters must be positive
                        ");
                }
430
431
                // set up counter, parameter and path generator
432
                double pathCounter = 0;
433
                double deltaT = exerciseT / numberTimeStepsPerPath;
434
                MCPaths path = new MCPaths(r, kappaStar, thetaStar, sigma, rho, v);
436
                // generate paths of length numberTimeStepsPerPaths, evaluate
437
                    payoff on each
                for (double i = 0; i < numberPaths; i++)</pre>
438
440
                    double min = S;
                    double holder = S;
441
                    for (double j = 0; j <= exerciseT; j += deltaT)</pre>
442
                    {
443
                        // take a step with appropriate time change and start point
```

```
holder = path.PathGenerator(deltaT, holder, 1);
445
446
                         // keep track of minimum
447
                         if (holder < min)
448
                              min = holder;
449
                     }
450
451
                     pathCounter += holder - min;
452
454
                // average and discount
455
                return Math.Exp(-r * exerciseT) * (pathCounter / numberPaths);
456
457
458
       }
459
460 }
```

#### MCPaths.cs class

```
1 using System;
2 using MathNet.Numerics.Distributions;
3 using System.Collections.Generic;
4 using System.Linq;
5 using System.Text;
6 using System. Threading. Tasks;
8 namespace HestonCalibrationAndPricing
9 {
       /// <summary>
10
       /// This class is used to create the Monte Carlo paths which will be used
11
           to price options within the Heston
       /// model in the class OptionsMC
12
       /// </summary>
13
14
      public class MCPaths
15
          private double r;
16
17
          private double rho;
          private double kappaStar;
18
          private double thetaStar;
19
          private double sigma;
20
          private double v;
22
          public MCPaths(double r, double kappaStar, double thetaStar, double
23
              sigma, double rho, double v)
24
              if (r <= 0 || sigma <= 0 || v <= 0)
25
26
              {
                  throw new System.ArgumentException("r, sigma, v must be
27
                      positive");
              }
28
              this.r = r; this.rho = rho; this.kappaStar = kappaStar;
30
31
              this.thetaStar = thetaStar; this.sigma = sigma; this.v = v;
32
33
           /// <summary>
34
           /// Returns a simulated future price for a risky asset within the
35
               Heston model.
           /// </summary>
36
           37
                asset price.</param>
           /// <param name = "S">The initial asset price.</param>
38
           /// <param name = "numberTimeStepsPerPath">The number of steps we wish
                the scheme to take to reach time T.</param>
           /// <returns>Simulated asset price.</returns>
41
          public double PathGenerator (double T, double S, int
              numberTimeStepsPerPath)
```

```
if (T <= 0 || S <= 0 || numberTimeStepsPerPath <= 0)
43
               {
44
                   throw new System.ArgumentException("Parameters must be positive
45
                       ");
46
47
               //sample from normal dist
48
               double[] x1 = new double[numberTimeStepsPerPath];
               Normal.Samples(x1, 0, 1);
50
               double[] x2 = new double[numberTimeStepsPerPath];
51
               Normal.Samples(x2, 0, 1);
52
53
               //set up parameters
               double tau = T / numberTimeStepsPerPath;
55
               double sqrtTau = Math.Sqrt(tau);
56
               double sqrtOneMinusRhoSquared = Math.Sqrt(1 - rho * rho);
57
               double alpha = (4 * kappaStar * thetaStar - sigma * sigma) / 8.0;
58
59
               double beta = -kappaStar / 2.0;
               double gamma = sigma / 2.0;
60
               // set up holder variables for Monte Carlo
62
               double y = Math.Sqrt(v);
63
               double s = S;
64
65
               // update holder variables iteratively according to MC scheme.
               for (int i = 0; i < numberTimeStepsPerPath; i++)</pre>
67
                   double deltaZ1 = sqrtTau * x1[i];
69
                   double deltaZ2 = sqrtTau * (rho * x1[i] +
70
                       sqrtOneMinusRhoSquared * x2[i]);
                   s = s + r * s * tau + y * s * deltaZ1;
71
                   double a = (y + gamma * deltaZ2) / (2 * (1 - beta * tau));
72
                   y = a + Math.Sqrt(a * a + alpha * tau / (1 - beta * tau));
73
               }
74
75
               return s;
76
          }
78
79
            /// <summary>
            /// Returns a simulated future price for a risky asset within the
80
                Heston model using anithetic sampling with a view to reducing
                variance.
            /// </summarv>
81
            /// <param name = "T">The future time at which we wish to simulate the
                 asset price.</param>
            /// <param name = "S">The initial asset price.</param>
83
            /// <param name = "numberTimeStepsPerPath">The number of steps we wish
                 the scheme to take to reach time T.</param>
            /// <returns>Simulated asset price.</returns>
           public double[] PathGeneratorAnithetic(double T, double S, int
86
               numberTimeStepsPerPath)
87
               if (T <= 0 || S <= 0 || numberTimeStepsPerPath <= 0)
88
                   throw new System.ArgumentException("Parameters must be positive
90
                       ");
               }
91
92
               // sample from normal dist
93
               double[] x1 = new double[numberTimeStepsPerPath];
94
               double[] x2 = new double[numberTimeStepsPerPath];
96
               Normal.Samples(x1, 0, 1);
97
               Normal.Samples(x2, 0, 1);
98
               //set up parameters
99
               double tau = T / numberTimeStepsPerPath;
```

{

