Modeling and Simulation of Aerospace Systems Flipped Class: Thermal Domain

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Overview of this presentation

1. Mechanism of Heat Exchange

- Conduction
- Convection
- Radiation

2. Problem Modeling

- Thermal Capacitance
- Static and Dynamic thermal systems
- Biot Number

Introduction

- Thermal systems **transfer or store energy** via temperature and heat flow.
- **Key effects**: conduction, convection, radiation, and heat storage.
- **Similarities** with mechanical, fluid, and electrical systems: Resistance, capacitance, circuit analysis, and dynamic response.
- **Unique aspects**:
  - Nonlinear, variable-coefficient, distributed-parameter models.
  - No thermal inductance.
- **Combination** of thermodynamics, fluid mechanics, and heat transfer.
- **Simplified analysis** using linear, lumped-parameter models.

Conduction: Fourier's Law

The **Fourier's law** is defined as follows:

$$\frac{Q_h}{A} = -k_{\underline{p},\underline{d},T}\,\underline{\nabla}T$$

Where $k_{\underline{p},\underline{d},T}$ is the **thermal conductivity**. In general, it is a second-order tensor function of: **point**: $\underline{p}$, **direction**: $\underline{d}$ and **temperature**: T

Assuming:

- *Isotropic medium*
- *Homogeneous medium*
- *No dependance on T*

$$k_{\vec{p},\vec{d},T} = k_t = const.$$

By considering only the x direction, Fourier's law becomes:

$$\boxed{\frac{Q_h}{A} = -k_t\,\frac{dT}{dx}}$$

General Conduction Equation

Considering a **closed system** with no mechanical work, the **First Law of Thermodynamic** can be simplified into:

$$\frac{\partial U}{\partial t} = \sum Q + U$$

With the **Reynold's Transport Theorem**, **Divergence Theorem** and application of the **Fourier's Law**:

$$\rho c \frac{\partial T}{\partial t} = \nabla \left(k_{\underline{p},\underline{d},T} \nabla T \right) + \frac{U}{V}$$

If the conductivity is a constant:

$$\rho c \frac{\partial T}{\partial t} = k_t \nabla^2 T + \frac{U}{V}$$

Includes three second-order space derivatives and a derivative in time.

Conduction for 1D plate and axial rod

Considering the one-dimensional Fourier's law, assuming costant k_t and uniform Q_h:

$$\frac{Q_h}{A} = -k_t \frac{dT}{dx} \longrightarrow \int_0^l \frac{Q_h}{A} dx = -\int_{T_1}^{T_2} k_t \, dT$$

$$\frac{Q_h}{A} L = -k_t(T_2 - T_1) \longrightarrow Q_h = \frac{k_t A}{L} \Delta T \longrightarrow \Delta T = \frac{L}{k_t A} Q_h$$

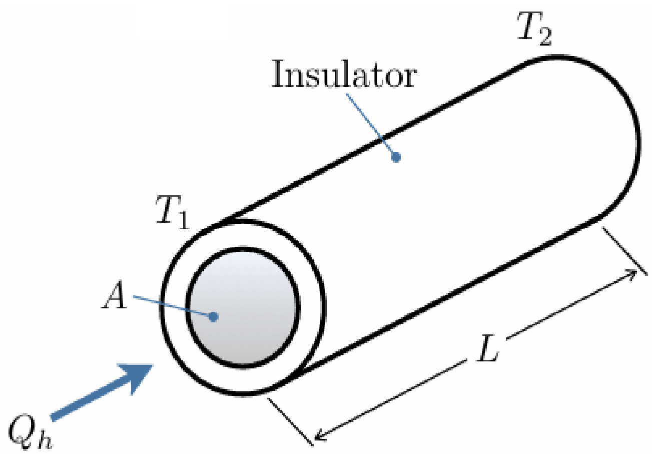
Effort

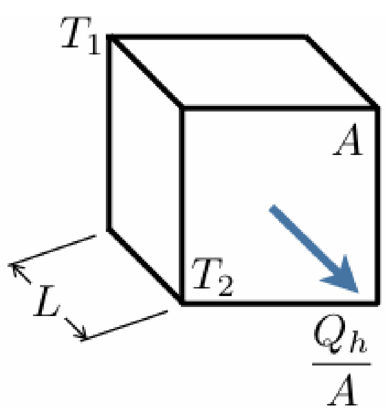
Thermal resistance

Flow

Thermal resistance due to conduction (axial case):

