Simulation studies of characteristics of Full Availability Group with generalized traffic classes

Adam Kaliszan
Faculty of Electronics and Telecommunications
Poznan University of Technology
Poznan, Poland
adam.kaliszan@put.poznan.pl

Abstract—**This paper presents the results of simulation investigations into Full Availability Group to which any type of traffic classes is offered. In such traffic classes, call streams and service streams are described by different, non-exponential, probability distribution. In this paper, the dependencies between call intensity, system state and system capacity are analysed. The possibilities of approximation of a multi-dimensional non-Markovian system by one-dimensional Markovian system are also considered in the paper.**

Keywords—Full Availability Group; non-Poisson streams; non-Markovian process; generalized probability distribution

I. INTRODUCTION

Analytical methods for modeling telecommunications systems are commonly used, both when designing new systems and managing them. However, these models are subject to criticism due to the fact that call arrival and service processes are described only by an exponential distribution.

There are two reasons for using exponential distribution. First of all, it perfectly reflects the call arrival and service process[*1*][*2*].The other reason is connected with the fact that the processes in with the time intervals between events are described using exponential distribution are Markovian processes[*3*]. Such processes allow state probabilities to be determined relatively easy by analytical methods.

An important aspect of Markov processes is their reversibility, which allows approximation of multidimensional processes by one-dimensional process [*4*]. This property has enabled the development of many methods for analytical modelling of multi-service systems: Full Availability Group FAG [4]-[6], Full Availability Group with reservation [7]-[12], Limited Availability Group [13]-[15], threshold systems [16]-[20], overflow systems [21]-[28] and multi-rate queuing systems [29]-[32].

The paper includes research into systems in which call arrival and service processes are described by distributions different from the (exponential distribution). Such systems are called systems with non-Markovian traffic classes. The method of simulation investigations is described in [24] though in this paper the problem of the dependence between the intensity $\lambda(n)$ of the call arrival, system state n, and system capacity V is not considered. In the case of exponential distribution, this dependence is constant regardless of the number of serviced calls, free resources, and the total system's capacity. Such a property allows the determine probability distribution based on the analysis of a relevant Markov process. This work shows, that it is possible to determine the states probabilities $p(n)$ in a single-rate systems based on call arrival intensity $\lambda(n)$ in state n and call service intensity $\mu(n)$, if both intensities were determined based on the simulation experiment. Then, it is checked if it is possible to determine the occupancy distribution in multi-rate systems based on call arrival and call service intensities. For this purpose, the occupancy probabilities, call arrival and service intensities were determined using the simulation experiment. Next those parameters were substituted for equations describing approximated one-dimensional Markov process for multi-rate systems.

The remaining part of the paper is organised as follows. In Section II, FAG (Full Availability Group) with Erlang and generalized (non-Markovian) traffic classes is described. Section III introduces the method of simulation investigations of state probabilities$p(n)$ and relations between the call arrival intensity $\lambda(n)$, system state n and system capacity V. In this section, the results of the simulation experiments for single-rate systems are presented. Section IV describes the simulation experiments for multi-rate systems. The relative error between Markovian and non-Markovian models for chosen systems is calculated in order to check possibilities for application of the equations, that assume, that the system is reversible. Section V concludes the paper.

II. MODEL OF THE FULL AVAILABILITY GROUP

FAG is a basic model in multi-service traffic engineering theory [6]. In this paper, the FAG has been extended to include traffic classes with any call arrival and service stream.

A. Full Availability Group with multirate traffic

Let us assume that m traffic classes, that correspond to the services offered in a given network are offered to the FAG under investigation. In FAG, a call of a given class is always admitted for service if only the group has unoccupied resources required for this call to be serviced. After call service termination the occupied FAG resources are released.

Each call class is characterized by the following parameters: the number of resources required for servicing a call of a given class, time intervals between consecutive calls and distribution of service time.

