Weekly Assignment 4
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Question 1

Consider an experiment where we roll two 6-sided dice. Let random variable Y be the sum of the values rolled. The sample space is $\{(1,1),(1,2),(1,3),...,(6,6)\}$ and recall that a random event is a subset of the sample space.

Problem part a

What random event corresponds to $Y = 2$?

Solution part a

Minimum value each dice can produce is 1, so to get a sum of 2, both dice rolled can only produce 1 and not higher. Hence,

$$Y = \{(1,1)\}$$

Problem part b

What event corresponds to $Y = 3$?

Solution part b

To get a sum of 3, one dice have to produce 1 while the other produce 2. Hence

$$Y = \{(1,2),(2,1)\}$$

Problem part c

What event corresponds to $Y = 4$?

Solution part c

To get a sum of 4, consider the following cases:

- First dice rolled produce 1:
  Then, second dice rolled must produce 3.
- First dice rolled produce 2:
  Then, second dice rolled must produce 2.
- First dice rolled produce 3:
  Then, second dice rolled must produce 1.

So,

$$Y = \{(1,3),(2,2),(3,1)\}$$

Problem part d

Now let X be the indicator random variable associated with the event $\{(1,1),(2,2),(3,3)\}$. What is the probabilities that $X = 1$?

Solution part d

$$\begin{aligned} P(X) &= \frac{|X|}{|S|} \\ &= \frac{3}{6^2} \\ &= \frac{1}{12} \\ &\approx 0.0833 \end{aligned}$$

Question 2

Let X represent the difference between the number of heads and the number of tails obtained when a coin is tossed 3 times.

Problem part a

What are the possible values of X ?

Solution part a

Consider the following possible cases:

- Getting 3 Heads and 0 tails:
  Difference is 3
- Getting 2 Heads and 1 tails:
  Difference is 1
- Getting 1 Heads and 2 tails:
  Difference is -1
- Getting 0 Heads and 3 tails:
  Difference is -3

So, the possible values of X are $\{-3, -1, 1, 3\}$.

Problem part b

What is $P(X = -3)$?

Solution part b

Since a coin has two sides and tossing three times will give $2^3 = 8$ possible sequence results, hence $|S| = 8$. $P(X = -3) =$ Probability of getting $\{T, T, T\}$ OR 3 Tails. So,

$$\begin{aligned} P(X = -3) &= \frac{\big|(X = -3)\big|}{|S|} \\ &= \frac{1}{8} \\ &= 0.1250 \end{aligned}$$

Problem part c

What is $P(X = -1)$?

Solution part c

$P(X=-1) =$ Probability of getting 2 Tails,i.e $\{(T,T,H),(T,H,T),(H,T,T)\}$. So,

$$\begin{aligned} P(X=-1) &= \frac{\big|(X=-1)\big|}{|S|} \\ &= \frac{3}{8} \\ &= 0.3750 \end{aligned}$$