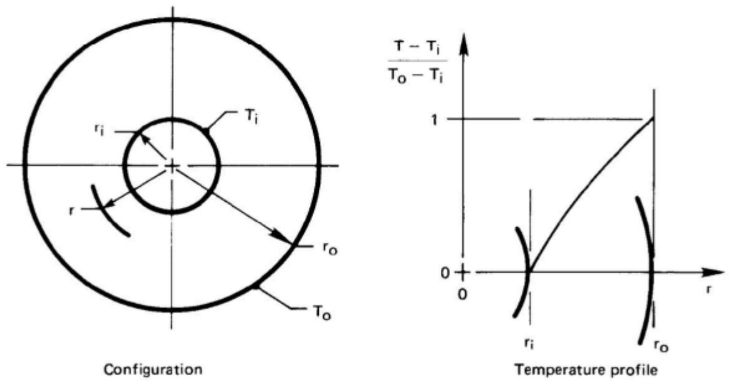
$$\boxed{R = \frac{L}{k_t A}}$$





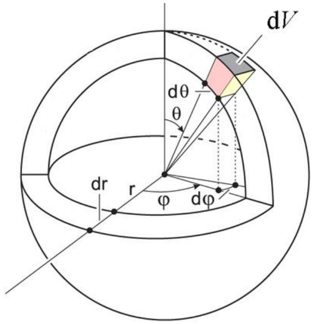
Conduction in Cylindrical and Spherical Coordinates

Thermal resistance due to radial conduction:



$$R = \frac{\ln(r_o/r_i)}{2\pi k_t L}$$

Thermal resistance due to spherical conduction:

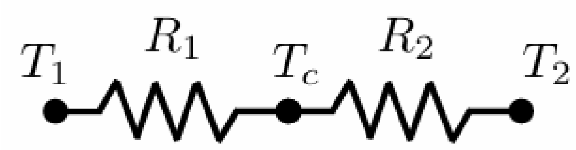


$$R = \frac{r_o - r_i}{4\pi k_t r_o r_i}$$

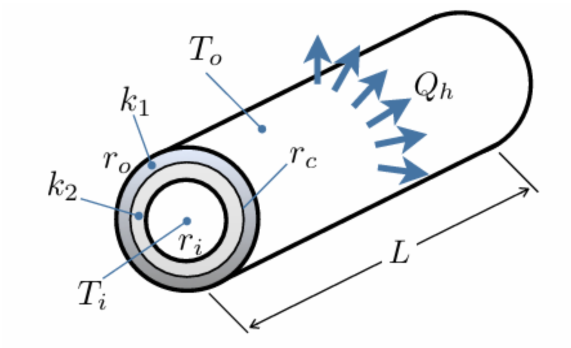
Conduction: resistance modeling

In both cases of **radial conduction in a cylinder** and **axial conduction through a flat plate**, it is possible to model the problem considering a **series of resistance**:

$$T_1 - T_2 = R_{eq} Q_h$$



In which R_{eq} is just the summation of the resistances in series, using the previous formulas depending on the case. For example, considering two different coating materials:



