# Spatial Data and Analysis in R

## A PRISM Workshop

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# Outline

# Why Are We Here?

- Tobler's first law of geography:

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- Tobler's first law of geography:
- "Everything is related to everything else, but near things are more related than distant things"
- We want to quantify how the spatial relationship between our observations affect our inferences

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- I will be introducing *spatial statistics* with a touch of *GIS*
- I will not be discussing *GIS* in depth, nor will I discuss remote sensing *at all*

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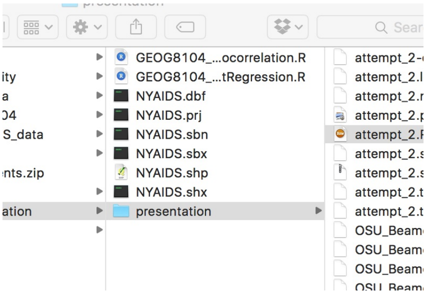
- Many kinds of spatial data: Points, Lines, Polygons, Raster data
- Today, we are working with polygon data

# Prepping our data

- Spatial data come in *shapefiles* which are really mini-databases



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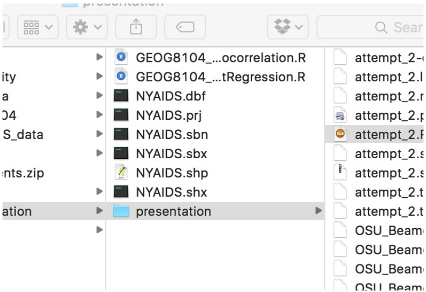

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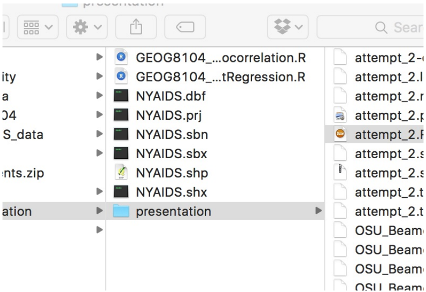
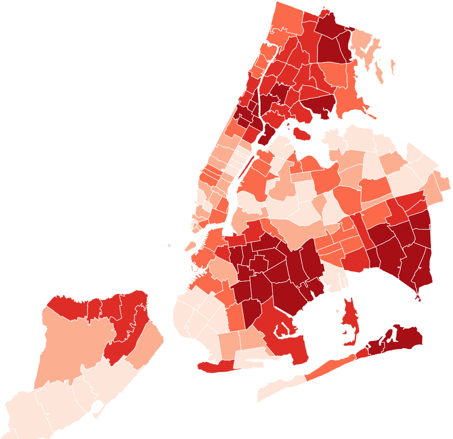
- Six parts, all combine to create a map to represent data

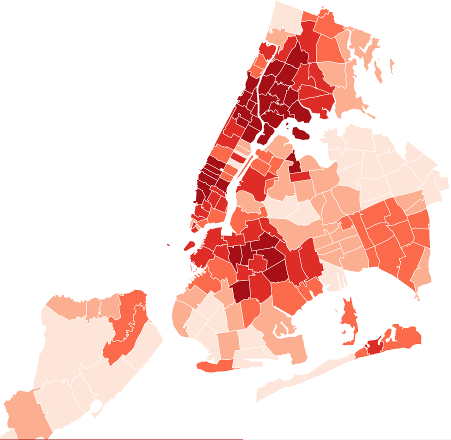


# Loading Our data

# Percent Black



# AIDs Rate per 1000 people



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- Observations with more similar values tend to occur more closely together
- Most common test: Moran's I

# Formula for Moran's I

$$I = \frac{n}{\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}} \frac{\displaystyle\sum_{i=1}^{n} \sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\displaystyle\sum_{i=1}^{n} (x_i - \bar{x})^2},$$

where $w_{ij}$ is the weight between observation i and $j$

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- Can be specified in a variety of ways, the simplest of which is binary (”contiguity”): 1 if observations share a boundary, 0 if they do not
- The default in $R$ is "row standardized," where $w_{ij} = \frac{i}{\sum j}$

# Creating a weights matrix in *R*

```
library(rgdal)
library(spdep)
library(sp)
library(spatstat)
file_path <- "/Users/adamlauretig/data/Prism_presentation/NYAIDS_data"
ny <- readOGR(dsn = file_path,
              layer = "NYAIDS", verbose=FALSE)
nygal <- poly2nb(ny) #Create the neighborhood object
nyQ1.gal <- nb2listw(nygal, zero.policy=T) #Create the weights object
```

