Spatial Data and Analysis in R
A PRISM Workshop

Adam Lauretig

The Ohio State University

Outline

Introduction

Spatial Data Prep

Spatial Autocorrelation

Regression

Spatial Regression

Discussion

Why Are We Here?

- Tobler's first law of geography:

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Why Are We Here?

- Tobler's first law of geography:
- "Everything is related to everything else, but near things are more related than distant things"
- We want to quantify how the spatial relationship between our observations affect our inferences

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- I will not be discussing *GIS* in depth, nor will I discuss remote sensing *at all*

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- ▶ Many kinds of spatial data: Points, Lines, Polygons, Raster data
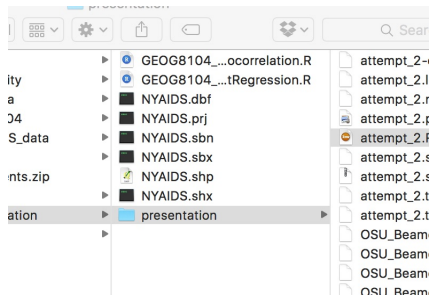- ▶ Today, we are working with polygon data

Prepping our data
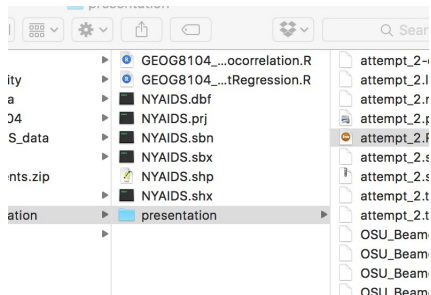
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- ORDBMS - Linking spatial and attribute data
- Six parts, all combine to create a map to represent data
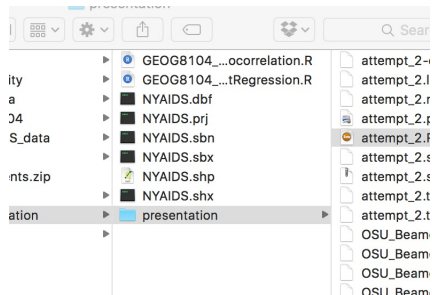
Loading Our data

Percent Black
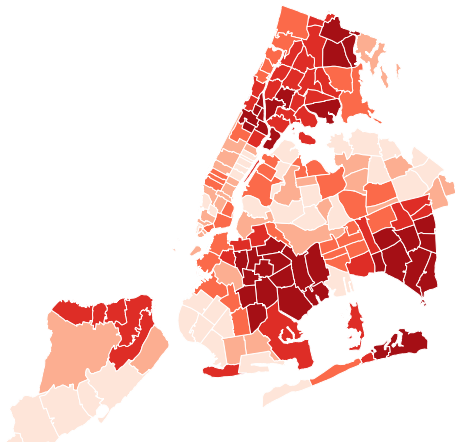
AIDs Rate per 1000 people
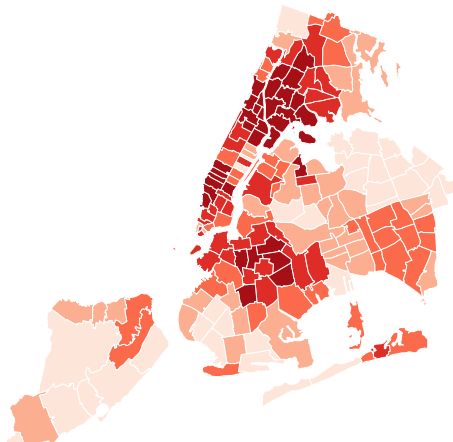
Measuring Spatial Autocorrelation

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- ▶ Observations with more similar values tend to occur more closely together
- ▶ Most common test: Moran's I

Formula for Moran's I

$$I = \frac{n}{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}} \frac{\sum_{i=1}^{n}\sum_{j=1}^{n} w_{ij}(x_i - \bar{x})(x_j - \bar{x})}{\sum_{i=1}^{n}(x_i - \bar{x})^2},$$

where w_{ij} is the weight between observation i and j

The Weights Matrix W

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- It allows us to measure the effect neighboring observations j have on our observation of interest i
- Can be specified in a variety of ways, the simplest of which is binary ("contiguity"): 1 if observations share a boundary, 0 if they do not
- The default in R is "row standardized," where $w_{ij} = \frac{i}{\sum j}$

