8.07 Lecture Slides 1
September 4, 2019

INTRO TO 8.07

PLACE OF E&M IN PHYSICS

REVIEW OF VECTORS

Website

http://web.mit.edu/8.07/www

We will use the Gradebook on Stellar to record and present grades, but otherwise we will use only the website at the URL above.

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Information about the course is posted under the tab "General Info".

Course Staff

Lecturer: Me (Alan Guth), guth@ctp.mit.edu, Room 6-322.

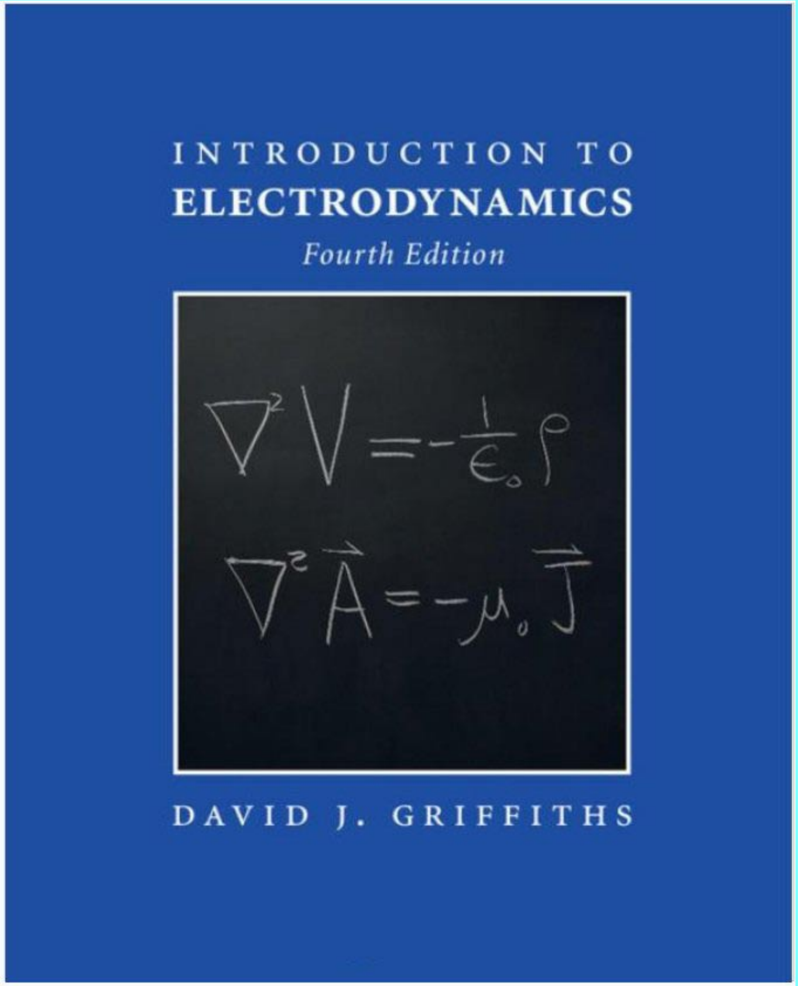
Recitation Instructor: Marin Soljačić, soljacic@mit.edu, Room 6C-419.

Teaching Assistant: Yitian Sun, yitians@mit.edu.

Required Textbook

Introduction to Electrodynamics, Fourth Edition (Cambridge University Press, 2017; originally published by Pearson, 2013—the CUP and Pearson versions are identical), by David J. Griffiths.

Be sure to get the 4th edition.



Grading

Two In-Class Quizzes: 40%

The quizzes will tentatively be on the following dates:

Monday, October 7, 2019

Wednesday, November 6, 2019

If you have a problem of any kind with either of these dates, you should email me (Alan Guth) as soon as possible.

Final Exam: 35%

3-hour exam, during the final exam period.

Problem Sets: 25%

Problem Sets

About 1 problem set per week, about 10 altogether, mostly due on Fridays at 4:45 pm.

