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PERFORMANCE EVALUATION OF TWO-DEGREE-OF-FREEDOM PLANAR PARALLEL ROBOTS

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Abstract—We utilize a geometric model of the solution space to obtain analytical relationships between the link lengths of two-degree-of-freedom parallel planar manipulators and performance criteria based on the global conditioning and global velocity indices. The model is used to develop graphical charts that are useful for analysis and design of the mechanisms.

INTRODUCTION

Two-degree-of-freedom parallel planar manipulators (2-DOF PPMs) are an important class of robotic mechanisms that can follow arbitrary planar curves. Because of their usefulness in applications, these mechanisms have attracted the attention of researchers who have investigated their workspace, mobility, and methods for analysis and design [1–7]. Gosselin and Angeles [8] proposed a performance criterion, called the global conditioning index, for the kinematic optimization of robotic manipulators. This criterion is based on evaluating the condition number of the Jacobian over the manipulator workspace. Because the condition number is a measure of invertibility of the Jacobian matrix, the global conditioning index is valuable for the design of robotic manipulators.

Although extensive research has been directed toward the analysis of 2-DOF PPMs, there has not been an effective way to relate performance criteria and the link lengths of these mechanisms. Previously, a geometric model of the solution space was derived for 2-DOF PPMs and used to study performance characteristics [9, 10]. We shall use that approach to investigate relationships between the link lengths of two-degree-of-freedom parallel planar manipulators and performance criteria based on the global conditioning and global velocity indices.

SOLUTION SPACE

In this section we review the solution space technique developed by the authors [9, 10]. Consider a 2-DOF PPM as shown in Fig. 1. Because the link lengths may have a wide range of possible values, it is convenient to avoid explicit use of the physical sizes of the mechanisms during analysis and design. We shall define normalized parameters of the mechanisms and construct a geometric model of the solution space as follows.

Let

$$r_i = R_i/L \quad (i = 1, 2, 3, \dots, 5), \tag{1}$$

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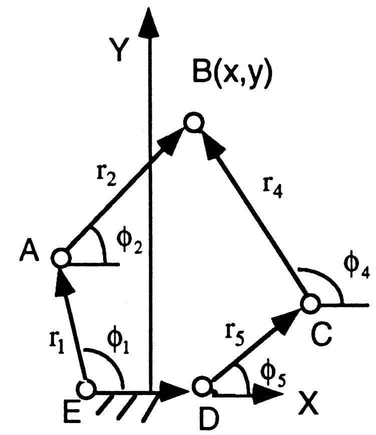

Fig. 1. 2-DOF planar parallel manipulator.

$$L = \frac{1}{4}\sum_{i=1}^{5} R_i \tag{2}$$

where R_i is the length of link i, r_i is the normalized, nondimensional length of link i, and L is the average link length of the manipulator. Since Gosselin[11] has shown that the parallel manipulator should be symmetric, we obtain

$$R_5 = R_1 \text{ and } R_4 = R_2; \quad r_5 = r_1 \text{ and } r_4 = r_2. \tag{3}$$

These conditions show that the two input links should have equal lengths and the two coupling bars also should have equal lengths. Therefore, the sum of the link lengths is

$$2r_1 + 2r_2 + r_3 = 4. \tag{4}$$

If the manipulator can be assembled the normalized link lengths satisfy

$$0 < r_i < 2 \quad (i = 1, 2, 4, 5) \text{ and } 0 \le r_3 < 2. \tag{5}$$

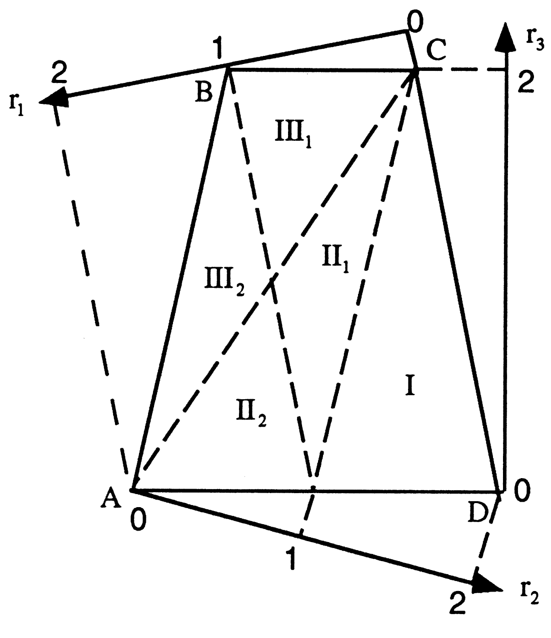

Fig. 2. Geometric model of the solution space.

