Proceedings of the 27th Conference
on Decision and Control
Austin, Texas • December 1988

ON THE IDENTIFICATION OF THE INERTIAL PARAMETERS OF ROBOTS

M. GAUTIER and W. KHALIL

Laboratoire d'Automatique de Nantes UA C.N.R.S 4.823
E.N.S.M. 44072 NANTES CEDEX
FRANCE

ABSTRACT

This paper presents a new algorithm for the identification of the inertial parameters and friction coefficients of robots . The algorithm does not require to mesure or to calculate the joint accelerations. The identification model is based on the energy theorem, it is linear in the robot parameters and is easy to calculate. An example of a two degree of freedom robot is presented.

1. INTRODUCTION

The motion of a robotic mechanical system is described by a set of highly coupled and non linear equations. Dynamic control techniques have been proposed to take into account the exact equations of motion [1], ..., [4]. The computational complexity problem of these algorithms is supposed now as solved [5],...,[8] . A major problem to implement these algorithms on real robots is that they need accurate identification of the inertial parameters of the links and the load. Recently, some papers [9],...,[13] have the common idea to estimate the inertial parameters by the use of a dynamic model linear with respect to the inertial parameters. The identification of the inertial parameters are carried out by the use of the standard least squares techniques. However, in these algorithms the joint accelerations are assumed available, measurable, or must be obtained by the derivation of the joint velocities which makes the results noisy.

The aim of this paper is to present a new solution for the identification problem. We propose here to use a model linear in the inertial parameters. The proposed model is based on the energy theorem, it is function of the joint positions and velocities, therefore the problem of measuring or calculating the joint accelerations has been avoided. Furthermore the computation of this model is more simple than the dynamic equations model.

2. DESCRIPTION OF THE ROBOT

The robot to be considered in this paper is an open loop mechanism. The description of the system will be carried out by the use of the modified Denavit - Hartenberg notation [14]. The coordinate frame $\mathbf{R}_i$ $(\mathbf{X}_i,\mathbf{Y}_i,\mathbf{Z}_i)$ is assigned fixed with respect to link i. The $\mathbf{Z}_i$ axis is along the axis of joint i, the $\mathbf{X}_i$ axis is along the common perpendicular of $\mathbf{Z}_i$ and $\mathbf{Z}_{i+1}$. The frame i coordinate will be defined with respect to the frame i-1 by the matrix ${}^{i-1}\mathbf{T}_i$ which is function of the parameters (α_i, d_i, θ_i, r_i), Figure 1, such that :

$$ {}^{i-1}\mathbf{T}_i = \begin{bmatrix} {}^{i-1}\mathbf{A}_i & {}^{i-1}\mathbf{P}_i \\ 000 & 1 \end{bmatrix} = \begin{bmatrix} C\theta_i & -S\theta_i & 0 & d_i \\ S\theta_i C\alpha_i & C\theta_i C\alpha_i & -S\alpha_i & -r_i S\alpha_i \\ S\theta_i S\alpha_i & C\theta_i S\alpha_i & C\alpha_i & r_i C\alpha_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (1)$$

where:
${}^{i-1}\mathbf{A}_i$ defines the orientation of frame i with respect to frame i-1,
${}^{i-1}\mathbf{P}_i$ defines the position of the origin of frame i with respect to frame i-1.

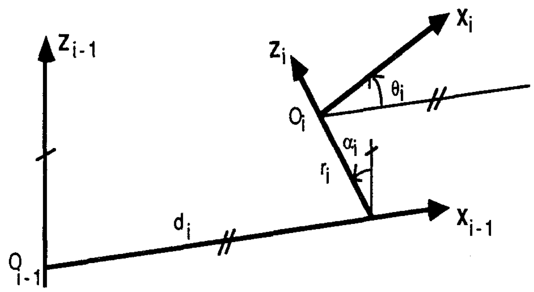


Figure 1

The joint variable i will be given as :

$$q_i = \bar{\sigma}_i \theta_i + \sigma_i r_i \quad (2)$$

with $\sigma_i = 0$ for i rotational, $\sigma_i = 1$ for i translational, and $\bar{\sigma}_i = (1 - \sigma_i)$.

