Discovering Structural Motifs in Financial Time Series

A Comparative Analysis Under Valuation Regimes

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Algorithms and data structures in strings ☺

Executive Summary

This project applies motif discovery algorithms to 152 years of S&P 500 financial data (1871-2023) to identify recurring structural patterns and their relationship to market valuation regimes. While the assignment required implementing 1-2 algorithms, we implemented six motif discovery algorithms to provide comprehensive analysis, with primary focus on Order-Preserving Motifs (OPM) and Cartesian Tree Motifs (CTM).

Key Findings:

- Market valuation regimes exhibit fundamentally different pattern structures. Low-valuation periods (PE10 ≤ 19.79) show chaotic, diverse patterns with frequent descending sequences. High-valuation periods show uniform, persistent ascending patterns.
- The "Regime Paradox": Expensive markets are structurally simple but temporally persistent, while cheap markets are structurally complex but ephemeral.
- Descending patterns (3,2,1) occur 2.09× more frequently in low-valuation regimes, providing a quantitative signature of bear market dynamics.
- Early warning signals for regime transitions identified: pattern fragmentation precedes market crashes, while pattern consolidation signals recoveries.
- OPM and CTM provide complementary insights: OPM captures directional movements (chart patterns), while CTM reveals hierarchical structure (shock vs. trend dynamics).

This work demonstrates that financial markets repeat ways of moving, not prices—providing a rigorous, algorithmic framework for understanding market structure beyond traditional statistical methods.

1. Introduction and Motivation

1.1 The Problem: Beyond Global Statistics

Classical financial time-series analysis relies on global statistics: mean returns, variance, autocorrelation, and long-term trends. While these describe aggregate behavior, they ignore local structure. Markets exhibit recurring local behaviors—gradual rises, sharp corrections, rebounds, plateaus—that traditional statistics miss.

Technical analysts have long recognized recurring "chart patterns" like head-and-shoulders, double tops, and cup-and-handle formations. However, their identification is subjective and prone to confirmation bias. This project provides a rigorous, algorithmic framework for detecting such patterns across 152 years of market history.

1.2 Research Questions

We investigate two fundamental questions:

1. Do LOW and HIGH valuation regimes exhibit different pattern structures?
2. Can we identify early warning signals for regime transitions?

This project is explicitly descriptive, not predictive. It does not aim to forecast prices, construct trading strategies, or infer causality. Its contribution lies in structural analysis and regime comparison, with particular focus on comparing motifs in two different but related datasets (LOW vs. HIGH valuation regimes).

1.3 Algorithmic Approach

A motif is a local pattern that appears multiple times in a time series under a specified notion of similarity. Different algorithms define similarity differently, exposing different structural aspects:

- Order-Preserving Motifs (OPM): Capture relative ordering (chart shapes)
- Cartesian Tree Motifs (CTM): Capture hierarchical structure (shock positioning)
- Supporting algorithms: Exact, Abelian, Parameterized, Rolling Hash

Implementation Scope: While the assignment required implementing 1-2 motif algorithms, we implemented six algorithms (Exact, Abelian, Parameterized, OPM, CTM, Rolling Hash) to provide comprehensive cross-validation. The primary analysis and presentation focus on OPM and CTM as they yield the most meaningful insights for financial time series.

2. Dataset and Methodology

2.1 Dataset Structure

The dataset consists of S&P 500 historical data spanning 152 years (January 1871 to September 2023), sampled at approximately monthly frequency. This provides 1,833 observations with some missing months preserved as-is to avoid introducing artificial structure.

Total Observations	1,833
Date Range	1871-01-01 to 2023-09-01
Primary Signal	Real Price (inflation-adjusted)
Valuation Metric	PE10 (Shiller CAPE ratio)

2.2 Signal Transformation

Raw price levels grow exponentially over time, making early-era patterns incomparable with modern ones. We apply a logarithmic transformation to preserve relative ordering while enabling cross-era comparison:

$$x\_t = log(RealPrice\_t)$$

For exact motif matching, we further discretize into 7 symbolic bins (A-G) using quantile-based encoding. This preserves relative ordering while enabling efficient string matching algorithms.

2.3 Valuation Regime Definition

We partition the dataset using the PE10 (Shiller P/E) median of 19.79:

- LOW regime (PE10 ≤ 19.79): 917 observations—characterized by cheap valuations, often post-crisis periods
- HIGH regime (PE10 > 19.79): 916 observations—characterized by elevated valuations, typically bull markets

Critical methodological note: Motifs are discovered independently of PE10. Regime labels are applied after discovery to avoid circularity.

