

```
> Digits:=400;
> Z:=proc(s) option remember; Digits:=400; evalf(Zeta(s)) end;
    Z:=proc(s) option remember; Digits:=400; evalf( $\zeta(s)$ ) end proc
```

(1)

```
> d:=proc(n) local k; option remember; Digits:=400; add((-1)^k*
    binomial(n,k)/Z(k),k=2..n); end;
d:=proc(n)
```

(2)

```
    option remember;
```

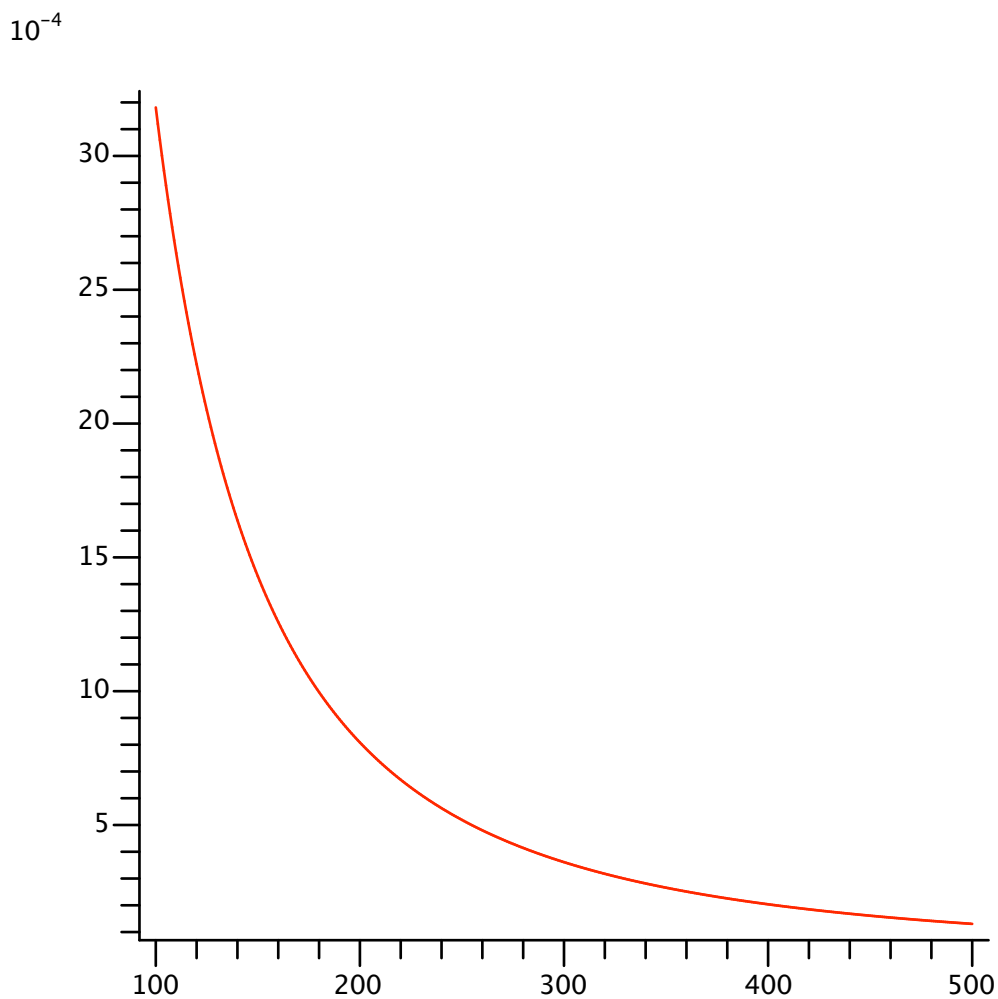
```
    local k;
```

```
    Digits:=400;
```