Each problem set will have a Part A and a Part B, to be handed in separately, each to go to one of our two graders.

The first problem set will appear today on the website.

The problem sets will not all be worth the same number of points. Your grade will be the total number of points you earn, compared to the maximum possible. That is, problem sets with more points will count a little more than the others.

All Problem Sets Required But with Flexibility

All problem sets will be required, none will be dropped.

Reason: Psets will be an integral part of the course, so you will miss something significant if you blow one of them off.

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However: I understand that you are all have very overfilled lives. That's what MIT students are like! So, to make up for not dropping any of the problem sets, I will be very generous with extensions. If you are having an unusually busy week (or if you are sick, have a family crisis, or have a fight with your roommate), just send me an email describing the situation, and ask me for an extension.

If the solutions are posted before you turn in your problem set, you are on your honor not to look at the solutions, or discuss the problems with anyone who has (other than me or the other course staff), until you have turned it in.

Extra Credit

Some of the the problem sets will offer additional problems for extra credit. We will keep track of the extra credit grades separately.

At the end of the course I will consult with Marin Soljačić and Yitian Sun to set grade cuts based solely on the regular coursework. We will try to make sure that the grade cuts are reasonable with respect to this data set.

Then the extra credit grades will be added, allowing the grades to change upwards accordingly.

Finally, we will look at each student's grades individually, and we might decide to give a higher grade to some students who are slightly below a borderline. Students whose grades have improved significantly during the term, and students whose average has been pushed down by single low grade, will be the ones most likely to be boosted.

Homework Policy

I regard the problem sets primarily as an educational experience, rather than a mechanism of evaluation.

Your are encouraged — even strongly encouraged — to work on the homework in groups. I will be setting up a Class Contact webpage to help you find each other. But, you are each expected to **write up** your own solutions, even if you found those solutions as a group project.

8.07 Problem Solutions from previous years are strictly off limits, but other sources — textbooks, webpages — are okay, as long as you rewrite the solution in your own words.

A homework problem that appears to be copied from another student, from a previous year's solution, or copied from some other source without rewording might be given zero credit. Except in blatant cases, the first time you will be given a chance to redo it.

Remember that this homework policy does not apply to other classes.

Course Content

We expect to discuss all the chapters in Griffiths' book except the last *Electrodynamics and Relativity*.

These are:

1. Vector Analysis
2. Electrostatics
3. Potentials
4. Electric Fields in Matter
5. Magnetostatics
6. Magnetic Fields in Matter
7. Electrodynamics
8. Conservation Laws
9. Electromagnetic Waves
10. Potentials and Fields
11. Radiation

Two "Beyond-Griffiths" Topics

1. **The Dirac Delta Function:** Griffiths tells us that the Dirac delta function is not a function at all, but rather a **generalized function**, or **distribution**. We will learn what this means!

   We will also learn more about how to use Dirac delta functions. For example, the charge distribution corresponding to an ideal electric dipole can be described in terms of the derivative of a delta function.

   The material on Dirac delta functions will be presented in lecture, and in several "guided" homework problems.

Two "Beyond-Griffiths" Topics, cont.

2. **Spherical Harmonics:** The course will include spherical harmonics (which Griffiths does not). Spherical harmonics arise for example in describing the electric field of a localized charge distribution, in the region outside that distribution.

   We will introduce a novel approach involving traceless symmetric tensors, which I claim gives a much better understanding of the properties of spherical harmonics, and it is also useful in solving some kinds of problems.

   The material on spherical harmonics will be presented in several sets of supplementary lecture notes.

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PART II:
PLACE OF E&M IN PHYSICS

Four Fundamental Interactions of Nature

Strong Interactions:

Bind the quarks inside of protons, neutrons, and other strongly interacting particles.

The residual forces act to give forces between protons and neutrons, and other strongly interacting particles.

Electromagnetic Interactions:

Four Fundamental Interactions of Nature

Strong Interactions:

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Electromagnetic Interactions:

Well, this is what we'll be talking about all term.