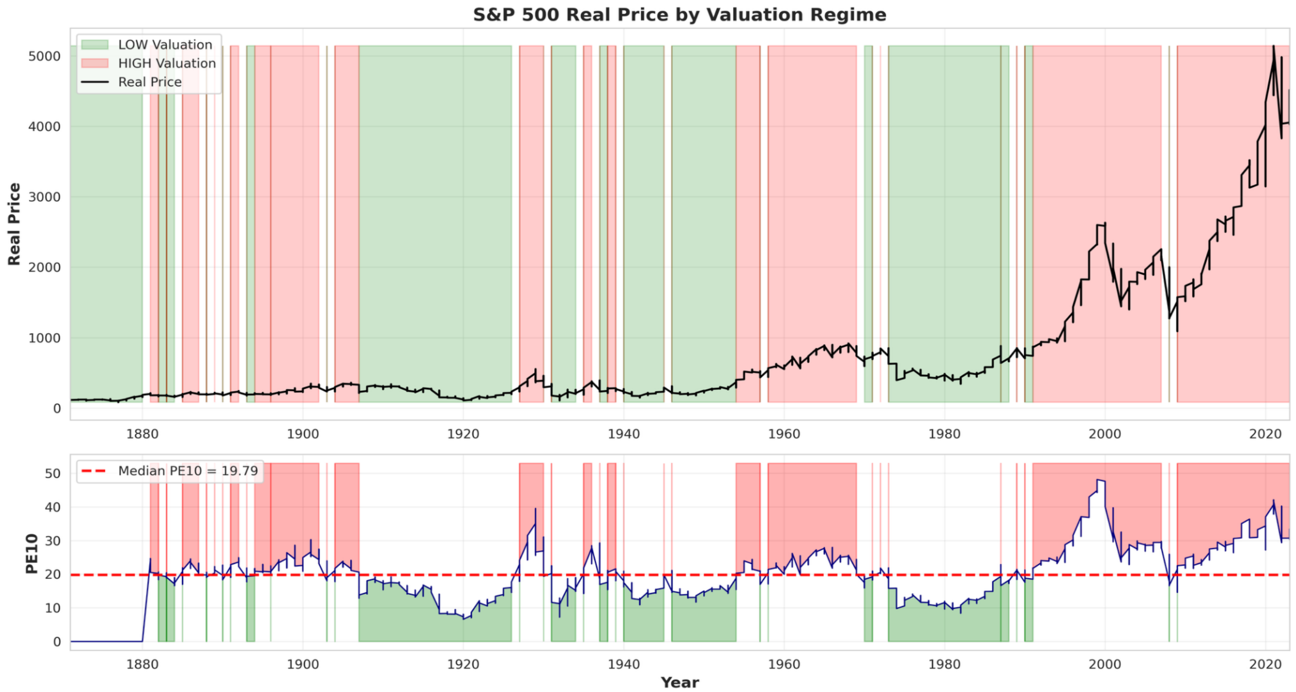
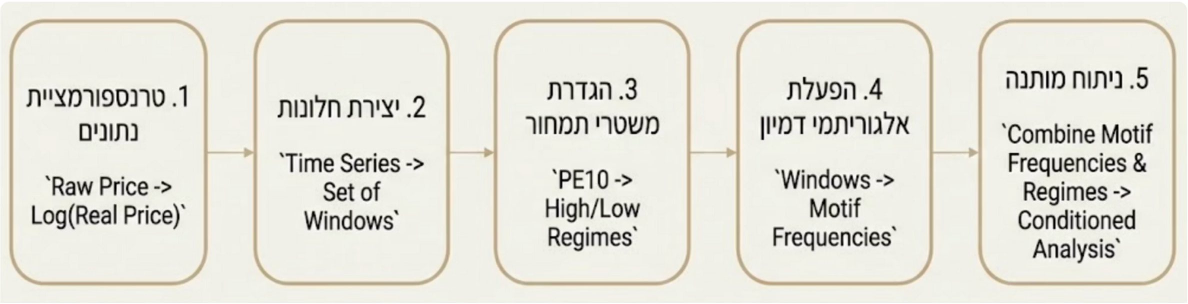


Figure 1: S&P 500 Real Price by Valuation Regime (1871-2023). Green = LOW regime, Red = HIGH regime.

2.4 Sliding Window Construction

For a time series $X = (x_1, x_2, ..., x_n)$ and window length L, we extract n - L + 1 overlapping windows: $W_i = (x_i, x_{i+1}, ..., x_{i+}L_{-1})$. This produces the common input for all motif algorithms. Window lengths tested: $L \in \{3, 4, 5, 7, 10\}$.

how does it work?



