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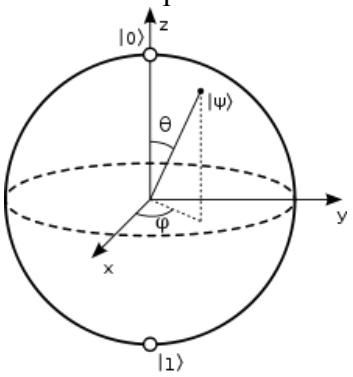
Ion Trap Quantum Computers 8/12/2011

The theory of computer science existed before the first silicon transistor was produced by Texas instruments in 1954. The Turing machine introduced by Alan Turning 1936 provided a formal theory of computation, within the framework algorithms could be specified and complexity theory could be investigated. People understood the notion of information being stored and processed inside of a mechanical computer but they did not yet have the technology to implement such a machine on a useful scale.

Eventually the transistor would provide a way to consistently and compactly store, access, and modify information in the form of binary bits. The information age saw an explosion of computational power and the implementation of many algorithms that were already specified theoretically. Currently we may be on the verge of a similar transition, theorists have developed a model of quantum computation and some important algorithms have already been written. What remains to be solved is the implementation of the quantum equivalent of the transistor, a way to store and modify quantum bits (qubits).

This paper will examine some of the possible routes toward implementation of a quantum computer. Specifically it will describe progress being made on developing an ion trap quantum computer as well as the current problems and limitations with this method (due to decoherence and scalability). There is plenty of reason for optimism as many of the problems with implementing this approach have each been successfully tackled in isolation which might eventually lead to a working system which ties in all of the components.

A bit is either on or off, there are only two possibilities. A qubit is more complex and is best understood from a quantum mechanical perspective. The qubit is described by a quantum state in a two-dimensional vector space over the complex numbers. The two states in which a qubit can be measured in are its basis vectors and are conventionally written in Dirac notation as |0> and |1>. A pure qubit state is a linear superposition of the basis states, in general state ψ will be $|\psi\rangle = \alpha|0\rangle + \beta|1\rangle$, where α and β are probability amplitudes. A measurement of the system will yield |0> with probability α^2 and |1> with probability β^2. The coefficients can be complex numbers and a normalization condition restricts the state of the qubit to the surface of the "Bloch Sphere" shown below.



Representation of a qubit. The probability amplitudes are given by $\alpha = \cos\left(\frac{\theta}{2}\right)$ and $\beta = e^{i\phi}\sin\left(\frac{\theta}{2}\right)$.

What makes qubits so special is the phenomenon of entanglement, which is observed in dealing with the interactions between multiple qubit systems. When talking about a system of multiple qubits, the dimension of the vector space is no longer 2. The dimension needed to describe the physical reality is increased exponentially with the number of qubits in the system. The proper mathematical tool to describe multiple qubits is the tensor product . If I had a qubit $\psi = |0>$ and another qubit $\phi = |1>$ the system of the two qubits is described by $\psi \otimes \phi = |0> \otimes |1>$ which to simply notation can be written |01>. Entanglement is nonlocal phenomenon that allows for sets of qubits to be highly correlated such that the observation of one qubit reveals information about another qubit that was not observed (no matter where that qubit is).

Some states can be more entangled than others, the following state (of a two qubit system) is minimally entangled because the measurement of the first qubit reveals no information about the second

$\frac{1}{2}(|00> + |01> + |10> + |11>)$, whereas the following state (one of the bell states) is maximally entangled (measuring one qubit reveals what the measurement of the other will be).

$\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle).$ Entanglement is a necessary ingredient of many quantum computations that can't be simulated efficiently with the resources of a classical computer. Some of these quantum algorithms have as much as an exponential speed up over their classical counter parts.

Probably the most well known problem that quantum computation can speed up is the problem of factoring. RSA, an encryption scheme used ubiquitously in web communication relies on the fact that factoring integers is a "hard" problem in the sense that doing so requires more than a polynomial amount of time. It has yet to be proven that factoring is indeed a problem that can't be solved with a classical polynomial time algorithm but in 1994 Peter Schor provided a quantum algorithm that relied on period finding to factor integers in polynomial time. This algorithm has provided significant motivation in the search for a physical realization of a quantum computer, which if found could implement Schor's algorithm and fundamentally change the field of cryptography.

With at least one application of quantum computers now pretty clear, it becomes important to commit to researching how to implement a particular solution for a quantum computer. Over the past 15 years there has been research into a number of possible different implementations for the qubit and corresponding quantum computer each with distinct advantages and disadvantages. I list here some of the more prominent approaches (taken from Wikipedia) along with links to more information.