```
double sqrtTau = Math.Sqrt(tau);
101
                double sqrtOneMinusRhoSquared = Math.Sqrt(1 - rho * rho);
102
                double alpha = (4 * kappaStar * thetaStar - sigma * sigma) / 8.0;
103
                double beta = -kappaStar / 2.0;
104
                double gamma = sigma / 2.0;
105
106
                // set up holder variables, now two sets due to anithetic sampling
107
                double y = Math.Sqrt(v);
108
                double y1 = Math.Sqrt(v);
                double s = S;
110
                double s1 = S;
111
112
                // update both sets of holder variables iteratively, using the
113
                   negative of normal sample for anitheic path
                for (int i = 0; i < numberTimeStepsPerPath; i++)</pre>
114
                    double deltaZ1 = sqrtTau * x1[i];
116
                    double deltaZ2 = sqrtTau * (rho * x1[i] +
117
                        sqrtOneMinusRhoSquared * x2[i]);
                    s = s + r * s * tau + y * s * deltaZ1;
118
                    s1 = s1 + r * s1 * tau - y1 * s1 * deltaZ1;
                    double a = (y + gamma * deltaZ2) / (2 * (1 - beta * tau));
120
                    double aa = (y1 - gamma * deltaZ2) / (2 * (1 - beta * tau));
121
                    y = a + Math.Sqrt(a * a + alpha * tau / (1 - beta * tau));
122
123
                    y1 = aa + Math.Sqrt(aa * aa + alpha * tau / (1 - beta * tau));
124
125
                // return array of end points of the two paths.
                double[] sArray = { s, s1 };
127
                return sArray;
128
129
130
131
132 }
```

#### Calibrator.cs class

```
using System;
2 using System.Collections.Generic;
3 using System.Linq;
4 using System. Text;
5 using System.Threading.Tasks;
8 namespace HestonCalibrationAndPricing
10
      public class CalibrationFailedException : Exception
11
12
           public CalibrationFailedException()
13
15
16
           public CalibrationFailedException(string message)
17
               : base(message)
18
           }
19
      }
20
22
      public struct MarketData
23
24
           public double K;
           public double T;
25
           public double Price;
27
29
      public enum CalibrationOutcome
30
```

```
NotStarted,
31
32
           FinishedOK.
           FailedMaxItReached,
33
34
           {\tt FailedOtherReason}
      };
35
36
       /// <summary>
37
       /\!/\!/ This class is used to calibrate the parameters of the Heston Model to
38
           real world data
       /// </summary>
39
      public class Calibrator
40
41
           private const double defaultAccuracy = 1.0e-15;
42
43
           private const int defaultMaxIts = 1000;
           private double accuracy;
44
           private int maxIts;
46
           private List<MarketData> marketList;
47
48
           private double r;
           private double S;
49
           private CalibrationOutcome outcome;
51
52
           private double[] calibratedParams;
53
54
           public Calibrator()
55
56
               accuracy = defaultAccuracy;
               maxIts = defaultMaxIts:
58
               marketList = new List < MarketData > ();
59
60
               r = 0;
               S = 0;
61
           }
62
63
           public Calibrator(double r, double S, int maxIts, double accuracy)
64
65
               if (r \le 0 \mid | S \le 0 \mid | maxIts \le 0)
66
               {
                    throw new System.ArgumentException("Parameters must be positive
68
                        ");
               }
69
70
               this.accuracy = accuracy;
               this.maxIts = maxIts;
72
               marketList = new List<MarketData>();
               this.r = r;
74
               this.S = S;
75
76
           }
77
           /// <summary>
           /// Sets the parameters which will be used as a starting point by the
79
               calibrator
           /// </summary>
80
           public void SetGuessParameters(double kappaStar, double thetaStar,
81
               double sigma, double rho, double v)
           {
82
               if (sigma <= 0 || v <= 0)
               {
84
                    throw new System.ArgumentException("Sigma, v must be positive")
85
               }
86
               Options e = new Options(r, S, kappaStar, thetaStar, sigma, rho, v);
88
               calibratedParams = e.ParamsAsArray();
89
           }
90
91
           /// <summary>
```

