In[68]:=
```
(* Convergence of Fourier Series to a function depending on number of terms*)
```

In[69]:=
```
NScProd[f_, g_] := (1 / (2 * Pi)) * NIntegrate[f Conjugate[g], {x, -Pi, Pi}];
NFcoef[f_, n_] := Table[NScProd[f, Exp[ⅈ k x]], {k, -n, n}];
cf[f_, n_] = Chop[NFcoef[f, n]];
fct[c_, k_] := (k - 1 - (Length[c] - 1) / 2);
Fseries[c_] := Sum[c⟦k⟧ Exp[ⅈ fct[c, k] x], {k, 1, Length[c]}];


FConvPlot[f_, n_] := Module[{c, F, plt},
  c = NFcoef[f, n];
  F = Fseries[c];
  plt = Plot[{F, f}, {x, -π, π}, Frame → True, GridLines → Automatic,
    PlotStyle → {Red, Blue}, PlotLabel → "Convergence of Fourier Series"];


  Return[plt]
 ]
Off[Table::iterb]
```

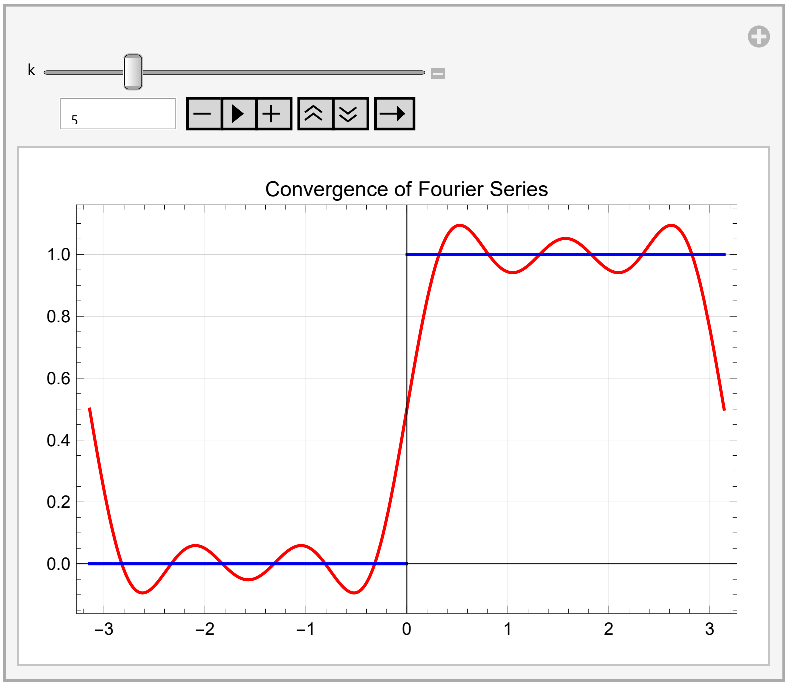
In[76]:=
```
(* Functions to evaluate *)
f5[x_] := 1 + Sin[x] + 10 Cos[x] + 100 Sin[2 x] + 1000 Cos[3 x];
p1[x_] := x - Floor[(x + Pi) / (2 * Pi)] (2 Pi);
f1[x_] := If[p1[x] < 0, 0, 1];
f3[x_] := Abs[Cos[x / 2]];
f4[x_] := 2 Sin[Cos[x]] + 3 Cos[Sin[x]];
```

In[81]:=
```
(* 1: Heaviside Function *)
```

In[82]:=
```
Manipulate[FConvPlot[f1[x], k], {k, 1, 20, 2}]

Off[NIntegrate::slwcon, NIntegrate::ncvb];
```

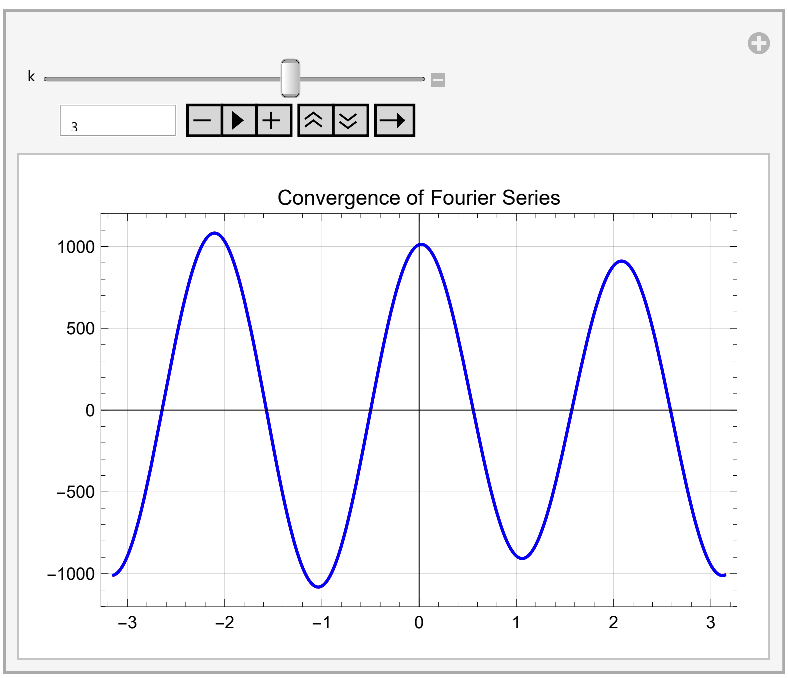
Out[82]=



In[84]:=
```
(* Notice Gibbs' Phenomenon at large k*)
```

In[85]:=
```
(* 2: Simple Periodic Function *)
Manipulate[FConvPlot[f5[x], k], {k, 1, 4, 1}]
```