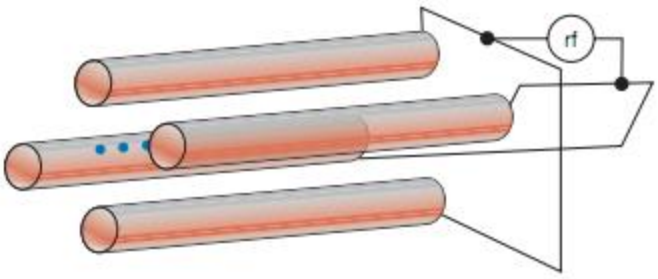
- Trapped ion quantum computer; qubit implemented by the internal state of trapped ions
- Superconductor-based quantum computers; qubit implemented by the state of small superconducting circuits (Josephson junctions)
- Optical lattices; qubit implemented by internal states of neutral atoms trapped in an optical lattice
- Nuclear magnetic resonance; on molecules in solution (liquid-state NMR), qubit provided by nuclear spins within the dissolved molecule

- Cavity quantum electrodynamics (CQED); qubit provided by the internal state of atoms trapped in and coupled to high-finesse cavities
- Optics-based quantum computer (Quantum optics); qubits realized by appropriate states of different modes of the electromagnetic field, e.g
- Diamond-based quantum computer; qubit realized by the electronic or nuclear spin

For the remainder of this paper I will focus on the current research regarding trapped ion quantum computers. The idea behind an ion trap quantum computer is to use two close together (hyperfine) internal energy levels of an ion as qubit states, states which have a very small energy separation. With ions confined into some sort of trap, a method is needed to reliably transition between the internal states of the qubit. A laser can be used to excite the transition between states in the ion but only if the ion is held approximately in place by the trap. Since the ion is a charged particle it would be ideal to have it trapped in an electric field but the problem is that a collection of point charges cannot be maintained in a stable stationary equilibrium only by electrostatic interaction, as seen from Earnshaw's theorem. The force on a charge at location x (F(x)) resulting from inverse-square forces (from a potential which satisfies Laplace's equation) will be divergenceless ($\nabla \cdot F = 0$). This law means that there can be no absolute extrema in the field, so no stable equilbria for an ion to be trapped in.

Since a static electric charge configuration won't work other ways must be found. Two such methods to confine an ion in a small space are the Penning trap and Paul trap. The Penning trap uses a combination of static electric and strong magnetic fields to confine the ion. In this trap, the ions undergo a distorted cyclotron motion. The goal should be for the qubits to move as little as possible. The favored trap for a proposed ion quantum computer proposed by J. I. Cirac and P. Zoller in their 1995 paper [2] was a Paul trap.

In a Paul ion trap there is no magnetic field, instead the set up is a combination of static and oscillating electric fields. Picture below is a carton of a Paul trap the 4 rods are electrodes which are wired to a sinusoidal RF voltage.



For a negatively charged ion when the electrodes are negative the ion will be pushed to center, but from Earnshaw's theorem this is not a stable equilibrium, the fast switching of the electrodes to a positive voltage and then back again $V_0\cos(\omega_T t)$ will damp the ions motion, and because of its inertia (if the switching is done fast enough) With the ions only confined to the middle axis, in order to eliminate the last degree of freedom two end cap electrodes are used with potential U_0.

The motion of the ions along the axis will behave according to the solutions of the Mathieu equation.

$$\frac{d^2u}{d\xi^2} + [a_u - 2q_u\cos(2\xi)]u = 0$$

Near the axis of the trap the potential is approximately.

$$\Phi \simeq \frac{V_0}{2}\cos\Omega_T t\left(1+\frac{x^2-y^2}{R^2}\right)+\frac{U_0}{2}\left(\frac{2z^2-x^2-y^2}{d^2}\right)$$

Where R is equal to the distance from the axis to the surface of the electrode and d is a characteristic axial dimension of the static electrodes.

This gives rise to harmonic ponderomotive potentials [3] in X ,Y directions as well as confinement along the Z axis. The complete effect is an approximate 3d Harmonic potential of the general form $U = \frac{1}{2}m\omega_x^2x^2 + \frac{1}{2}m\omega_y^2y^2 + \frac{1}{2}m\omega_z^2z^2$. Where the frequencies not along the z axis ω_x and ω_y can be approximated as $\sqrt{\omega_p^2 - \frac{\omega_z^2}{2}}$ with $\omega_p \equiv \frac{qV_0}{\sqrt{2}\,\Omega_T m R^2}$. These oscillatory motions away from the z axis are not ideal; there should be as little energy as possible in these modes. This is achieved by choosing parameters of the setup to minimize ω_x and ω_y. The ions will be able to oscillate in these directions at multiples of these frequencies, it will be important to insure that the ions are oscillating at the lowest possible multiple (the ground state) of these frequencies in x and y in order to minimize decoherence errors in qubit calculations. In order to prepare ions that are in the ground state in these directions laser cooling techniques are used, which I talk about later on in this paper.