A model of the solution space can be constructed in the trapezoid ABCD of Fig. 2. Within this region we can analyze 2-DOF PPMs.

GLOBAL CONDITIONING INDEX

A useful performance measure for manipulator control is the global conditioning index which Gosselin [8] defined as the inverse of the condition number of the Jacobian matrix integrated over the reachable workspace and divided by the volume of the workspace,

$$\eta = A/B \tag{6}$$

$$A = \int_w \left(\frac{1}{\kappa}\right) \mathrm{d}w \text{ and } B = \int_w \mathrm{d}w, \tag{7}$$

where η is the global conditioning index, κ is the condition number of the Jacobian matrix evaluated for a given pose in the manipulator workspace W, and B is the area of the workspace. A large value of η ensures that the manipulator can be precisely controlled.

The condition number of 2-DOF PPMs is defined as

$$\kappa = \|\mathbf{J}\| \|\mathbf{J}^{-1}\| \tag{8}$$

where $[J]$ is the Jacobian matrix of the 2-DOF PPM,

$$[J] = \begin{bmatrix} J_{11} & J_{12} \\ J_{21} & J_{22} \end{bmatrix}, \tag{9}$$

and

$$\begin{aligned} J_{11} &= \Delta \sin\phi_4 \sin(\phi_1 - \phi_2) \\ J_{12} &= -\Delta \sin\phi_2 \sin(\phi_5 - \phi_4) \\ J_{21} &= -\Delta \cos\phi_4 \sin(\phi_1 - \phi_2) \\ J_{22} &= \Delta \cos\phi_2 \sin(\phi_5 - \phi_4) \\ \Delta &= r_1/\sin(\phi_2 - \phi_4) \end{aligned}$$

$$\|\mathbf{J}\| = \mathrm{sqrt}[\mathrm{tr}(\mathbf{JNJ}^{\mathrm{T}})], \tag{10}$$

where $\|\mathbf{J}\|$ is a frame-invariant norm, n is the dimension of the square matrix $\mathbf{J}$, and

$$[N] = \frac{1}{n}[I].$$

From Equations (8)–(10), we obtain:

$$\frac{1}{k} = \frac{2 \mid \sin(\phi_1 - \phi_2) \cdot \sin(\phi_5 - \phi_4) \cdot \sin(\phi_2 - \phi_4) \mid}{\sin^2(\phi_1 - \phi_2) + \sin^2(\phi_5 - \phi_4)}. \tag{11}$$

From Equations (6), (7) and (11), the global conditioning index for the manipulators can be calculated.

GLOBAL VELOCITY INDEX

Velocity is another useful criterion for 2-DOF PPMs. For a manipulator configuration with a given pose, the end-effector may have different velocities because it can move with different orientations. We shall determine extreme values of the end-effector velocity for a manipulator configuration with a given pose in the workspace.

We define the global maximum and minimum velocity indices to be the extreme values of the end-effector velocities integrated over the reachable workspace and divided by the volume of the workspace. The global velocity index is a measure of robot speed.

Extreme values

From Fig. 1, we obtain:

$$(V_B) = [J](V_{in}), \tag{12}$$

where (V_B) is the end-effector velocity at point B, (V_{in}) is the input (angular) velocity $(V_{in}) = (\phi_1 \phi_5)^T$, and $[J]$ is the Jacobian matrix. From Equation (12), we obtain:

$$\| V_B \|^2 = (V_{in})^T [J]^T [J](V_{in}). \tag{13}$$

Let

$$\| V_{in} \|^2 = (V_{in})^T (V_{in}) = 1. \tag{14}$$

From Equations (12) and (13), we define the Lagrangian function as:

$$L_V = (V_{in})^T [J]^T [J](V_{in}) - \lambda_V[(V_{in})^T (V_{in}) - 1], \tag{15}$$

where λ_V is a scalar valued Lagrangian multiplier. From Equation (15), necessary conditions for extreme values of the end-effector velocities are as follows:

$$\frac{\partial L_V}{\partial \lambda_V} = 0 : (V_{in})^T (V_{in}) - 1 = 0, \tag{16}$$

$$\frac{\partial L_V}{\partial V_{in}} = 0 : [J]^T [J](V_{in}) - \lambda_v (V_{in}) = 0. \tag{17}$$