In the FAG model, the number of resources d_i that are required for servicing class i call is expressed in the so-called Allocation Units (AU). While constructing multi-rate models for telecommunication systems it is assumed that the AU is the greatest common divisor (or its multiplicity) of all bit rates of calls offered to the system.

In the basic model of FAG, the time intervals between successive calls of a given class are described by exponential distribution. Similarly, the service time of a particular call is also described by exponential distribution. A traffic class in which call arrivals and service times undergo the exponential distribution is called Erlang class.

A FAG with a large number of Erlang traffic has been widely addressed in the literature of the subject [4]. In these models, the mean call intensity is described by the parameter λ_i and mean service time is described by the parameter μ_i. Thus the offered traffic intensity can be written in the following way:

$$A_i = d_i \frac{\lambda_i}{\mu_i} \tag{1}$$

From the engineering point of view, the intensity of offered traffic A_i, the number of required AUs d_i and the capacity of FAG V expressed in AUs are the most important parameters used in any service process analysis.

The model of FAG with Erlang traffic classes is shown in Fig. 1. The analytical methods elaborated in literature allow the occupancy distribution in FAG with Erlang traffic classes to be determined. Such a distribution determines the probability of *0*, *1*, …, V AUs being serviced. On the basis of the probability distribution, the QoS parameters can be then calculated, e.g. the blocking probability.

B. Full Availability Group with generalized multirate traffic

Let us consider again FAG with Erlang traffic class. In this kind of traffic, the exponential distribution describes the time

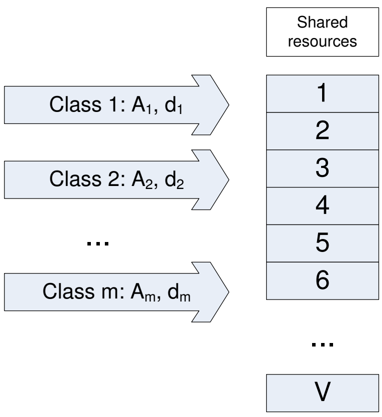

Fig. 1. Full Availability Group with Erlang traffic classes

intervals between neighbouring calls and time duration of call service. Probability density function (PDF) of the exponential distribution is defined on the basis of the average intensity λ:

$$f(t) = \lambda e^{-\lambda t} \tag{2}$$

The intensity λ is reversely proportional to the expected value of time duration between consecutive events. If we assume that $E_a(t)$ determines the expected value of the time between successive calls and $E_s(t)$ determines the expected value of the service time of a single call service, then the offered traffic intensity (1) can be rewritten in the following way:

$$A_i = d_i \frac{E_s(t)}{E_a(t)} \tag{3}$$

Equation (3) can be used to determine the intensity of traffic where call arrival and service processes are characterized by different distributions.

Let us consider an application of gamma distribution for description of time intervals between events. In order to determine PDF for this distribution, the expected value $E(t)$ and the variance $D(t)$ are required [33]:

$$f(t) = \frac{t^{\propto-1} e^{-\frac{t}{\beta}}}{\beta^{\propto} \Gamma(\propto)}, \tag{4}$$

Where $\propto$ and β parameters depend on $E(t)$ and $D(t)$:

$$\propto = \frac{E(t)^2}{D(t)}, \beta = \frac{\mathrm{D(t)}}{\mathrm{E(t)}} \tag{5}$$

For a PDF of the Pareto distribution the following dependencies can be taken into account:

$$\mathrm{V}_{t>\beta}\, f(t) = \frac{\propto \beta^{\propto}}{t^{\propto+1}}, \tag{6}$$

$$\propto = 1 + \sqrt{1 + \frac{D(t)}{E(t)^2}}, \beta = \frac{\mathrm{E(t)}\sqrt{1 + \frac{D(t)}{E(t)^2}}}{1 + \sqrt{1 + \frac{D(t)}{E(t)^2}}} \tag{7}$$

Fig. 2 presents a model of a FAG with generalized traffic. In this model, each class $i (1 \le i \le m)$ of offered traffic is described by the following parameters: the number d_i of required AUs for a single call service, PDF for call arrival stream $f_{a,i}(t)$ and service stream $f_{s,i}(t)$ respectively, expected values $E_{a,i}(t)$, $E_{s,i}(t)$ and variances $D_{a,i}(t)$, $D_{s,i}(t)$ of particular distributions.