The third, z dimension, of motion along the axis is what will be utilized as a "bus" to entangle specific qubits. The harmonic potential along this axis yields a fundamental frequency of $\omega_z = \sqrt{\frac{2qU_0}{md^2}}$. For two ions in the trap the normal mode frequencies are ω_z and $\sqrt{3}\omega_z$, for three ions the possible normal modes are ω_z, $\sqrt{3}\omega_z$, and $\sqrt{5.4}\omega_z$, for a trap with more ions the normal mode frequencies are calculated numerically [4].

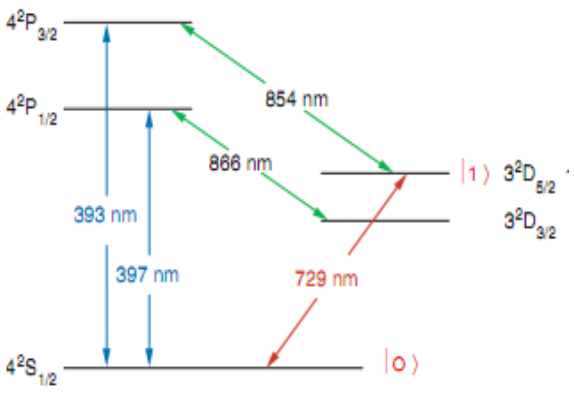
With a knowledge of how a specific ion is confined using the trap, the other important aspect for a quantum computer is how to store and interact with data in the form of qubits. With multiple ions along the axis of the trap Cirac and Zoller proposed two components that would factor into the state of an individual ion qubit, the internal hyperfine energy state of the ion and the vibrational mode that qubit was oscillating at. With a string of ions contained along the axial direction of the trap, they will mutually repel each other via a column force.

The ions will vibrate harmonically along the axis in some linear combination of the normal mode frequencies similar to a series of pendulums connected by springs. These axial oscillations are characterized by the number of phonons in the vibrational mode (they are quantized). The lowest energy mode corresponds to all the ions translating back and forth

together, higher order modes (meaning more energy in the system) happen when adjacent ions move in opposite directions. The state of axial center of mass (COM) motion is described by a ladder of vibrational eigenstates |n> with each state having energy $\hbar\omega_z(n+\frac{1}{2})$ with n specifying the number of phonons in the vibrational state. An important point to note is that even when the

ions are cooled to their vibrational ground state they are still oscillating and have some amount of energy.

In order to alter and measure the state of the qubits, tightly focused laser beams are used.



I will discuss the requirements and intricacies of such a system in a following section. Each Ion's internal hyperfine state must be able to represent an isolated qubit with the possibility of being in the state |0>, |1>, or some superposition of these. The lasers in the system need to be able to apply unitary transformation to arbitrarily rotate the state of this qubit in a two-dimensional Hilbert space. In an experiment with a calcium ion [5], when the ion is in the $4^2S_{1/2}$ energy state this represents a |0> for the qubit, the ion being in the $3^2D_{5/2}$ represents the |1> for the qubit.

When a single, periodically varying, electromagnetic field propagating along z applied to a trapped ion is driven at the resonant frequency between the ion's internal states, a change to the ion's wave function yields a solution of sinusoidal "Rabi oscillations" between these internal states. The qubit evolves according to $|0> \rightarrow \cos(\theta)|0> + e^{i\varphi}\sin(\theta)|1>$ where θ is proportional to the drive duration and φ is a phase factor.

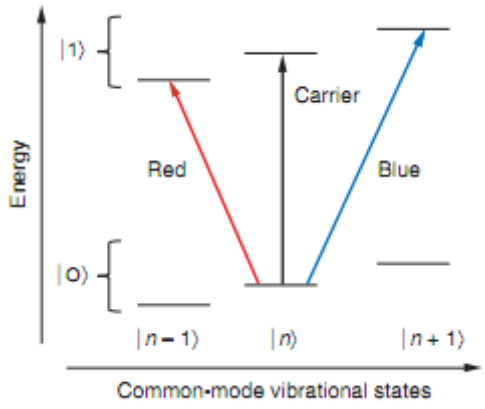
To cause the ion to completely transition from the |0> to |1> state a laser pulse is aimed at the ion, if the ion is completely in the |0> state (no superposition) then a π-pulse ($\theta = \pi$) will cause half a Rabi rotation and result in the pure |1> state, now if the full π-pulse is not applied, but instead is cut short half way through (π/2 pulse) then the electron will be left in an equal superposition state $\frac{1}{\sqrt{2}}(|0> + |1>)$.