$$R_{eq} = \frac{\ln(r_o/r_c)}{2\pi k_1 L} + \frac{\ln(r_c/r_i)}{2\pi k_2 L}$$

Convection: basics

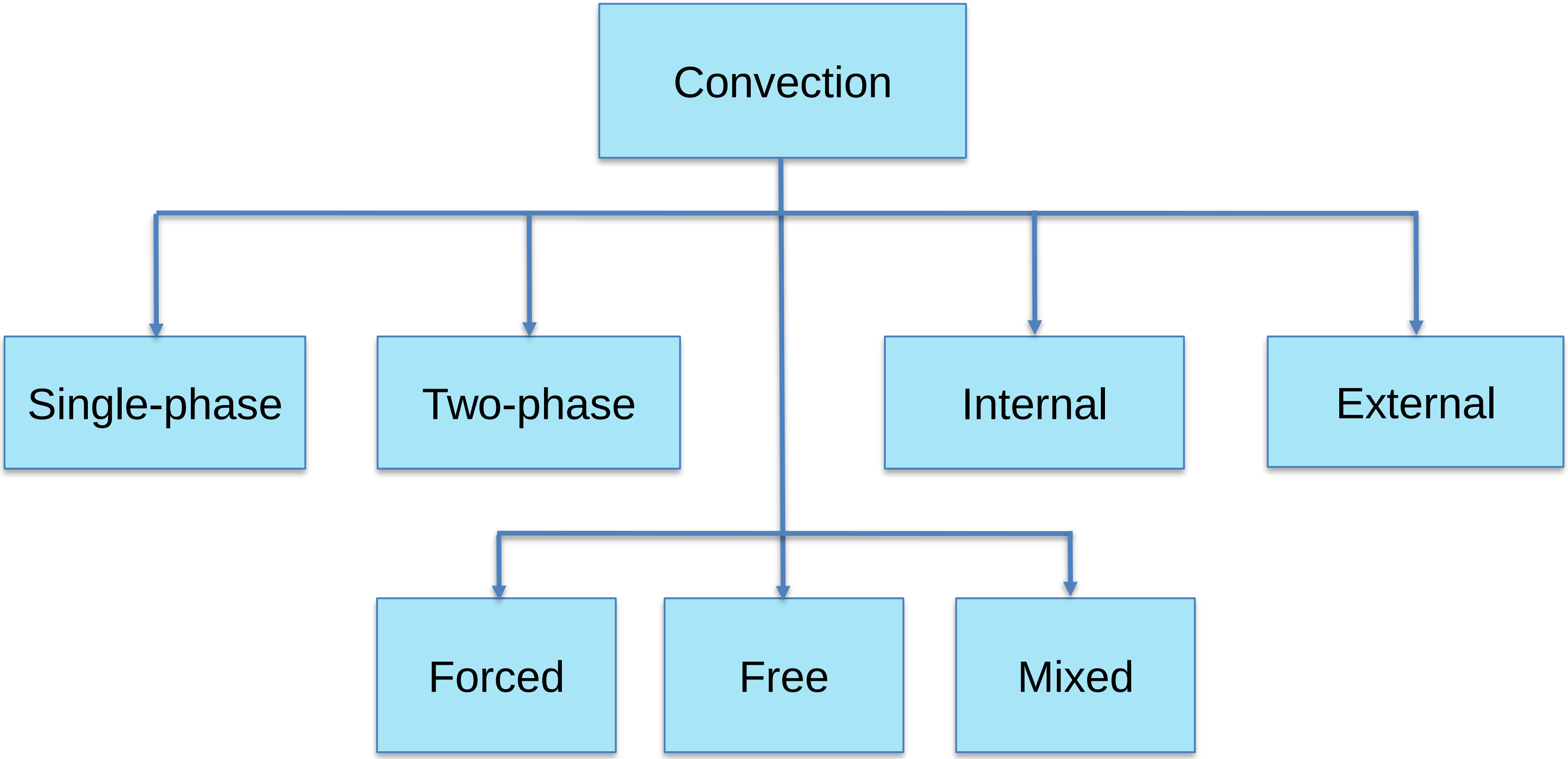
- Heat transfer by convection occurs when **macroscopic mass motion** (advection) is present in addition to **energy diffusion**.

- It is possible only in states of matter that permits internal mass movement, such as **fluids** (liquids, gases, plasma).

- While convection enhances heat transfer but it also complicates the process significantly more than conduction alone, primarily due to the complexities of fluid dynamics!

Convection: classification

Heat transfer by convection can be **classified** following different lines:



Convection: Newton's law

It is ruled by the **Newton's law**:

$$\boxed{\frac{Q_h}{A} = h\,(T_s - T_\infty)}$$

Where:

- T_s is the temperature of the surface of the solid
- T_∞ is the temperature of the free-steam fluid
- *Convection coefficient h* is a function of:
  - fluid properties (thermal conductivity, density, specific heat)
  - fluid velocity field
  - area of the solid-fluid interface across which heat transfer occurs