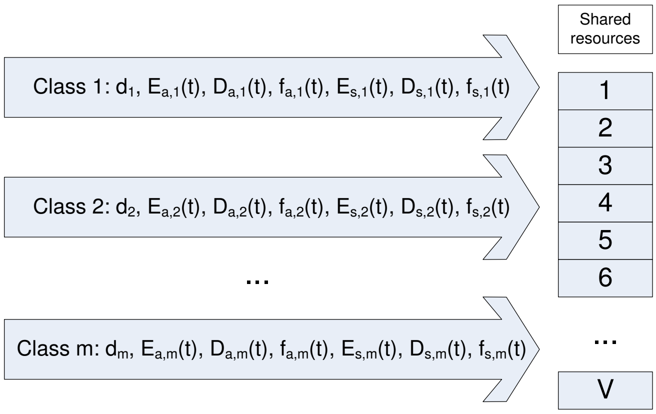

Fig. 2. Full Availability Group with generalized traffic

III. SIMULATION EXPERIMENT FOR SINGLE SERVICE SYSTEMS.

A simulation investigation allows us to determine the states possibilities (the probability of occupancy a specified number of resources) for any call arrival and service process. The Probability $p(n)$ of occupancy n AUs in a simulation experiment is determined according to the total time $T(n)$ of the process being in state n in the time of observation T (i.e. the total time of a simulation experiment):

$$p(t) = \frac{T(n)}{T} \tag{8}$$

The capability of determining the occupancy distribution for any processes in the system provides a significant advantage in a simulation study as compared to available analytical methods that only allow effective modelling of systems with Markov processes. In the case of Poisson streams, the time interval between events is exponential and independent of the intervals between the events preceding.

As a consequence, the intensity of events is constant and independent of the state of the process.

In the case of distributions, different from exponential, the intensities $\lambda(n)$ and $\mu(n)$ can be determined according to a simulation investigation.

The call arrival and service processes in a single-rate system are reversible because there is only one possible transition between neighboring states. Let $X(n)$ determines the number of transitions from state n state $n+1$, and $Y(n+1)$ determines the number of transitions from state $n+1$ to state n for very large number of passes so we can write:

$$X(n) = Y(n+1) \tag{9}$$

The intensity $\lambda(n)$ of transitions from state n to state $n+1$ depends on number of transitions and duration time of state n:

$$\lambda(n) = \frac{X(n)}{T(n)} \tag{10}$$

Analogically, call service intensity $\mu(n+1)$ (i.e. intensity of transitions from state $n+1$ to the state n) can be determined in the following way:

$$\mu(n+1) = \frac{Y(n+1)}{T(n+1)} \tag{11}$$

According to (8), (10), (11) it is possible to estimate the relations between: the number of passages, intensity of transitions and state probability:

$$X(n) = Tp(n)\lambda(n) \tag{12}$$

$$Y(n+1) = Tp(n+1)\mu(n+1) \tag{13}$$

Substituting (12), (13) to (9), we get an equation that describes the Markov process:

$$p(n)\lambda(n) = p(n+1)\mu(n+1) \tag{14}$$

Equation (14) is not sufficient for an analytical determination of the occupancy distribution, because the information about the intensities $\lambda(n)$ and $\mu(n+1)$ is required. For Erlang traffic, these intensities can be determined using the following formulas:

$$\mu(n) = n\mu \tag{15}$$

$$\lambda(n) = \lambda \tag{16}$$

In the case of other distributions, the parameters $\lambda(n)$ and $\mu(n)$ depend on state n as well as on the system capacity V, which is shown in further part of this section. A number of simulations experiments for single-service systems were carried out to determine these relationship characteristics. The results are presented in Fig. 3-5. For simulation investigation, the simulator described in [34] was applied. Single-service systems with capacities between 15 and 20 AUs and offered traffic equal to 15 Erlangs were investigated. Picture 3 presents the results of the simulation investigation of a system with Erlang traffic class.