Weak Interactions:

Includes all processes involving neutrinos, but also some other processes, such as $K^0 \to \pi^+\pi^-$. The K^0 has a lifetime of 0.895×10^{-10} s, a typical weak interaction lifetime.

The decay of a free neutron, $n \to p + e^- + \bar{\nu}_e$, has a very unusual lifetime of 879 s (14.65 minutes). Reason: the proton is just a tiny bit lighter than the neutron, so very little energy is given off by the decay: $(m_n - m_p)/m_n \approx 0.00138$.

Gravity:

Gravitational force between two electrons is only about 10^{-42} times the Coulomb force.

Curiosity: the energy density of a gravitational field is **nega-tive!** The total energy of the universe is very small, with the negative energy of gravity approximately canceling the energy of everything else. The total energy is consistent with zero.

Theoretical Descriptions

Theoretical description of E&M:

Maxwell, 1864 — the classical equations of E&M, the main subject of this course. Unified the description of electricity and magnetism.

Quantum version: Quantum Electrodynamics, QED, developed in 1930's and 40's.

1965: Nobel Prize to Feynman, Schwinger, and Tomonaga.

Maxwell's equations still hold, but they have a different interpretation.

Further Unification: Electroweak Theory

Weak Interaction Theory:

1958: "V-A" theory, by Feynman and separately by Marshak and Sudarshan

"V-A" was a flawed theory: worked very well in 1st order perturbation theory, but gave infinity at 2nd order.

1967: Glashow-Weinberg-Salam theory.

1979: Nobel Prize to Glashow, Weinberg, and Salam.

Exchange particles:

E&M: γ (the photon), massless.

Weak interactions: W^+, W^-, Z.

Massive: $mc^2 = 80.4$ GeV (W's) and 91.2 GeV (Z). For comparison, mass of proton = 0.938 GeV.

New particle: the Higgs boson. Discovered in 2012 at CERN.

Theory of the Strong Interactions: QCD

A theory of the strong interactions, known as Quantum Chromodynamics (QCD), was developed in the 1960's and 70's, with important contributions from a number of different people, including MIT's Frank Wilczek.

It is based loosely on Quantum Electrodynamics, except that it has three kinds of charge instead of just one, and the particle analogous to the photon, called the gluon, is itself charged. Furthermore, the charges are larger in value than the electric charge of an electron, so perturbation theory is applicable only in rare circumstances. These differences make the theory much more difficult to understand than QED.

Despite the difficulties, physicists are convinced that QCD is correct, at least within its range of applicability.

The Standard Model of Particle Theory

The electroweak and strong interaction theories together are called the Standard Model of Particle Physics.

The model is hugely successful, successfully describing every particle physics experiment that has been done. It needs to be modified a little to account for the nonzero masses that neutrinos are now known to have.

Despite the success, BSM ("Beyond-Standard-Model") particle physics is a huge enterprise. Why? (1) Standard model (SM) does not include gravity. (2) SM cannot account for the "dark matter," which is known from astrophysics to make up $> 80\%$ of all matter. (3) The SM is very complicated and "ugly".

Gauge Theories

The electroweak and strong interaction theories are **gauge theories**, like E&M.

The "gauge theory" properties of E&M can be seen only when it is written in terms of the potentials $\vec{A}$ and V, rather than in terms of $\vec{E}$ and $\vec{B}$. They are related by

$$\vec{E} = -\vec{\nabla} V - \frac{\partial \vec{A}}{\partial t}$$

$$\vec{B} = \vec{\nabla} \times \vec{A} \; .$$

If $\Lambda(\vec{x}, t)$ is an arbitrary function of space and time, then the transformation

$$\vec{A}' = \vec{A} + \vec{\nabla}\Lambda$$

$$V' = V - \frac{\partial \Lambda}{\partial t}$$

leaves $\vec{E}$, $\vec{B}$, and all physical quantities unchanged. This property is hugely important in allowing the standard model to be formulated in a consistent way.