Creating a weights matrix in *R*

```
library(rgdal)
library(spdep)
library(sp)
library(spatstat)
file_path <- "/Users/adamlauretig/data/Prism_presentation/NYAIDS_data"
ny <- readOGR(dsn = file_path,
              layer = "NYAIDS", verbose=FALSE)
nygal <- poly2nb(ny) #Create the neighborhood object
nyQ1.gal <- nb2listw(nygal, zero.policy=T) #Create the weights object
```

Running the Moran's I

Moran I statistic	Expectation	Variance	p-value
0.68	-0.01	0.00	0.00

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- While the Moran's I detects clustering, the LISA detects *clusters*

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Moran Scatterplot AIDS Rate
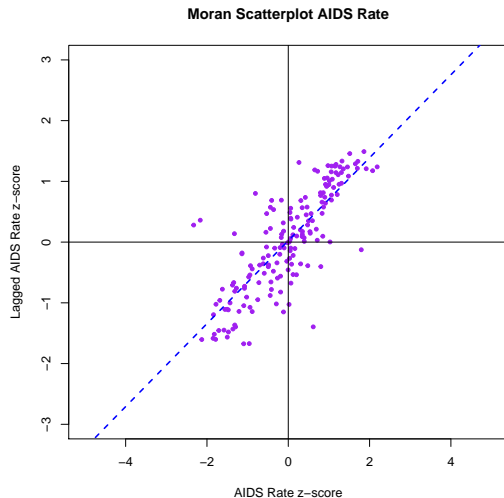
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LISA Map AIDS Rate; weights: Q1
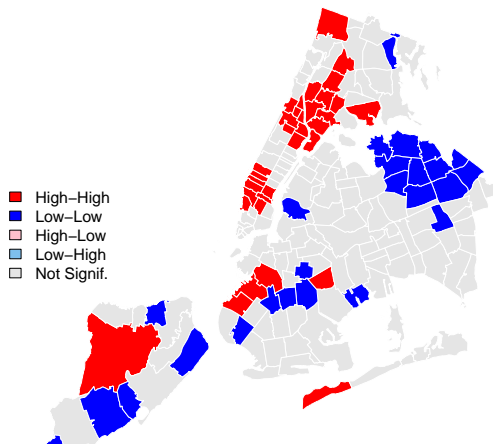
■ High–High
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□ Not Signif.

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- ▶ Addtionally, with spatial autocorrelation, our coefficients may be biased
- ▶ Two ways of handling this: Spatial Error Models, and Spatial Autoregressive models

The Math: Spatial Error Model

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- The full model: $y_i = x_i \beta + \sum_{j=1}^n w_{ij} e_j + \varepsilon_i$

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- Instead of lagging the error term, lag y, the DV
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- SAR vs. SEM

Plain OLS

	Model 1
(Intercept)	-3.05^{***}
	(0.80)
PctWht	-0.01^{***}
	(0.00)
PctHisp	0.02^{***}
	(0.00)
Gini	7.97^{***}
	(0.71)
PctHSEd	0.02^{**}
	(0.01)
PctFemHH	0.01
	(0.01)
R^2	0.57
Adj. R^2	0.55
Num. obs.	174
RMSE	0.54

$^{***}p < 0.001$, $^{**}p < 0.01$, $^{*}p < 0.05$

Table: Statistical models

Did we model out our autocorrelation?

Plot of Residuals against Fitted y
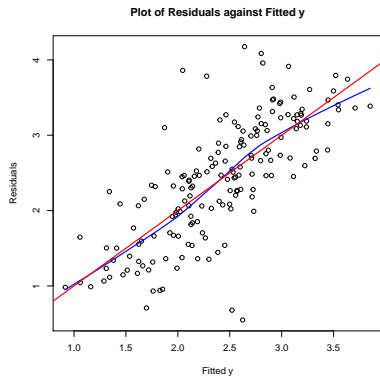
Did we model out our spatial autocorrelation?

Moran I statistic	Expectation	Variance	p-value
0.63	-0.01	0.00	0.00

No! This statistic didn't change much from before we ran our regression: .68 vs .63

Where is the residual autocorrelation?

LISA Map AIDS Rate; weights: Q1

- High–High
- Low–Low
- High–Low
- Low–High
- Not Signif.
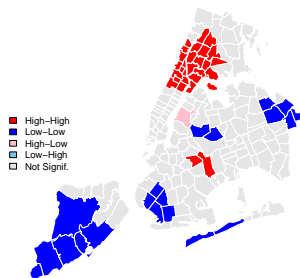
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Can we Model this Spatial Autocorrelation?