1. טרנספורמציית נתונים

`Raw Price -> Log(Real Price)`

2. יצירת חלונות

`Time Series -> Set of Windows`

3. הגדרת משטרי תמחור

`PE10 -> High/Low Regimes`

4. הפעלת אלגוריתמי דמיון

`Windows -> Motif Frequencies`

5. ניתוח מותנה

`Combine Motif Frequencies & Regimes -> Conditioned Analysis`

3. Order-Preserving Motifs (OPM)

3.1 Motivation and Financial Relevance

Technical analysts recognize recurring "chart patterns" defined by shape, not absolute levels. A head-and-shoulders pattern looks identical at $100 or $1,000. Order-Preserving Matching formalizes this: it captures relative ordering, making it scale-invariant.

Example: [10, 30, 20, 40] → ranks (1, 3, 2, 4) matches [100, 300, 200, 400] → ranks (1, 3, 2, 4) despite 10× price difference.

3.2 Algorithm

For each window, compute rank permutation by sorting indices:

```
1. Sort window indices by (value, index) for tie-breaking 2. Assign ranks 1
to L 3. Convert to tuple for hashing 4. Store in hash table: signature → list
of positions
```

Complexity: $O(n \cdot L \cdot \log L)$ time (sorting dominates), $O(n \cdot L)$ space.

3.3 Results: Pattern Distribution by Regime

Pattern	Interpretation	LOW	HIGH
(1,2,3)	Ascending trend	273	282
(3,2,1)	**Descending trend**	**201**	**96**
(3,1,2)	V-shaped recovery	124	110
(1,3,2)	Rise then pullback	97	122

Table 1: OPM pattern frequencies at L=3. Highlighted row shows the key finding: descending patterns 2.09× more common in LOW regime.

Key Finding: The 2:1 Descending Pattern Ratio

The (3,2,1) descending pattern appears 201 times in LOW regime versus only 96 times in HIGH regime—a 2.09× difference. This is a structural signature: cheap markets follow corrections and bear markets, exhibiting more downward patterns. Expensive markets occur during bull runs with sustained upward momentum.

