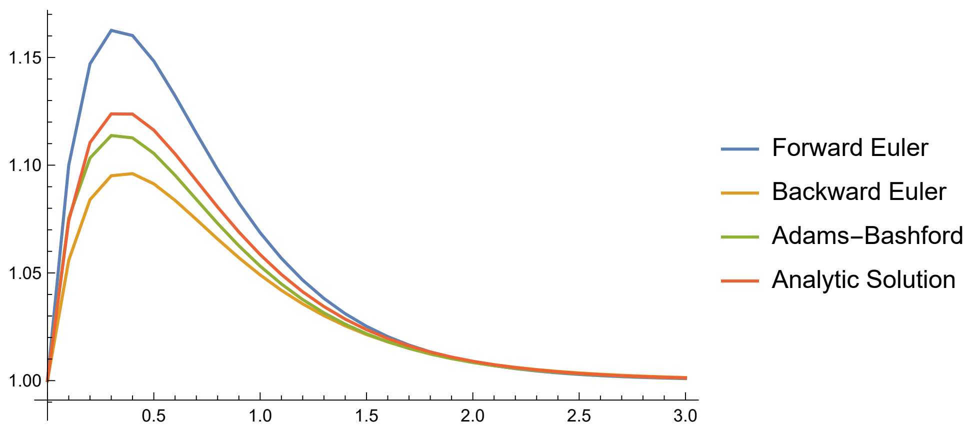
```
In[1]:= (* Backward Euler *)

In[2]:= bE[f_, dt_, T_, init_] := Module[{ret = {{0.0, init}}, y = init, t = 0.0, numsteps = T / dt},
          Do[y = ynew /. Solve[ynew == y + dt * f[ynew, t + dt], ynew][[1]];
           t = t + dt;
           ret = Append[ret, {t, y}], {i, 1, numsteps}];
          ret];

In[3]:= (* Forward Euler *)

        fE[f_, dt_, T_, init_] := Module[{ret = {{0.0, init}}, y = init, t = 0.0, numsteps = T/dt},
          Do[y = y + dt * f[y, t]; t = t + dt;
           ret = Append[ret, {t, y}], {i, 1, numsteps}];
          ret];

In[4]:= (* Adams-Bashforth 2-step *)

In[5]:= aB[f_, dt_, T_, init_] :=
         Module[{ret = {{0.0, init}}, y1 = init, y2, y, t = 0.0, numsteps = T / dt},
          Do[y2 = y1 + (3 / 2) * dt * f[y1, t] - (dt / 2) * f[y, t - dt];
           y = y1;
           y1 = y2;
           t = t + dt;
           ret = Append[ret, {t, y2}], {i, 1, numsteps}];
          ret];

In[6]:= (* Ex 1: f' = 2-2f+ Exp(-4t), f(0) = 1.
              Analytical Sol: f(t) = 1+ 1/2 Exp(-2t) -1/2 Exp(-4t)
                Each Numerical Solution has a step size h=
          0.1 and is calculated over domain (0,3).*)
```

In[7]:=

```
f[y_, t_] = 2 - 2 y + Exp[-4 t];
retaB = aB[f, 0.1, 3, 1];
retfE = fE[f, 0.1, 3, 1];
retbE = bE[f, 0.1, 3, 1];
sol =
  Transpose[{Range[0, 3, .1], Table[1 + 1 / 2 Exp[-2 t] - 1 / 2 Exp[-4 t], {t, 0, 3, .1}]}];
soly = Transpose[{ConstantArray[0, Length[sol]],
    Table[1 + 1 / 2 Exp[-2 t] - 1 / 2 Exp[-4 t], {t, 0, 3, .1}]}];
ListLinePlot[{retfE, retbE, retaB, sol},
 PlotLegends → {"Forward Euler", "Backward Euler", "Adams-Bashford", "Analytic Solution"}]
```