Controlling the state of an isolated qubit via unitary rotations is an important piece of building a quantum computer, but in order for the set up to be useful there also needs to be a way to couple qubits together (entanglement). In general with rotations on an isolated qubit and an operation that can couple two qubits together (such as a C-Not operation) the theoretical requirements for a universal quantum computation are met. So far I have only described a mechanism for rotations on a single qubit a method to entangle qubits is also needed.

In order to couple individual qubits together in the ion trap the physically oscillating center of mass mode must be utilized, using a different frequency laser to excite the trap's vibrational modes. The state of the entire ion trap system is given by the tensored product state $|q_1,q_2,\ldots q_j>|n>$ where the first j qubits (each belonging to a two dimensional vector space) refer to the internal states of the j ions (some superposition of |0> and |1>) and n corresponds to the common mode vibrational state of the whole system.

To see how the common mode |n> can be used to entangle the states of individual qubits consider the following situation. The internal energy difference of the ion between the |0> and |1> states corresponds to some frequency call it ω_0, the lowest common mode frequency at which all the ions oscillate back and forth along the axis is $\omega_1 < \omega_0$. If a laser is aimed at the system with a frequency ω_0 an internal state change can occur, but a transition can also be excited if the

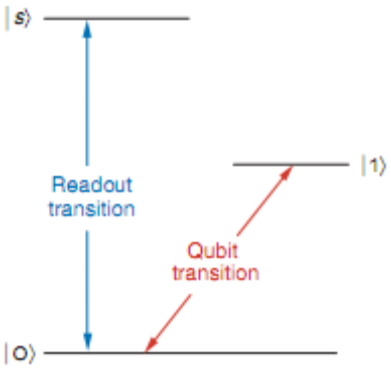
laser has frequency (called a sideband frequency) $\omega_0 \pm \omega_1$. I will refer to the higher of these sideband frequencies as the "blue sideband" and the lower as the "red sideband".



If we have a system of two ions initially in the ground state |0,0>|0> and we apply a π-pulse with a laser at the blue sideband frequency to the first ion we excite the ion to its internal |1> state and at the same time excite the system (both ions since the vibrational mode involves all ions on the axis) to the |1> mode. The state that results is |1,0>|1>, similarly if the π-pulse is applied to the second ion, then the system evolves to the |0,1>|1> state.

If the pulse is applied for half as long to the first ion (π/2-pulse) then the system evolves to a superposition made up equally of two pure states; the state in which the blue sideband transition has excited both the first ion and the vibrational mode, and the state in which the system is still in the ground state. $\frac{1}{\sqrt{2}}(|0,0>|0>+|1,0>|1>)$. If a red sideband π-pulse is now set to the second ion it will have no effect on the ground state making up the current superposition (since its energy is too low to excite a transition to the internal excited state of the ion). On the pure excited state however, it will bump up the second ion to the internal excited state by taking the extra energy from the vibrational mode, leaving the vibrational mode in the ground state.

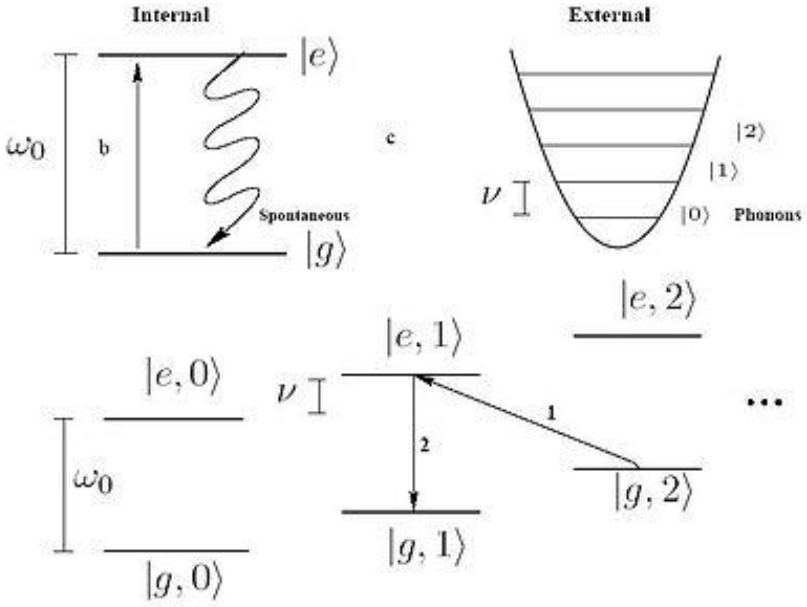
What is left is now the two qubit state $\frac{1}{\sqrt{2}}(|0,0>|0>+|1,1>|0>)$. This is a maximally entangled bell state. Two independent ions have been coupled together in the trap, when measuring one ion the result of a measurement on the second ion is automatically known. This procedure for coupling two ions together in the Paul trap is not restricted to just adjacent particles but can be used to couple any two particles in the trap because the vibrational mode excitation acts as a bus since it is shared among all the ions. Cirac and Zoller go further and discuss how they can get a C-Not from the two basic processes just introduced [2].