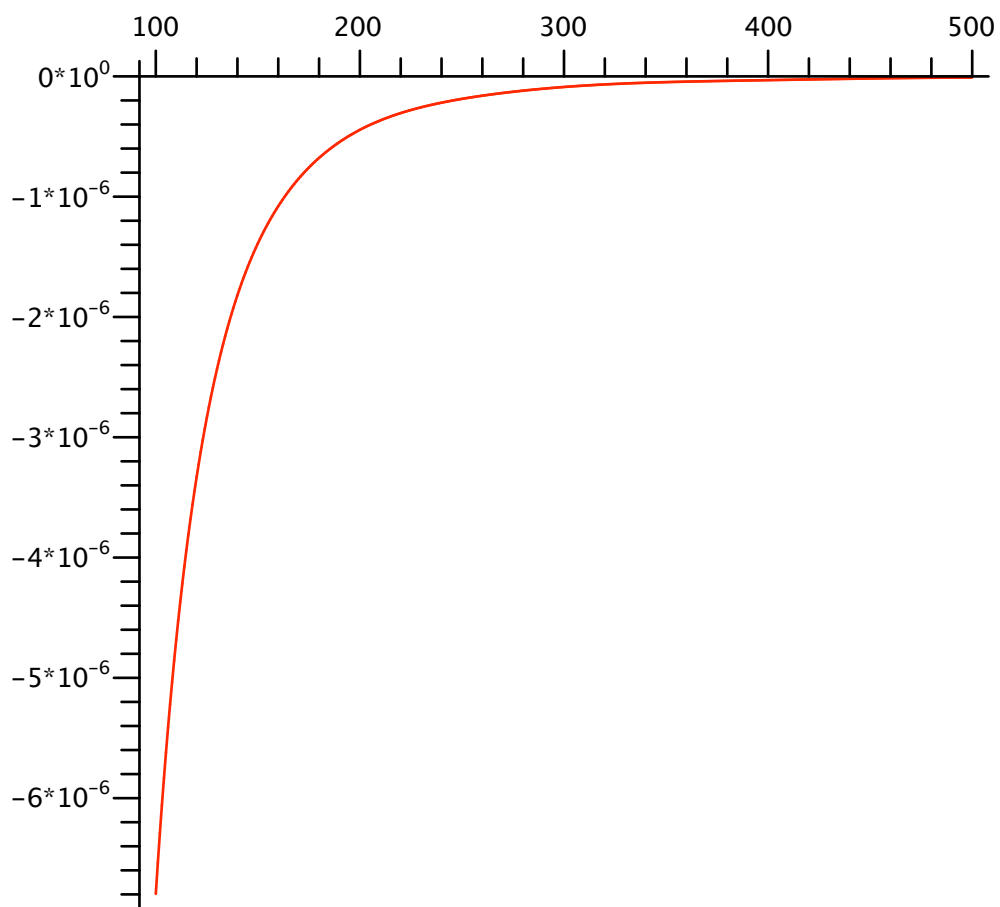
```
    add(( (-1)^k*binomial(n,k))/Z(k),k=2..n)
```

```
end proc
```

```
> plot([seq([n,2-d(n)],n=100..500)]);
```

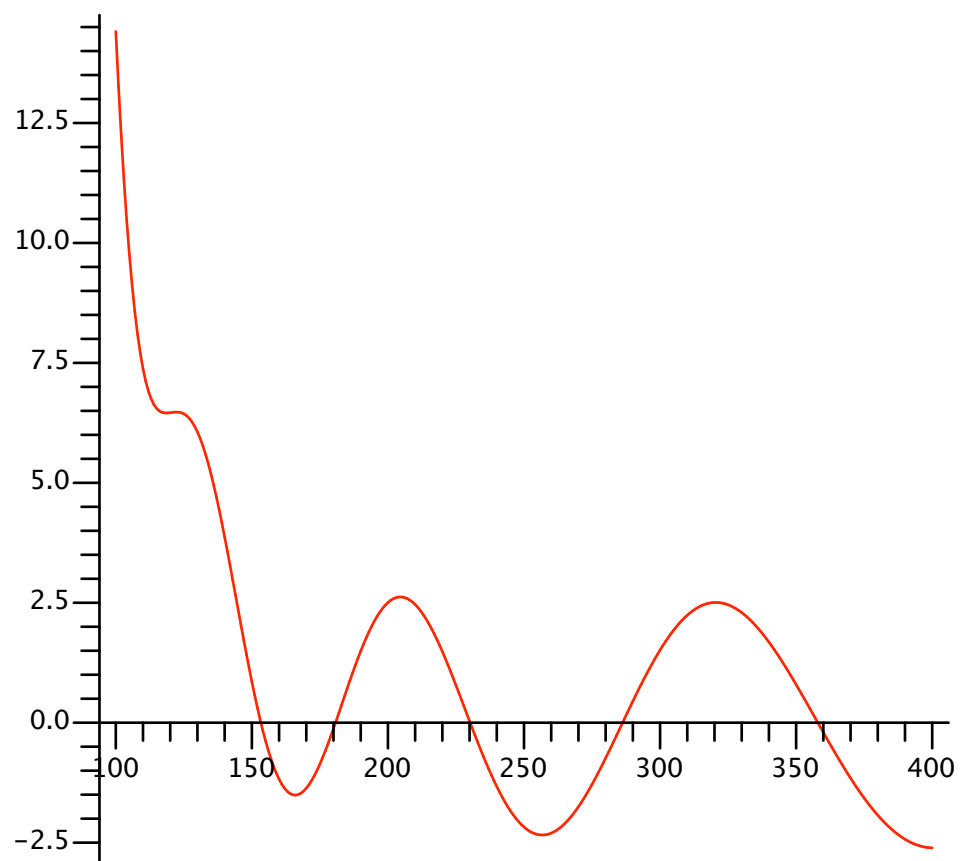


```
> plot([seq([n,2-d(n)+1/(Zeta(1, -2)*(n+1)*(n+2))],n=100..500)]);
```



