

```
> with(numtheory);
```

Warning, the protected name order has been redefined and unprotected

```
[GIgcd, bigomega, cfrac, cfracpol, cyclotomic, divisors, factorEQ, factorset, fermat, imagunit, index,
integral_basis, invcfrac, invphi, issqrfree, jacobi, kronecker,  $\lambda$ , legendre, mcombine, mersenne, migcdex,
minkowski, mipolys, mlog, mobius, mroot, msqrt, nearestp, nthconver, nthdenom, nthnumer, nthpow,
order, pdexpand,  $\phi$ ,  $\pi$ , pprimroot, primroot, quadres, rootsunity, safeprime,  $\sigma$ , sq2factor, sum2sqr,  $\tau$ , thue
]
```

```
> seq([j, phi(j)], j=1..12);
```

```
[1, 1], [2, 1], [3, 2], [4, 2], [5, 4], [6, 2], [7, 6], [8, 4], [9, 6], [10, 4], [11, 10], [12, 4]
```

```
> Z:=proc(s) local olddigits; option remember; olddigits:=Digits;
Digits:=400; evalf(Zeta(s));
Digits:=olddigits; %%; end;
```

```
Z := proc(s)
local olddigits;
option remember;
olddigits := Digits;
Digits := 400;
evalf(Zeta(s));
Digits := olddigits;
`%%`;
end proc;
```

```
> dnphi:=proc(n) local k, olddigits;
option remember;
olddigits:=Digits; Digits:=400;
add(binomial(n,k)*(-1)^k*Z(k-1)/Z(k), k=3..n);
Digits:=olddigits; %%;
end;
```

```
>
```

```
dnphi := proc(n)
local k, olddigits;
option remember;
olddigits := Digits;
Digits := 400;
add((binomial(n, k)*(-1)^k*Z(k - 1))/(Z(k)), k = 3 .. n);
Digits := olddigits;
`%%`;
end proc;
```

```
> resid:=proc(s0) option remember;
series(Zeta(s-1)/Zeta(s)*GAMMA(n+1)/(GAMMA(n-s+1)/GAMMA(-s)), s=s0, 3);
coeff(%, s-s0, -1); simplify(%); end;
```

```
resid := proc(s0)
option remember;
series((Zeta(s - 1)*GAMMA(n + 1)*GAMMA(-s))/(Zeta(s)*GAMMA(n - s + 1)), s = s0, 3);
coeff(%, s - s0, -1);
simplify(%);
end proc;
```

```
> resid(2),resid(0),resid(-2);
```

$$-\frac{3(-1+n)n(-3\pi^2+4\gamma\pi^2-12\zeta(1,2)+2\Psi(-1+n)\pi^2)}{2\pi^4}, \frac{-1}{6}, \frac{1}{120\zeta(1,-2)(n+1)(n+2)}$$

```
> asympt(%,n):convert(%,polynom):expand(%,n):collect(%,n);
```

$$\left(-\frac{3\ln(n)}{\pi^2}+\frac{9}{2\pi^2}-\frac{6\gamma}{\pi^2}+\frac{18\zeta(1,2)}{\pi^4}\right)n^2+\left(\frac{6\gamma}{\pi^2}-\frac{18\zeta(1,2)}{\pi^4}+\frac{3\ln(n)}{\pi^2}\right)n-\frac{5}{4\pi^2}-\frac{1}{4\pi^2n}-\frac{1}{40\pi^2n^2}+\frac{1}{40\pi^2n^3}$$

```
> evalf(%);
```

$$(-0.3039635508\ln(n)-0.0682070873)n^2+(0.5241524135+0.3039635508\ln(n))n-0.1266514795-\frac{0.02533029590}{n}-\frac{0.002533029590}{n^2}+\frac{0.002533029590}{n^3}$$

```
> testphi:=proc(N) dnphi(N)-subs(n=N,resid(2)+resid(0)+resid(-2)+resid(-4));
evalf(%); end;
```