```
/// Adds the details of a real world option to the list marketList of
93
                data which will be used for calibration.
            /// </summary>
94
           /// <param name="K">Observed option's strike price.</param>
95
           /// <param name="T">Observed option's maturity time.</param>  
96
           /// <param name="Price">Observed options price.</param>
97
           public void AddObservedOption(double K, double T, double Price)
98
99
               if (K <= 0 || T <= 0 || Price <= 0)
101
                    throw new System.ArgumentException("Parameters must be positive
102
103
                MarketData observedOption;
105
                observedOption.K = K;
                observedOption.T = T;
107
                observedOption.Price = Price;
108
109
                marketList.Add(observedOption);
110
           /// <summary>
112
           /// Calculates the mean squared error between the European call prices
113
               of an instance, options,
           /// of the class Options and the market prices found in marketList
114
           /// </summary>
115
           /// <param name="options">An instance of the class Options.</param>
116
           \verb|public| double CalculateMeanSquaredErrorBetweenModelAndMarket(Options)| \\
               options)
118
                double mse = 0;
119
               foreach (MarketData data in marketList)
120
121
                    double T = data.T;
122
                    double K = data.K;
123
                    double price = options.EuropeanCallPrice(T, K);
124
                    double diff = price - data.Price;
125
                    mse += diff * diff;
127
                return mse;
128
129
130
             /// <summary>
            /// This is the function which will be used by the calibrator
132
            /// </summary>
134
           public void CalibrationObjectiveFunction(double[] paramsarray, ref
               double func, object obj)
135
                Options european = new Options(r, S, paramsarray);
136
                func = CalculateMeanSquaredErrorBetweenModelAndMarket(european);
138
139
            /// <summary>
140
            /// Calibrates the model parameters to fit the market data as closely
141
                 as possible
            /// </summary>
142
           public void Calibrate()
144
145
                // set up for calibration
146
                outcome = CalibrationOutcome.NotStarted;
147
                double[] initialParams = new double[Options.numberParams];
149
                if (calibratedParams == null)
150
151
                    throw new System. Exception ("Please add an initial guess for
152
                        parameters");
```

```
}
153
154
                calibratedParams.CopyTo(initialParams, 0);
155
                double epsg = accuracy;
156
                double epsf = accuracy;
157
                double epsx = accuracy;
158
159
                double diffstep = 1.0e-6;
                int maxits = maxIts;
160
                double stpmax = 0.05;
161
162
163
                alglib.minlbfgsstate state;
                alglib.minlbfgsreport rep;
164
                alglib.minlbfgscreatef (5, initialParams, diffstep, out state);
165
                alglib.minlbfgssetcond(state, epsg, epsf, epsx, maxits);
                alglib.minlbfgssetstpmax(state, stpmax);
167
168
                // calibrate and return outcome, error
169
                alglib.minlbfgsoptimize(state, CalibrationObjectiveFunction, null,
170
                    null);
                double[] resultParams = new double[Options.numberParams];
171
                alglib.minlbfgsresults(state, out resultParams, out rep);
173
                System.Console.WriteLine("Termination type: {0}", rep.
174
                    terminationtype);
                System.Console.WriteLine("Num iterations {0}", rep.iterationscount)
175
                System.Console.WriteLine("{0}", alglib.ap.format(resultParams, 5));
176
178
                if (rep.terminationtype == 1
                     || rep.terminationtype == 2
179
                     || rep.terminationtype == 4)
180
                {
181
                     outcome = CalibrationOutcome.FinishedOK;
182
                     calibratedParams = resultParams;
183
                }
184
                else if (rep.terminationtype == 5)
185
186
                     outcome = CalibrationOutcome.FailedMaxItReached;
                     calibratedParams = resultParams;
188
189
                }
190
                else
191
                {
192
                    outcome = CalibrationOutcome.FailedOtherReason;
193
                     throw new CalibrationFailedException("Heston model calibration
194
                         failed badly.");
                }
195
            }
196
197
             /// <summary>
             /// Obtains the calibration status of the model, as well as the models \left(\frac{1}{2}\right)^{2}
199
                  pricing error.
             /// </summary>
200
            public void GetCalibrationStatus(ref CalibrationOutcome calibOutcome,
201
                ref double pricingError)
            ₹
202
                calibOutcome = outcome;
                Options m = new Options(r, S, calibratedParams);
204
                pricingError = CalculateMeanSquaredErrorBetweenModelAndMarket(m);
205
            }
206
207
             /// <summary>
208
             /// Creates an instance of the class Options with the calibrated
209
                 parameters.
             /// </summary>
210
             /// <returns > Calibrated model. </returns >
211
            public Options GetCalibratedModel()
```

### InterfaceFill.cs class