According to the theoretical properties of the exponential distribution, the intensity is constant. This is evidenced by the fact that the intensity presented in Fig. 3 is horizontal, independent of the state of the arrival and service process. Plotting of diagrams for systems with different capacities shows that intensity is independent on the system capacity.

In the area of states near 0 it can be observed a fluctuation of the intensity λ. This phenomenon results from the fact that probability of states near zero is very low (about one per million) as well as the number of registered transitions between these states. In consequence, the confidentiality intervals are increasing. The 95% confidence intervals are marked in Fig. 3-5.

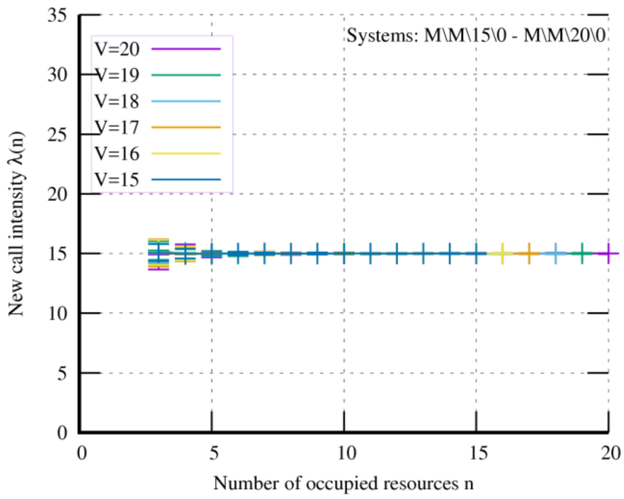

Fig. 3. Call arrival intensity for a class with streams described by exponential distribution

During the study, a number of experiments were carried out in which other distributions, different from the exponential distribution, i.e. gamma and Pareto, were considered.

Fig. 4 shows the relation between the call arrival intensity in state n and the system capacity in the system with service and arrival process described by the gamma distribution. The results indicate very clearly the dependence of the intensity $\lambda(n)$ on the state (the lines are not horizontal) and on the system capacity (lines has different shapes).

Fig. 5 shows the characteristics for the systems with call arrival and service process described using the Pareto distribution.

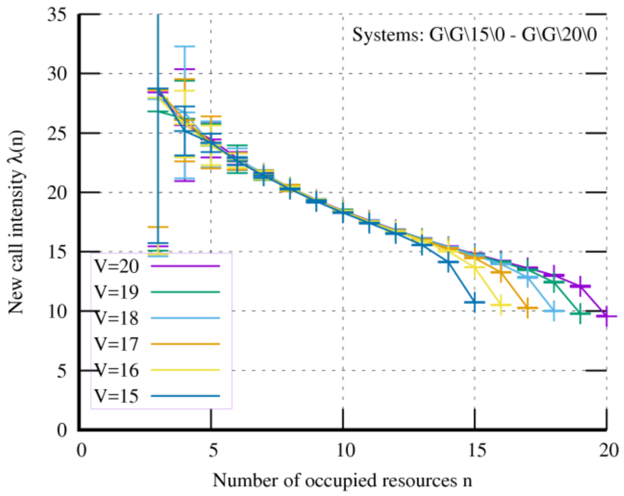

Fig. 4. Call arrival intensity for a class with streams described by gamma distribution

