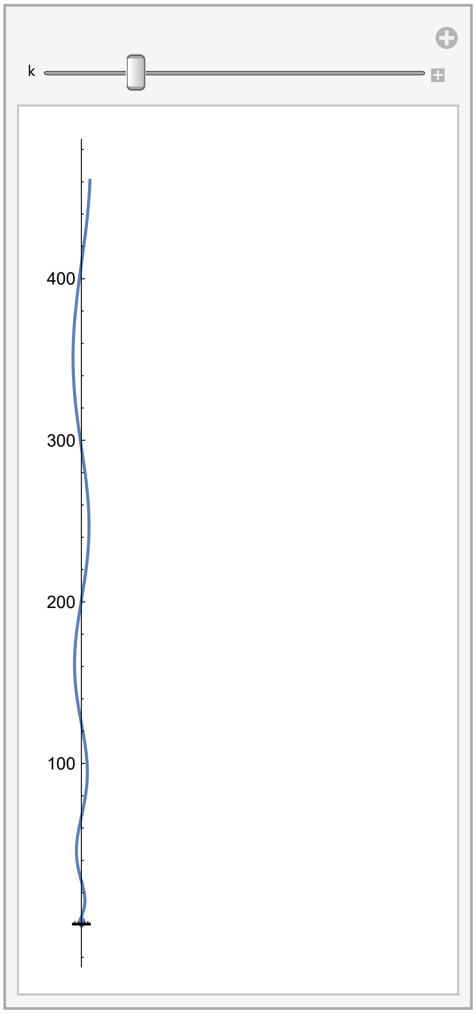
In[1]:=
```
(* Elastic
 Pendulum: instead of a traditional pendulum with a mass attached to a rigid bar,
consider a pendulum of a mass attached to a spring. We solve this
 via Lagrangian Equations and construct parametric equations,
just like a rigid pendulum (see DoublePendulum) *)
m = 1;
g = 9.8;
l0 = 1.5;
eq1[k_] := m * l''[t] == m (l0 + l[t]) * ((th'[t])^2) + m * g * Cos[th[t] - k * l[t]];
eq2[k_] := m * (l0 + l[t]) th''[t] + 2 m * l'[t] * th'[t] == -m * g * Sin[th[t]];
```

In[6]:=
```
epend[k_] := NDSolve[{eq1[k], eq2[k], l[0] == 1, l'[0] == 0, th[0] == Pi / 4, th'[0] == 0},
  {th[t], l[t]}, {t, 0, 100}]
```

In[7]:=
```
xpos[t_, k_] := l[t] * Sin[th[t]] /. epend[k][[1]];
ypos[t_, k_] := l[t] * Cos[th[t]] /. epend[k][[1]];
```

In[9]:=
```
(* This first model is not a spring pendulum,
but rather a general elastic non-rigid pendulum. Variable k is the 'spring' constant*)
```

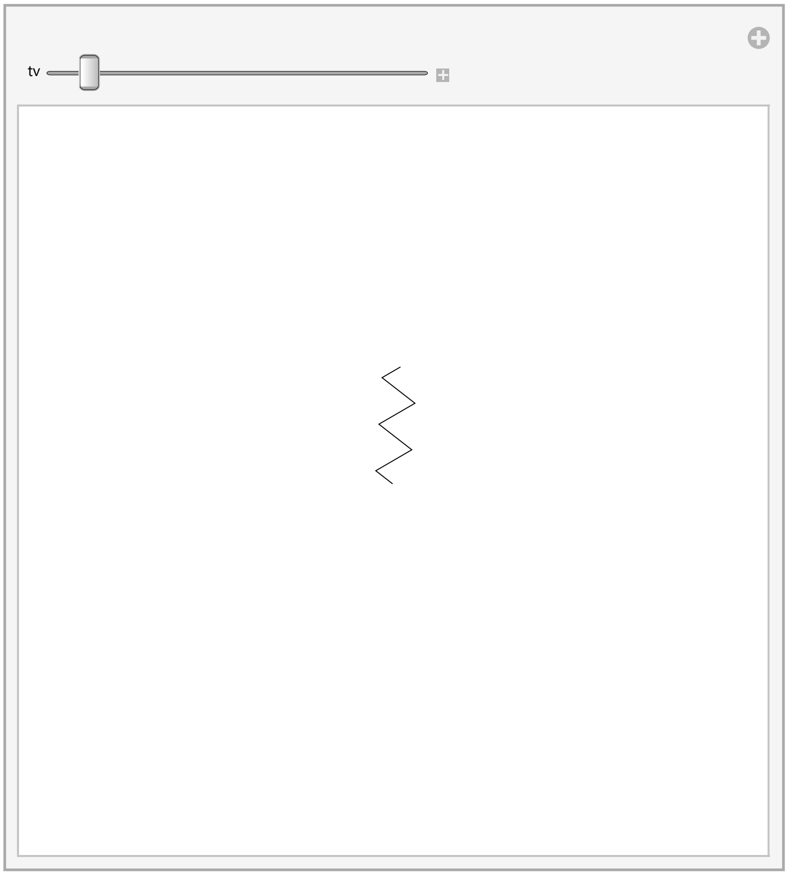
In[10]:=
```
Manipulate[ParametricPlot[
  Evaluate[{xpos[t, k] /. k → kv, ypos[t, k] /. k → kv}], {t, 0, 100}], {k, 0, 5}]
```