```
using System;
2 using System.Collections.Generic;
3 using System.Linq;
4 using System. Text;
5 using System.Threading.Tasks;
6 using HestonModel.Interfaces;
{\tt 8} \ \ {\tt namespace} \ \ {\tt HestonModel.InterfaceImplement}
9 {
       /// <summary>
10
      /// This class is used to implement several interfaces.
11
       /// </summary>
12
      \verb"public class InterfaceFill: IOption, IVarianceProcessParameters",
13
           IMonteCarloSettings, IAsianOption, IEuropeanOption,
           ICalibrationSettings
14
           double T;
           double kappa;
16
           double theta;
           double sigma;
18
           double v;
19
20
           double rho;
           int numberTrials;
21
           int numberTimeSteps;
           PayoffType p;
23
24
           IEnumerable < double > timeList;
25
           double K;
           double accuracy;
26
27
28
           public InterfaceFill(double T, double kappa, double theta, double sigma
30
                , double rho, double v, int numberTrials, int numberTimeSteps,
               PayoffType p, double[] timeArray, double K, double accuracy)
31
               this.T = T; this.kappa = kappa; this.theta = theta; this.sigma =
32
                   sigma;
               this.v = v; this.rho = rho; this.numberTrials = numberTrials; this.
33
                   numberTimeSteps = numberTimeSteps;
               this.p = p; timeList = timeArray.AsEnumerable(); this.K = K;
34
               this.accuracy = accuracy;
36
37
38
39
           double IOption.Maturity => T;
40
41
           double IVarianceProcessParameters.Kappa => kappa;
43
           double IVarianceProcessParameters.Theta => theta;
44
45
           double IVarianceProcessParameters.Sigma => sigma;
46
           double IVarianceProcessParameters.V0 => v;
48
49
           double IVarianceProcessParameters.Rho => rho;
50
51
```

```
int IMonteCarloSettings.NumberOfTrials => numberTrials;
52
53
           int IMonteCarloSettings.NumberOfTimeSteps => numberTimeSteps;
54
55
           IEnumerable < double > IAsianOption.MonitoringTimes => timeList;
56
57
           PayoffType IEuropeanOption.Type => p;
58
59
           double IEuropeanOption.StrikePrice => K;
61
           double ICalibrationSettings.Accuracy => accuracy;
62
63
           int ICalibrationSettings.MaximumNumberOfIterations => 1000;
64
65
      }
66 }
```

#### OtherInterfaceFill.cs class

```
using System;
2 using System.Collections.Generic;
3 using System.Linq;
4 using System. Text;
5 using System.Threading.Tasks;
6 using HestonModel.Interfaces;
{\tt 8} \ \ {\tt namespace} \ \ {\tt HestonModel.InterfaceImplement}
9 {
       /// <summary>
10
       /// This class is used to implement the interfaces IHestonModelParameters
11
           and ICalibrationResult
       /// </summary>
      public class OtherInterfaceFill : IHestonModelParameters,
13
           {\tt ICalibrationResult}
14
           double S;
15
           double r;
16
           IVarianceProcessParameters paramss;
17
           CalibrationOutcome c;
           double error;
19
20
           public OtherInterfaceFill(double T, double kappa, double theta, double
21
               sigma, double rho, double v, int numberTrials, int numberTimeSteps,
                double S, double r, CalibrationOutcome c, double error)
           {
22
               this.S = S; this.r = r;
23
               double[] TT = { 0 };
24
               InterfaceFill fill = new InterfaceFill(T, kappa, theta, sigma, rho,
25
                    v, numberTrials, numberTimeSteps, 0, TT, 100, 0);
               paramss = fill;
26
27
               this.c = c; this.error = error;
           }
28
29
30
           double IHestonModelParameters.InitialStockPrice => S;
31
           double IHestonModelParameters.RiskFreeRate => r;
32
33
           IVarianceProcessParameters IHestonModelParameters.VarianceParameters =>
34
                paramss;
35
           CalibrationOutcome ICalibrationResult.MinimizerStatus => c;
36
37
           double ICalibrationResult.PricingError => error;
38
39
40
      }
41 }
```

### AnotherInterfaceFill.cs class