Problem part d

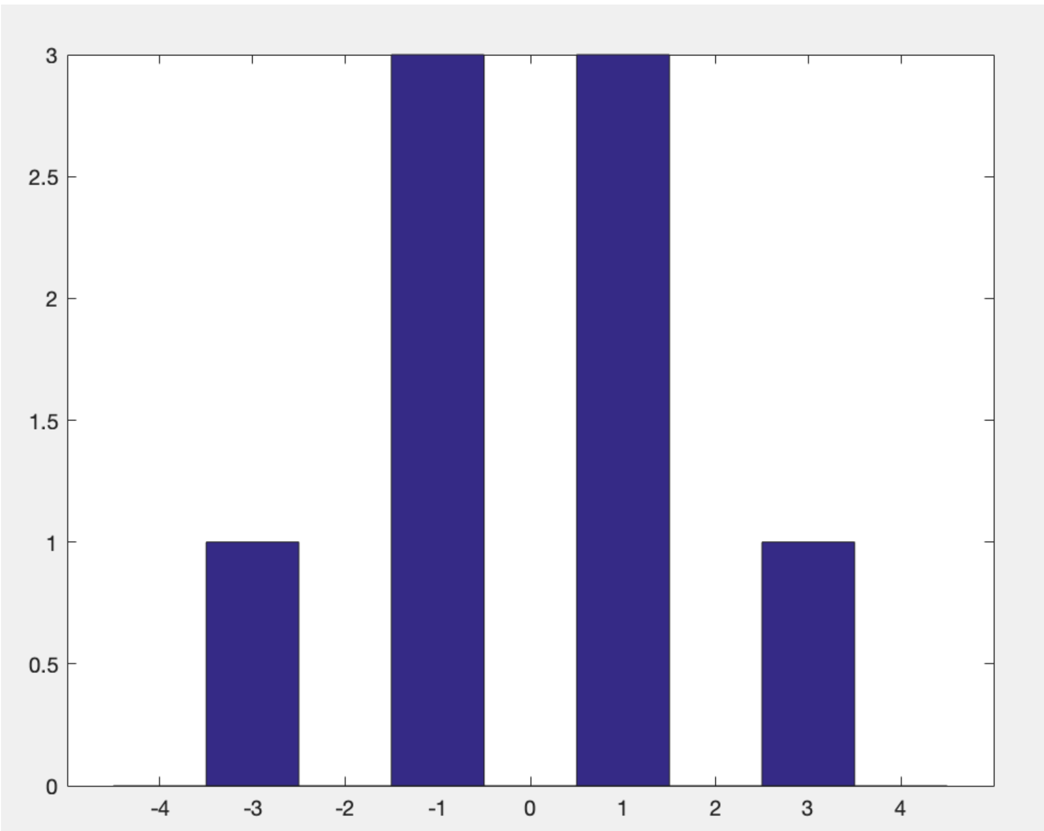
If the coin is assumed fair, calculate the PMF and CDF of X and plot a sketch of both.

Solution part d

Variable X takes values in $\{-3,-1,1,3\}$. To calculate its CDF, we can compute the respective PMF first:

- $P(X=-3) = \frac{1}{8}$
- $P(X=-1) = \frac{3}{8}$
- $P(X=1) = \frac{3}{8}$
- $P(X=3) = \frac{1}{8}$

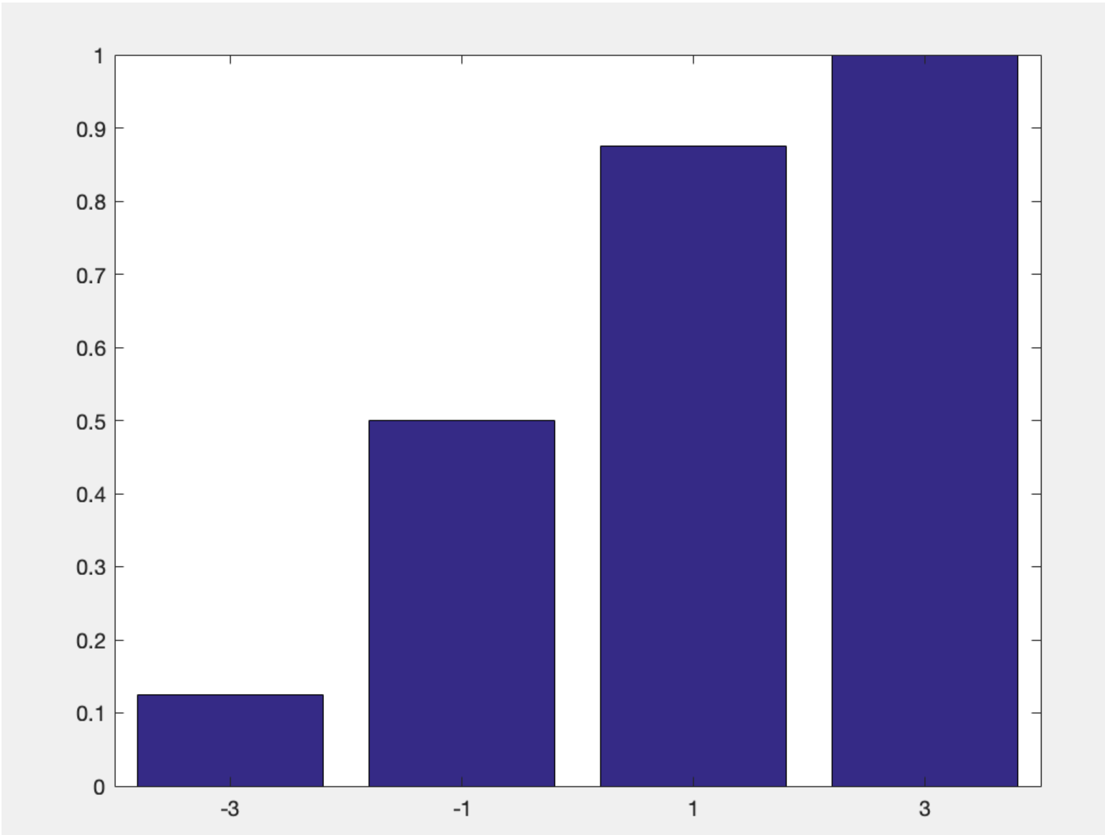
PMF diagram:



The random variable X has a CDF of:

$$P(X) = \begin{cases} 0.125 & x = -3 \\ 0.5 & x = -1 \\ 0.875 & x = 1 \\ 1 & x = 3 \end{cases}$$

CDF diagram:



Question 3

Four 6-sided dice are rolled. The dice are fair, so each one has equal probability of producing a value in $\{1, 2, 3, 4, 5, 6\}$. Let X =the minimum of the four values rolled. (It is fine if more than one of the dice has the minimal value.)

Problem part a

What is $P(X \geq 1)$?

Solution part a

Since the minimum value on a dice is 1, it doesn't matter how it is rolled, the result will always be more or equal than 1. Hence,

$$\begin{aligned} P(X \geq 1) &= \frac{6^4}{6^4} \\ &= 1 \end{aligned}$$

Problem part b

What is P $(X \geq 2)$?

Solution part b

For every dice roll, there is a $\frac{5}{6}$ chance of getting values higher than 1, namely 2 to 6. Hence, rolling 4 dice will have a probability of:

$$\begin{aligned} P(X \geq 1) &= (\frac{5}{6})^4 \\ &= \frac{625}{1296} \\ &\approx 0.4823 \end{aligned}$$

Problem part c

What is the CDF of X i.e. $P(X \leq k)$ for all values of k ?

Solution part c

To calculate the CDF for each $P(X \leq k)$ for each k, we can just reverse subtract 1 by the probability of not getting any values equal or less than k.

Let R be the event of not rolling values $\leq k$.

$$\begin{aligned}
P(X \leq 1) =& 1 - P(R) \\
=& 1 - (\frac{5}{6})^4 \\
=& 1 - \frac{625}{1296} \\
=& \frac{671}{1296} \\
P(X \leq 2) =& 1 - P(R) \\
=& 1 - (\frac{4}{6})^4 \\
=& 1 - \frac{16}{81} \\
=& \frac{65}{81} \\
P(X \leq 3) =& 1 - P(R) \\
=& 1 - (\frac{3}{6})^4 \\
=& 1 - \frac{1}{16} \\
=& \frac{15}{16} \\
P(X \leq 4) =& 1 - P(R) \\
=& 1 - (\frac{2}{6})^4 \\
=& 1 - \frac{1}{81} \\
=& \frac{80}{81} \\
P(X \leq 5) =& 1 - P(R) \\
=& 1 - (\frac{1}{6})^4 \\
=& 1 - \frac{1}{1296} \\
=& \frac{1295}{1296} \\
P(X \leq 6) =& 1 - P(R) \\
=& 1 - (\frac{0}{6})^4 \\
=& 1 - 0 \\
=& 1
\end{aligned}$$

$$P(X) = \begin{cases} \frac{671}{1296} & 1 \leq x < 2 \\ \frac{65}{81} & 2 \leq x < 3 \\ \frac{15}{16} & 3 \leq x < 4 \\ \frac{80}{81} & 4 \leq x < 5 \\ \frac{1295}{1296} & 5 \leq x < 6 \\ 1 & 6 \leq x \end{cases}$$