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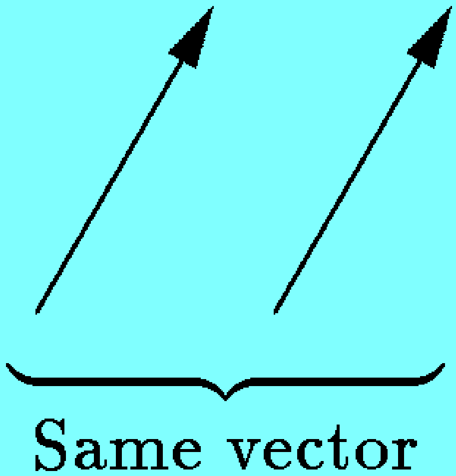
REVIEW OF VECTORS

Definition

DEFINITION: A *vector* is a quantity that has magnitude and direction.

Examples: displacement, velocity, acceleration, force, momentum, electric and magnetic fields.

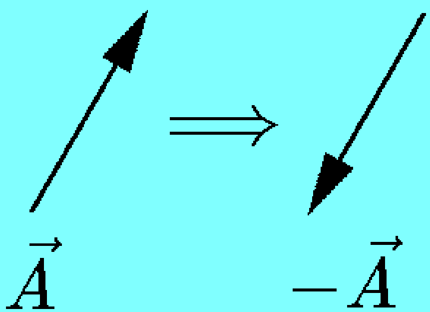
Vectors do not have position:



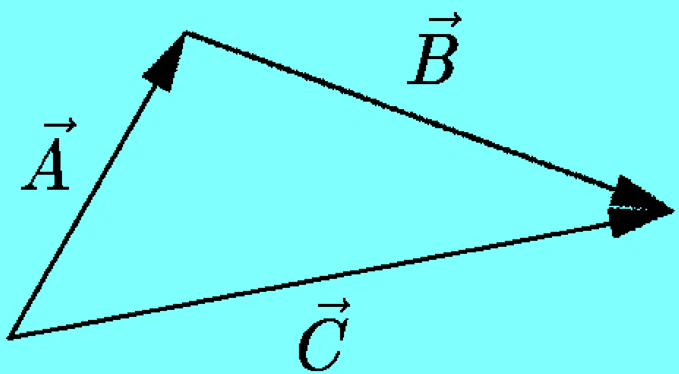
Operations

Magnitude: $\|\vec{\boldsymbol{A}}\| \equiv$ magnitude of $\vec{\boldsymbol{A}}$. (Here $\equiv$ means "is defined to be".) Often A is used to denote $\|\vec{\boldsymbol{A}}\|$.

Negation:

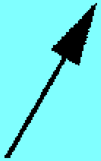
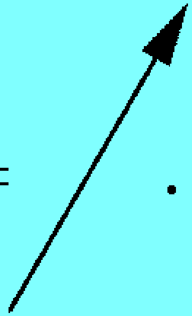


Addition:



$\vec{A} + \vec{B} = \vec{C}$

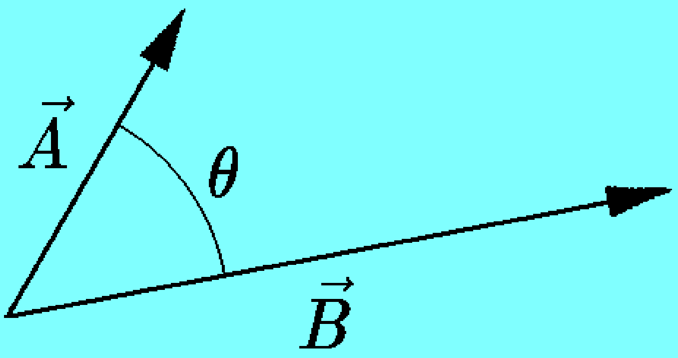
Subtraction: $\vec{A} - \vec{B} \equiv \vec{A} + (-\vec{B})$.

Multiplication by a scalar: $2 \times$  $=$  .

Property— Distributive: $a(\vec{A} + \vec{B}) = a\vec{A} + a\vec{B}$.