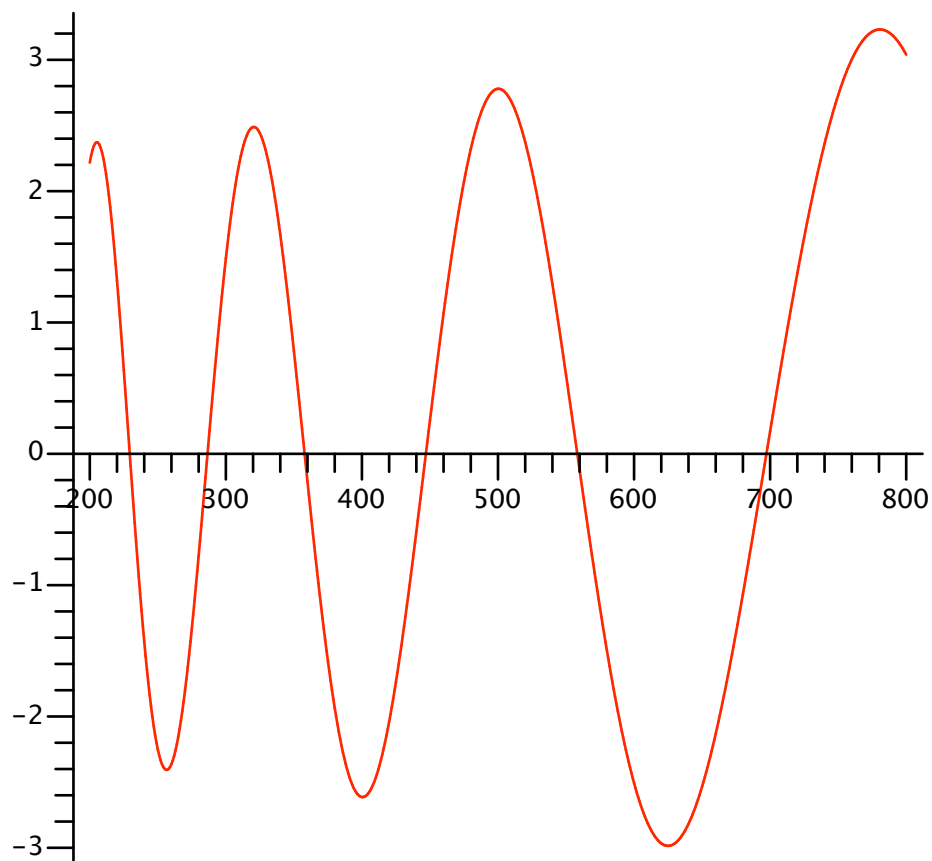
```
>
> plot([seq([n, 2-d(n)+1/(Zeta(1, -2)*(n+1)*(n+2))+6/(Zeta(1, -4)*
(n+1)*(n+2)*(n+3)*(n+4))], n=100..400)]);
```

$10^{-9}$



```
> plot([seq([n, 2-d(n)+1/(Zeta(1, -2)*(n+1)*(n+2))+6/(Zeta(1, -4)*  
(n+1)*(n+2)*(n+3)*(n+4))+120/(Zeta(1, -6)*(n+1)*(n+2)*(n+3)*(n+4)*  
(n+5)*(n+6))], n=200..800)]);
```