- ▶ One option is to use fixed effects
- ▶ But this removes something interesting – the spatial relationship
- ▶ Another option is to explicitly model the spatial relationship

Spatial Autoregressive Model

	Model 1
(Intercept)	-1.52^{*}
	(0.63)
PctWht	-0.01^{***}
	(0.00)
PctHisp	0.01^{**}
	(0.00)
Gini	4.48^{***}
	(0.64)
PctHSEd	0.01
	(0.01)
PctFemHH	0.00
	(0.01)
ρ	0.54^{***}
	(0.06)
Num. obs.	174
Parameters	8
Log Likelihood	-101.93
AIC (Linear model)	285.72
AIC (Spatial model)	219.85
LR test: statistic	67.86
LR test: p-value	0.00

$^{***}p < 0.001, {}^{**}p < 0.01, {}^{*}p < 0.05$

Table: SAR Model output

Spatial Error Model

	Model 1
(Intercept)	−0.28
	(0.88)
PctWht	−0.01**
	(0.00)
PctHisp	0.01**
	(0.00)
Gini	4.25***
	(0.90)
PctHSEd	0.01
	(0.01)
PctFemHH	0.01
	(0.01)
λ	0.66***
	(0.06)
Num. obs.	174
Parameters	8
Log Likelihood	-107.85
AIC (Linear model)	285.72
AIC (Spatial model)	231.70
LR test: statistic	56.01
LR test: p-value	0.00

***$p < 0.001$, **$p < 0.01$, *$p < 0.05$

Table: SEM model output

Comparing Findings - OLS

Comparing Regression Results

Coefficient estimates and 95% C.I. shown
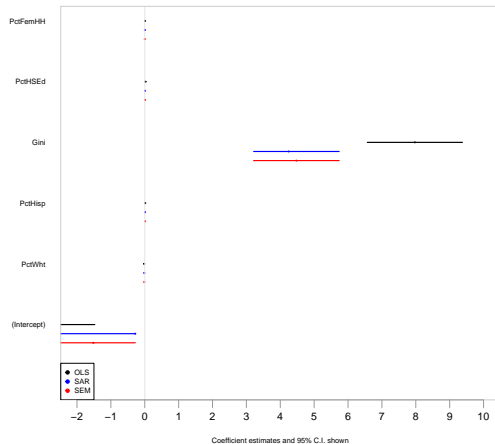
Did These Resolve our Spatial Autocorrelation?

Moran I statistic	Expectation	Variance	p-value
-0.10	-0.01	0.00	0.08

YES!

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$$E(y_i | y_{-i}) = x_i \beta + \sum_{j=1}^{n} + c_{ij}[y_j - x_i \beta]$$

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$$\mathrm{var}(y_i|y_{-i}) = \sigma_i^2$$

Conditional Autoregressive Model (CAR) (cont'd)

► This defines a joint multivariate normal distribution with variance

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$$\Sigma_Y = (I - C)^{-1} \sum_C$$

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$$\Sigma_{CAR} = \sigma^2 (I - C)^{-1} V_C$$

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$$= \sigma^2 V_{CAR}$$

SAR vs. CAR

- SAR:

$$y_i \sim N(0, (I - W)^{-1} D^{\sim} (I - W)'^{-1})$$
$$\Sigma_{SAR} = \sigma^2 (I - W)^{-1} V_\epsilon (I - W')^{-1}$$

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- CAR only requires the variance to estimate, SAR requires the full variance-covariance matrix
- CAR is computationally less intensive
- CAR: local/first order dependency
- CAR: fit with MCMC – can fit GLMs
- SAR: global/higher order dependency
- SAR: fit with MLE

Fitting a CAR (1)

	Median	2.5%	97.5%	n.sample	% accept	n.effective	Geweke.diag
(Intercept)	-0.60	-2.35	1.08	18000.00	100.00	2417.70	0.50
PctWht	-0.01	-0.02	-0.01	18000.00	100.00	2973.70	1.40
PctHisp	0.01	0.01	0.02	18000.00	100.00	2592.00	-0.80
Gini	3.98	2.10	5.97	18000.00	100.00	1628.90	-0.30
PctHSEd	0.02	0.00	0.03	18000.00	100.00	3252.90	-0.90
PctFemHH	0.01	-0.01	0.03	18000.00	100.00	2006.10	0.20
nu2	0.01	0.00	0.06	18000.00	100.00	1599.50	0.70
tau2	0.50	0.34	0.65	18000.00	100.00	3907.90	-0.80
rho	0.81	0.61	0.94	18000.00	44.10	5196.60	0.90