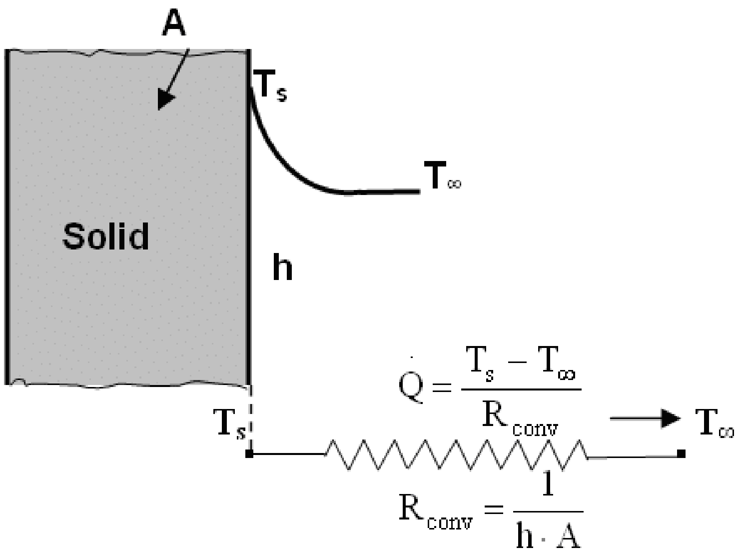
$$h = h\left(c_{Pf},\, \rho_f,\, \mu_f,\, \lambda_f,\, D_C,\, \overset{w}{g\,\rho_{f0}\,k_{Pf}\,(T_f - T_w)}\right)$$

Typically, the computation of *h* requires detailed CFD approach or experiments

Convective Resistance

Thermal resistance due to convection:

$$\boxed{R = \frac{1}{hA}}$$



Conditions of heat transfer	$h\ [W/(m^2K)]$
Gases in **free** convection	2-20
Liquids in **free** convection	50-1000
Gases in **forced** convection	25-300
Liquid in **forced** convection	1000-40000

Radiation: black body

Stefan-Boltzman Law for an ideal **black body**:

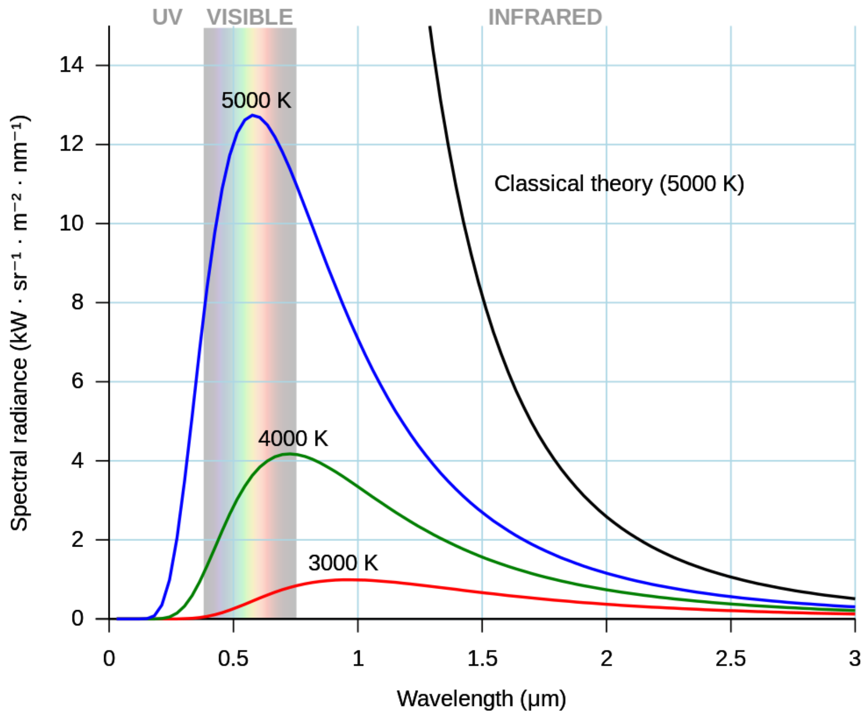
$$\boxed{\frac{Q_h}{A} = \sigma\, T^4}$$

Where $\sigma = 5.67 \times 10^{-8}\ \frac{W}{m^2 K^4}$

Where a black body is an ideal body that:

- **absorbs all incident radiation**, regardless of frequency.
- if at thermal equilibrium, **emits thermal radiation at every frequency isotropically**.