```
using System;
2 using System.Collections.Generic;
3 using System.Linq;
4 using System.Text;
5 using System. Threading. Tasks;
6 using HestonModel.Interfaces;
{\tt 8} \  \, {\tt namespace} \  \, {\tt HestonModel.InterfaceImplement}
9 {
10
       /// <summary>
      /// This class is used to implement the interface IHestonCalibrationResult.
11
12
      /// </summary>
      public class AnotherInterfaceFill : IHestonCalibrationResult
13
14
           IHestonModelParameters paramms;
           double error;
16
           CalibrationOutcome c;
18
           public AnotherInterfaceFill(double T, double kappa, double theta,
               double sigma, double rho, double v, int numberTrials, int
               numberTimeSteps, double S, double r, CalibrationOutcome c, double
               error)
           {
20
               this.c = c; this.error = error;
               OtherInterfaceFill fill = new OtherInterfaceFill(T, kappa, theta,
22
                   sigma, rho, v, numberTrials, numberTimeSteps, S, r, c, error);
               paramms = fill;
23
24
           IHestonModelParameters IHestonCalibrationResult.Parameters => paramms;
26
           CalibrationOutcome ICalibrationResult.MinimizerStatus => c;
28
29
           double ICalibrationResult.PricingError => error;
31
```

## FinalInterfaceFill.cs class

```
1 using System;
2 using System.Collections.Generic;
3 using System.Linq;
4 using System.Text;
{\scriptstyle 5} using System.Threading.Tasks;
6 using HestonModel.Interfaces;
{\tt 8} \ \ {\tt namespace} \ \ {\tt HestonModel.InterfaceImplement}
9 {
       /// <summarv>
10
       /// This class is used to implement the interface IOptionMarketData when it
11
             takes IEuropeanOption.
       /// </summary>
12
       public class FinalInterfaceFill : IOptionMarketData<IEuropeanOption>
14
            double price;
15
16
            IEuropeanOption option;
17
            public FinalInterfaceFill(double price, double T, PayoffType p, double
18
               K)
19
                this.price = price;
double[] TT = { 0 };
20
21
                InterfaceFill fill = new InterfaceFill(T, 0, 0, 0, 0, 0, 0, 0, p,
                    TT, K, 0);
```

#### Heston.cs

```
using System;
2 using System.Collections.Generic;
3 using HestonModel.Interfaces;
4 using HestonCalibrationAndPricing;
5 using System.Linq;
_{6} using HestonModel.InterfaceImplement;
8 namespace HestonModel
9 {
10
      /// <summary>
11
      /// This class will be used for grading.
12
      /// Don't remove any of the methods and don't modify their signatures. Don'
           t change the namespace.
       /// Your code should be implemented in other classes (or even projects if
           you wish), and the relevant functionality should only be called here
           and outputs returned.
       /// You don't need to implement the interfaces that have been provided if
15
          you don't want to.
       /// </summary>
      public static class Heston
17
18
           /// <summary>
19
           /// Method for calibrating the heston model.
20
           /// </summary>
21
           /// <param name="guessModelParameters">Object implementing
22
               IHestonModelParameters interface containing the risk-free rate,
               initial stock price
           /// and initial guess parameters to be used in the calibration.</param>
23
           /// <param name="referenceData">A collection of objects implementing
24
               IOptionMarketData < IEuropeanOption > interface. These should contain
               the reference data used for calibration.</param>
           /// <param name="calibrationSettings">An object implementing
25
               ICalibrationSettings interface.</param>
           /// < \mathtt{returns} > \mathtt{Object} \ \mathtt{implementing} \ \mathtt{IHestonCalibrationResult} \ \mathtt{interface}
26
               which contains calibrated model parameters and additional
               diagnostic information </returns>
           public static IHestonCalibrationResult CalibrateHestonParameters(
27
               IHestonModelParameters guessModelParameters, IEnumerable <
               IOptionMarketData < IEuropeanOption >> referenceData,
               ICalibrationSettings calibrationSettings)
28
               // set up calibrator
29
               Calibrator cal = new Calibrator(guessModelParameters.RiskFreeRate,
                   guessModelParameters.InitialStockPrice, calibrationSettings.
                   MaximumNumberOfIterations, calibrationSettings.Accuracy);
               \verb|cal.SetGuessParameters| (\verb|guessModelParameters|. Variance Parameters|.
31
                   Kappa, guessModelParameters.VarianceParameters.Theta,
                   guessModelParameters.VarianceParameters.Sigma,
                        guessModelParameters.VarianceParameters.Rho,
                        guessModelParameters.VarianceParameters.V0);
33
               // add market data
34
35
               foreach (IOptionMarketData < IEuropeanOption > data in referenceData)
               {
36
```