$10^{-9}$



```
> resid:=proc(s0) local kern, s; 1/Zeta(s)*GAMMA(N+1)/(GAMMA(N-s+1)
/GAMMA(-s)); series(%,s=s0,4); coeff(%,s-s0,-1); simplify(%); end;
resid := proc(s0) (3)
```

```
local kern, s;
( $\Gamma(N+1) * \Gamma(-s)$ )/ ( $\zeta(s) * \Gamma(N-s+1)$ );
series(%, s=s0, 4);
coeff(%, s-s0, -1);
simplify(%)
```

```
end proc
```

```
>
```

```
> resid(0);
```

2

(4)

```
> resid(-2);lprint(%);
```

$$\frac{1}{\zeta(1, -2) (N+1) (N+2)}$$

```
1/(Zeta(1, -2)*(N+1)*(N+2))
```

```
> resid(-4);
```

```

> lprint(%);
6/(Zeta(1, -4)*(N+1)*(N+2)*(N+3)*(N+4))
> resid(-6):subs(N=n,%);lprint(%);
120

$$\frac{120}{\zeta(1, -6) (n+1) (n+2) (n+3) (n+4) (n+5) (n+6)}$$

120/(Zeta(1, -6)*(n+1)*(n+2)*(n+3)*(n+4)*(n+5)*(n+6))
> ?ZetaZeroes
> zz0:=proc() local olddigits, res; option remember;
olddigits:=Digits; Digits:=20; res:=fsolve(abs(Zeta(1/2+I*t)),t=
14..15); Digits:=olddigits; res; end;
zz0:=proc() (5)
option remember;
local olddigits, res;
olddigits := Digits;
Digits := 20;
res := fsolve(abs(Zeta(1/2 + I*t)), t = 14..15);
Digits := olddigits;
res
end proc
> zz0();
14.134725141734693790 (6)
> f:=proc(n) local olddigits, res; olddigits:=Digits; Digits:=20;
1/Zeta(1, 1/2.+I*zz0())*GAMMA(n+1.)/(GAMMA(n-1/2-I*zz0())/GAMMA(
-1/2-I*zz0()));res:=evalc(%); evalf(%);
Digits:=olddigits; res; end;
f:=proc(n) (7)
local olddigits, res;
olddigits := Digits;
Digits := 20;
(GAMMA(n + 1.)*GAMMA(-1/2 + I*zz0()))/(Zeta(1, 1/2.
+ I*zz0())*GAMMA(n - 1/2 - I*zz0()));
res := evalc('%');
evalf('%');
Digits := olddigits;
res
end proc
> for j from 240 to 400 by 10 do j,f(j) od;
240, 1.5745358080918481227 10-7 - 2.3979470065966618001 10-7 I
250, 2.7494826161098368634 10-7 - 1.1984694588702337038 10-7 I
260, 3.0996017590564946385 10-7 + 4.5238164972450041018 10-8 I
270, 2.5412973189486136337 10-7 + 2.0545599770500087671 10-7 I
280, 1.2488527041691127597 10-7 + 3.1685249487201663704 10-7 I

```

$$\begin{aligned}
290, & -4.3115009435023943122 \cdot 10^{-8} + 3.5195360718765399742 \cdot 10^{-7} I \\
300, & -2.0896141215727940497 \cdot 10^{-7} + 3.0390739403188504575 \cdot 10^{-7} I \\
310, & -3.3564598746270378080 \cdot 10^{-7} + 1.8501862553324535879 \cdot 10^{-7} I \\
320, & -3.9734439974661284412 \cdot 10^{-7} + 2.1439164409524316388 \cdot 10^{-8} I \\
330, & -3.8302012467727615837 \cdot 10^{-7} - 1.5392136126908048498 \cdot 10^{-7} I \\
340, & -2.9653103063544132964 \cdot 10^{-7} - 3.0844258010211749176 \cdot 10^{-7} I \\
350, & -1.5403912612074243861 \cdot 10^{-7} - 4.1550377522592198160 \cdot 10^{-7} I \\
360, & 2.0208825914718944186 \cdot 10^{-8} - 4.5816493778462533707 \cdot 10^{-7} I \\
370, & 1.9867128810793138548 \cdot 10^{-7} - 4.3066092403570476859 \cdot 10^{-7} I \\
380, & 3.5500006314087227193 \cdot 10^{-7} - 3.3794771219409683295 \cdot 10^{-7} I \\
390, & 4.6762453788554482888 \cdot 10^{-7} - 1.9377827718434541889 \cdot 10^{-7} I \\
400, & 5.2211295114674863711 \cdot 10^{-7} - 1.7880934857494418087 \cdot 10^{-8} I
\end{aligned}$$

(8)