# Running the Moran's I

| Moran I statistic | Expectation | Variance | p-value |
|---|---|---|---|
| 0.68 | -0.01 | 0.00 | 0.00 |

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- Measure how similar a value is compared to neighboring values
- While the Moran's I detects clustering, the LISA detects *clusters*

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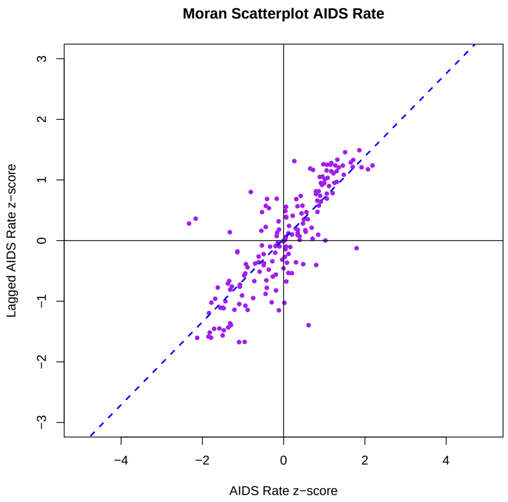

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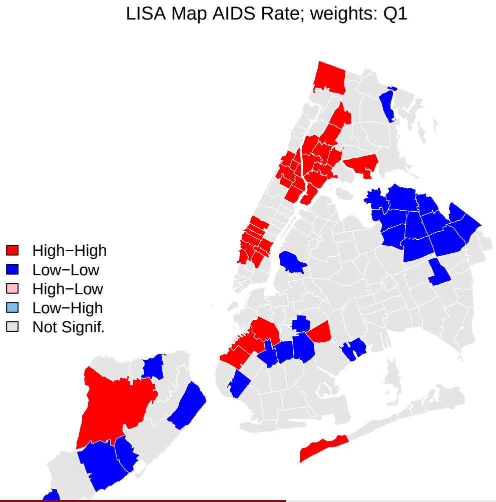

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- Addtionally, with spatial autocorrelation, our coefficients may be biased
- Two ways of handling this: Spatial Error Models, and Spatial Autoregressive models

# The Math: Spatial Error Model

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- $\varepsilon$ is the residual of residuals, with $\sum_{\varepsilon} = \sigma^2 I$
- The full model: $y_i = x_i\beta + \sum_{j=1}^{n} w_{ij}e_j + \varepsilon_i$

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- $y_i = x_i\beta + \sum_{j=1}^{n} w_{ij}y_j + \varepsilon_i$
- SAR vs. SEM

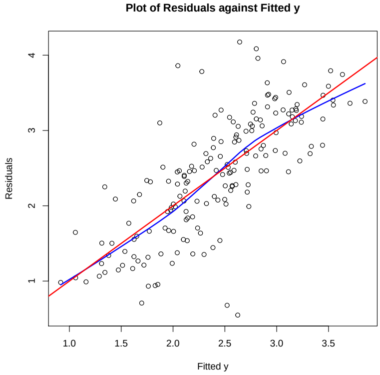
# Plain OLS

| | Model 1 |
|---|---|
| (Intercept) | $-3.05^{***}$ |
| | (0.80) |
| PctWht | $-0.01^{***}$ |
| | (0.00) |
| PctHisp | $0.02^{***}$ |
| | (0.00) |
| Gini | $7.97^{***}$ |
| | (0.71) |
| PctHSEd | $0.02^{**}$ |
| | (0.01) |
| PctFemHH | 0.01 |
| | (0.01) |
| $R^2$ | 0.57 |
| Adj. $R^2$ | 0.55 |
| Num. obs. | 174 |
| RMSE | 0.54 |

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table: Statistical models

# Did we model out our autocorrelation?

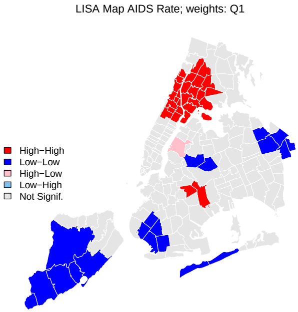


# Did we model out our spatial autocorrelation?

| Moran I statistic | Expectation | Variance | p-value |
|---|---|---|---|
| 0.63 | -0.01 | 0.00 | 0.00 |

No! This statistic didn't change much from before we ran our regression: .68 vs .63

# Where is the residual autocorrelation?



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- One option is to use fixed effects
- But this removes something interesting – the spatial relationship
- Another option is to explicitly model the spatial relationship