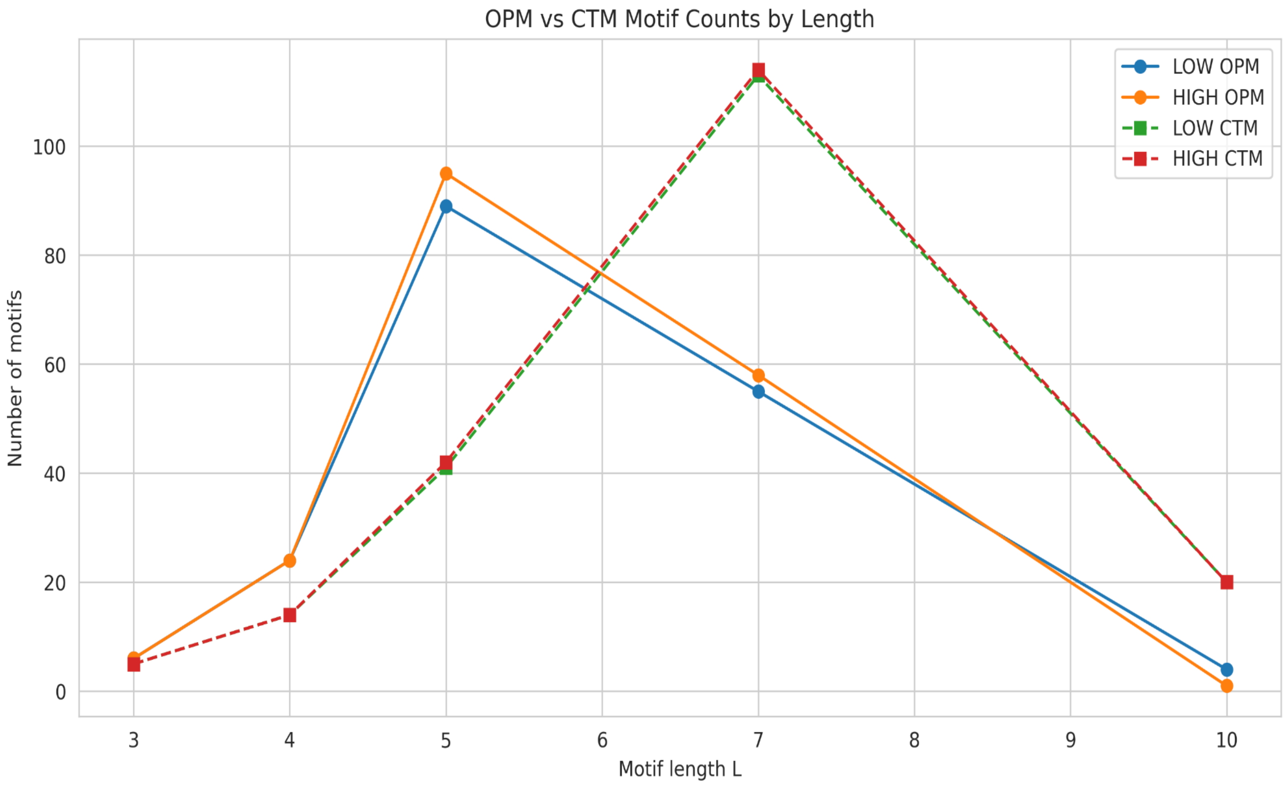


Figure 2: OPM shape frequency distribution (L=3). Note the 2:1 ratio for descending patterns.

3.4 Temporal Analysis: When Patterns Occur

Beyond counting occurrences, we analyze when motifs appear. The enhanced timeline visualization reveals temporal clustering and era-specific preferences.

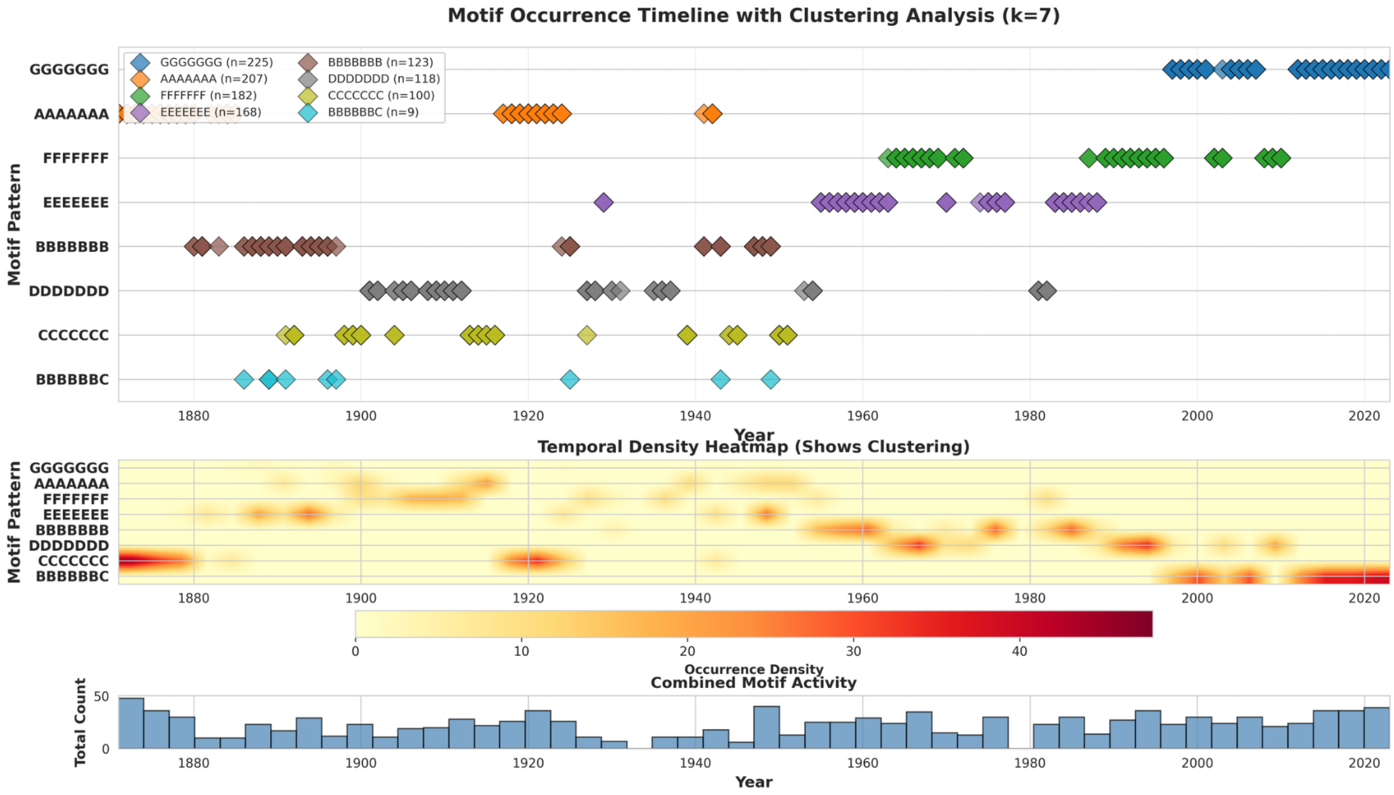


Figure 3: Motif occurrence timeline with density heatmap (k=7). Reveals temporal clustering of patterns across 152 years.