In particular, a black body emits depending on its temperature, as shown on the right.



Radiation: gray opaque body

Stefan-Boltzman Law for a **gray opaque body**:

$$\boxed{\frac{Q_h}{A} = \varepsilon\, \sigma\, T^4}$$

Where ε is the **emissivity** which defines how much a surface differs from a **black body**. By definition, a black body in **thermal equilibrium** has an ε =1.

From **balance of energy**:

$$\alpha + \rho + \tau = 1$$

Where

- α: Absorbivity
- ρ: Reflectivity
- τ: Transmissivity

Kirchhoff's law: $\alpha = \varepsilon$

Thermal Capacitance

Thermal capacitance is the measurable physical quantity that characterizes the **amount of heat** required to change a substance's **temperature by a given amount**:

$$C = \frac{Q}{\Delta T} = \left[\frac{kJ}{kg\ K}\right]$$

C can be also defined as the product of the **specific heat capacity** per mass unit and the **mass of the object**.

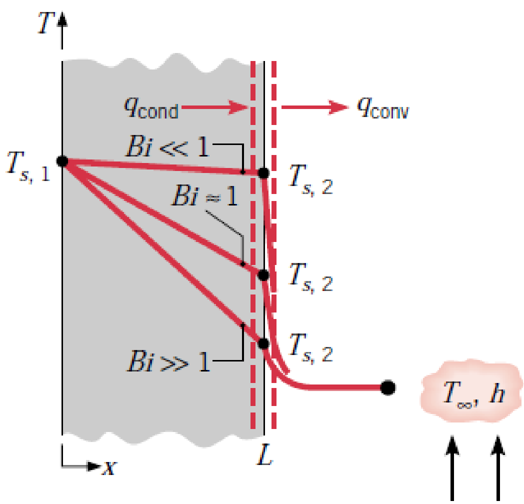
$$Q_h = MC_p\frac{dT}{dt} \longrightarrow C = MC_p$$

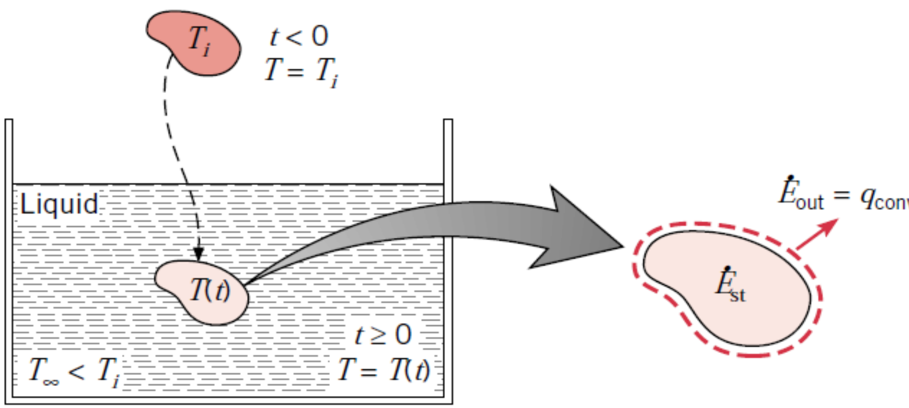
Static *vs* Dynamic

Aspect	Static Thermal Analysis	Dynamic Thermal Analysis
Capacitance	Low storage	High storage
Modeling	Algebraic Equations	Diffusion equation (2nd-order PDE)
Modeling approach	Simple laws of conduction, convection and radiation	Lumped-parameter modeling possible (ODEs)
Capacitance model		**Single** or **multiple-lumped** models depending on convection and conduction

Biot Number

When temperature gradients within the solid may be neglected?





