Multifractal Measures

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Abstract

The study of multifractal measures.

1 Intro

to be written

2 The Sierpinski Gasket, The Pascal Triangle

It is known that if one considers the divisibility of the entries appearing in Pascal's Triangle (the tree of binomial coefficients), that one gets shapes that are pre-cursors to Sierpinski's Gasket [?]. Consider the p-adic expansion of an integer n:

$$n=\sum_{m=0}^{\infty}a_m p^m$$

with $a_m \in \{0,1,...,p-1\}$. Then the number of entries in the n'th row of the Pascal Triangle not divisible by p is given by

$$h_p(n)=\prod_{m=0}^{\infty}(1+a_m)$$

The figure 1 shows a graph of this value as a function of n.

Let us define the sum of the digits in the expansion as

$$\sigma_p(n)=\sum_{m=0}^{\infty}a_m$$

Then, for $p=2$, we can write

$$h_2(n)=2^{\sigma_2(n)}$$

Let us now consider all values of $n<2^M$ for some positive integer M.

Figure 1: Divisibility in Pascal's Triangle

Blah blah blah. Show $h_2(n)$

2.1 Fibonacci Numbers and Binomial Coefficients

Here's another relation. Fibonacci Numbers are a kind of integer-version of the Farey Fractions; the one goes with the other. Fibonacci numbers occur in sums of binomials:

$$F_{n+1} = \sum_{m=0}^{\lfloor n/2 \rfloor} \left(\begin{array}{c} n-m \\ m \end{array} \right)$$

where we defined the Fibonacci sequence as $F_{n+1} = F_n + F_{n-1}$ and $F_0 = 0$ and $F_1 = 1$.