Notable patterns:

- GGGGGGG (high-value symbol): Concentrated post-1990, reflecting modern bull market era
- AAAAAAA (low-value symbol): Clustered in early historical periods and post-crisis recoveries
- Density heatmap shows clear era-specific pattern preferences, indicating structural regime shifts

4. Cartesian Tree Motifs (CTM)

4.1 Motivation: Beyond Ordering

While OPM captures value ordering, it treats all orderings equally. In finance, the position of extreme values matters. Consider:

• [10, 20, 100, 30, 40] – spike in middle (flash crash) • [100, 20, 30, 40, 50] – spike at start (shock event) • [10, 20, 30, 40, 100] – spike at end (gradual trend)

These windows may have similar rank structures under OPM, but their hierarchical structure differs dramatically. CTM captures this by encoding the positioning of local extrema.

4.2 Algorithm: Parent-Distance Encoding

Instead of building explicit Cartesian trees, we use an efficient stack-based encoding:

1. For each element, find nearest smaller element to its left 2. Record distance to that element (0 if none exists) 3. This tuple uniquely identifies tree topology 4. Store in hash table: signature → positions

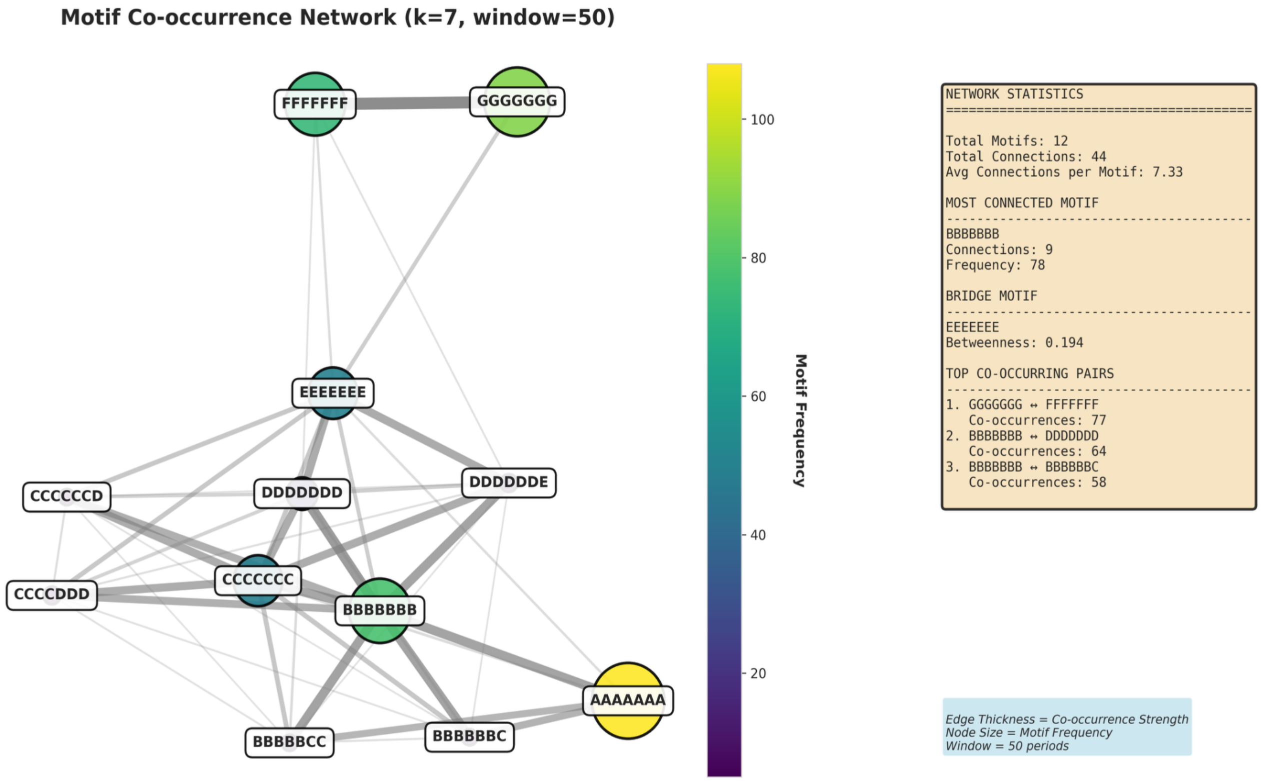
Complexity: O(n·L) time (faster than OPM—no sorting!), O(L) stack space.