# Spatial Autoregressive Model

| | Model 1 |
|---|---|
| (Intercept) | $-1.52^{*}$ |
| | (0.63) |
| PctWht | $-0.01^{***}$ |
| | (0.00) |
| PctHisp | $0.01^{**}$ |
| | (0.00) |
| Gini | $4.48^{***}$ |
| | (0.64) |
| PctHSEd | 0.01 |
| | (0.01) |
| PctFemHH | 0.00 |
| | (0.01) |
| $\rho$ | $0.54^{***}$ |
| | (0.06) |
| Num. obs. | 174 |
| Parameters | 8 |
| Log Likelihood | -101.93 |
| AIC (Linear model) | 285.72 |
| AIC (Spatial model) | 219.85 |
| LR test: statistic | 67.86 |
| LR test: p-value | 0.00 |

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table: SAR Model output

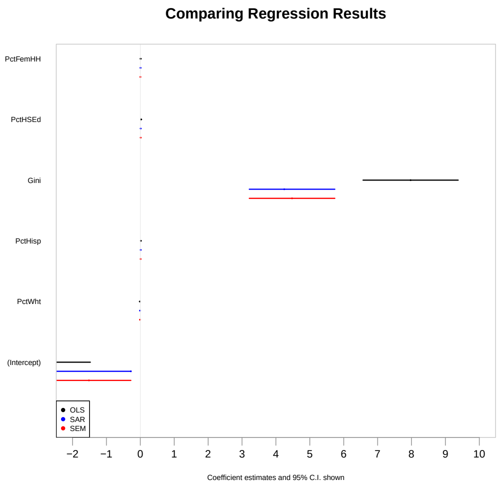
# Spatial Error Model

| | Model 1 |
|---|---|
| (Intercept) | $-0.28$ |
| | $(0.88)$ |
| PctWht | $-0.01^{**}$ |
| | $(0.00)$ |
| PctHisp | $0.01^{**}$ |
| | $(0.00)$ |
| Gini | $4.25^{***}$ |
| | $(0.90)$ |
| PctHSEd | $0.01$ |
| | $(0.01)$ |
| PctFemHH | $0.01$ |
| | $(0.01)$ |
| $\lambda$ | $0.66^{***}$ |
| | $(0.06)$ |
| Num. obs. | 174 |
| Parameters | 8 |
| Log Likelihood | -107.85 |
| AIC (Linear model) | 285.72 |
| AIC (Spatial model) | 231.70 |
| LR test: statistic | 56.01 |
| LR test: p-value | 0.00 |

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table: SEM model output

# Comparing Findings - OLS



# Did These Resolve our Spatial Autocorrelation?

| Moran I statistic | Expectation | Variance | p-value |
|---|---|---|---|
| -0.10 | -0.01 | 0.00 | 0.08 |

YES!

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- $$\mathrm{var}(y_i|y_{-i}) = \sigma_i^2$$

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- $$= \sigma^2 V_{CAR}$$

# SAR vs. CAR

- SAR:

$$y_i \sim N(0, (I - W)^{-1} D^{\sim} (I - W)'^{-1})$$

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- CAR is computationally less intensive
- CAR: local/first order dependency
- CAR: fit with MCMC – can fit GLMs
- SAR: global/higher order dependency
- SAR: fit with MLE

# Fitting a CAR (1)

| | Median | 2.5% | 97.5% | n.sample | % accept | n.effective | Geweke.diag |
|---|---|---|---|---|---|---|---|
| (Intercept) | -0.60 | -2.35 | 1.08 | 18000.00 | 100.00 | 2417.70 | 0.50 |
| PctWht | -0.01 | -0.02 | -0.01 | 18000.00 | 100.00 | 2973.70 | 1.40 |
| PctHisp | 0.01 | 0.01 | 0.02 | 18000.00 | 100.00 | 2592.00 | -0.80 |
| Gini | 3.98 | 2.10 | 5.97 | 18000.00 | 100.00 | 1628.90 | -0.30 |
| PctHSEd | 0.02 | 0.00 | 0.03 | 18000.00 | 100.00 | 3252.90 | -0.90 |
| PctFemHH | 0.01 | -0.01 | 0.03 | 18000.00 | 100.00 | 2006.10 | 0.20 |
| nu2 | 0.01 | 0.00 | 0.06 | 18000.00 | 100.00 | 1599.50 | 0.70 |
| tau2 | 0.50 | 0.34 | 0.65 | 18000.00 | 100.00 | 3907.90 | -0.80 |
| rho | 0.81 | 0.61 | 0.94 | 18000.00 | 44.10 | 5196.60 | 0.90 |