For reading out the state of the system one must find a short-lived excited state |s> which can only be excited from one of the two hyperfine states, the |0> state, since |s> is not stable the ion will decay back down to |0> and fluoresce in the process. Because the |s> state decays so quickly if the ion is measured to be in the |0> state a huge number of photons will be detected, if this huge number of photons is not seen than the ion was measured to be in the |1> state since the |s> transition was not excited. The readout (being a quantum measurement) will destroy any quantum information contained in the qubit and provide a pure value either |0> or |1> with nearly 100% efficiency [7].

Before any quantum computation can occur, the ions need to be put into their ground state. Laser cooling is used to bring the ions into their ground state through a combination of

techniques known as Doppler cooling, and resolved sideband cooling. Doppler cooling is used as a preliminary first step on the way to forcing the ions into their |n=0> motional state. The process involves shining light whose frequency is slightly lower than the electronic transition of the ion on both sides of the linear trap. The light that is moving in the direction toward the ion will appear to have a higher frequency (due to the Doppler Effect) and will thus get absorbed, while the light from the other side will appear to have a lower frequency and thus never get absorbed. Due to conservation of momentum the ion will always lose its momentum because it will only absorb photons traveling in its opposite direction. When the ion spontaneously emits a photon to conserve energy it will be in a random direction and the net effect of the collision will be to lower the energy of the ion.



Doppler cooling will cool the ion to about 1mK with an expected value of n from a measurement, $\langle n \rangle \geq 1$, still above the ground state. Resolved sideband cooing is used to further cool the ion. Sideband cooling works by using a laser tuned to the red sideband ($\omega = \omega_0 - \nu$, where ω_0 is the internal energy difference in states and ν is the frequency of the excited motional mode) talked about earlier. From exciting the atom using the red sideband a transition $|g,n\rangle \rightarrow |e,n-1\rangle$ will occur (g being ground internal state and n being the motional mode along the trap axis). Spontaneous emission from the ion will bring it back down from the ground state with the transition $|e,n-1\rangle \rightarrow |g,n-1\rangle$. When the steps are repeated eventually $|g,0\rangle$ is reached.

Now the long lifetime of the excited state is actually a hindrance since only about one energy state can be lowered every second. If the ion starts out in say n=30 then the competing thermal influences will far overpower the cooling sideband process. To speed up the cooling an alternate decay route can be used. Not only is a laser used at the red sideband frequency but also another to pump to a slightly higher (less stable) P-state from which the ion will decay rapidly to the ground state.

One of the most important design considerations for the ion trap is how to individually address the ions for logic operations with the laser beam. This is formidable task because the separation of the ions is extremely small, 10μm for 2 ions and smaller separations as the number of ions increase [3]. The simplest method for targeting individual ions is a Gaussian beam pulse which has a distribution of $I(r) = \frac{2P}{\pi\omega_0^2} e^{-\frac{2r^2}{\omega_0^2}}$ where P is the power of the beam and $\omega_0 \cong \lambda(\pi * NA)$ with λ being the beams wavelength. The numerical aperture(NA) of the beam is thus described by $\tan\theta = NA$. So for large enough aperture it would appear you could focus a beam to a spot the size of the wavelength, but this is hard to do in practice. $\omega_0 = 5$ μm will result in a relative beam intensity of 1.8% 10 μm away(at the adjacent ion's position). Also imperfections in the beam window and effects from diffraction will blur the predicted laser intensity further. This means that focused Gaussian beams have the potential for significant intensity fluctuations of the selected ion and can even unintentionally affect adjacent ions. Details about more effective than this basic Gaussian beam method for ion manipulation in a Paul trap are given by Wineland, et al.

The description of these trapped ion qubits having both motional and internal states that can be individually accessed and modified by laser pumping is greatly idealized, the reasons such a system has not been implemented on a large scale generally fall into the category of decoherence. There are three separate categories where decoherence can appear in this system, decoherence in the ion motion, decoherence in the internal state of the ion, and decoherence from the applied fields and lasers.

Decoherence is the loss of the alignment in phase angles between components of a state in a superposition; decoherence leads to the observation of classical physics because of the appearance of wave function collapse. Decoherence is essentially a loss of information from a system into the wider environment. Since no system can really ever be completely isolated there will always be some sort of coupling with the energy states of its environment.

Decoherence can be looked at from the point of view of the effect on a density matrix representing the quantum system. The effect of decoherence is the decay of the off diagonal terms in the partial trace of the total system + environment matrix traced over the environment. Decoherence over some characteristic time period will convert the density matrix from a pure state to a reduced mixture giving the appearance of collapse, this will pick out certain environmentally induced states.

