MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II — September 30, 2019
Prof. Alan Guth

PRACTICE PROBLEMS FOR QUIZ 1

PROBLEM 1: VECTOR CALCULUS PRACTICE 1

Griffiths 1.54:

Problem 1.54 Check the divergence theorem for the function

$$\mathbf{v} = r^2 \cos\theta \, \hat{\mathbf{r}} + r^2 \cos\phi \, \hat{\boldsymbol{\theta}} - r^2 \cos\theta \sin\phi \, \hat{\boldsymbol{\phi}},$$

using as your volume one octant of the sphere of radius R (Fig. 1.48). Make sure you include the *entire* surface. [*Answer:* $\pi R^4/4$]

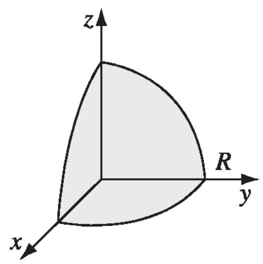

FIGURE 1.48

PROBLEM 2: VECTOR CALCULUS PRACTICE 2

Griffiths 1.55:

Problem 1.55 Check Stokes' theorem using the function $\mathbf{v} = ay\,\hat{\mathbf{x}} + bx\,\hat{\mathbf{y}}$ (a and b are constants) and the circular path of radius R, centered at the origin in the xy plane. [*Answer:* $\pi R^2(b - a)$]

PROBLEM 3: ELECTRIC FIELDS 1

Griffiths 2.16:

Problem 2.16 A long coaxial cable (Fig. 2.26) carries a uniform *volume* charge density ρ on the inner cylinder (radius a), and a uniform *surface* charge density on the outer cylindrical shell (radius b). This surface charge is negative and is of just the right magnitude that the cable as a whole is electrically neutral. Find the electric field in each of the three regions: (i) inside the inner cylinder ($s < a$), (ii) between the cylinders ($a < s < b$), (iii) outside the cable ($s > b$). Plot $|\mathbf{E}|$ as a function of s.

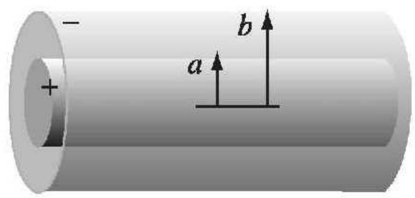

FIGURE 2.26

PROBLEM 4: ELECTRIC FIELDS 2

Griffiths 2.18. It's a good idea to solve Griffiths 2.12 first.

Problem 2.12 Use Gauss's law to find the electric field inside a uniformly charged solid sphere (charge density ρ). Compare your answer to Prob. 2.8.

Problem 2.18 Two spheres, each of radius R and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed so that they partially overlap (Fig. 2.28). Call the vector from the positive center to the negative center $\mathbf{d}$. Show that the field in the region of overlap is constant, and find its value. [*Hint:* Use the answer to Prob. 2.12.]

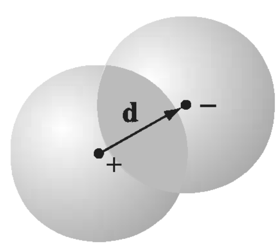

FIGURE 2.28

PROBLEM 5: ELECTRIC POTENTIAL 1

Griffiths 2.52, part (a). The easy way to do this is to consider the two line charges in isolation, then use superposition to combine them. Part (b) is fairly tedious, so it is not recommended.

Problem 2.52 Two infinitely long wires running parallel to the x axis carry uniform charge densities $+\lambda$ and $-\lambda$ (Fig. 2.54).

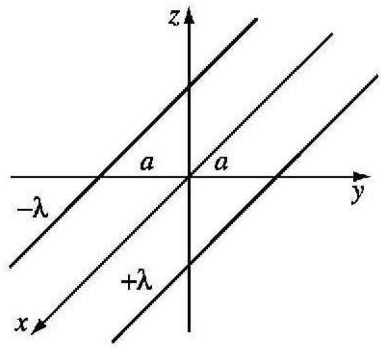

FIGURE 2.54

(a) Find the potential at any point (x, y, z), using the origin as your reference.

(b) Show that the equipotential surfaces are circular cylinders, and locate the axis and radius of the cylinder corresponding to a given potential V_0.

PROBLEM 6: ELECTRIC POTENTIAL 2

Griffiths 2.54, parts (b), (c), (d).

Problem 2.54 Imagine that new and extraordinarily precise measurements have revealed an error in Coulomb's law. The *actual* force of interaction between two point charges is found to be

$$\mathbf{F} = \frac{1}{4\pi\epsilon_0}\frac{q_1 q_2}{\imath^2}\left(1 + \frac{\imath}{\lambda}\right)e^{-(\imath/\lambda)}\hat{\boldsymbol{\imath}},$$

where λ is a new constant of nature (it has dimensions of length, obviously, and is a huge number—say half the radius of the known universe—so that the correction is small, which is why no one ever noticed the discrepancy before). You are charged with the task of reformulating electrostatics to accommodate the new discovery. Assume the principle of superposition still holds.