Table: uninformative priors

Fitting a CAR (2)

	Median	2.5%	97.5%	n.sample	% accept	n.effective	Geweke.diag
(Intercept)	-0.81	-2.48	0.86	18000.00	100.00	11033.20	1.40
PctWht	-0.01	-0.02	-0.01	18000.00	100.00	13600.70	0.20
PctHisp	0.01	0.01	0.02	18000.00	100.00	13780.40	0.30
Gini	4.03	2.12	5.93	18000.00	100.00	7508.30	-2.00
PctHSEd	0.02	0.00	0.03	18000.00	100.00	15248.50	-0.70
PctFemHH	0.01	-0.01	0.03	18000.00	100.00	10575.60	1.30
nu2	0.08	0.06	0.13	18000.00	100.00	10732.90	-1.30
tau2	0.25	0.16	0.38	18000.00	100.00	10628.20	2.70
rho	0.91	0.75	0.98	18000.00	43.80	10169.70	0.50

Table: weakly informative priors

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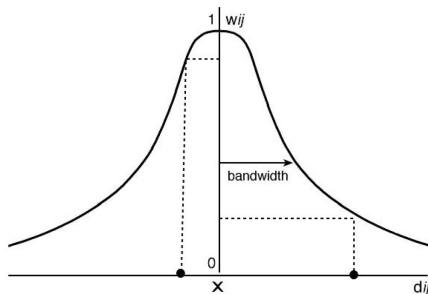
Geographically Weighted Regression

- A non-parametric method for inference with non-stationary spatial data
- Non-stationarity: spatial variation is not constant across the data
- Use kernel regression to model local spatial variation
- A weighted moving window regression:

$$Y(x) = \alpha(u_i, v_i) + \sum_k \beta_k(u_i, v_i) x_k + e_i$$

Geographically Weighted Regression (Cont'd)

▶ A Kernel as a way of weighting observations which are closer together
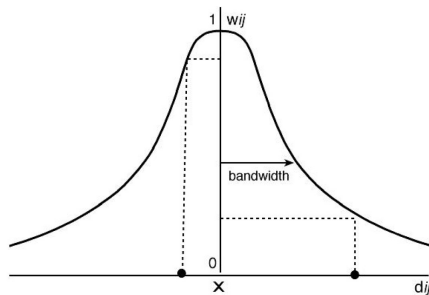
The concept of a kernel

x regression point	w_{ij} is the weight of data point j at regression point i
• data point	d_{ij} is the distance between regression point i and data point j

Geographically Weighted Regression (Cont'd)

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- Usually, fit adaptively – different "sizes" depending on the density of the data
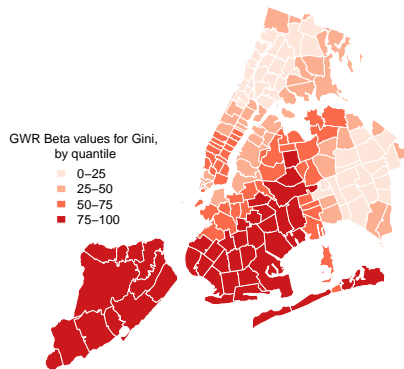
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Fitting a Geographically Weighted Regression

- Usually, we represent the results from a GWR visually

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- Danger: overfitting the data
- Extensions: GWR + Ridge regression, GWR GLMs, GWPCA

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- Check once more for spatial autocorrelation, in your residuals
- If there's still autocorrelation, run a spatial model
- One final check for autocorrelation in your residuals

Where Can we Apply These Methods in Political Science

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- Conflict/Political Violence data (ex: ACLED)

Where Can we Apply These Methods in Political Science

- ▶ Voting and Political Behavior patterns (Data available at the Census Tract level (or less))
- ▶ Agricultural/industrial data (economic output)
- ▶ Conflict/Political Violence data (ex: ACLED)
- ▶ Anything with an address or lon/lat coordinates can be georeferenced

Other tools

- Spatial/Spatio-temporal scan statistics

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- Kernel Density Estimation

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Other tools

- Spatial/Spatio-temporal scan statistics
- Kernel Density Estimation
- Kriging/Geostatistics
- Spatio-temporal approaches

Resources

- Yuri Zhukov's Spatial Workshop:
  `http://www.people.fas.harvard.edu/~zhukov/spatial.html`

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