Out[85]=

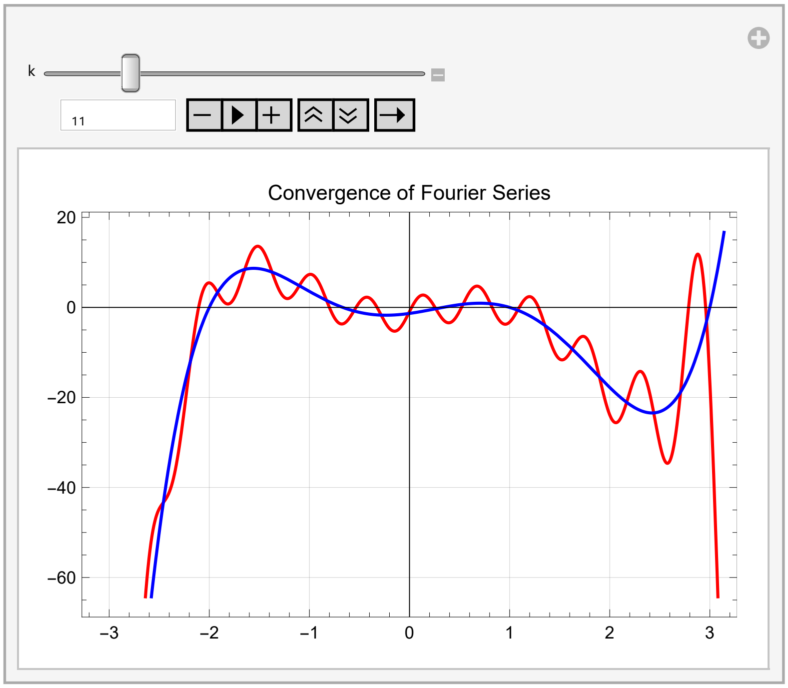


In[86]:=
```
(* Converges perfectly even at small k, since our function is a smooth periodic*)
```

In[87]:=
```
(* 3: Simple Polynomial *)
```

In[88]:=
```
f9[x_] = (x - 1) (x - 1 / 3) (x + 2 / 3) (x + 2) (x - 3);
Manipulate[FConvPlot[f9[x], k], {k, 1, 50, 2}]
```

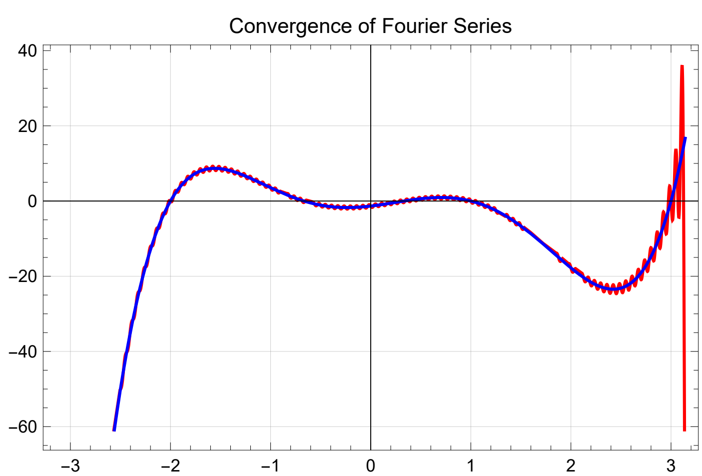
Out[89]=



In[90]:=
```
(* Will not converge for a non-periodic function. With a large number of coefficients,
models may approach convergence for a small interval,
but not for the entire function, as seen below. *)
```

In[91]:=
```
FConvPlot[f9[x], 100]
```

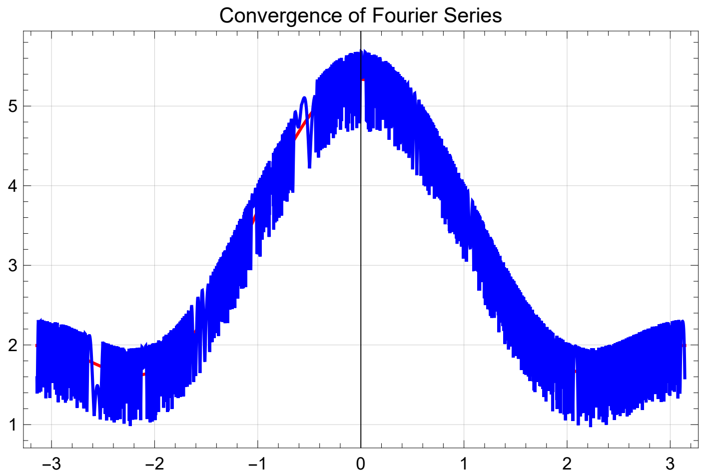
Out[91]=



In[92]:=
```
(* 4: Complex Construction of Periodic Functions*)
f6[x_] := f4[x] + f3[f5[x]];
```