From Equation (17), λ_v is an eigenvalue of the matrix $[J]^T[J]$. Therefore, the maximum and minimum eigenvalues of $[J]^T[J]$ are:

$$\begin{aligned} \lambda_{V_{max}} &= \frac{1}{2}[v_{11} + v_{22} + \sqrt{(v_{11} - v_{22})^2 + 4v_{12}v_{21}}] \\ \lambda_{V_{min}} &= \frac{1}{2}[v_{11} + v_{22} + \sqrt{(v_{11} - v_{22})^2 + 4v_{12}v_{21}}], \end{aligned} \tag{18}$$

where

$$[J]^T[J] = \begin{bmatrix} v_{11} & v_{12} \\ v_{21} & v_{22} \end{bmatrix}, \tag{19}$$

and

$$\begin{aligned} v_{11} &= J_{11}^2 + J_{21}^2 \\ v_{12} &= J_{11}J_{12} + J_{21}J_{22} \\ v_{21} &= J_{11}J_{12} + J_{21}J_{22} \\ v_{22} &= J_{12}^2 + J_{22}^2, \end{aligned} \tag{20}$$

and J_{ij} can be obtained from Equation (9). By definition of eigenvalues and utilizing Equations (16) and (17), one obtains:

$$\| V_B \|^2 = (V_{in})^T [J]^T [J](V_{in}) = \lambda_V. \tag{21}$$

Thus, if the input velocity is a unit vector satisfying Equation (14), and using Equations (18) and (21), the extreme values of the velocity can be evaluated as follows:

$$\| V_{B\max} \| = \sqrt{\lambda_{V\max}} \text{ and } \| V_{B\min} \| = \sqrt{\lambda_{V\min}} \tag{22}$$

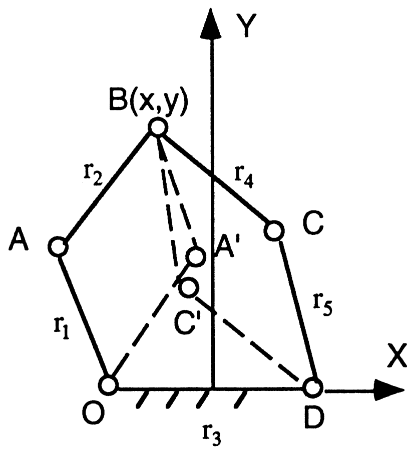

Fig. 3. Configurations for 2-DOF PPMs.

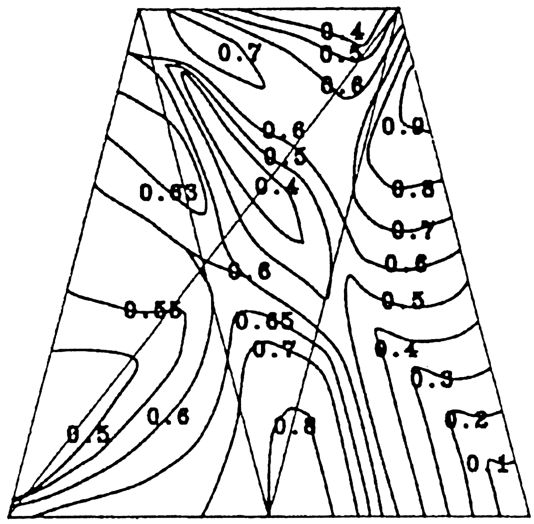

Fig. 4. Constant loci of the global conditioning index.