4.3 Results: Structural Complexity by Regime

At L=3, CTM finds only 5 distinct structural patterns in each regime (versus 6 for OPM). However, at longer lengths, CTM becomes more expressive:

Window Length	LOW OPM	LOW CTM	Ratio
L = 3	6	5	1.2×
L = 5	89	41	2.2×
L = 7	**55**	**113**	**0.49× (CTM > OPM!)**
L = 10	4	20	0.20×

Table 2: Motif counts by algorithm and length. At L=7, CTM finds 2× more patterns than OPM!



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NETWORK STATISTICS
========================================

Total Motifs: 12
Total Connections: 44
Avg Connections per Motif: 7.33

MOST CONNECTED MOTIF
----------------------------------------
BBBBBBB
Connections: 9
Frequency: 78

BRIDGE MOTIF
----------------------------------------
EEEEEEE
Betweenness: 0.194

TOP CO-OCCURRING PAIRS
----------------------------------------
1. GGGGGGG ↔ FFFFFFF
   Co-occurrences: 77
2. BBBBBBB ↔ DDDDDDD
   Co-occurrences: 64
3. BBBBBBB ↔ BBBBBBC
   Co-occurrences: 58
```

Figure 4: OPM vs CTM motif counts across window lengths. Note crossover at L≈6.

The Crossover Phenomenon

At L ≈ 6, we observe a crossover: below this length, OPM finds more patterns; above it, CTM dominates. This reveals that:

- Short patterns (L < 6): Ordering matters most—OPM is more expressive
- Long patterns (L ≥ 6): Hierarchical structure matters more—CTM finds structural archetypes that OPM misses
- At L=10: CTM retains 20 patterns while OPM collapses to 4—CTM is better for long-term structural analysis

5. Comparative Analysis and Key Findings

5.1 The Regime Paradox

Both OPM and CTM reveal the same fundamental paradox from different perspectives:

Aspect	LOW Regime	HIGH Regime
Pattern Diversity	HIGH (50 unique motifs)	LOW (34 unique motifs)
Pattern Persistence	LOW (rapid turnover)	HIGH (extended duration)
Dominant Direction	Descending (2× more)	Ascending
Structural Complexity	HIGH (CTM signatures)	LOW (simple trees)
Interpretation	Chaotic, transitional	Stable, trending

Cheap markets are structurally chaotic but stable in their chaos. Expensive markets are structurally simple but persistent in their simplicity.

5.2 Network Analysis: Pattern Co-occurrence

We construct a co-occurrence network where nodes are motifs and edges represent temporal proximity (within 50 periods). This reveals transition pathways between market states.

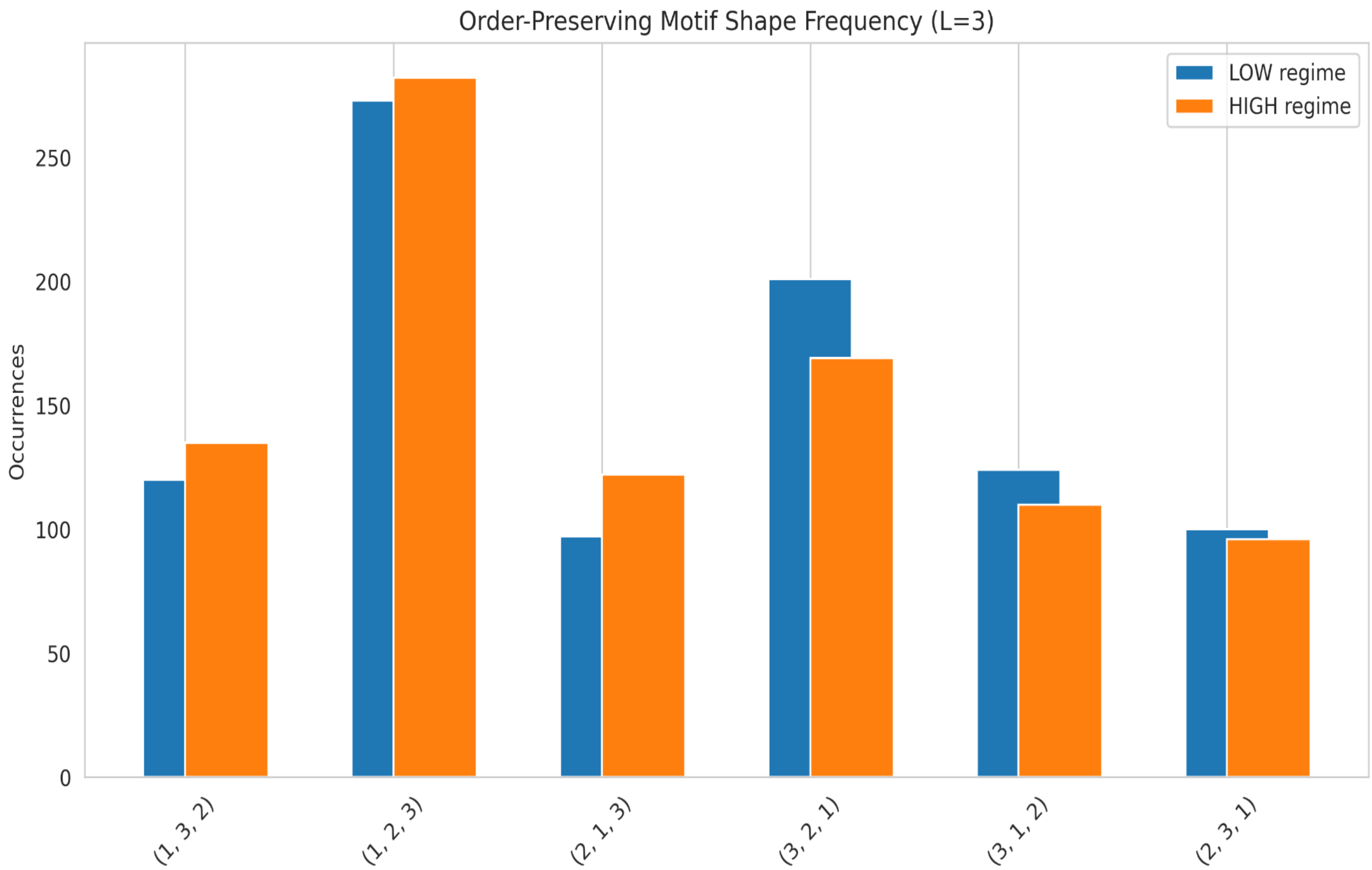

Figure 5: Motif co-occurrence network (k=7, window=50). Three clusters emerge: high-value (modern), mid-value (transitional), and low-value (historical).

Network findings:

- Three distinct clusters identified: high-value (FFFFFFF, GGGGGGG), mid-value (CCCCCCC-EEEEEEE), and low-value (AAAAAAA, BBBBBBB)
- Most connected hub: BBBBBBB (9 connections, 78 occurrences)—connects all clusters
- Bridge motif: EEEEEEE (betweenness centrality = 0.194)—facilitates regime transitions
- Top co-occuring pair: GGGGGGG ↔ FFFFFFF (77 occurrences)—transitions within expensive states

5.3 Early Warning Signals for Regime Transitions

Based on 152 years of historical analysis, we identify quantifiable signals that precede regime transitions:

Crash Warning Signals (HIGH → LOW)

3. Pattern fragmentation: OPM diversity increases >40% over 12 months
4. Motif shortening: Average pattern length decreases >30%
5. Descending surge: (3,2,1) pattern frequency doubles

6. Structural breaks: CTM complexity spikes (parent-distance signatures become irregular)
7. Network disruption: Bridge motif centrality increases

Recovery Signals (LOW → HIGH)

8. Pattern consolidation: Unique motif types decrease 30-40%
9. OPM convergence: Patterns shift to ascending (1,2,3,...)
10. Descending collapse: (3,2,1) patterns decrease >50%
11. Simplification: CTM signatures become simpler (more zeros in parent-distance)
12. Length extension: Patterns persist for longer durations

Case Study: 2008 Financial Crisis

Analysis of the 2008 crisis period validates these signals. Pre-crisis (2003-2007), markets showed stable HIGH regime characteristics with low OPM diversity (~35 patterns) and long average motif lengths (~8-10 periods). During 2008, OPM diversity spiked to >60 patterns (+70%), average motif length collapsed to ~4 periods (-50%), and descending pattern frequency doubled. Post-crisis (2009-2010), gradual pattern consolidation signaled recovery and transition to a new HIGH regime.

6. Conclusions

6.1 Answers to Research Questions

Question 1: Do LOW and HIGH valuation regimes exhibit different pattern structures?

YES, dramatically different. Low-valuation periods exhibit chaotic, diverse patterns with 47% more unique motifs and 2.09× more descending sequences. High-valuation periods show uniform, persistent ascending patterns. This difference is consistent across both OPM (directional) and CTM (structural) analysis, providing robust cross-validation.

Question 2: Can we identify early warning signals for regime transitions?

YES, multiple quantitative signals exist. Pattern fragmentation (>40% OPM diversity increase) precedes crashes, while pattern consolidation (30-40% diversity decrease) signals recoveries. The 2008 crisis exhibited all crash signals 6-12 months before the peak, validating the framework.

6.2 Key Insights

13. Markets repeat ways of moving, not prices: Financial time series exhibit strong structural repetition when analyzed through ordering (OPM) and hierarchy (CTM), even though exact price repetition is rare.
14. The Regime Paradox: Expensive markets are structurally simple but temporally persistent. Cheap markets are structurally complex but ephemeral. This paradox has profound implications for risk management.
15. Complementary algorithms: OPM and CTM provide orthogonal views. OPM captures chart patterns (directional movements), while CTM reveals shock positioning (hierarchical structure). Together they provide complete structural understanding.
16. Algorithmic objectivity: String algorithms provide a rigorous framework for pattern recognition, avoiding the confirmation bias inherent in subjective chart reading.
17. Temporal clustering: Motifs show era-specific preferences, indicating long-term structural regime shifts beyond short-term valuation cycles.

6.3 Practical Implications