(a) What is the electric field of a charge distribution ρ (replacing Eq. 2.8)?

(b) Does this electric field admit a scalar potential? Explain briefly how you reached your conclusion. (No formal proof necessary—just a persuasive argument.)

(c) Find the potential of a point charge q—the analog to Eq. 2.26. (If your answer to (b) was "no," better go back and change it!) Use ∞ as your reference point.

(d) For a point charge q at the origin, show that

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} + \frac{1}{\lambda^2}\int_{\mathcal{V}} V\, d\tau = \frac{1}{\epsilon_0}q,$$

where $\mathcal{S}$ is the surface, $\mathcal{V}$ the volume, of any sphere centered at q.

(e) Show that this result generalizes:

$$\oint_{\mathcal{S}} \mathbf{E} \cdot d\mathbf{a} + \frac{1}{\lambda^2}\int_{\mathcal{V}} V\, d\tau = \frac{1}{\epsilon_0}Q_{\text{enc}},$$

for *any* charge distribution. (This is the next best thing to Gauss's Law, in the new "electrostatics.")

(f) Draw the triangle diagram (like Fig. 2.35) for this world, putting in all the appropriate formulas. (Think of Poisson's equation as the formula for ρ in terms of V, and Gauss's law (differential form) as an equation for ρ in terms of $\mathbf{E}$.)

(g) Show that *some* of the charge on a conductor distributes itself (uniformly!) over the volume, with the remainder on the surface. [*Hint:* $\mathbf{E}$ is still zero, inside a conductor.]

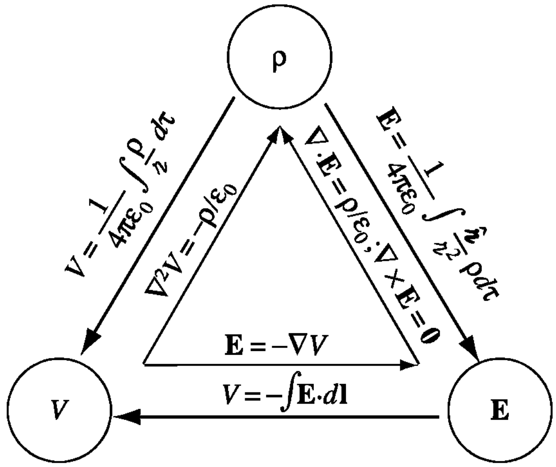

FIGURE 2.35

PROBLEM 7: ELECTRIC FIELDS AND CHARGE DENSITIES

Griffiths 2.55. Add the following thought: What is the charge density corresponding to the field $\vec{E} = \frac{1}{3}ar\hat{\boldsymbol{r}}$? Can you reconcile this with the situation described in Griffiths?

Problem 2.55 Suppose an electric field $\mathbf{E}(x, y, z)$ has the form

$$E_x = ax, \qquad E_y = 0, \qquad E_z = 0$$

where a is a constant. What is the charge density? How do you account for the fact that the field points in a particular direction, when the charge density is uniform? [This is a more subtle problem than it looks, and worthy of careful thought.]

PROBLEM 8: LAPLACE OPERATOR

Griffiths 3.3:

Problem 3.3 Find the general solution to Laplace's equation in spherical coordinates, for the case where V depends only on r. Do the same for cylindrical coordinates, assuming V depends only on s.

PROBLEM 9: IMAGE CHARGE

Griffiths 3.10. See Griffiths Section 3.2.2 for a discussion of how to calculate surface charge densities.

Problem 3.10 A uniform line charge λ is placed on an infinite straight wire, a distance d above a grounded conducting plane. (Let's say the wire runs parallel to the x-axis and directly above it, and the conducting plane is the xy plane.)

(a) Find the potential in the region above the plane. [*Hint:* Refer to Prob. 2.52.]

(b) Find the charge density σ induced on the conducting plane.

PROBLEM 10: SEPARATION OF VARIABLES: RECTANGULAR PIPE

Griffiths 3.15.

Problem 3.15 A rectangular pipe, running parallel to the z-axis (from $-\infty$ to $+\infty$), has three grounded metal sides, at $y = 0$, $y = a$, and $x = 0$. The fourth side, at $x = b$, is maintained at a specified potential $V_0(y)$.

(a) Develop a general formula for the potential inside the pipe.

(b) Find the potential explicitly, for the case $V_0(y) = V_0$ (a constant).