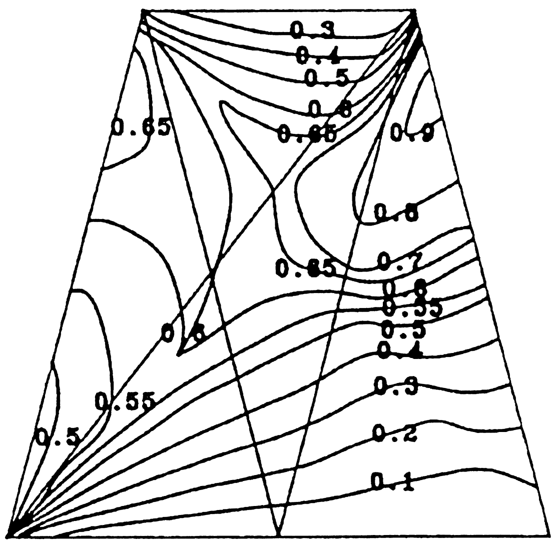

Fig. 5. Constant loci of the global conditioning index.

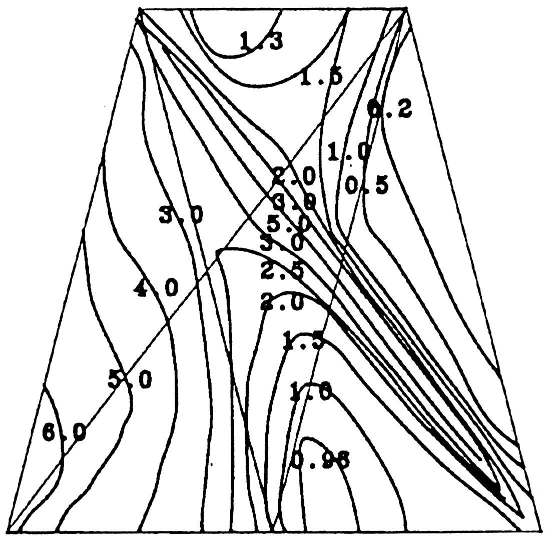

Fig. 6. Constant loci of the global maximum velocity index.

Global velocity index

Because [*J*] is a function of the manipulator configuration, the extreme values of the end-effector velocity are not suitable measures of manipulator velocity. We therefore define the global velocity index

$$\gamma_{\max} = \frac{\mathrm{C}_{\max}}{\mathrm{B}}, \tag{23}$$

$$\gamma_{\min} = \frac{\mathrm{C}_{\min}}{\mathrm{B}}, \tag{24}$$

where $\gamma_{\min}$ and $\gamma_{\min}$ are the global maximum and minimum velocity indices, respectively, B is calculated from Equation (7), and

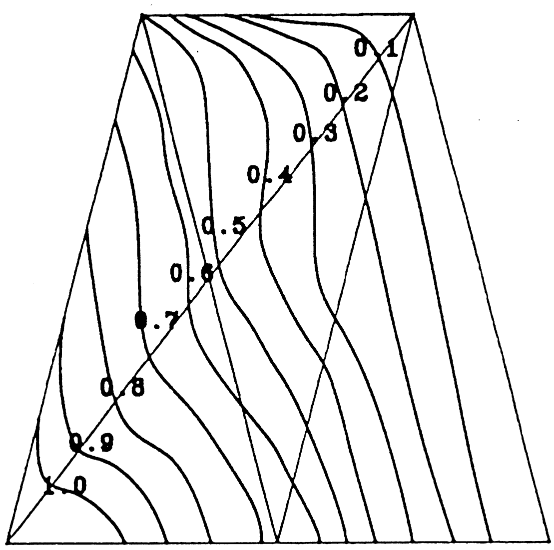

Fig. 7. Constant loci of the global minimum velocity index.

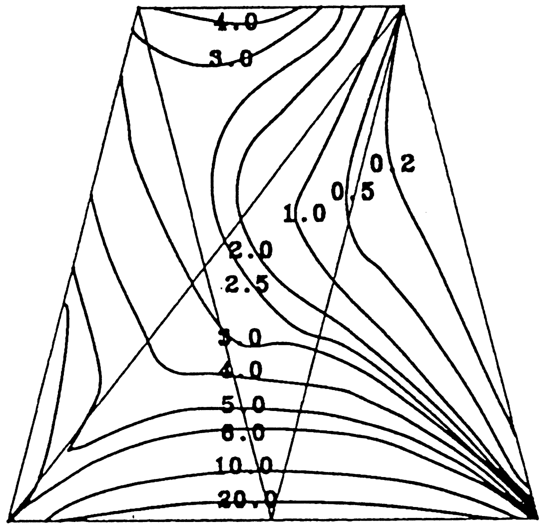

Fig. 8. Constant loci of the global maximum velocity index.