```
cal.AddObservedOption(data.Option.StrikePrice, data.Option.
37
                       Maturity, data.Price);
               }
38
39
               \ensuremath{//} calibrate, get error, outcome, parameters
40
               cal.Calibrate();
41
42
               double error = 0:
               HestonCalibrationAndPricing.CalibrationOutcome outcome =
43
                   {\tt HestonCalibrationAndPricing.CalibrationOutcome.NotStarted;}
               cal.GetCalibrationStatus(ref outcome, ref error);
44
45
               Options e = cal.GetCalibratedModel();
46
               double[] paramArray = e.ParamsAsArray();
47
               CalibrationOutcome outcome1 = (CalibrationOutcome)outcome;
49
               // implement and return IHestonCalibrationResult
               AnotherInterfaceFill fill = new AnotherInterfaceFill(0, paramArray[
                   Options.kappaIndex], paramArray[Options.thetaIndex], paramArray
                   [Options.sigmaIndex], paramArray[Options.rhoIndex], paramArray[
                   Options.vIndex], 0, 0, 0, 0, outcome1, error);
               return fill;
           }
53
54
           /// <summary>
55
           /// Price a European option in the Heston model using the Heston
56
               formula. This should be accurate to 5 decimal places
           /// </summary>
57
           /// <param name="parameters">Object implementing IHestonModelParameters
                interface, containing model parameters.</param>
           /// <param name="europeanOption">Object implementing IEuropeanOption
59
               interface, containing the option parameters.</param>
           /// <returns>Option price</returns>
60
           public static double HestonEuropeanOptionPrice(IHestonModelParameters
61
               parameters, IEuropeanOption europeanOption)
62
               Options eur = new Options(parameters.RiskFreeRate, parameters.
63
                   InitialStockPrice,
                  parameters. VarianceParameters. Kappa, parameters.
                      {\tt Variance Parameters. Theta, parameters. Variance Parameters.}
                      Sigma,
                  parameters.VarianceParameters.Rho, parameters.VarianceParameters
65
                      .VO);
               if (europeanOption.Type == 0)
67
                   \verb"return" eur. European Call Price (european Option. Maturity",
69
                       europeanOption.StrikePrice);
               }
70
               else
71
                   return eur. EuropeanPutPrice (europeanOption. Maturity,
                       europeanOption.StrikePrice);
73
           }
74
           /// <summary>
75
           /// Price a European option in the Heston model using the Monte-Carlo
               method. Accuracy will depend on number of time steps and samples
           /// </summary>
           /// <param name="parameters">Object implementing IHestonModelParameters
78
                interface, containing model parameters.</param>
           /// <param name="europeanOption">Object implementing IEuropeanOption
79
               interface, containing the option parameters.</param>
           /// <param name="monteCarloSimulationSettings">An object implementing
               IMonteCarloSettings object and containing simulation settings.</
               param >
           /// <returns>Option price</returns>
81
           \verb|public| static| double| HestonEuropeanOptionPriceMC(IHestonModelParameters)|
82
                parameters\,,\,\,IEuropeanOption\,\,europeanOption\,,\,\,IMonteCarloSettings
```