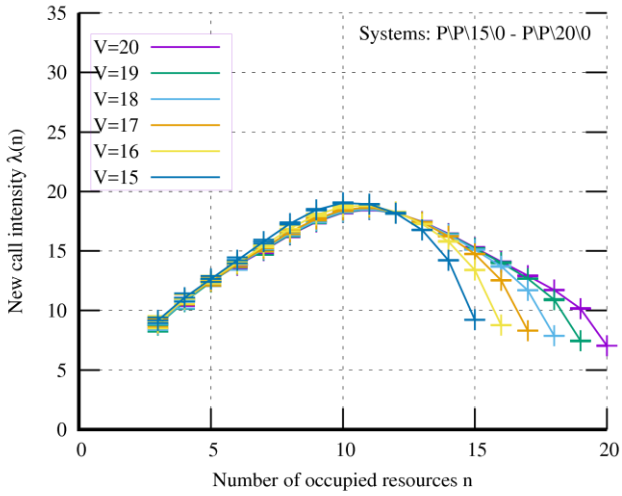

Fig. 5. Call arrival intensity for a class with streams described by Pareto distribution

In all experiments, the dependence of call arrival and service processes on the system capacity was observed. The consequence of this dependence is the change in the proportion between the probabilities of the state, which makes it impossible to create tables of occupancy distributions for the non-Markovian traffic classes in a simple way.

IV. Simulation experiment for multi service systems.

Let us now consider a multi-rate model of a FAG. Using simulation experiments, it is possible to estimate the states probabilities, mean intensity $\lambda_i(n)$ of call of class $i (1 \leq i \leq m)$ arrival in state n and mean call of class i in state n service intensity $\mu_i(n)$. In the case of reversible processes, the relation between these parameters can be written in three possible ways:

- determining the balance between the total intensity of the input stream to the state n and output stream from this state:

$$\sum_{i=1}^{m}\left(p(n-d_i)\lambda_i(n-d_i)+p(n+d_i)\mu_i(n+d_i)\right) = p(n)\sum_{i=1}^{m}\left(\lambda_i(n)+\mu_i(n)\right) \tag{17}$$

- determining the balance between the intensity of outgoing streams from state n to higher states and streams that came from higher states to state n:

$$\sum_{m}^{i=1} p(n+d_i)\mu_i(n+d_i) = p(n)\sum_{m}^{i=1}\lambda_i(n) \tag{18}$$

- defining the balance between the intensity of outgoing streams from state n to lower states and the streams entering the state n that came from lower states:

$$\sum_{m}^{i=1} p(n-d_i)\lambda_i(n-d_i) = p(n)\sum_{m}^{i=1} \mu_i(n) \tag{19}$$

In order to verify the applicability of (17), (18), (19) simulation experiments for multi-service systems were carried out. On the basis of the obtained results, the values of relative errors ε were determined according to the following equations:

$$\varepsilon_1 = \left|1 - \frac{p(n)\sum_{m}^{i=1}(\lambda_i(n)+\mu_i(n))}{\sum_{m}^{i=1} p(n+d_i)\mu_i(n+d_i) + p(n-d_i)\lambda_i(n-d_i)}\right| \tag{20}$$

$$\varepsilon_2 = \left|1 - \frac{p(n)\sum_{m}^{i=1}\lambda_i(n)}{\sum_{m}^{i=1} p(n+d_i)\mu_i(n+d_i)}\right| \tag{21}$$

$$\varepsilon_3 = \left|1 - \frac{\sum_{m}^{i=1} p(n-d_i)\lambda_i(n-d_i)}{p(n)\sum_{m}^{i=1}\mu_i(n)}\right| \tag{22}$$

Simulation experiments were carried out for a FAG with the capacity of 20 AUs and the total traffic intensity equal to 14 Erlangs. The group was offered three traffic classes, with call request 1, 2 and 3 AUs, respectively. The proportions of offered traffic intensity $A_1{:}A_2{:}A_3$were 1:1:1. In the very beginning, the relative errors ε_1, ε_2, ε_3 for a reversible system with Erlang traffic classes were compared. The results are presented in Fig. 6. It is possible to observe that ε_1 is about two orders lower as compared to ε_2 and ε_3. In most analytical methods for classes with exponential distributions based on (19) are accurate. Taking into account the confidence intervals of the parameters ($\lambda_i(n)$, $\mu_i(n)$, $p(n)$) determined during the simulation experiments, it is possible to find proper value (within the confidence intervals), while substituted to the (20), (21), (22) gives the result equal to 0.