Biot Number

Biot Number: $B_i = \frac{Thermal\ resistance\ for\ conduction\ inside\ a\ body}{Thermal\ resistance\ for\ convection\ on\ the\ surface} = \frac{h\ L_C}{k}$

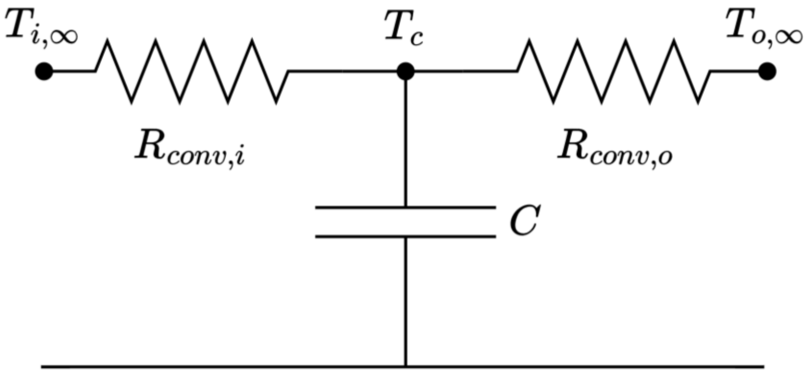
Where L_C is a characteristic length (thickness for flat plate, diameter/6 for sphere, thickness/2 for a fin and diameter/4 for a long cylinder).

If $B_i < 0.1$	If $B_i > 0.1$
Conduction negligible compared to convection	Conduction **not** negligible
Thermal gradient inside body negligible	Thermal gradient inside body **not** negligible
Single-lumped capacitance modeling	**Multiple-lumped capacitance modeling**

Case: $B_i < 0.1$

Single lumped capacitance:

- Temperature gradient in solid negligible
- Model with single lumped capacitance, heat capacity of solid and convection heat transfer

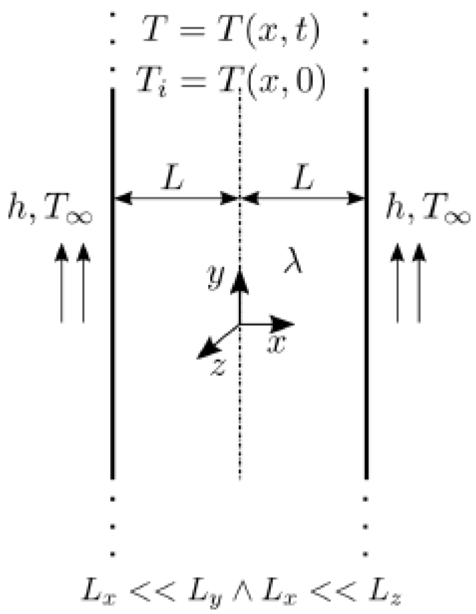


From energy balance on the internal node, the solution is the following **first-order ODE**:

$$MC_p\frac{dT_c}{dt} + 2T_chA = hA(T_{i,\infty} + T_{o,\infty})$$

Case: $B_i > 0.1$

1D Transient Analysis:



Cartesian coord.

$$\frac{\partial T}{\partial t} = \alpha \frac{\partial^2 T}{\partial x^2} \qquad 0 < x < L;\ t > 0$$

$$\left.\frac{\partial T}{\partial x}\right|_0 = 0 \qquad \text{symm. BC; } T_i \text{ homog.}$$

$$-\lambda \left.\frac{\partial T}{\partial x}\right|_L = h\left[T(L,t) - T_\infty\right] \qquad \text{conv. BC}$$

$$T(x,0) = T_i \qquad \text{initial cond.}$$

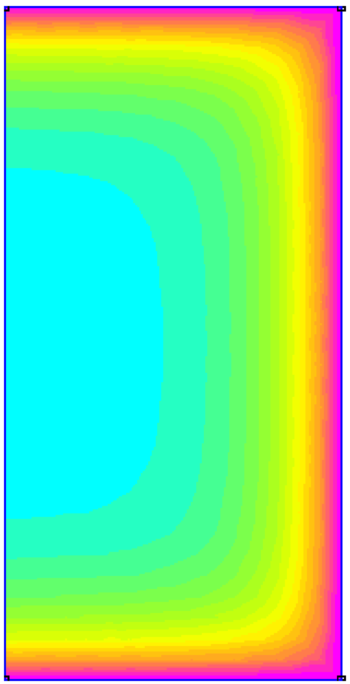
$$\boxed{\theta(\tilde{x}, \mathrm{Fo}) = \sum_{n=1}^{\infty} C_n \exp\left(-\lambda_n^2 \mathrm{Fo}\right) \cos(\lambda_n \tilde{x})}$$