3- DEFINITION OF THE INERTIAL PARAMETERS

Let us denote:
m_j the mass of link j
${}^j\mathbf{mS}_j$ the first moment of link j about the origin of frame j, referred to frame j, its elements are denoted as: $[\ mX_j \ \ mY_j \ \ mZ_j]^T$
${}^j\mathbf{J}_j$ the inertia tensor of link j about the origin of frame j, its elements are denoted :

$$ {}^j\mathbf{J}_j = \begin{bmatrix} XX_j & XY_j & XZ_j \\ XY_j & YY_j & YZ_j \\ XZ_j & YZ_j & ZZ_j \end{bmatrix} \quad (3)$$

Ia_j the rotor actuator inertia referred to joint side.
The inertial parameters of link j will be represented by the vector $\mathbf{X}^j$, denoted as :

$$\mathbf{X}^j = [\ XX_j\ XY_j\ XZ_j\ YY_j\ YZ_j\ ZZ_j\ mX_j\ mY_j\ mZ_j\ m_j\ Ia_j\ ]^T \quad (4)$$

The inertial parameters of the robot will be represented by the (11nx1) vector $\mathbf{X}$, denoted as :

$$\mathbf{X} = [\ \mathbf{X}^{1T} \quad \mathbf{X}^{2T} \quad \dots \mathbf{X}^{nT}\ ]^T \quad (5)$$

4-CALCULATION OF THE ENERGY OF THE ROBOT

In this section we give the relations to be used to calculate the kinetic and potential energy of the robot, we see that they are linear in the foregoing defined inertial parameters. In section 5 we give the model to be used in the identification process.

4-1 Calculation of the kinetic energy

The kinetic energy of the robot can be calculated as [15]:

$$E=\sum_{j=1}^{n} \frac{1}{2}\left({}^{j}\omega_j^T\, {}^{j}J_j\, {}^{j}\omega_j + m_j\, {}^{j}V_j^T\, {}^{j}V_j + 2\, {}^{j}V_j^T({}^{j}\omega_j \times {}^{j}mS_j)\right) + \frac{1}{2}\dot{q}_j^2\, Ia_j \qquad (6)$$

where:

${}^{j}V_j$ the velocity of the origin of the link j fixed frame, referred to frame j,

${}^{j}\omega_j$ the angular velocity of link j, referred to frame j.

${}^{j}\omega_j$ and ${}^{j}V_j$ can be calculated by the following recursive equations [8, 15]:

$${}^{j}\omega_j = {}^{j}A_{j-1}\, {}^{j-1}\omega_{j-1} + \bar{\sigma}_j Z_0 \dot{q}_j \qquad (7)$$

$${}^{j}V_j = {}^{j}A_{j-1}\, {}^{j-1}V_{j-1} + {}^{j}A_{j-1}[{}^{j-1}\omega_{j-1} \times {}^{j-1}P_j] + \sigma_j Z_0\, \dot{q}_j \qquad (8)$$

where :

$$Z_0 = [0 \quad 0 \quad 1]^T$$

4-2 Calculation of the potential energy

The potential energy is calculated as [8,15]:

$$U = -\sum_{j=1}^{n} \left(g^T (m_j\, {}^{0}P_j + {}^{0}A_j\, {}^{j}mS_j)\right) \qquad (9)$$

where g is the acceleration of gravity.

4-3 The linearity of the energy in the inertial parameters

From equations (6),...,(9) we deduce that E and U are linear in the inertial parameters. Thus we can write :

$$E = \sum_{i=1}^{m} \frac{\partial E}{\partial X_i} X_i = \sum_{i=1}^{m} DE_i\, X_i \qquad (10)$$

$$U = \sum_{i=1}^{m} \frac{\partial U}{\partial X_i} X_i = \sum_{i=1}^{m} DU_i\, X_i \qquad (11)$$

where X_i is an inertial parameter, DE_i is function of $\mathbf{q}$, $\dot{\mathbf{q}}$, and the geometric parameters of the robot, and DU_i is function of $\mathbf{q}$ and the geometric parameters. We supose that the geometric parameters are known accurately which is generally the case.

5- THE IDENTIFICATION MODEL

In order to identify the inertial parameters of the links and the payload we make use of the energy theorem, which states that the work of the forces which are applied to a system and which are not derived of a potential is equal to the change of the total energy of the system, thus:

$$\int_{t1}^{t2} \mathbf{T}^T\, \dot{\mathbf{q}}\, dt = (E(t2) + U(t2)) - (E(t1) + U(t1)) = L(t2) - L(t1) \qquad (12)$$

where :

$\mathbf{T}$ is the torques or forces vector not deriving of a potential,
E(ti) is the kinetic energy at time ti,
U(ti) is the potential energy at time ti,
L(ti) is the total energy at time ti, thus

$$L(ti) = E(ti) + U(ti) \qquad (13)$$

T can be obtained as :

$$\mathbf{T} = \Gamma + \Gamma_f \qquad (14)$$

where:

Γ is the motor torques or forces vector,

Γ_f is the friction forces vector, it is given as the sum of coulomb and viscous friction :

$$\Gamma_f = \Gamma_{fs} + \Gamma_{fv} \qquad (15)$$

Let the coulomb and viscous friction coefficients be denoted by FS_j, FV_j respectively, thus:

$$\Gamma_{fsj} = -FS_j\, \mathrm{sign}(\dot{q}_j) \qquad (16)$$

$$\Gamma_{fvj} = -FV_j\, \dot{q}_j \qquad (17)$$

where sign(.) denotes the sign function.
As E and U are linear in the inertial parameters then we can write :

$$L(t2) - L(t1) = \mathbf{DL}^T \mathbf{X} \qquad (18)$$

where :

$$\mathbf{DL}^T = [DL_1(t2) - DL_1(t1) \;\ldots\; DL_m(t2) - DL_m(t1)] \qquad (19)$$

and

$$DL_i(tk) = DE_i(tk) + DU_i(tk)$$

Let the friction coefficients be defined as :

$$\mathbf{FS} = [FS_1 \ldots FS_n]^T \qquad (20)$$

$$\mathbf{FV} = [FV_1 \ldots FV_n]^T \qquad (21)$$

let :

$$\mathbf{DFS}^T = \left[\int_{t1}^{t2} |\dot{q}_1|\, dt \;\ldots\; \int_{t1}^{t2} |\dot{q}_n|\, dt\right] \qquad (22)$$

$$\mathbf{DFV}^T = \left[\int_{t1}^{t2} \dot{q}_1^2\, dt \;\ldots\; \int_{t1}^{t2} \dot{q}_n^2\, dt\right] \qquad (23)$$

Equation (12) can be rewritten as :

$$y = \mathbf{DL}^T \mathbf{X} + \mathbf{DFS}^T \mathbf{FS} + \mathbf{DFV}^T \mathbf{FV} \qquad (24)$$

where:

$$y = \int_{t1}^{t2} \dot{\mathbf{q}}^T\, \Gamma\, dt \qquad (25)$$

The vector $\mathbf{X}$ can be extended, to include the friction parameters, it is defined as :

$$\mathbf{X}^T = [\mathbf{X}^T \quad \mathbf{FS}^T \quad \mathbf{FV}^T] \qquad (26)$$

so:

$$y = \mathbf{d}^T \mathbf{X} \qquad (27)$$

where:

$$d^T = [\,DL^T \quad DFS^T \quad DFV^T] \tag{28}$$

Relation (27) represents a linear equation in the inertial parameters of the links, actuators and friction parameters, it is function of the joint positions, of the velocities and of the input joint torques. Most of the robots are provided by position and velocity sensors, and the joint torques constitute the input signal to the motors which are known.
To identify **X** a sufficient number of equations can be obtained by calculating the relation (27) between different intervals of times. From (27) and (28) the k^{th} equation corresponding to an interval of time $(t1,t2)_k$, will be represented by :

$$y_k = \mathbf{d}_k^T \mathbf{X} \tag{29}$$

where:

$$\mathbf{d}_k^T = [\,\mathbf{DL}_k^T \quad \mathbf{DFS}_k^T \quad \mathbf{DFV}_k^T] \tag{30}$$

Practical calculation of the coefficients DU and DE

Assuming that:

$${}^j\omega_j = [\omega_1 \ \omega_2 \ \ \omega_3\,]_j^T, {}^jV_j = [V_1 \ V_2 \ \ V_3\,]_j^T \tag{31}$$

and:

$$\mathbf{DE}^j = \left[\frac{\partial E}{\partial XX}\frac{\partial E}{\partial XY}\frac{\partial E}{\partial XZ}\frac{\partial E}{\partial YY}\frac{\partial E}{\partial YZ}\frac{\partial E}{\partial ZZ}\frac{\partial E}{\partial mX}\frac{\partial E}{\partial mY}\frac{\partial E}{\partial mZ}\frac{\partial E}{\partial m}\frac{\partial E}{\partial Ia}\right]_j^T \tag{32}$$

$$\mathbf{DE}^j = [\,DE_1 \ \ DE_2 \ \ldots \ DE_{10} \ \ DE_{11}\,]^T \tag{33}$$

From the kinetic energy and velocity relations we obtain that :

$$\begin{aligned}
&DE_1 = \frac{1}{2}\omega_1{}^2 \cdot DE_2 = \omega_1\omega_2 \cdot DE_3 = \omega_1\omega_3 \cdot \\
&DE_4 = \frac{1}{2}\omega_2{}^2 \cdot DE_5 = \omega_2\omega_3 \cdot DE_6 = \frac{1}{2}\omega_3{}^2 \cdot \\
&DE_7 = \omega_3 V_2 - \omega_2 V_3,\ DE_8 = \omega_1 V_3 - \omega_3 V_1, \\
&DE_9 = \omega_2 V_1 - \omega_1 V_2 \cdot DE_{10} = \frac{1}{2}\,\mathbf{V}^T\mathbf{V}, \\
&DE_{11} = \frac{1}{2}\,\dot{q}_j^2
\end{aligned} \tag{34}$$

In relations (32) ... (34) all the components are belonging to link or joint j.
From the expression of U, given by (9), we can obtain :

$$[DU_7 \ DU_8 \ DU_9]_j = -\mathbf{g}^T\ {}^0\mathbf{A}_j \ ,\ DU^j_{10} = -\,\mathbf{g}^T\ {}^0\mathbf{P}_j \tag{35}$$

The DU_i^j are equal to zero for $i=1,\ldots,6$ and $i = 11$.

