8.07 Lecture Slides 16
November 4, 2019

ELECTRIC FIELDS IN MATTER

Announcements

Quiz 2 will be given on Wednesday, November 13, two weeks from today. Problem Set 6 is due this Friday, 11/1/19, and Problem Set 7 will be due the next Friday, 11/8/19. The quiz will include material through Problem Set 7.

Office hour modifications:

Yitian Sun is away this week.
Wednesday, 5:00–6:00 pm: office hour by me, Room 6-322 (as usual).
Thursday, 5:30–6:30 pm: office hour by me, Room 8-320.
Friday, 1:30–2:30 pm: office hour by me, Room 8-320.
Friday, 2:30–3:30 pm: office hour by Marin, Room 8-320.
Tuesday (11/12/19): 3:00–4:00 pm: office hour by Marin, Room 6C-419.
Tuesday (11/12/19): 4:30–5:30 pm: office hour by me, Room 6-322.

Tentative: Review Session, Tuesday evening, 11/12/19, by Yitian. Time?

Added after class: we decided that the review session, with Yitian, will be held at 7:30 pm on Tuesday, 11/12/19.

Bound Charges

Matter can become "polarized," meaning that it acquires a nonzero density of dipoles.

$$\boxed{\vec{\boldsymbol{P}}(\vec{\boldsymbol{r}}\,) = \text{dipole moment per unit volume.}}$$

$\vec{\boldsymbol{P}}(\vec{\boldsymbol{r}}\,)$ is just a particular way of describing a distribution of charge. In principle, one can equivalently use $\rho(\vec{\boldsymbol{r}}\,)$

Given $\vec{\boldsymbol{P}}(\vec{\boldsymbol{r}}\,)$, what is $\rho(\vec{\boldsymbol{r}}\,)$?

Answer:

$$\boxed{\rho_b(\vec{\boldsymbol{r}}\,) = -\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{P}}(\vec{\boldsymbol{r}}\,)\ ,}$$

and on the surface of a polarized material,

$$\boxed{\sigma_b = \vec{\boldsymbol{P}} \cdot \hat{\boldsymbol{n}}\ ,}$$

where $\hat{\boldsymbol{n}}$ is the outward unit normal.

Derivations of Bound Charge Equations

1. Derivation using potential $V(\vec{r})$, as done by Griffiths.
2. Derivation based on counting the dipoles that are cut by the boundary of a given region, as done in The Feynman Lectures.
3. Method using δ-functions.

Electrostatic Field Equations in Matter

"Vacuum" Equations:

$$\vec{\nabla} \cdot \vec{E} = \frac{\rho}{\epsilon_0} \ , \qquad \vec{\nabla} \times \vec{E} = 0 \ .$$

These are the fundamental equations, which are **always** true, as long as all charges are included in ρ.

Bound and Free charges:

Write $\rho = \rho_{\rm free} + \rho_{\rm bound} \equiv \rho_f + \rho_b$. ρ_b is any charge described by the polarization density $\vec{P}$.

The Electric Displacement $\vec{D}$:

Define $\boxed{\vec{\boldsymbol{D}} \equiv \epsilon_0 \vec{\boldsymbol{E}} + \vec{\boldsymbol{P}}\ .}$ Then

$$\boxed{\vec{\boldsymbol{\nabla}} \cdot \vec{\boldsymbol{D}} = \rho_f\ .}$$

Note, however, that $\vec{\boldsymbol{\nabla}} \times \vec{\boldsymbol{D}}$ is not necessarily zero.

Boundary Conditions:

$$E^{\perp}_{\text{above}} - E^{\perp}_{\text{below}} = \frac{\sigma}{\epsilon_0} \ ,$$

$$\vec{\boldsymbol{E}}^{||}_{\text{above}} - \vec{\boldsymbol{E}}^{||}_{\text{below}} = 0 \ .$$

$$D^{\perp}_{\text{above}} - D^{\perp}_{\text{below}} = \sigma_f \ ,$$

$$\vec{\boldsymbol{D}}^{||}_{\text{above}} - \vec{\boldsymbol{D}}^{||}_{\text{below}} = \vec{\boldsymbol{P}}^{||}_{\text{above}} - \vec{\boldsymbol{P}}^{||}_{\text{below}} \ ,$$

where "above" = outside the polarized material, and "below" means inside, $\perp$ = perpendicular to interface, $\|$ means parallel.

Linear Dielectrics

For many substances, called linear dielectrics, to a good approximation:

$$\boxed{\vec{\boldsymbol{P}} = \epsilon_0 \chi_e \vec{\boldsymbol{E}} \ ,}$$

where $\vec{\boldsymbol{E}}$ is the macroscopic $\vec{\boldsymbol{E}}$-field, and χ_e is a property of the material called the electric susceptibility. χ_e is dimensionless. Then

$$\boxed{\vec{\boldsymbol{D}} = (1 + \chi_e)\epsilon_0 \vec{\boldsymbol{E}} \equiv \epsilon \vec{\boldsymbol{E}} \ ,}$$

where

$$\boxed{\frac{\epsilon}{\epsilon_0} = 1 + \chi_e \equiv \epsilon_r}$$

is called the **dielectric constant**, or sometimes the **relative permittivity**. ϵ is the **permittivity**, and ϵ_0 is the permittivity of free space, or the permittivity of the vacuum.

Linear Dielectrics and Laplace's Equation

I should have said, but didn't, that:

For a linear dielectric for which ϵ is independent of position, then

$$\boxed{\vec{\nabla} \times \vec{D} = \vec{\nabla} \times (\epsilon \vec{E}) = \epsilon \vec{\nabla} \times \vec{E} = 0 \ ,}$$

and

$$\boxed{\nabla^2 V = -\vec{\nabla} \cdot \vec{E} = -\frac{1}{\epsilon} \vec{\nabla} \cdot \vec{D} = -\frac{\rho_f}{\epsilon} \ .}$$

So, in particular, if $\rho_f = 0$, then $\nabla^2 V = 0$.

Example of a Problem with Linear Dielectrics

We partially did Example 4.7 from Griffiths, which involves a homogeneous linear dielectric material that is placed in an otherwise uniform electric field:

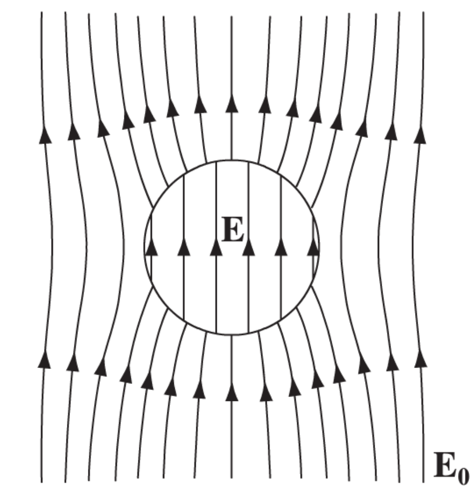

FIGURE 4.27

I should have justified $\nabla^2 V = 0$, inside the dielectric, by the argument on the previous slide.