```
{
               OptionsMC option = new OptionsMC(parameters.RiskFreeRate,
84
                    europeanOption.StrikePrice, parameters.VarianceParameters.Kappa
                    parameters. VarianceParameters. Theta, parameters.
85
                        VarianceParameters.Sigma, parameters.VarianceParameters.Rho
                    parameters.VarianceParameters.V0, parameters.InitialStockPrice)
87
               if(europeanOption.Type == 0)
88
               {
89
                    return option.EuropeanCallOptionPriceMCAnitheticParallel(
                        {\tt europeanOption.Maturity, monteCarloSimulationSettings.}
                        {\tt NumberOfTimeSteps}, \ {\tt monteCarloSimulationSettings}.
                        NumberOfTrials):
               }
91
92
               else
                    return option.EuropeanPutOptionPriceMCAnitheticParallel(
93
                        europeanOption.Maturity, monteCarloSimulationSettings.
                        NumberOfTimeSteps, monteCarloSimulationSettings.
                        NumberOfTrials);
           }
94
95
           /// <summary>
           /// Price a Asian option in the Heston model using the
97
           /// Monte-Carlo method. Accuracy will depend on number of time steps
               and samples </summary>
           /// <param name="parameters">Object implementing IHestonModelParameters
99
                interface, containing model parameters.</param>
           /// <param name="asianOption">Object implementing IAsian interface,
100
               containing the option parameters.</param>
           /// <param name="monteCarloSimulationSettings">An object implementing
101
               IMonteCarloSettings object and containing simulation settings.</
               param>
           /// <returns>Option price</returns>
102
           \verb|public| static| double| HestonAsianOptionPriceMC(IHestonModelParameters|) \\
               parameters, IAsianOption asianOption, IMonteCarloSettings
               monteCarloSimulationSettings)
           {
104
               OptionsMC asian = new OptionsMC(parameters.RiskFreeRate,
105
                    asianOption.StrikePrice, parameters.VarianceParameters.Kappa,
                   parameters.VarianceParameters.Theta, parameters.
106
                       VarianceParameters.Sigma, parameters.VarianceParameters.Rho,
107
                   parameters.VarianceParameters.V0, parameters.InitialStockPrice);
108
               if (asianOption.Type == 0)
109
               ₹
110
                    return asian.PriceAsianCallMC(asianOption.MonitoringTimes.
                        ToArray(), asianOption.Maturity,
                    monteCarloSimulationSettings.NumberOfTrials,
112
                        {\tt monteCarloSimulationSettings.NumberOfTimeSteps);}
               }
113
               else
                    return asian.PriceAsianPutMC(asianOption.MonitoringTimes.
115
                        ToArray(), asianOption.Maturity,
                    {\tt monteCarloSimulationSettings.NumberOfTrials}\ ,
116
                        monteCarloSimulationSettings.NumberOfTimeSteps);
           }
117
118
           /// <summary>
           /// Price a lookback option in the Heston model using the
120
           /// a Monte-Carlo method. Accuracy will depend on number of time steps
121
               and samples </summary>
           /// <param name="parameters">Object implementing IHestonModelParameters
122
                interface, containing model parameters.</param>
```

monteCarloSimulationSettings)

```
/// <param name="maturity">An object implementing IOption interface and
123
                                                     containing option's maturity
                                    /// <param name="monteCarloSimulationSettings">An object implementing
124
                                                 IMonteCarloSettings object and containing simulation settings.</
                                               param>
                                    /// <returns > Option price </returns >
125
                                    \verb|public| static| double| HestonLookbackOptionPriceMC(IHestonModelParameters|) | The analysis of the property of the propert
126
                                                    parameters, IOption maturity, IMonteCarloSettings
                                                 monteCarloSimulationSettings)
                                    {
127
                                                 OptionsMC lookback = new OptionsMC(parameters.RiskFreeRate,
128
                                                              {\tt parameters.VarianceParameters.Kappa}\ ,
                                                                                                      {\tt parameters.VarianceParameters.Theta,\ parameters.}
129
                                                                                                                   VarianceParameters.Sigma, parameters.
                                                                                                                   VarianceParameters.Rho,
                                                                                                      parameters.VarianceParameters.V0, parameters.
                                                                                                                   InitialStockPrice);
131
                                                 \tt return\ lookback.PriceLookbackCallMC(maturity.Maturity,
132
                                                              monteCarloSimulationSettings.NumberOfTrials,
                                                              monteCarloSimulationSettings.NumberOfTimeSteps);
                                   }
133
                      }
134
135 }
```