6. SIMPLIFICATION OF THE MODEL

The number of inertial parameters to be identified seems to be 11 parameters per link. However, some of these parameters have no effect on the dynamic model, nor on the energy model, and some others may appear in linear combinations. The parameters affecting the dynamic model separately and the regrouped parameters constitute the set of identifiable parameters and they are only needed to the control applications.The determination of this set of parameters is essential for the robustness of the identification process [12].

In [16] we have presented a direct method, consisting of explicit and closed form relations, which lead to classify the set of identifiable inertial parameters directly. The following facts are obtained :

- if joint j is rotational, then the parameters mZ_j and m_j, can be regrouped to the parameters of the link j–1, furthermore the parameter XX_j, (or YY_j), can be regrouped to the parameters of link j-1 and YY_j, (or XX_j) .

- if joint j is translational, all the parameters of $\mathbf{J}_j$ will be combined to the parameters of $\mathbf{J}_{j-1}$,

- if joint 1 is rotational the parameters XX_1, XY_1, XZ_1, YY_1, YZ_1, mZ_1, m_1 have no effect on the dynamic model, while Ia_1 can be regrouped to ZZ_1, moreover, if the axis of the first joint is along the direction of the acceleration of gravity then the parameters mX_1, mY_1, have no effect also.

- if joint 1 is translational the parameters XX_1, XY_1, XZ_1, YY_1, YZ_1, ZZ_1, mX_1, mY_1, mZ_1, have no effect on the dynamic model, while Ia_1 can be regrouped to m_1.

- some supplementary inertial parameters may be also eliminated. In [16] we presented how to determine them.

- the number of minimum inertial parameters is equal or less than :

$8n_r + 5n_t - 4 - \bar{\sigma}_1$
where:

n_r = number of rotational joints = $\sum \bar{\sigma}_j$

n_t = number of translational joints = $\sum \sigma_j$

7. IDENTIFICATION ALGORITHM

In the previous sections the canonical linear model on the parameters has been obtained as :

$$y_k = \mathbf{d}_k^T\ \mathbf{X} \quad \text{for } k=1,\ldots,r \tag{36}$$

In the following we will first discuss a deterministic approach and then a stochastic approach .

7-1 Deterministic approach.

Let us assume that $\mathbf{q}$, $\dot{\mathbf{q}}$, Γ are known without noise. Calculating the expression (36) for k= 1, ..., r, a linear system of r equations in m unknowns is obtained:

$$\mathbf{y}_r = \mathbf{h}_r\,\mathbf{X} + \mathbf{w} \tag{37}$$

where:

w is the observation errors vector,

$$\mathbf{y}_r = [\,y_1 \ldots y_r\,]^T \tag{38}$$

$$\mathbf{h}_r = [\,\mathbf{d}_1 \ldots \mathbf{d}_r\,]^T \tag{39}$$

The least squares solution $\mathbf{X}_r$ of (37) is chosen to minimize the mean square observation error, given by:

$$\mathbf{X}_r = \underset{\mathbf{X}}{\text{Argmin}}\ [(\mathbf{y}_r - \mathbf{h}_r\mathbf{X})^T(\mathbf{y}_r - \mathbf{h}_r\mathbf{X})] \tag{40}$$

with r > m, the solution is given as:

$$\mathbf{X}_r = (\mathbf{h}_r^T\,\mathbf{h}_r)^{-1}\,\mathbf{h}_r^T\,\mathbf{y}_r \tag{41}$$

In order that a solution can be obtained, $\mathbf{h}_r^T\mathbf{h}_r$ must be positive definite. Consequently the $\mathbf{h}_r$ columns must be independent. A necessary condition is that $y_k=\mathbf{d}_k^T\mathbf{X}$ is a canonical model and that good exciting trajectories are used. Owing to the known numerical inversion problems, the recursive solution is generally used [17].

7-2 Choice of the exciting trajectories.

Trajectories must be chosen such that $\mathbf{h}_r^T\mathbf{h}_r$ is positive definite. Classical methods used in generating robot motion such as bang-bang, third or five order polynomial trajectory functions cannot be used because joint velocities are generally proportionnal, due to the

synchronization of the joints movements.
For instance, it can be seen on the 2 planar rotational joints robot that $DE_1=1/2\dot{q}_1^2$ and $DE_2=1/2(\dot{q}_1+\dot{q}_2)^2$ are proportionnal if $\dot{q}_1$ and $\dot{q}_2$ are proportionnal also. So the corresponding columns of $\mathbf{h}_r$ are not independent and the parameters X_1 and X_2 cannot be identified separately. To overcome this difficulty, a different type of generator has been chosen for each joint. The method proposed in [18] can also be applied here easily owing to the relatively simplicity of our model.