Out[10]=


In[11]:=
```
(* Now model the pendulum using an actual spring *)
spring[r_ : {1, 0}, l_ : {0, 0}, n_ : 5, w_ : 0.1] :=
  Line@Transpose[{r - l, -Cross[r - l]}.{(# - 1) / (2 n), Re[I^#] w / Norm[r - l]} + l] &@
   Range[2 n + 1];
```

In[12]:=
```
Manipulate[Graphics[Evaluate[spring[{xpos[t, 3], ypos[t, 3]}]] /. t → tv,
  PlotRange → {{-2, 2}, {-2, 2}}], {tv, 0, 10}]
```

Out[12]=



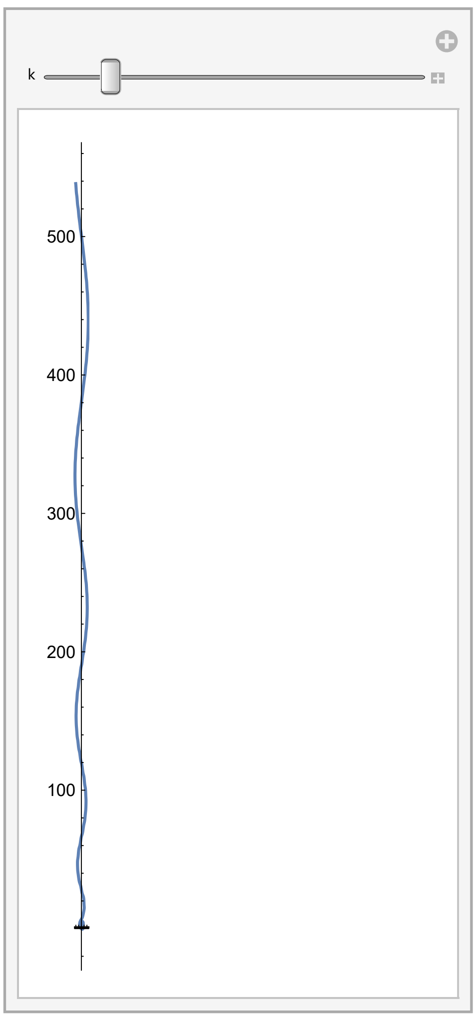
In[13]:=
```
(* Add damping to the spring model *)
eq2damp[k_] := m * (l0 + l[t]) th''[t] + 2 m * 0.5 * l'[t] * th'[t] == -m * g * Sin[th[t]];

ependdamp[k_] := NDSolve[{eq1[k], eq2damp[k], l[0] == 1,
   l'[0] == 0, th[0] == Pi / 4, th'[0] == 0}, {th[t], l[t]}, {t, 0, 100}]

xdpos[t_, k_] := l[t] * Sin[th[t]] /. epend[k][[1]];
ydpos[t_, k_] := l[t] * Cos[th[t]] /. epend[k][[1]];

Manipulate[ParametricPlot[
  Evaluate[{xdpos[t, k] /. k → kv, ydpos[t, k] /. k → kv}], {t, 0, 100}], {k, 0, 5}]
```