Out[13]=



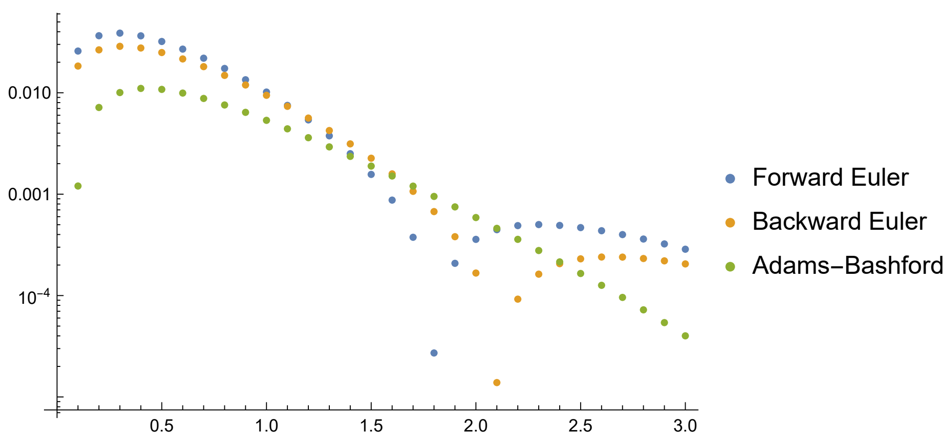
In[14]:=

```
(* Log Plot of Error for each Method *)
```

In[15]:=

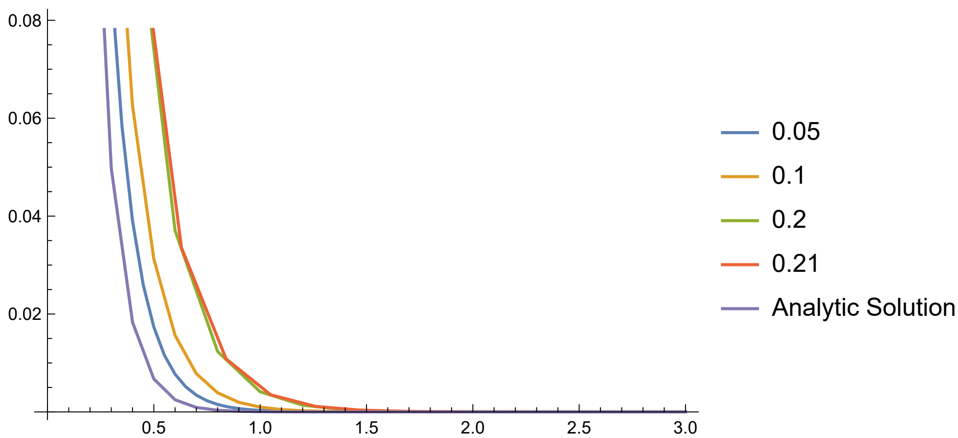
```
ListLogPlot[{Abs[retfE - soly], Abs[retbE - soly], Abs[retaB - soly]},
 PlotLegends → {"Forward Euler", "Backward Euler", "Adams-Bashford"}]
```

Out[15]=



In[16]:=
```
(* Ex 2: y' = -10y, y(0) = 1
      Solved Numerically with Backwards Euler in different step sizes*)
ode[y_, t_] = -10 y;
besol005 = bE[ode, 0.05, 3, 1];
besol01 = bE[ode, 0.1, 3, 1];
besol02 = bE[ode, 0.2, 3, 1];
besol021 = bE[ode, 0.21, 3, 1];
ansol = Transpose[{Range[0, 3, .1], Table[Exp[-10 t], {t, 0, 3, .1}]}];
ListLinePlot[{besol005, besol01, besol02, besol021, ansol},
 PlotLegends → {"0.05", "0.1", "0.2", "0.21", "Analytic Solution"}]
```