7-3 Stochastic approach.

Now we consider noisy measurements of $\mathbf{q}$, $\dot{\mathbf{q}}$, so

$$\tilde{\mathbf{q}} = \mathbf{q} + v_p \; ; \; \tilde{\dot{\mathbf{q}}} = \dot{\mathbf{q}} + v_s \tag{42}$$

$$\tilde{\Gamma} = \Gamma + v_t \tag{43}$$

where:
v_p and v_s, are zero mean white noise processes.

Γ is the effective torque (or force) applied to the joints.

$\tilde{\Gamma}$ is the measured joint torque which is given by the exciting torque (see Fig. 2).

v_t represents a noise due to amplifiers, motors, and gears

Thus from (28) we can write:

$$\tilde{\mathbf{d}} = \mathbf{d}\,(\tilde{\mathbf{q}}, \tilde{\dot{\mathbf{q}}}) \tag{44}$$

$$y = \mathbf{d}^T \mathbf{X} + v_m \tag{45}$$

where v_m represents a noise due to modeling error.

$$\tilde{y} = \int_{t1}^{t2} \tilde{\dot{\mathbf{q}}}^T \, \tilde{\Gamma} \; dt \tag{46}$$

$$\tilde{y} = y + v_i$$

$$\tilde{y} = \mathbf{d}^T \mathbf{X} + v_m + v_i = \tilde{\mathbf{d}}^T \mathbf{X} + \mathbf{d}^T \mathbf{X} - \tilde{\mathbf{d}}^T \mathbf{X} + v_m + v_i \tag{47}$$

thus:

$$\tilde{y} = \tilde{\mathbf{d}}^T \mathbf{X} + v_e \tag{48}$$

where:

$$v_e = \mathbf{d}^T \mathbf{X} - \tilde{\mathbf{d}}^T \mathbf{X} + v_m + v_i \tag{49}$$

v_e is a random variable depending on v_p, v_s, v_m, v_t but it is very difficult to calculate an explicit expression for it. It can be seen that v_e is correlated with $\mathbf{d}_k$.

Using notations of equations (37) to (39), we can write:

$$\mathbf{y}_r = \mathbf{h}_r \mathbf{X} + \mathbf{w} \tag{50}$$

In this case, $\mathbf{y}_r$ and $\mathbf{w}$ are realizations of the random vectors $\mathbf{Y}_r$ and $\mathbf{W}$, where as $\mathbf{h}_r$ is a realization of a random matrix $\mathbf{H}_r$. The matrices $\mathbf{H}_r$ and $\mathbf{W}$ are correlated.

$\mathbf{X}$ is the deterministic vector of parameters to be estimated.
$\mathbf{h}_r$ is a non deterministic matrix and it is not possible to take of the results of linear estimation such as the Markov estimator.

The solution in most of papers is to use the least squares estimator $\hat{\mathbf{X}}_r$. Let us calculate the bias of this estimator whose realization is $\hat{\mathbf{x}}_r$.

$$\hat{\mathbf{x}}_r = \underset{\mathbf{X}}{\text{Argmin}}[\, (\mathbf{y}_r - \mathbf{h}_r \mathbf{X})^T (\mathbf{y}_r - \mathbf{h}_r \mathbf{X}) \,] \tag{51}$$

$$E\,(\mathbf{X} - \hat{\mathbf{X}}_r) = E\,[\,(\mathbf{H}_r^T \mathbf{H}_r)^{-1} \mathbf{H}_r^T \mathbf{W}\,] \tag{52}$$

where E is the expectation operator.

Because of the dependence of $\mathbf{W}$ and $\mathbf{H}_r$, the estimator is biased, but the bias is impossible to calculate explicitly.
The only way to conclude on the efficiency of this estimator is to study it by simulation, for a given robot with given noise measurements.

8.EXAMPLE

We consider in this example a two joints planar robot.

8-1 Description of the robot and its model

The geometric parameters are given in the following table :

i	σ_i	α_i	d_i	θ_i	r_i
1	0	0	0	q_1	0
2	0	0	L1	q_2	0

We suppose that the joints are directly driven and that the friction effects are neglected.