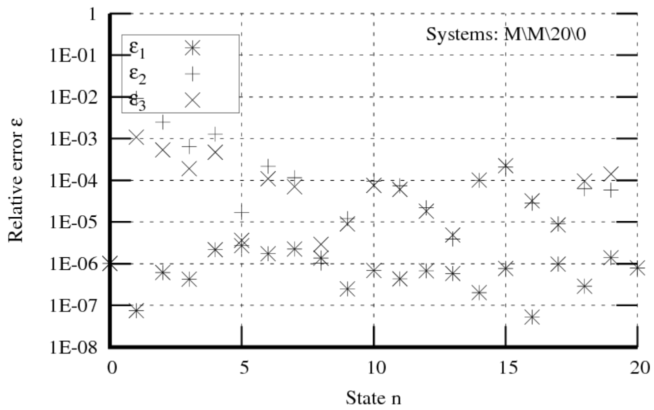

Fig. 6. A comparison of relative errors of the equations that are describing the balance between the states in a system with Erlang traffic classes

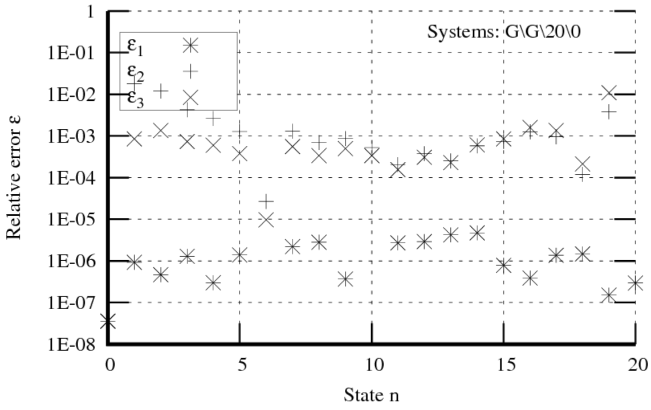

Fig. 7. A comparison of relative errors of the equations that are describing the balance between the states in a system with streams described by gamma distributions

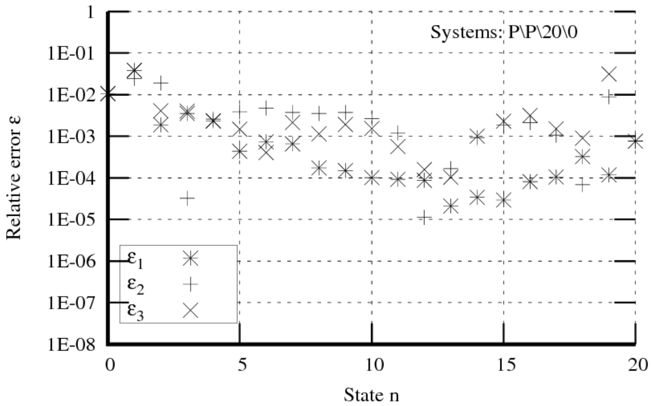

Fig. 8. A comparison of relative errors of the equations that are describing the balance between the states in a system with streams described using Pareto distributions

In order to check if it is possible to use (19) in systems with non-exponential distributions, simulation investigation were carried out. The relative errors for systems with gamma (Fig. 7) and Pareto (Fig. 8) streams are presented in the paper. The values of relative errors for these systems are comparable to the errors that appear in systems with Markov processes. It is difficult to state clearly whether the discrepancies in these systems result from the assumption of the process reversibility or from inaccuracy of the simulation experiment. During the simulation tests, the duration of the experiment was extended as long as the values of relative errors were decreasing. Regardless of the reasons for the errors, it can be clearly stated that it is possible to assume the reversibility of the processes in the approximates one-dimensional chain for FAG with multi-rate traffic, regardless of the distribution describing the time intervals between events.