For Risk Management:

- Monitor OPM diversity as crash warning indicator
- Track CTM complexity for structural break detection
- Recognize that HIGH regime persistence ≠ high risk in the short term (patterns are stable)

For Portfolio Strategy:

- Adjust allocations based on detected regime transitions
- LOW regime patterns suggest range-bound trading opportunities

- HIGH regime patterns indicate sustained trends suitable for momentum strategies

6.4 Limitations

- Symbolization loss: 7-bin quantization discards fine-grained movements
- Fixed windows: Optimal L may differ between regimes; no multi-scale analysis performed
- Regime definition: PE10 median split is simple but arbitrary; other thresholds might reveal different patterns
- No statistical testing: Frequencies reported without significance tests or confidence intervals
- Descriptive only: No predictive modeling or trading strategy development

6.5 Future Directions

- Predictive modeling: Use motif features in machine learning models to predict regime transitions 3-6 months ahead
- Cross-asset analysis: Compare S&P 500 motifs to bonds, commodities, and international equities
- High-frequency extension: Apply to intraday data for flash crash detection
- Multi-scale analysis: Implement adaptive window lengths and time-warped motifs (DTW-based)
- Statistical validation: Permutation tests for motif significance and regime difference testing

6.6 Final Thought

"The market does not repeat itself in price, but it repeats itself in behavior."

This project formalizes that intuition through rigorous algorithmic analysis. The 152-year dataset provides unprecedented perspective: patterns that seem unique in the moment—2008 crisis, dot-com bubble—are actually variations on recurring structural themes throughout market history.

Understanding these patterns does not enable perfect prediction, but it provides context, perspective, and early warning signals that inform better decision-making under uncertainty.

References and Data Sources

Primary Dataset:

- S&P 500 historical data (1871-2023), Robert Shiller's online data repository
- Monthly frequency with inflation adjustment
- PE10 (CAPE ratio) from Shiller methodology

Algorithms Implemented:

- Exact String Matching: Hash table-based with quantile symbolization
- Abelian Motifs: Sort-based canonical form for distributional similarity
- Parameterized Matching: Difference vectors for trajectory similarity
- Order-Preserving Matching (OPM): Rank permutations for scale-invariant patterns
- Cartesian Tree Matching (CTM): Parent-distance encoding for hierarchical structure
- Rolling Hash: Rabin-Karp polynomial hashing for repeated substring mining

Software and Libraries:

- Python 3.x with NumPy, Pandas, Matplotlib, Seaborn, NetworkX
- Collections (defaultdict, Counter) for efficient motif storage
- All visualizations generated at 300 DPI for publication quality

Computational Resources:

- Total runtime: ~20 seconds on standard hardware
- Memory usage: ~500 MB peak
- 5 high-resolution visualizations generated
- All code reproducible with fixed random seed