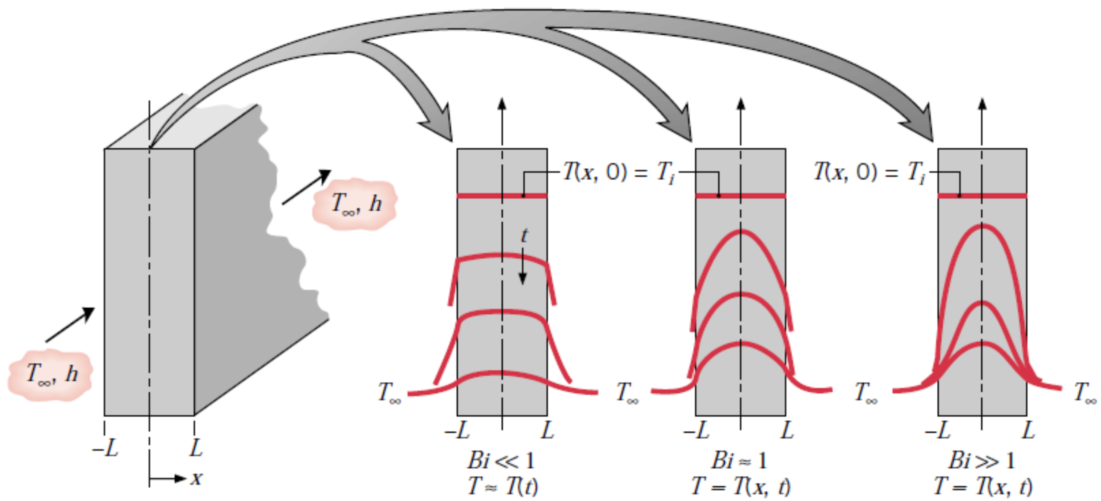
$$C_n = \frac{4 \sin \lambda_n}{2\lambda_n + \sin(2\lambda_n)}$$

$$\lambda_n \tan \lambda_n = \mathrm{Bi}$$

Case: $B_i > 0.1$

1D Transient Analysis:

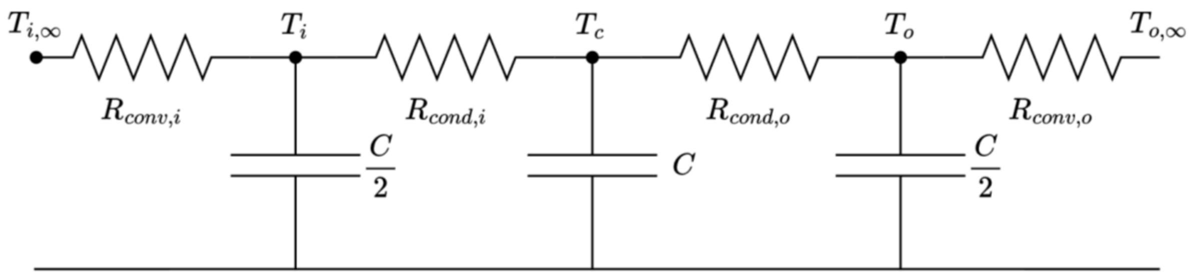




Case: $B_i > 0.1$

Multiple-lumped capacitance modeling:

- Conduction resistance present
- Multiple capacitance lumps
- Thermal properties assumed homogeneous
- Large Biot number requires division into several nodes



$$C/2\dot{T}_i = hA(T_{i,\infty} - T_i) - 1/R(T_i - T_c)$$

$$C\dot{T}_c = 1/R(T_i - T_c) - 1/R(T_c - T_o)$$

$$C/2\dot{T}_o = 1/R(T_c - T_o) - hA(T_o - T_{o,\infty})$$

Biot Number

The Biot number could be generalized, by redefining h depending on mechanisms of thermal energy transport:

- **Conduction*** →

  (*only for heat flux)

$$h_{cond} = \frac{1}{R_{cond}} = \frac{k}{L}$$

- **Convection** →

$$h_{conv} = f(N_u, R_e, P_r)$$

- **Radiation** →

$$h_{rad} = \epsilon\sigma(T_w^2 + T_r^2)(T_w + T_r)$$

Bibliography

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- *John H. Lienhard IV* - A Heat Transfer Notebook;
- *Frank P.Incropera* - Fundamentals of Heat and Mass Transfer.