There is a fundamental form of radiation decoherence that effects the ions' motional state transitions. If the relaxation of a harmonic oscillator associated with a mode of the radiation is coupled to the environment. This type of decoherence has been directly observed in cavity QED experiments [6] and points to the fact that the rate of relaxation of superposition states increases with the separation of states in Hilbert space and will thus prohibit the existence of some complicated "large" states except if done in short time scales. Decoherence for the COM mode

of ion motion is understood by considering that the dipole associated with the oscillating charged ion(s)is radioactively coupled to the thermal fields of the environment.

Decoherence in the motions of the ions effects the common mode value of the system, generally some type of heating will cause unexpected or uncontrolled alterations to the motional states of the ions. A couple of different sources can be used to account for these excess heats that can decohere the motional states on various time scales.

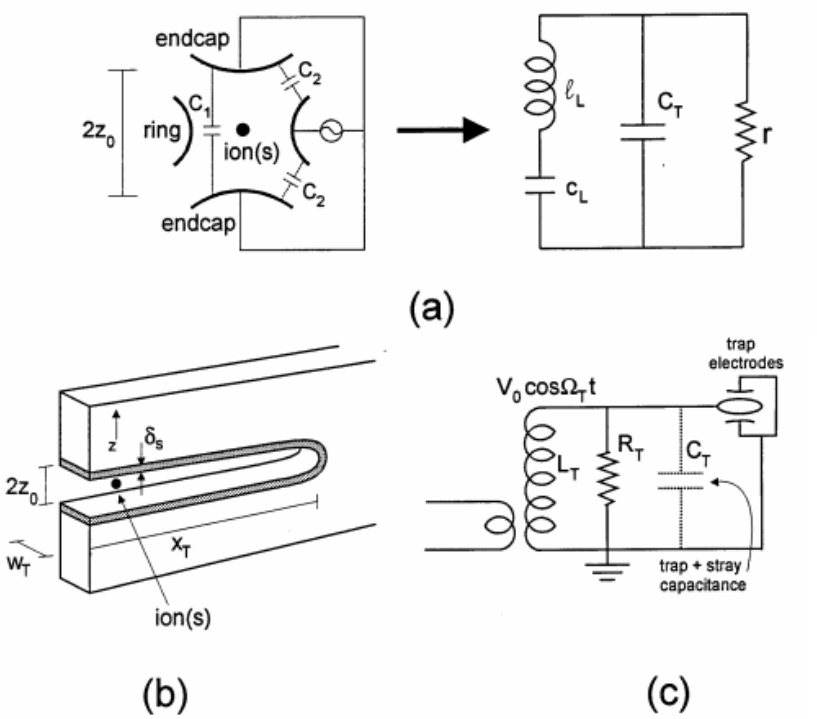


Fig. 6. Schematic diagrams of the lumped circuit equivalents for the trap electrodes and trapped ion(s). (a) The left part of the diagram shows, schematically, the electrodes for a Paul trap with hyperbolic electrodes and a collection of trapped ions. On the right is shown the corresponding lumped circuit equivalent; C_T represents the inter-electrode capacitance (the combined effects of the capacitances shown in the left part of the figure); r represents the resistive losses in the electrodes and connecting wires, l_L and c_L represent the equivalent inductance and capacitance for the COM mode of oscillation in the z (vertical) direction [103]. (b) A schematic diagram of the endcaps electrode for the trap of Jefferts et al. [211] which was used in the NIST experiments (the ring is not shown). Induced currents in the z (vertical) direction are assumed to follow a path indicated by shading; the resistance in this path represents r in part (a) of the figure. (c) The rf potential between ring and endcaps electrode (or between pairs of rods as indicated in Fig. 1) is typically generated with a resonant rf step-up transformer. The resistance in this transformer can, in principle, couple to the ion motion as discussed in the text.

The RF fields used to trap the ions to the radial axis can inadvertently cause excess excitation of the ions leading to micromotion of the ions and thus decoherence through heating in the following ways.

If there is a displacement (or misalignment) of one of the 4 electrodes towards one of the endcaps then the impedance between the ring and endcaps (represented in the equivalent circuit fig c) will created an induced ion current between the ring and endcaps. The electric field associated with this noisy oscillation can heat the ion.

Rf heating can also result from the coulomb interaction between the ions. In the trap the other two dimensions of modes (in X and Y) have the potential to be excited in a nonlinear way from the driving RF fields. Experiments at IBM and Munich [8] have shown chaotic motion even for two trapped ions resulting from inhomogenetities in the exciting field. The heating from this effect can be controlled and made negligible if the mode frequencies in the other dimensions are not factors of the RF frequency and if the other modes are cooled to their linear regimes.