Dot Product

Dot product of two vectors: $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} \equiv |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}| \cos\theta$, where θ is the angle between $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$:



always tail to tail!

Properties:

Rotational invariance. The value of the dot product does not change if both of the vectors are rotated together.

Cummutative: $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \vec{\boldsymbol{B}} \cdot \vec{\boldsymbol{A}}$.

Distributive: $\vec{\boldsymbol{A}} \cdot (\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}}) = \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{C}}$.

Scalar multiplication: $(a\vec{\boldsymbol{A}}) \cdot \vec{\boldsymbol{B}} = a(\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}})$.

Why Cosθ?

Query: Why $\cos\theta$??? If I defined a Guth-dot product by

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}\Big|_{\text{Guth}} \equiv |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}| \sin\theta \ ,$$

and hired a really good advertising agency, could my product (note the pun!) compete?

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Answer: Maybe a really good advertising agency can do anything, but I would have a serious marketing problem. My dot product would not be distributive. In fact, one can show that if $\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}}$ obeys rotational invariance, commutativity, the scalar multiplication law, and the distributive law, then

$$\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = \text{const}|\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|\cos\theta \ .$$

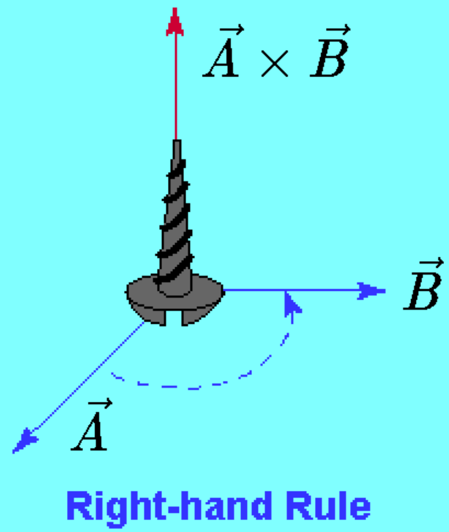
We'll come back to this later.

The Vector Cross-Product $\vec{A} \times \vec{B}$

Cross product of two vectors:

$$\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} \equiv |\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}| \sin\theta\, \hat{n} \ ,$$

where $\hat{n}$ is a unit vector perpendicular to $\vec{\boldsymbol{A}}$ and perpendicular to $\vec{\boldsymbol{B}}$. The choice of the two (opposite) directions that are perpendicular to both $\vec{\boldsymbol{A}}$ and $\vec{\boldsymbol{B}}$ is determined by the right-hand rule:



(Source: Modified from chortle.ccsu.edu/vectorlessons/vch12/rightHandRule.gif.)

Properties:

Rotational invariance: If both vectors are rotated by the same rotation R, then the result of the cross product is also rotated by R.

Anticommutative: $\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} = -\vec{\boldsymbol{B}} \times \vec{\boldsymbol{A}}$.

Distributive: $\vec{\boldsymbol{A}} \times (\vec{\boldsymbol{B}} + \vec{\boldsymbol{C}}) = \vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}} + \vec{\boldsymbol{A}} \times \vec{\boldsymbol{C}}$.

Scalar multiplication: $(a\vec{\boldsymbol{A}}) \times \vec{\boldsymbol{B}} = a(\vec{\boldsymbol{A}} \times \vec{\boldsymbol{B}})$.

Why Sinθ?

Query: Why $\sin\theta$???

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Answer: Again, the function of θ is required for rotational invariance and distributivity. If we assume that the cross product is rotationally invariant, and obeys the distributive law and the scalar multiplication law on both the right and the left (i.e., $(\vec{\boldsymbol{A}}+\vec{\boldsymbol{B}})\times\vec{\boldsymbol{C}}=\vec{\boldsymbol{A}}\times\vec{\boldsymbol{C}}+\vec{\boldsymbol{B}}\times\vec{\boldsymbol{C}}$ and $\vec{\boldsymbol{A}}\times(a\vec{\boldsymbol{B}})=a(\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}})$ as well as the identities on the previous slide), then one can show that

$$\vec{\boldsymbol{A}}\times\vec{\boldsymbol{B}}=\mathrm{const}|\vec{\boldsymbol{A}}||\vec{\boldsymbol{B}}|\sin\theta\,\hat{n}\ .$$

I'll come back to this.