The Canonical model is given as:

$$y_k = \mathbf{d}_k^T \; \mathbf{X} \tag{53}$$

where:
$\mathbf{X} = [\, X1 \;\; X2 \;\; X3 \;\; X4\,]^T$

$X1 = ZZ1R = ZZ1 + m2.L1.L1$
$X2 = ZZ2$
$X3 = mX2$
$X4 = mY2$

We have taken the initial time of each equation as t1=0, and since $DL_i(0) = 0$, for i=1,...,4, then the k[th] equation can be written as:

$$y_k = \int_0^{tk} \dot{q}^T \, \Gamma \; dt \tag{54}$$

$$\mathbf{d}_k^T = [\, DL_1(tk) \quad DL_2(tk) \quad DL_3(tk) \quad DL_4(tk) \,]$$

where:

$$DL_1 = \frac{1}{2} \dot{q}_1^2$$

$$DL_2 = \frac{1}{2} [\dot{q}_1 + \dot{q}_2]^2$$

$$DL_3 = L1\, [\dot{q}_1 + \dot{q}_2]\, \dot{q}_1 \cos(q_2)$$

$$DL_4 = -L1\, [\dot{q}_1 + \dot{q}_2]\, \dot{q}_1 \sin(q_2)$$

Noisy measurements are introduced as:

$$\tilde{q} = q + v_p \; ; \; \tilde{\dot{q}} = \dot{q} + v_s \;\; , \;\; \tilde{\Gamma} = \Gamma + v_t$$

Figure 2. illustrates the simulation block diagram.
The exciting torques are computed using a third order polynomial trajectory generator for the first joint and a sinus trajectory generator for the second joint assuming an initial estimation $\mathbf{X}_d$ of $\mathbf{X}$.

In our simulation we ta.e X_d=X and we suppose that v_t equals zero. The simulation block diagram can be simplified as shown in Figure 3.
The following data correspond to an experimental robot developed in our laboratory:

- Joint positions **q** are measured by encoders with NPT points per revolution. v_p is due to quantification error. It can be modelized by a uniform zero-mean and white noise between bounds $-\Delta q/2$ and $+\Delta q/2$, with $\Delta q=2\pi/NPT$.
For the prototype NPT=14400 points and bounds =2.18 10^{-4} rd.
- Joint velocities $\dot{q}$ are measured by tachometers characterised by their ripple $\Delta\dot{q}$.

v_s is modelized by a zero-mean white gaussian noise with a standard deviation $\sigma_s=\Delta\dot{q}/4$.

With: $\Delta\dot{q}=0.02\ \dot{q}_{max}$

and $\dot{q}_{1max}$=1rd/s, $\dot{q}_{2max}$=2rd/s,
we get :
σ_{1s}=0.005 rd/s and σ_{2s}=0.01 rd/s.
The values of X are:

X1=1.702 kg.m^2 ; X2=0.05644 kg.m2 ; X3=0.15 kg.m
X4=0.075 kg.m.

8-2 Results.

Figures 4, ... ,9 show trajectories and parameters estimations. The recurrence is about 100 points with a sampling period of 0.0622s. It can be seen that the estimations converge after 50 periods with a bias less than 10%. Relative error after 100 iterations is found to be:

for X1: 5.6 10^{-4}, for X2: 0.016, for X3: 0.011, for X4: 0.019

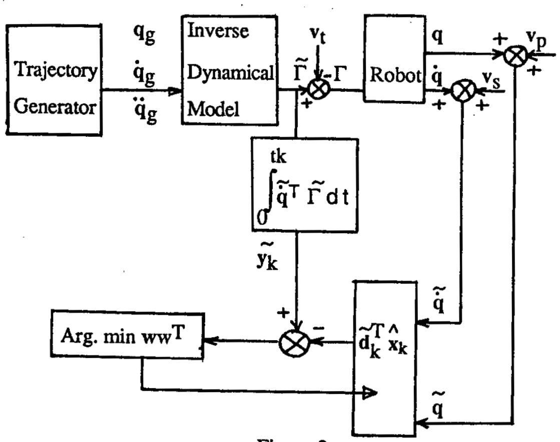


Figure 2.

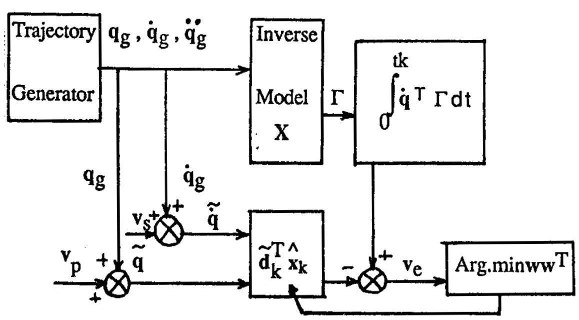


Figure 3.