Rf heating also can happen if the charge on the ions is too strong. These heating resonances are caused by higher order terms in the trap potential. One last way are RF fields can

contribute to heating is from stray electric fields leading to coherent motion of the ions at the RF frequency. Stray static electric fields can arise from variations (patches) on the electrode surfaces. These patches occur because of the finite crystalline size of the electrode material, or from buildup of stray charge that gets caught and clustered onto the charged electrodes. This stray field will end up shifting the equilibrium position of the ions in the trap. If the position of the ions is off the center axis then resultant motions in the x and y direction of the form shown below

$$x(t) \simeq (\Delta x + A_x \cos(\omega_x t + \phi_x))\left[1 + \frac{q_x}{2}\cos(\Omega_T t)\right],$$

$$\Delta x = \frac{qE_{sx}}{m\omega_x^2},$$

$$y(t) \simeq (\Delta y + A_y \cos(\omega_y t + \phi_y))\left[1 + \frac{q_y}{2}\cos(\Omega_T t)\right],$$

$$\Delta y = \frac{qE_{sy}}{m\omega_y^2},$$

will occur. This leads to changes in the sideband frequencies needed to excite the motional modes. Its possible to control these static fields by first heating the electrodes or by bombarding them with electrons [3].

To minimize interference from background gas an ion trap experiment will be performed in high vacuum. Even if the vacuum lowers the pressure to 10^{-8}Pa inelastic and elastic collisions with environmental gas can still be a problem. Inelastic collisions can change the internal states of the trapped ions while elastic collisions can alter the kinetic energy of the ions and lead to heating. For inelastic collisions there is the possibility of a background particle chemically reacting with the ion creating a completely different ion, or a neutral background atom giving up an electron and neutralizing the ion, both cases ruining the qubit. The probability of these types of chemical reactions with the such low pressure and with the ion laser cooled to have very little energy are extremely low and lead to trap lifetimes of several hours (effectively negligible). Inelastic collisions are more likely since such a collision does not to penetrate the angular momentum barrier but is still a relatively rare event leading to a ion lifetimes on the order of 10s of seconds.

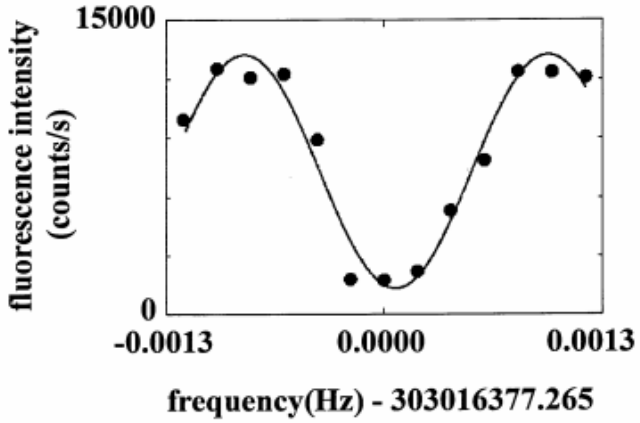


Fig. 7. Ramsey signal of the $2s\ ^2S_{1/2}(M_I = -1/2,\ M_J = +1/2) \rightarrow 2s\ ^2S_{1/2}(-3/2, +1/2)$ hyperfine transition in $^9Be^+$ at a magnetic field of 0.8194 T). This resonance was obtained using a free precession time of 550 s. The data are the result of one sweep (one measurement per frequency point). The fluctuations in the data were due to the instability of the reference oscillator used to take the spectrum. These kinds of measurements indicated that the coherence time for superpositions between the two hyperfine states (which could be used as qubit levels) was longer than 10 min [76].

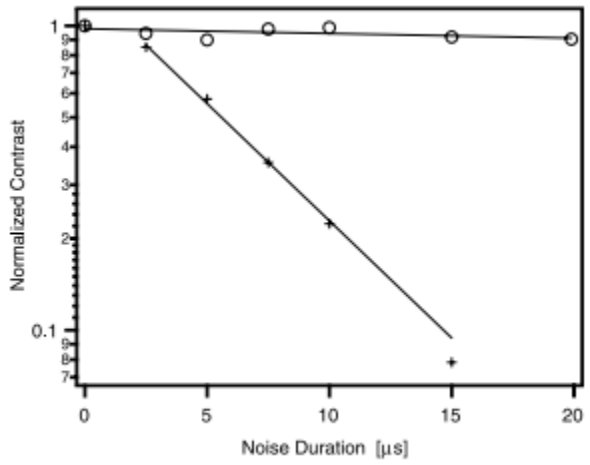
Another important aspect of these ion trap setups is the transition of internal energy ion states. The reason ion traps are a promising candidate in the first place is because of high resolution and long storage times of hours or more with minimal perturbations from electric and magnetic fields (which can be controlled). One experiment showing coherence times of a specific hyperfine transition in Be is particularly promising. The hyperfine transition is continuously driven and it is reasonably resistant to decoherence, with the main fluctuations being attributed to fluctuations in the driving oscillator.