Table: uninformative priors

# Fitting a CAR (2)

| | Median | 2.5% | 97.5% | n.sample | % accept | n.effective | Geweke.diag |
|---|---|---|---|---|---|---|---|
| (Intercept) | -0.81 | -2.48 | 0.86 | 18000.00 | 100.00 | 11033.20 | 1.40 |
| PctWht | -0.01 | -0.02 | -0.01 | 18000.00 | 100.00 | 13600.70 | 0.20 |
| PctHisp | 0.01 | 0.01 | 0.02 | 18000.00 | 100.00 | 13780.40 | 0.30 |
| Gini | 4.03 | 2.12 | 5.93 | 18000.00 | 100.00 | 7508.30 | -2.00 |
| PctHSEd | 0.02 | 0.00 | 0.03 | 18000.00 | 100.00 | 15248.50 | -0.70 |
| PctFemHH | 0.01 | -0.01 | 0.03 | 18000.00 | 100.00 | 10575.60 | 1.30 |
| nu2 | 0.08 | 0.06 | 0.13 | 18000.00 | 100.00 | 10732.90 | -1.30 |
| tau2 | 0.25 | 0.16 | 0.38 | 18000.00 | 100.00 | 10628.20 | 2.70 |
| rho | 0.91 | 0.75 | 0.98 | 18000.00 | 43.80 | 10169.70 | 0.50 |

Table: weakly informative priors

# Geographically Weighted Regression

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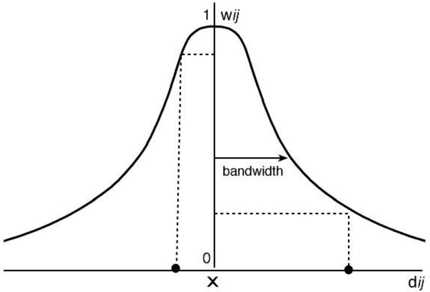
- A non-parametric method for inference with non-stationary spatial data
- Non-stationarity: spatial variation is not constant across the data
- Use kernel regression to model local spatial variation
- A weighted moving window regression:

$$Y(x) = \alpha(u_i, v_i) + \sum_k \beta_k(u_i, v_i)x_k + e_i$$

# Geographically Weighted Regression (Cont'd)

- A Kernel as a way of weighting observations which are closer together

The concept of a kernel




× regression point   $w_{ij}$ is the weight of data point $j$ at regression point $i$

• data point   $d_{ij}$ is the distance between regression point $i$ and data point $j$

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- A Kernel as a way of weighting observations which are closer together
- Usually, fit adaptively – different "sizes" depending on the density of the data

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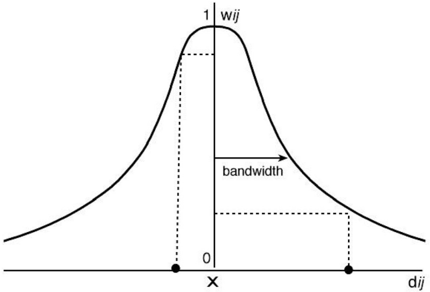
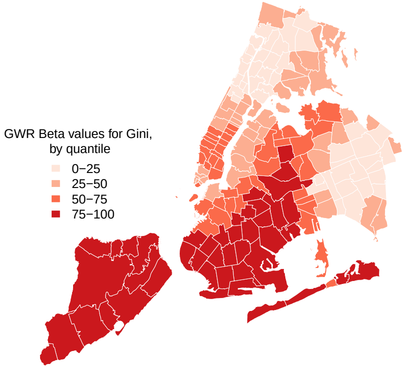



x regression point   wij is the weight of data point j at regression point i

• data point   dij is the distance between regression point i and data point j

# Fitting a Geographically Weighted Regression

- Usually, we represent the results from a GWR visually



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- Danger: overfitting the data
- Extensions: GWR + Ridge regression, GWR GLMs, GWPCA

# Workflow for Spatial Regression

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- In addition to normal EDA, do some ESDA (Exploratory Spatial Data Analysis), mapping out variables
- Check for spatial autocorrelation
- Run your normal regression, with the variables you think are necessary

# Workflow for Spatial Regression

- In addition to normal EDA, do some ESDA (Exploratory Spatial Data Analysis), mapping out variables
- Check for spatial autocorrelation
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- Check for spatial autocorrelation
- Run your normal regression, with the variables you think are necessary
- Check once more for spatial autocorrelation, in your residuals
- If there's still autocorrelation, run a spatial model
- One final check for autocorrelation in your residuals

# Where Can we Apply These Methods in Political Science

- Voting and Political Behavior patterns (Data available at the Census Tract level (or less))

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- Agricultural/industrial data (economic output)
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- Anything with an address or lon/lat coordinates can be georeferenced

# Other tools

- Spatial/Spatio-temporal scan statistics

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# Resources

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