35. (2 points) Show that the Subset Sum problem is NP-hard using the fact that the Vertex Cover problem is NP-complete. Recall that the input for the Subset Sum problem is positive integers $x_1, \ldots, x_n, L$ the problem is to determine if there is a subset of the x_i's with aggregate sum L. Recall that the input to the Vertex Cover problem is an undirected graph $G = (V, E)$ and a positive integer k and the problem is to determine if G has a vertex cover of size k. Recall that a vertex cover is a collection of vertices such that every edge is incident to at least one of these vertices.

    Hint: Try to roughly mimic the proof that we did in class that Subset Sum is NP-hard using the fact that 3SAT is NP-complete.

36. (4 points)

    (a) The input to the 3-Single-SAT problem consists of a collection of triples of literals (which is a Boolean variable or a negation of a Boolean variable). So for example, and input with four clauses is:
    $$\{\{x, \bar{y}, z\}, \{\bar{x}, \bar{y}, z\}, \{w, y, z\}, \{x, \bar{w}, z\}\}$$
    The problem is to determine whether there is an truth assignment to the variables that makes exactly one literal per each clause true. Show that the 3-Single-SAT problem is NP-hard using the fact that 3SAT is NP-complete.

    (b) The input to the triangle problem is a subset W of the Cartesian product $X \times Y \times Z$ of sets X, Y and Z, each of cardinality n. The problem is to determine if there is a subset U of W such that 1) every element of X is in exactly one element of U, 2) every element of Y is in exactly one element of U, and 3) every element of Z is in exactly one element of U. Here's a story version of the same problem. You have disjoint collections of n pilots, n copilots, and n flight engineers. For each possible triple of pilot, copilot, and flight engineer, you know if these three people are compatible or not. You goal is to determine if you can assign these $3n$ people to n flights so that every flight has one pilot, one copilot, and one flight engineer that are compatible. Show that this problem is NP-hard using the fact that 3-Single-SAT is NP-complete.

37. (6 points) We consider a generalization of the Fox, goose and bag of beans puzzle `http://en.wikipedia.org/wiki/Fox,_goose_and_bag_of_beans_puzzle`

    The input is a graph G an integer k. The vertices of G are objects that the farmer has to transport over the river, there are an edge between two objects if they can not be left alone together on the same size of the river. The goal is to determine if a boat of size k is sufficient to safely transport the objects across the river. The size of the boat is the number of objects that the farmer can haul in the boat.

    Show that this problem is NP-hard using a reduction from vertex cover.