This evidence for long coherence times seems promising however the results were obtained under special circumstances and the use of these ions for quantum logic poses some state transition decoherence problems. Radioactive decay of the excited ions can be a factor in internal ion decoherence with decay rate resulting in lifetimes of about 35s. The biggest contributions to decoherence in the internal energy states of the trapped ions come from fluctuating electric and magnetic fields.

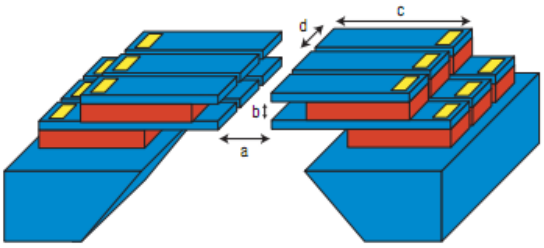
Now that sources of decoherence have been discussed there are two main approaches to combating the various problems. Error correcting codes and decoherence free subspaces (DFS), with the most effective set ups combing the methods of both techniques. Error correcting codes are a method of recovering intended qubits after some error (either phase or sign flip) has already happened. A DFS is a subspace of the system which is invariant under non-unitary transformations; it's a subspace where the system is decoupled from the environment.

For a specific type of decoherence that produce what are known as collective phase errors there are known DFS' invariant to these errors. Collective phase errors result from the majority of decoherence producing processes discussed above since the ion's are so close together that the decoherence affects their phases in the same way. The basis qubits $|0\rangle$ and $|1\rangle$ will transform in the following way: [4]$|0\rangle \rightarrow |0\rangle, |1\rangle \rightarrow e^{i\phi}|1\rangle$. This transformation is performed by the rotation operator $R_z(\phi) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\phi} \end{pmatrix}$.

Kielpinski[14] demonstrated a decoherence-free (to collective de-phasing) quantum memory on one qubit by encoding that qubit onto a DFS which was a pair of trapped Be ions. In the experiment the ambient storage time of a Be qubit is compared to the encoded DFS state which appears to increase store time by and order of magnitude. In addition to the test done with ambient storage a separate experiment is done with simulated environmental noise and produces similar results.

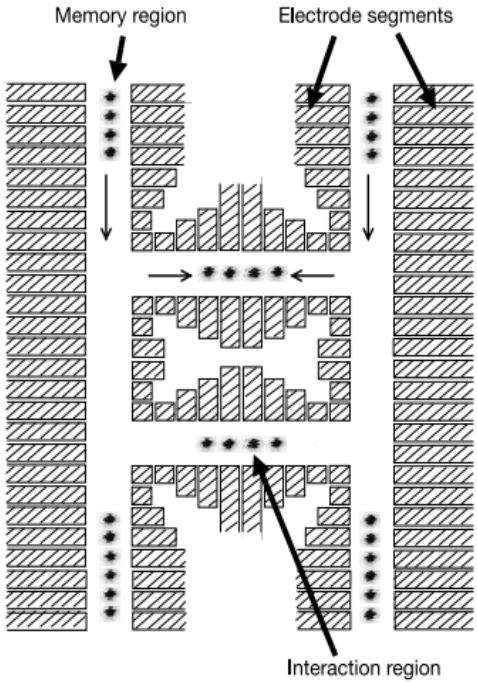


The DFS used is $|\varphi_- >= \frac{|\downarrow\uparrow> - i|\uparrow\downarrow>}{\sqrt{2}}$, $|\varphi_+ >= \frac{|\downarrow\uparrow> + i|\uparrow\downarrow>}{\sqrt{2}}$ which forms the basis for the encoded "logical qubit" and is invariant under collective de-phasing.



If eventually decoherence can be tamed, the goal is obviously to scale up the number of qubits so as to be able to perform meaningful quantum algorithms, such as Shor's algorithm on large primes. One huge step toward achieving that goal was performed at the University of Michigan [11]where researchers were able to report the operation of a micrometer-scale ion trap, fabricated on a monolithic chip using semiconductor micro-electromechanical systems (MEMS) technology. The natural host for a scalable system is an integrated ion-trap chip, which will hopefully lead to similar level of development as the silicon chip did for classical computers.

Most of the schemes to address individual ions with laser pulses are by far the most effective with only two ions to address between instead of many. This makes the idea of having thousands of ions in a single trap (or even more) unrealizable. This leads to the consideration of systems where the quantum logic is compartmentalized and performed in many "accumulators" with only 2 or 3 qubits. Pairs of ions could be held in different regions of the same trap structure or the data might even be transferred from one register to another by some sort of optical means. The issues involved with building quantum architecture are addressed in this paper by Kielpinski[12].



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