$$C_{\max} = \int_{w} \| V_{B\max} \| \mathrm{d}w \text{ and } C_{\min} = \int_{w} \| V_{B\min} \| \mathrm{d}w. \qquad (25)$$

ATLASES OF THE GLOBAL CONDITIONING INDEX

From Fig. 1 and Equation (11), the end-effector position at the locations (x, y) and $(-x, y)$ have the same value of the condition number. The reason for this is that the values of $\phi_1-\phi_2$ and $\phi_5-\phi_4$ at position (x, y) are equal to $\phi_5-\phi_4$ and $\phi_1-\phi_2$ at position $(-x, y)$, respectively. Furthermore, the value of $\phi_2-\phi_4$ does not change at these two positions. This implies that the values of the condition number are symmetric about the y-axis in the workspace of 2-DOF PPMs. Consequently, we only need to evaluate the condition number in the half space.

From Fig. 3, we see that the value of the condition number depends on the configuration of the manipulator. Although there are four distinct configurations (OABCD, OABC′D, OA′BC, OA′BC′D), only OABCD and OABC′D must be considered since the other configurations have

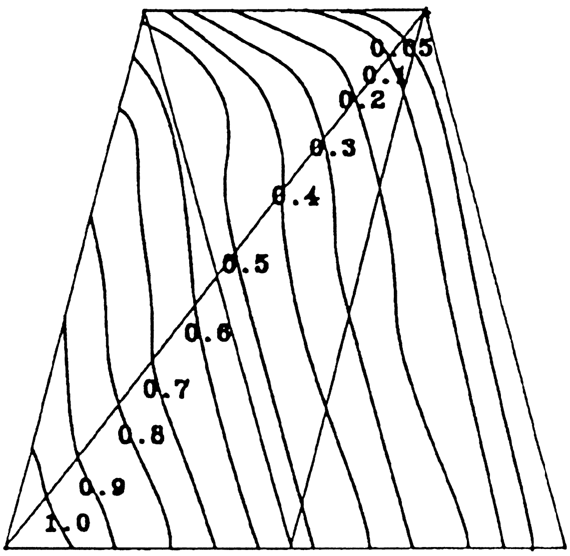

Fig. 9. Constant loci of the global minimum velocity index.

the same values of the global conditioning index

$$\eta_{\mathrm{OABCD}} = \eta_{\mathrm{OA'BC'D}}, \tag{26}$$

$$\eta_{\mathrm{OABC'D}} = \eta_{\mathrm{OA'BCD}}, \tag{27}$$

Using Equation (6), values of the global conditioning index are computed and loci of constant values are displayed in Figs 4 and 5. These figures show values of the global conditioning index for different configurations.

ATLASES OF THE GLOBAL VELOCITY INDICES

From Fig. 3 and Equation (18), we see that the values of $\gamma_{\max}$ and $\gamma_{\min}$ depend on the Jacobian matrix $[J]$ which is a function of the manipulator configuration. Although there are four distinct configurations (OABCD, OABC′D, OA′BCD, OA′BC′D), only OABCD and OABC′D are must be considered because the other two configurations have the same values of $\gamma_{\max}$ and $\gamma_{\min}$. Using Equations (23) and (24), the values $\gamma_{\max}$ and $\gamma_{\min}$ are evaluated and constant loci are plotted in Figs 6–9.

Figures 6 and 7 show the distribution of $\gamma_{\max}$ and $\gamma_{\min}$ loci for configurations OABCD and OA′BC′D within the solution space, respectively. Figures 8 and 9 show the distribution of $\gamma_{\max}$ and $\gamma_{\min}$ loci for configurations OA′BCD and OABC′D within the solution space, respectively. Figures 6–9 illustrate the relationship between the global velocity indices, $\gamma_{\max}$ and $\gamma_{\min}$, and the link lengths.

CONCLUSIONS

We have utilized a geometric model of the solution space to obtain analytical relationships between the link lengths of 2-DOF PPMs and performance measures based on the global conditioning index and global velocity indices. Constant values of the indices have been plotted in the solution space of the manipulator. Because these performance measures describe the global behavior of the manipulator they are useful for analysis and design.

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