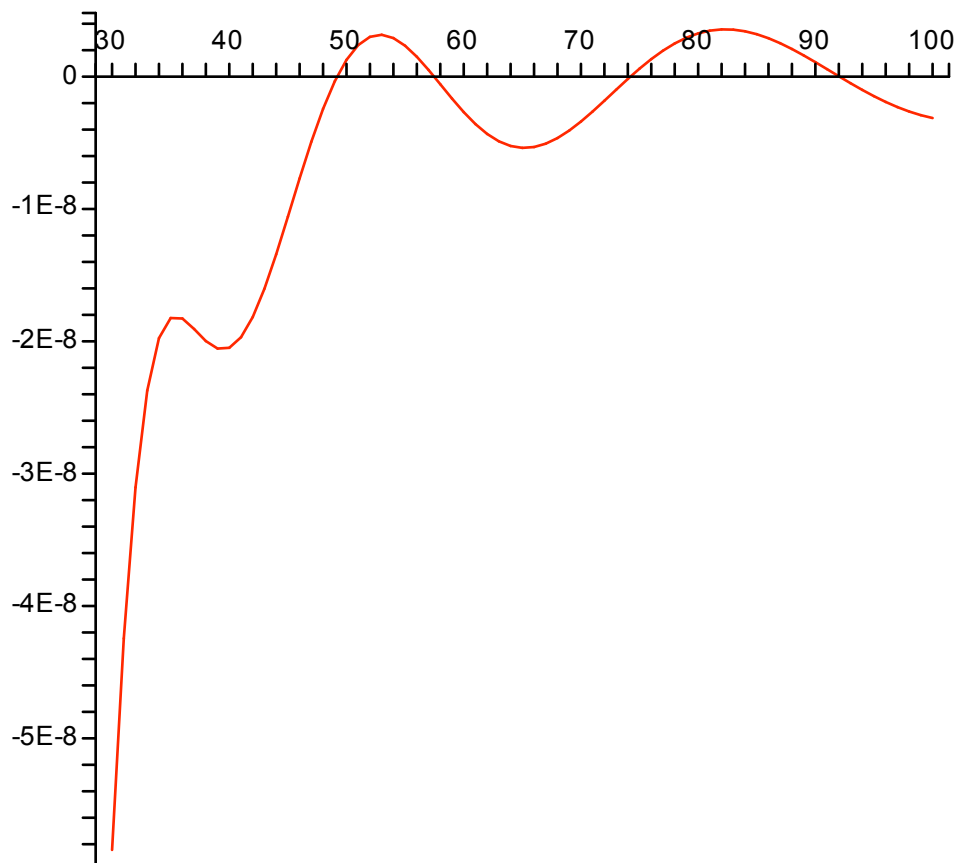
```
testphi := proc(N) dnphi(N) - subs(n = N, resid(2) + resid(0) + resid(-2) + resid(-4)); evalf(%); end proc;
```

```
> Digits:=40; for j from 3 to 20 do j,testphi(j) od;
```

Digits := 40

```
3, -0.004222824816256311806720977281464253310990
4, -0.001141203187201244253408207387117651210444
5, -0.0003832426533358249656043504917521345409445
6, -0.0001751523500647257948880619047913344279873
7, -0.0000967032838394763772669519174404138670384
8, -0.0000556770071498373743824624197128766294576
9, -0.0000317375823453140874113377394419404635043
10, -0.0000181516968113052471040442548443440636006
11, -0.00001084291440872160276136939708176621571136
12, -0.00000699443550367406058377047250359491728776
13, -0.00000486839077693432330008838755766780861182
14, -0.00000354375456414018719639095237000979733364
15, -0.00000260419668787410501091348674921552350413
16, -0.00000189032849750529809391861461866305376222
17, -0.00000134746735543879145168509114813177857423
18, -9.5043926514249113654592005739016758326 10-7
19, -6.7550244271212808994297136896534977872 10-7
20, -4.9524279959891561755536486123158999842 10-7
```

```
> plot([seq([j,testphi(j)],j=30..100)]);
```



```
> resid(2)+resid(0)+resid(-2)+resid(-4);
```

$$-\frac{3(-1+n)n(-3\pi^2+4\gamma\pi^2-12\zeta(1,2)+2\Psi(-1+n)\pi^2)}{2\pi^4}-\frac{1}{6}+\frac{1}{120\zeta(1,-2)(n+1)(n+2)}-\frac{1}{42\zeta(1,-4)(n+1)(n+2)(n+3)(n+4)}$$

```
> diff(Zeta(s),s); # Maple's notation
```

$\zeta(1,s)$

```
>
```