FIGURE 4: JOINT POSITIONS
JOINT 1: QG1(NO NOISE),QB1(WITH NOISE)
JOINT 2: QG2 ,QB2

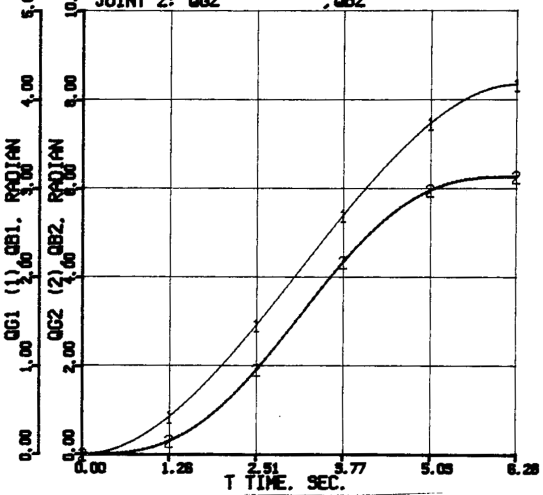


FIGURE 5: JOINT VELOCITIES
JOINT 1: QPG1(NO NOISE),QPB1(WITH NOISE)
JOINT 2: QPG2 ,QPB2

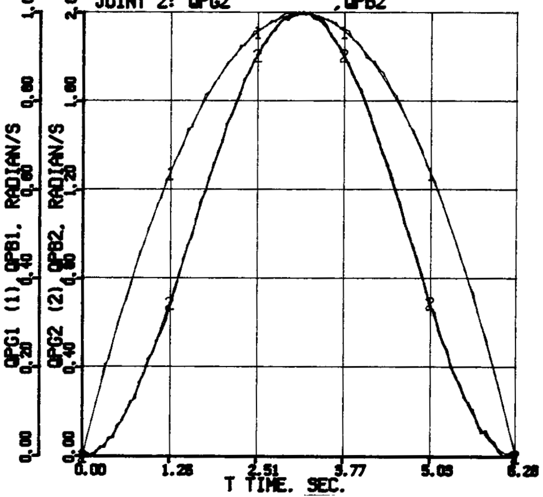


FIGURE 8: XOBJ=REAL VALUE OF ZZ1R
XEST=ESTIMATION WITHOUT NOISE
XESTB=ESTIMATION WITH NOISE

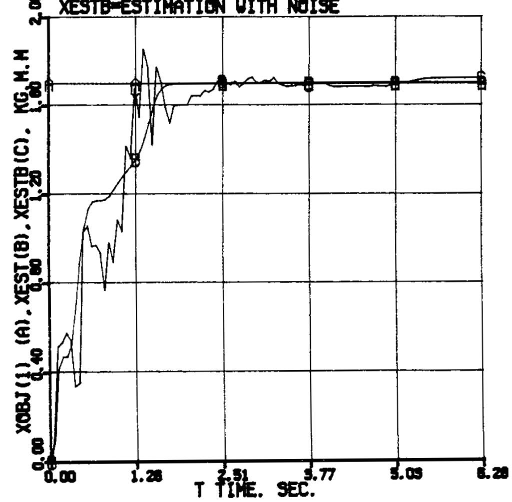


FIGURE 7: XOBJ=REAL VALUE OF ZZ2
XEST=ESTIMATION WITHOUT NOISE
XESTB=ESTIMATION WITH NOISE

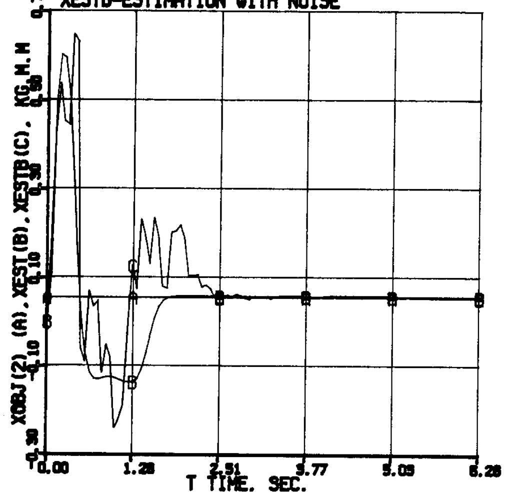


FIGURE 8: XOBJ=REAL VALUE OF MX2
XEST=ESTIMATION WITHOUT NOISE
XESTB=ESTIMATION WITH NOISE

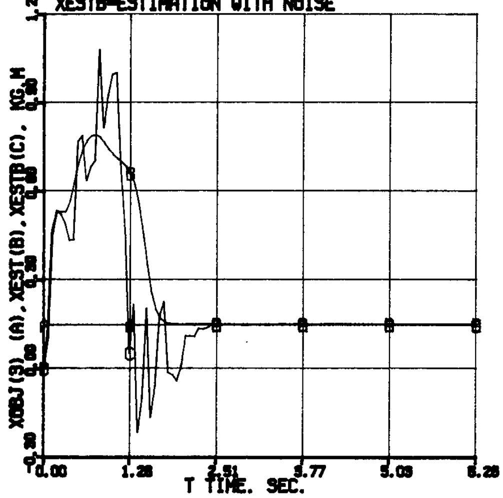
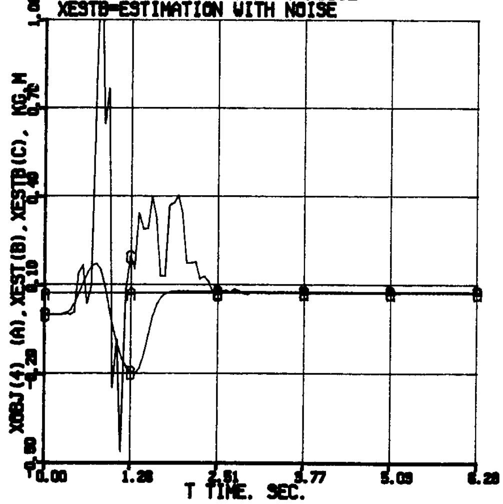


FIGURE 9: XOBJ=REAL VALUE OF MY2
XEST=ESTIMATION WITHOUT NOISE
XESTB=ESTIMATION WITH NOISE



9- CONCLUSION

This paper presents a new algorithm for the identification of the inertial parameters and friction coefficients of robots. The algorithm is computationaly simple, particularly it does not require not to mesure or to calculate the joint accelerations. The identification model is based on the energy theorem providing a model linear in the links and motors parameters. An example of a two degree of freedom robot is presented, the results are applied directly to any robot,.

REFERENCES

[1] W.Khalil, A.Liegeois, A.Fournier, "Commande Dynamique de Robots", RAIRO Automatique , Systems Analysis and Control, Vol.13, N°2, 1979, pp. 189-201.

[2] J.Y.S.Luh, M.W.Walker, R.Paul, "Resolved-acceleration control of mechanical manipulators", IEEE trans. Auto. Control, vol. AC-25,1980, pp.468-474.

[3] O.Khatib, " A Unified Approach for motion and force control: The operational space formulation", IEEE J. of Robotics and Automation,Vol. 3 (1), 1987, pp. 43-53.

[4] C.S.G.Lee, M.J.Chung, "An Adaptive control strategy for mechanical Manipulators", IEEE Transaction on Automatic Control, Vol.AC-29, N°9, 1984, pp.837-840.

[5] T. Kanade, P. Khosla, N. Tanaka, "Real-time control of CMU direct-drive arm II using customized inverse dynamics", Proc. 23 th CDC .1984, pp. 1345-1352.