Out[17]=



In[18]:=

In[19]:=
```
(* Model a Coupled Pendulum System: two pendulums,
with rigid bars connecting masses to fixed points in parallel,
in which their masses are connected to each other via a spring *)

th10 = Pi / 4;
th20 = Pi / 8;
L = -3.5;
k = 3;

w1 := Sqrt[g / L];
w2[k_] := Sqrt[(g / L) + 2 * (k / m)];
```

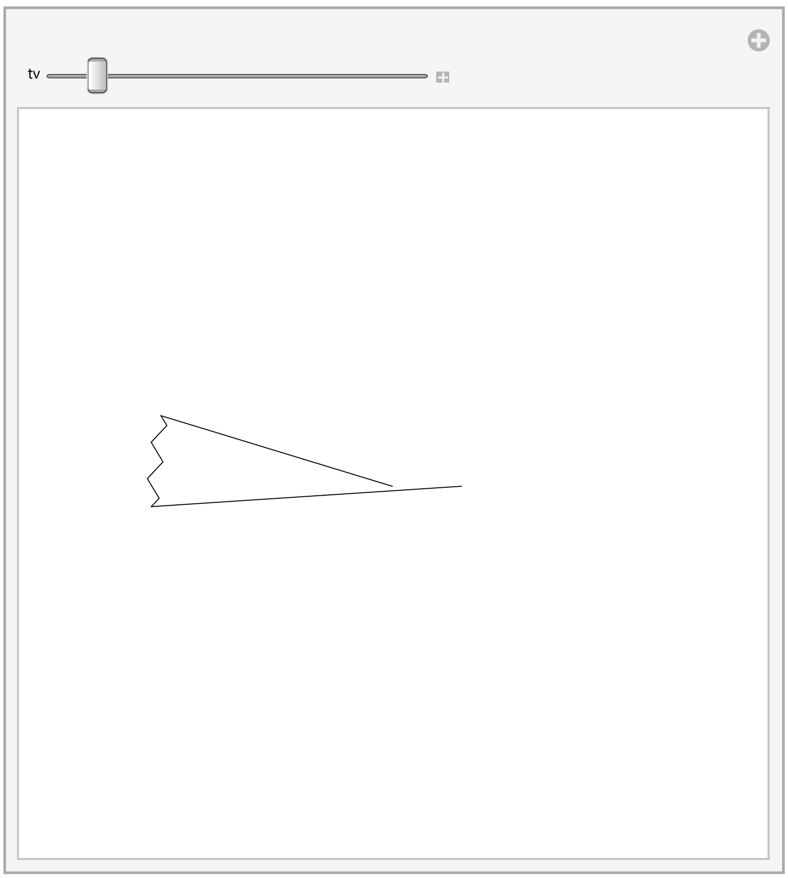
In[25]:=
```
A = th10 + th20;
B = th10 - th20;

th1[t_] :=  0.5 * A * Cos[w1 * t] + 0.5 * B * Sin[w2[k] * t];
th2[t_] := 0.5 * A * Cos[w1 * t] - 0.5 * B * Sin[w2[k] * t];

xc1pos[t_] := L * Sin[th1[t]];
yc1pos[t_] := L * Cos[th1[t]];
xc2pos[t_] := L * Sin[th2[t]];
yc2pos[t_] := L * Cos[th2[t]];

Manipulate[
 Graphics[{Evaluate[spring[{xc2pos[t], yc2pos[t]}, {xc1pos[t], yc1pos[t]}]] /. t → tv,
   Line[{{1, 0}, Evaluate[{xc2pos[t] /. t → tv, yc2pos[t] /. t → tv}]}],
   Line[{{0, 0}, Evaluate[{xc1pos[t] /. t → tv, yc1pos[t] /. t → tv}]}]},
  PlotRange → {{-5, 5}, {-5, 5}}], {tv, 0.1, 10}]
```

Out[33]=



In[34]:=

In[35]:=