V. Conclussions

The results of the conducted simulation tests indicate the possibility of an approximation of non-Markovian processes with reversible Markovian processes. Assuming that the process is reversible, there is potential possibility to simplify the analytical model. The relative error resulting from the assumption of the reversibility of the process in the investigated systems does not exceed one percent, so it is admissible in engineering practice.

However, the application of equilibrium equations in practice is very complex due to the inability to efficiently determine the characteristics $\lambda_i(n)$ and $\mu_i(n)$ analytically.

In further works attempts will be made to adequately approximate the parameters, enabling the construction of an analytical model of FAG model with non-Markovian traffic classes.

REFERENCES

[1] A. K. Erlang, "Solution of some problems in the theory of probabilities of significance in automatic telephone exchanges," Elektrotechnikeren, vol. 13, 1917.

[2] T. Bonald, J. Roberts, "Internet and the Erlang Formula," ACM Computer Communications Review, vol. 42, no. 1, 2012.

[3] P. A. Gagniuc, "Markov Chains: From Theory to Implementation and Experimentation," John Wiley & Sons, 2017.

[4] J. Kaufman, "Blocking in a Shared Resource Environment," IEEE Transactions on Communications, vol. 29, no. 10, 1981.

[5] J. W. Roberts, "A service system with heterogeneous user requirements," Performance of Data Communications Systems and Their Applications, 1981.

[6] M. Głąbowski, M. Stasiak, and J. Weissenberg, "Properties of Recurrent Equations for the Full-Availability Group with BPP Traffic," Mathematical Problems in Engineering, vol. 2012, 2012.

[7] J.W. Roberts, "Teletraffic Models for the Telecom 1 Integrated Service Network," Montreal, 1983.

[8] M. Głąbowski, M. Stasiak, "A simple approximation of the link model with reservation by a one-dimensional Markov chain," Performance Evaluation, vol. 41, no. 2-3, 2000.

[9] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Asymmetric Convolution Algorithm for Blocking Probability Calculation in Full-Availability Group with Bandwidth Reservation," IET Circuits, Devices & Systems, vol. 2, no. 1, 2008.

[10] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Modeling product-form state-dependent systems with BPP traffic," Performance Evaluation, vol. 67, no. 3, 2010.

[11] Q. Huang, K. Ko, and V. B. Iversen, "A new convolution algorithm for loss probablity analysis in multiservice networks," Performance Evaluation, vol. 68, no. 1, 2011.

[12] I. D. Moscholios, M. D. Logothetis, J. S. Vardakas, and A. C. Boucouvalas, "Performance metrics of a multirateresource sharing teletraffic model with finite sources under the threshold and bandwidth reservation policies," IET Networks, vol. 4, no. 3, 2015.

[13] M. Stasiak, "Blocking probability in a limited-availability group carrying mixture of different multichannel traffic streams," Annales Des Télécommunications, vol. 48, no. 1-2, 1993.

[14] M. Głąbowski, M. Stasiak, "Multi-rate model of the group of separated transmission links of various capacities," Lecture Notes in Computer Science, vol. 3124, 2004.

[15] M. Glabowski, S. Hanczewski, M. Stasiak, and J. Weissenberg, "Modeling Erlang's Ideal Grading with Multi-rate BPP Traffic," Mathematical Problems in Engineering, vol. 2012, 2012.

[16] J. S. Kaufman, "Blocking shared with retrials in a completely resource environment," Performance Evaluation, vol. 15, no. 2, 1992.