[6] W.Khalil, J.F.Kleinfinger, "Une modelisation performante pour la commande dynamique de robots", RAIRO, APII, Vol.6, 1985, pp.561-574.

[7] W.Khalil, J.F.Kleinfinger, M.Gautier, "Reducing the computational burden of the dynamic model of robots", Proc. IEEE Conf. on Robotics and Automation, 1986, pp.525–531.

[8] W.Khalil, J.F.Kleinfinger, "Minimum operations and minimum parameters of the dynamic models of tree structure robots", IEEE J. Robotics and Automation, Vol.RA-3(6), Dec. 1987,pp.517-526.

[9] E.Ferreira," Contribution à l'identification des paramètres et à la commande dynamique adaptative des robots manipulateurs". Thèse Doct. Ing., 1984, Toulouse, France.

[10]C. H. An, C.G. Atkeson, J.M. Hollerbach, " Estimation of inertial parameters of rigid body links of manipulators". Proc. 24th Conf. on Decision and control, 1985, pp. 990-995.

[11] A.Mukerjee, D.H.Ballard, "Self-calibration in robot manipulators". Proc. IEEE Conf. on Robotics and Automation, 1985, pp. 1050-1057.

[12] P.K.Khosla,T.Kanade, "Parameter identification of robot dynamics " . Proc. 24th Conf. on Decision and Control. 1985, pp. 1754-1760.

[13]M. Gautier," Identification of Robots dynamics", Proc. IFAC, Symp. on Theory of Robots ,Vienne, 1986, pp.351-356.

[14] W.Khalil, J.F.Kleinfinger, "A new geometric notation for open and closed loop robots" Proc. IEEE Conf. on Robotics and Automation, 1986,pp.1174–1180.

[15] W.Khalil, E. Dombre, "Modélisation et commande des robots manipulateurs".Hermès, Paris, 1988.

[16] M. Gautier,W.Khalil, " A Direct determination of minimum inertial parameters of robots", IEEE Conf. on Robotics and Automation, Philadelphia, April 1988.

[17] G.J.Bierman, "Factorization methods for discrete sequential estimation" Academic Press, 1977.

[18] B. Armstrong, "On finding `exciting` trajectories for identification experiments involving systems with non-linear dynamics". Proc. IEEE Conf. on Robotics and Automation, 1987, pp.1131-1139.