MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Physics Department

Physics 8.07: Electromagnetism II — December 7, 2019
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PRACTICE PROBLEMS FOR THE FINAL EXAM, PART A

DUE DATE: None. But the final exam will include two problems that will be taken verbatim, or at least almost verbatim, from Problem Sets 8 or 9, or from the practice problems for the final exam. This is the first part of the practice problems for the final exam.

READING ASSIGNMENT: Griffiths: Chapter 10 (*Potentials and Fields)*. The final exam will include questions relevant to Chapter 10, and to selected topics in Chapter 11 (*Radiation*), but for both chapters you will only be responsible for those topics that are discussed in lecture, or which appear in the problem sets or practice problems.

PROBLEM 1: THE PRESSURE OF SUNLIGHT *(10 points)*

Griffiths Problem 9.10 (p. 400). Although you will find a tiny pressure, note that the energy flux from the sun is actually very large.

Griffiths' text: The intensity of sunlight hitting the earth is about 1300 W/m^2. If sunlight strikes a perfect absorber, what pressure does it exert? How about a perfect reflector? What fraction of atmospheric pressure does this amount to?

PROBLEM 2: GAUGE CHOICES

Griffiths Problem 10.6 (p. 442).

In Chapter 5, I showed that it is always possible to pick a vector potential whose divergence is zero (Coulomb gauge). Show that it is always possible to choose $\vec{\nabla}\cdot\vec{A} = -\mu_0\epsilon_0(\partial V/\partial t)$, as required for the Lorenz gauge, assuming you know how to solve the inhomogeneous wave eqation (Eq. 10.16). Is it always possible to pick $V = 0$? How about $\vec{A} = 0$?

PROBLEM 3: RETARDED POTENTIALS AND THE LORENZ GAUGE CONDITION

Griffiths Problem 10.10 (p.448).

Confirm that the retarded potentials satisfy the Lorenz gauge condition. [*Hint*: First show that

$$\vec{\nabla}\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) = \frac{1}{\mathscr{r}}(\vec{\nabla}\cdot\vec{J}) + \frac{1}{\mathscr{r}}(\vec{\nabla}'\cdot\vec{J}) - \vec{\nabla}'\cdot\left(\frac{\vec{J}}{\mathscr{r}}\right) \ ,$$

where $\vec{\nabla}$ denotes derivatives with respect to $\vec{r}$, and $\vec{\nabla}'$ denotes derivatives with respect to $\vec{r}\,'$. Next, noting that $\vec{J}(\vec{r}\,', t - \mathscr{r}/c)$ depends on $\vec{r}\,'$ both explicitly and through $\mathscr{r}$, whereas it depends on $\vec{r}$ only through $\mathscr{r}$, confirm that

$$\vec{\nabla}\cdot J = -\frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot(\vec{\nabla}\mathscr{r}) \ , \qquad \vec{\nabla}'\cdot\vec{J} = -\frac{\partial\rho}{\partial t_r} - \frac{1}{c}\frac{\partial\vec{J}}{\partial t_r}\cdot(\vec{\nabla}'\mathscr{r}) \ .$$

Use this to calculate the divergence of $\vec{A}$ (Eq. 10.26).]

PROBLEM 4: A POINT CHARGE MOVING ON THE x AXIS

Griffiths Problem 10.20 (p. 462).

Suppose a point charge q is constrained to move along the x axis. Show that the fields at points on the axis to the *right* of the charge are given by

$$\vec{\boldsymbol{E}} = \frac{q}{4\pi\epsilon_0}\frac{1}{\mathcal{R}^2}\left(\frac{c+v}{c-v}\right)\hat{\boldsymbol{x}} \ , \qquad \vec{\boldsymbol{B}} = 0 \ .$$

What are the fields on the axis to the *left* of the charge?

PROBLEM 5: RADIATION RESISTANCE

Griffths Problem 11.3 (p. 472).

Find the **radiation resistance** of the wire joining the two ends of the dipole. (This is the resistance that would give the same average power loss—to heat—as the oscillating dipole in *fact* puts out in the form of radiation.) Show that $R = 790(d/\lambda)^2\,\Omega$, where λ is the wavelength of the radiation. For the wires in an ordinary radio (say, $d = 5$ cm), should you worry about the radiative contribution to the total resistance?

PROBLEM 6: A ROTATING ELECTRIC DIPOLE

Griffiths Problem 11.4 (p. 473).

A *rotating* electric dipole can be thought of as the superposition of two *oscillating* dipoles, one along the x axis, and the other along the y axis (Fig. 11.7), with the latter out of phase by 90°:

$$\vec{\boldsymbol{p}} = p_0[\cos(\omega t)\,\hat{\boldsymbol{x}} + \sin(\omega t)\,\hat{\boldsymbol{y}}] \ .$$

Using the principle of superposition and Eqs. (11.18) and (11.19) (perhaps in the form suggested by Prob. 11.2), find the fields of a rotating dipole. Also find the Poynting vector and the intensity of the radiation. Sketch the intensity profile as a function of the polar angle θ, and calculate the total power radiated. Does the answer seem reasonable? (Note that power, being *quadratic* in the fields, does not satisfy the superposition principle. In this instance, however, it *seems* to. How do you account for this?)

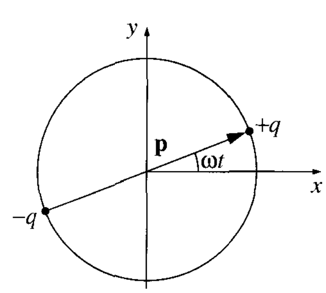

Figure 11.7

PROBLEM 7: RADIATION AND THE BOHR ATOM

Griffiths Problem 11.14 (p. 487).

> In Bohr's theory of hydrogen, the electron in its ground state was supposed to travel in a circle of radius 5×10^{-11} m, held in orbit by the Coulomb attraction of the proton. According to classical electrodynamics, this electron should radiate, and hence spiral in to the nucleus. Show that $v \ll c$ for most of the trip (so you can use the Larmor formula), and calculate the lifespan of Bohr's atom. (Assume each revolution is essentially circular.)

PROBLEM 8: THREE EXAMPLES OF RADIATION REACTION

Griffiths Problem 11.17 (p. 491).

(a) A particle of charge q moves in a circle of radius R at a constant speed v. To sustain the motion, you must, of course, provide a centripetal force mv^2/R; what *additional* force (F_e) must you exert, in order to counteract the radiation reaction? [It's easiest to express the answer in terms of the instantaneous velocity $\vec{\boldsymbol{v}}$.] What power (P_e) does this extra force deliver? Compare P_e with the power radiated (use the Larmor formula).

(b) Repeat part (a) for a particle in simple harmonic motion with amplitude A and angular frequency ω ($\vec{\boldsymbol{w}}(t) = A\cos(\omega t)\hat{\boldsymbol{z}}$). Explain the discrepancy.

(c) Consider the case of a particle in free fall (constant acceleration g). What is the radiation reaction force? What is the power radiated? Comment on these results.