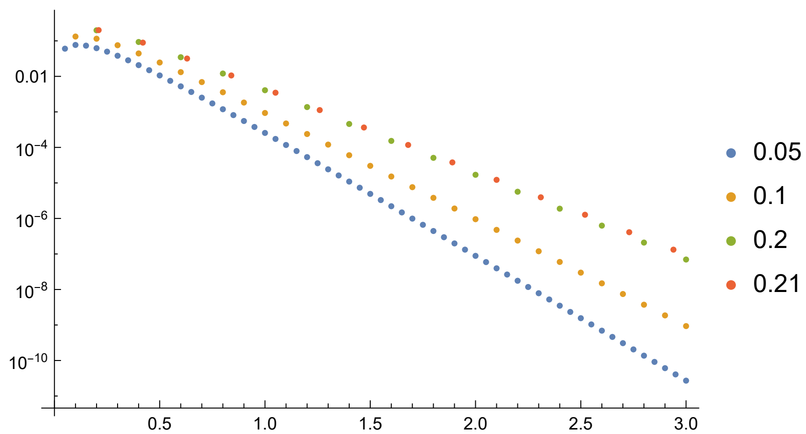
Out[22]=



In[23]:=
```
(* Log Plot of Error per Step Size*)
```

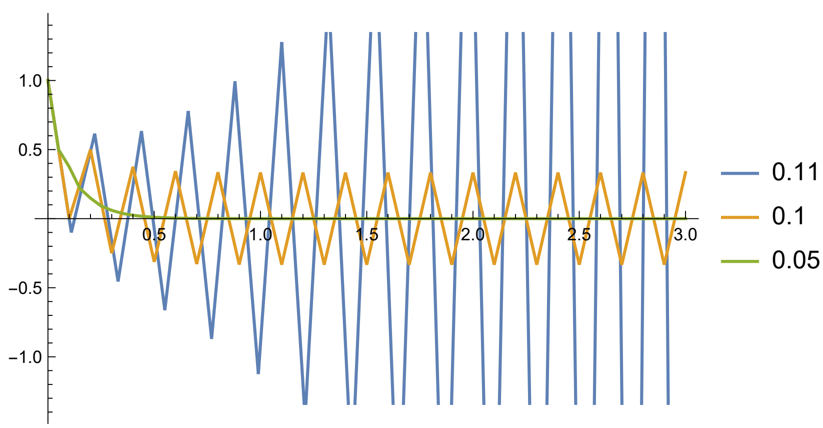
In[24]:=
```
ansoly1 =
  Transpose[{ConstantArray[0, Length[besol005]], Table[Exp[-10 t], {t, 0, 3, 0.05}]}];
ansoly2 =
  Transpose[{ConstantArray[0, Length[besol01]], Table[Exp[-10 t], {t, 0, 3, 0.1}]}];
ansoly3 =
  Transpose[{ConstantArray[0, Length[besol02]], Table[Exp[-10 t], {t, 0, 3, 0.2}]}];
ansoly4 =
  Transpose[{ConstantArray[0, Length[besol021]], Table[Exp[-10 t], {t, 0, 3, 0.21}]}];
ListLogPlot[{Abs[besol005 - ansoly1], Abs[ansoly2 - besol01], Abs[ansoly3 - besol02],
  Abs[besol021 - ansoly4]}, PlotLegends → {"0.05", "0.1", "0.2", "0.21"}]
```

Out[28]=



In[29]:=
```
(* Can we solve this ODE with an Adams-Bashforth method? *)
absol1 = aB[ode, 0.1, 3, 1];
absol2 = aB[ode, 0.05, 3, 1];
absol3 = aB[ode, 0.11, 3, 1];
ListLinePlot[{absol3, absol1, absol2}, PlotLegends → {"0.11", "0.1", "0.05"}]
```

Out[32]=



In[33]:=
```
(* Only if we use a small enough step size h <0.1*)
```