[17] I. D. Moscholios, M. D. Logothetis, and G. K. Kokkinakis, "Connection dependent threshold model: a generalization of the Erlang multiple rate loss model," Performance Evaluation, vol. 48, no. 1-4, 2002.

[18] M. Sobieraj, M. Stasiak, J. Weissenberg, and P. Zwierzykowski, "Analytical Model of the Single Threshold Mechanism with Hysteresis for Multi-service Networks," IEICE Transactions on Communications, vol. E95.B, no. 1, 2012.

[19] I. D. Moscholios, M. D. Logothetis, J. S.Vardakas, and A. C. Boucouvalas, "Congestion probabilities of elastic and adaptive calls in erlang-engset multirate loss models under the threshold and bandwidth reservation policies," Computer Networks, vol. 92, 2015.

[20] I. D. Moscholios, M. D. Logothetis, and A. C. Boucouvalas, "Blocking probabilities of elastic and adaptive calls in the erlangmultirate loss model under the threshold policy," Telecommunication Systems, vol. 62, no. 1, 2016.

[21] A. Zalesky, Z. Rosberg, M. Zukerman, and E. W. M. Wong, "A new method for approximating blocking probability in overflow loss networks," Computer Networks, vol. 51, no. 11, 2007.

[22] M. Glabowski, K. Kubasik, and M. Stasiak, "Modeling of systems with overflow multi-rate traffic," Advanced International Conference On Tellecomunication, 2007.

[23] K. Ko, V. B. Iversen, and Q. Huang, "Approximation of loss calculation for hierarchical networks with multiservice overflows," IEEE Transactions on Communications, vol. 56, no. 3, 2008.

[24] Q. Huang, Y. Huang, K. Ko, and V. B. Iversen, "Loss Performance Modeling for Hierarchical Heterogeneous Wireless Networks With Speed-Sensitive Call Admission Controll," IEEE Transactions on Vehicular Technology, vol. 60, no. 5, 2011.

[25] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Two-Dimensional Convolution Algorithm for Modelling Multiservice Networks with Overflow Traffic," Mathematical Problems in Engineering, vol. 60, no. 5, 2013.

[26] M.Wang, S. Li, E.Wong, and M. Zukerman, "Performance analysis of circuit switched multi-service multi-rate networks with alternative routing," Journal of Lightwave Technology, 2014.

[27] M. Głąbowski, A. Kaliszan, and M. Stasiak, "Modelling overflow systems with distributed secondary resources," Computer Networks, vol. 188, 2016.

[28] M. Głabowski, S. Hanczewski, and M. Stasiak, "Modelling of cellular networks with traffic overflow," Mathematical Problems in Engineering, 2015.

[29] S. Hanczewski, M. Stasiak, and J. Weissenber, "The Queueing Model of a Multiservice System with Dynamic Resource Sharing for Each Class of Calls," Computer Networks, vol. 370, 2013.

[30] S. Hanczewski, M. Stasiak, and J. Weissenber, "A Queueing Model of a Multi-Service System with State-Dependent Distribution of Resources for Each Class of Calls," IEICE Transactions, vol. E97-B, no. 8, 2014.

[31] M. Stasiak, "Queuing Systems for the Internet," IEICE Transactions on Communications, vol. E99-B, no. 6, pp. 1234-1242, 2016.

[32] S. Hanczewski, A. Kaliszan, and M. Stasiak, "Convolution Model of a Queueing System with the cFIFO," Mobile Information Systems, 2016

[33] A. Kaliszan, M. Stasiak, "The Possibilities and Limitations of the Application of the Convolution Algorithm for Modeling Network Systems," International Conference on Computer Networks, 2017

[34] S. Hanczewski, A. Kaliszan, "Simulation studies of queueing systems," International Symposium on Communication Systems, Networks & Digital Signal Processing, 2016