In[93]:=
```
FConvPlot[f6[x], 3]
```

Out[93]=

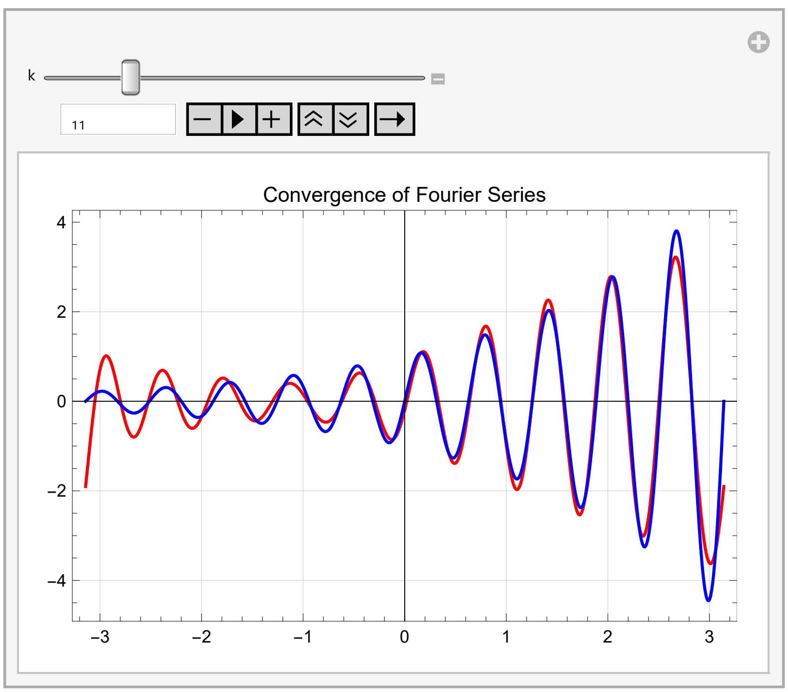


In[94]:=
```
(* Note this function is periodic,
but we are only displaying one period. Fourier series will eventually converge to f6,
but with a great number of terms and taking a long time to compute*)
```

In[95]:=
```
(* 5: Non-Periodic Function*)
```

In[96]:=
```
f7[x_] := Exp[0.5 x] * Sin[10 x];
Manipulate[FConvPlot[f7[x], k], {k, 1, 50, 2}]
```

Out[97]=

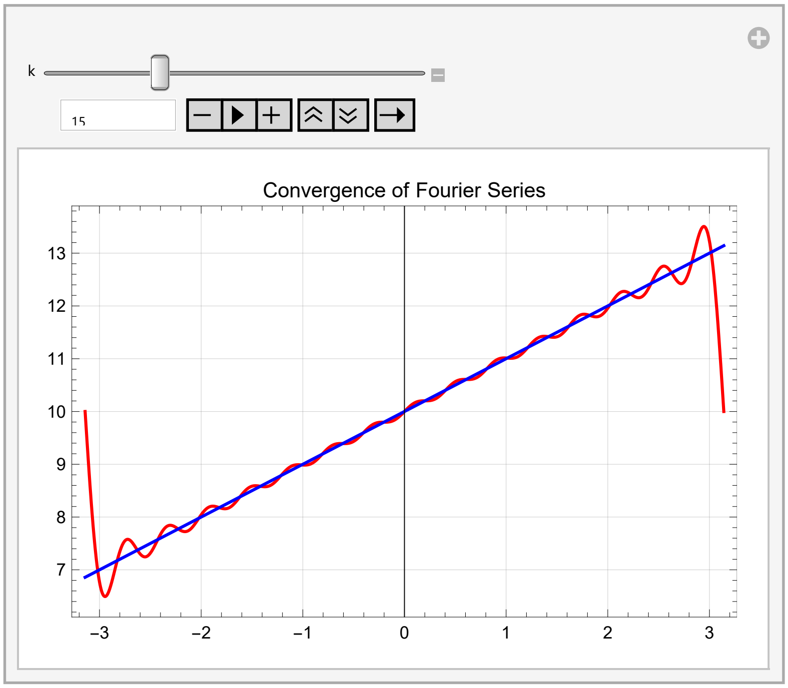


In[98]:=
```
(* Again, will not converge for a non-periodic function. As we can see here,
we can come much closer to convergence with a large number of terms,
but we will never converge over the entire function. *)
```

In[99]:=
```
(* 6: Linear Function *)
```

In[100]:=
```
Manipulate[FConvPlot[x + 10, k], {k, 1, 50, 2}]
```

Out[100]=



In[101]:=
```
(* Even an incredibly simple function like y= x+10,
we cannot create complete convergence as it is not periodic.*)
```