Component Notation

$$\vec{\boldsymbol{A}} = A_x \hat{\boldsymbol{x}} + A_y \hat{\boldsymbol{y}} + A_z \hat{\boldsymbol{z}} \ ,$$

where

$\hat{\boldsymbol{x}}$ is a unit vector in the x direction

$\hat{\boldsymbol{y}}$ is a unit vector in the y direction

$\hat{\boldsymbol{z}}$ is a unit vector in the z direction.

(Various notations are in use. Griffiths uses $\hat{\boldsymbol{x}}$, $\hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{z}}$, while many other books use $\hat{\boldsymbol{\imath}}$, $\hat{\boldsymbol{\jmath}}$, and $\hat{\boldsymbol{k}}$.)

Index Notation: Components by Number

Let $\hat{\boldsymbol{e}}_1 \equiv \hat{\boldsymbol{x}}$, $\hat{\boldsymbol{e}}_2 \equiv \hat{\boldsymbol{y}}$, and $\hat{\boldsymbol{e}}_3 \equiv \hat{\boldsymbol{z}}$, so

$$\vec{\boldsymbol{A}} = A_x \hat{\boldsymbol{x}} + A_y \hat{\boldsymbol{y}} + A_z \hat{\boldsymbol{z}} \ ,$$

can be written

$$\vec{\boldsymbol{A}} = A_1 \hat{\boldsymbol{e}}_1 + A_2 \hat{\boldsymbol{e}}_2 + A_3 \hat{\boldsymbol{e}}_3 \ ,$$

which can be abbreviated as $\vec{\boldsymbol{A}} = \displaystyle\sum_{i=1}^{3} A_i \hat{\boldsymbol{e}}_i$, which can be further abbreviated as $\vec{\boldsymbol{A}} = A_i \hat{\boldsymbol{e}}_i$. This final expression uses the "Einstein summation convention": if an index is repeated within one term, it is implicitly summed from 1 to 3. That is, $A_i B_i$ is summed, but $A_i + B_i$ is not (these are two different terms).

Vector Addition in Components

Vector Addition:

$$\vec{\boldsymbol{C}} = \vec{\boldsymbol{A}} + \vec{\boldsymbol{B}} \qquad \Longrightarrow \qquad C_i \hat{\boldsymbol{e}}_i = A_i \hat{\boldsymbol{e}}_i + B_i \hat{\boldsymbol{e}}_i$$

$$\Longrightarrow \qquad C_i \hat{\boldsymbol{e}}_i = (A_i + B_i) \hat{\boldsymbol{e}}_i$$

$$\Longrightarrow \qquad \boxed{C_i = A_i + B_i \ .}$$

Vector Dot Product in Components

Vector Dot Product:

$$\hat{\boldsymbol{e}}_i \cdot \hat{\boldsymbol{e}}_j = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases}$$

$$= \delta_{ij} \ ,$$

where δ_{ij} is called the Kronecker δ-function.

Then

$$\begin{aligned} \vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} &= A_i \hat{\boldsymbol{e}}_i \cdot B_j \hat{\boldsymbol{e}}_j \\ &= A_i B_j \hat{\boldsymbol{e}}_i \cdot \hat{\boldsymbol{e}}_j = A_i B_j \delta_{ij} \\ &= \sum_{i=1}^{3} A_i \left(\sum_{j=1}^{3} B_j \delta_{ij} \right) = \sum_{i=1}^{3} A_i \left(B_i \right) = A_i B_i \ . \end{aligned}$$

Thus,

$$\boxed{\vec{\boldsymbol{A}} \cdot \vec{\boldsymbol{B}} = A_i B_i \equiv A_x B_x + A_y B_y + A_z B_z \ .}$$