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Minimum Jerk Trajectory Control for Rehabilitation and Haptic Applications

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Abstract— **Smooth Trajectories are essential for safe interaction in between human and a haptic interface. Different methods and strategies have been introduced to create such smooth trajectories. This paper studies the creation of human-like movements in haptic interfaces, based on the study of human arm motion. These motions are intended to retrain upper limb movements of patients that lose manipulation functions following stroke. We present a model that uses higher degree polynomials to define a trajectory and control the robot arm to achieve minimum jerk movements. It also studies different methods that can be driven from polynomials to create more realistic human-like movements for therapeutic purposes.**

Keywords— **Human Machine Interaction, Minimum Jerk theory, Super Position, Target Selection, Therapy, Stroke, Rehabilitation, Haptic**

I. Introduction

THE study of human arm motion is essential for developing robot arms that interact with human subjects. A clear understanding of human arm motion, will aid for better interaction in between a machine and a human subject. The term trajectory refers to the configuration of the user's wrist in the space [1]. The biomechanics and neural control used to generate simple reaching movements toward static objects are extremely complicated, and are not completely understood. Various authors have shown the common kinematic and dynamic features of these types of movement [1], [2], [3], [4], [5]. They have demonstrated that, typical human arm movement has a straight path with a single peak and bell shaped velocity profile. Different approaches have been proposed and a summary of these approaches can be found in the study carried out by Wolpert et al. [6]. We use the empirical minimum jerk approach as it is simple to use in a real-time control context. The minimum jerk model, originally purposed by Hogan [7] for one-joint and Flash and Hogan [1] for multi-joint movements states that human movements tends to minimise the jerk parameter over the time of movement. Jerk is the rate of the change of acceleration with respect to time, that is the third time derivative of the position. According to minimum jerk theory, movement will have maximum smoothness when the parameter given by the equation (1) is minimised:

$$J = \int_0^d |\frac{d^3x}{dt^3}|^2 dt \tag{1}$$

Where d is the duration of the movement and x is the hand position at the time t. We have used this theory to provide smooth trajectories for a robot arm that interacts with stroke patients to deliver daily exercises [8], [9], [10]. In this paper, we focus on different mathematical approaches used in our project, leading to a smooth interaction between a robot and a stroke patient.

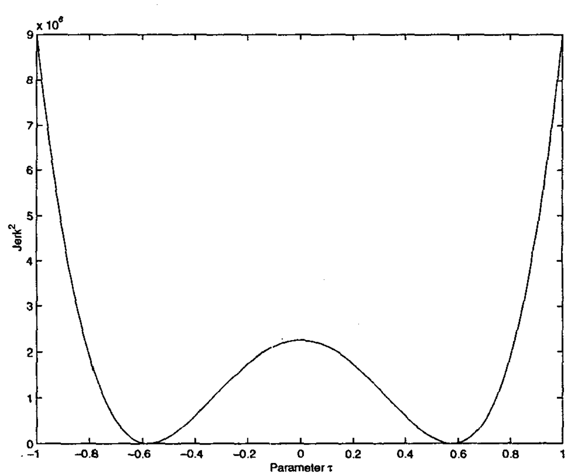

Fig. 1. Jerk Squared results for a straight-line trajectory, Minimum jerk theory states that by minimising the area under this curve, we can achieve a human-like trajectory.

II. Minimum jerk for polynomials

Polynomials are a common method for defining robot trajectory. Using polynomials, we are able to control the velocity and acceleration of the movement, as well as the position. Using polynomials in real time control applications has the advantage that we can change our trajectory in real time by redefining the polynomials or by superimposing a new trajectory over the previous one [11].

For straight-line movements between two points it is desirable to have accelerations that are zero at the beginning and end of the movement. A parameter τ is chosen such that:

$$-1 \leq \tau \leq 1$$

That can be scaled to the true time of movement at a later stage. This parameterisation is convenient since, for symmetrical movements, mid range position, velocity and acceleration occurs when $\tau = 0$. It also eases the calculation of the coefficients in a later stage. A polynomial with odd power is needed to ensure the acceleration at the beginning and end of the movements is zero. A 5^{th} order polynomial has been used elsewhere in the literature (eg. Hogan). We began with a 7^{th} order polynomial to allow for non-symmetric non-minimum jerk polynomials to coexist with symmetric minimum jerk polynomials:

$$p = a + b\tau + c\tau^2 + d\tau^3 + e\tau^4 + f\tau^5 + g\tau^6 + h\tau^7 \quad (2)$$

and its derivatives with respect to the parameter τ denoted as the more familiar p', p'', p'''.

These constraints apply to the start and end of the movement:

- Start and end positions are defined:

$$p|_{\tau=-1} = P_{start} \qquad p|_{\tau=1} = P_{end}$$

- Start and end velocities and accelerations are zero:

$$p'|_{\tau=-1} = 0 \qquad p'|_{\tau=1} = 0$$

$$p''|_{\tau=-1} = 0 \qquad p''|_{\tau=1} = 0$$

Applying these assumptions, our polynomial becomes:

$$p = a + b\tau + d\tau^3 + f\tau^5 + h\tau^7 \quad (3)$$

We can identify the coefficients of the polynomial as:

$$a = \frac{(p_{start} + p_{end})}{2} \quad (4)$$

$$p'|_{\tau=0} = v_{mid} = b \quad (5)$$

$$d = \frac{35}{16}\Delta_p - 3b \quad (6)$$

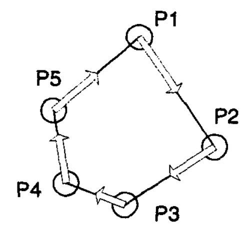

Fig. 2. point to point movement definition

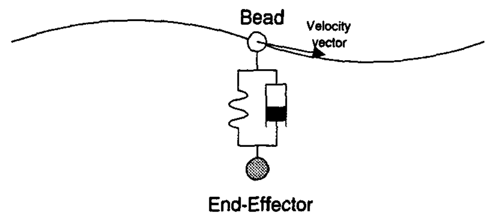

Fig. 3. Spring-damper combination was used to guide the user over the defined trajectory

$$f = 3b - \frac{21}{8}\Delta_p \quad (7)$$

$$h = \frac{15}{16}\Delta_p - b \quad (8)$$

where:

$$\Delta_p = p_{end} - p_{start} \quad (9)$$

Thus to achieve minimum jerk movement, we need to determine mid velocity to minimise the integral (Figure 1) given by the Equation 10:

$$J_{param} = \int_{-1}^{1} |p'''|^2 d\tau \quad (10)$$

It can be shown that to achieve the maximum smoothness, mid velocity should be given by the Equation 11:

$$b = \frac{15}{16}\Delta_p \quad (11)$$

Using Equations 8 and 11 it can be seen that achieving minimum jerk movement reduces the polynomial order to a 5^{th} order polynomial.

We used minimum jerk model and the polynomials shown here to implement our therapy modes. Target oriented therapies are defined using a point to point approach (Figure 2). Stroke patients often lack the ability to move their arm toward the targets or lack movement coordination, therefore haptic interface leads and helps the patient toward predefined targets.

We have used a virtual spring-damper combination (Figure 3) to help positioning the arm on the path defined by the minimum jerk polynomial. Polynomial coefficients are calculated between end-effector's current position (P1 in Figure 2) and targets position (P2 in Figure 2). Spring and damper coefficients can be matched to the patients therapy needs. This movement can be performed with different velocities and different amount of help provided by the virtual spring and damper. The key to this approach is the ability to constrain the movement of the 'bead' along the specified trajectory. Figure 4 shows a concept sketch of the current implementation of the of the Gentle/s system (see www.gentle.rdg.ac.uk for photographs and videos). We compared the experimental results achieved in a therapy session, with the results that we expected theoretically. The position data is given in a reference frame that has a principal axis aligned with the path of the movement. Figure 5 shows that there is some differences between the expected position and the position achieved. This difference is due to the weight of the patient's arm when relaxed, residual device friction and the loss of depth perception when viewing a 2D computer screen. For various clinical reasons a mechanism is used to compensate for the weight of the patient arm. This has the added benefit of allowing the robot (a 3DOF '$HapticMaster$' from FCS robotics, see www.fcs-robotics.com) to operate with in the load capacity of its force sensor.

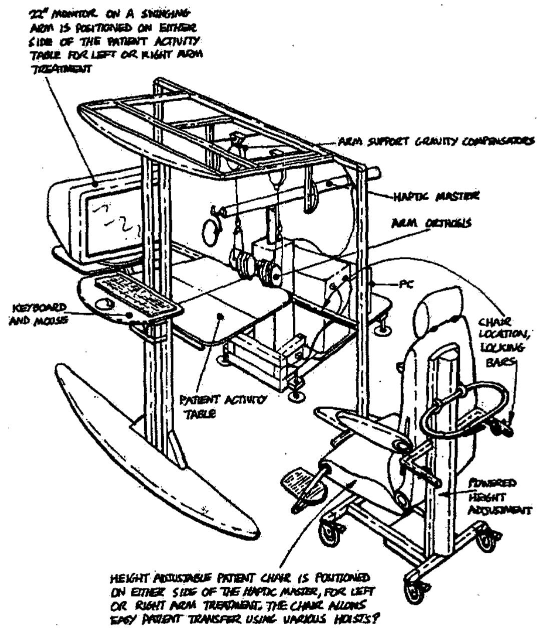

Fig. 4. Gentle/s system, Robot mediated therapy

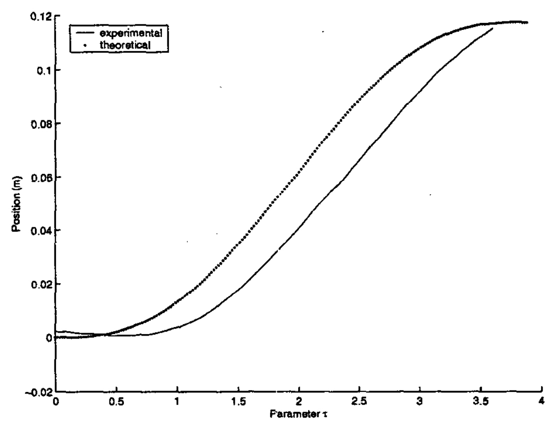

Fig. 5. Experimental results versus theoretical results

III. 'Up and Over' trajectory

The previous section shows minimum jerk trajectories for direct point to point movements. Actions in haptic and rehabilitation systems often involve lifting an object 'Up and Over' obstacles on a work surface. Achieving these trajectories requires an even order polynomial (Equation 12), that is no longer minimum jerk, for the vertical axis of the movement:

$$p = a + c\tau^2 + e\tau^4 + g\tau^6 \tag{12}$$

In order to lift the trajectory by amount of H and return to the start point, these additional assumptions are made:

- Start and end positions are the same:

$$P_{start} = P_{end}$$

- Movement is symmetric:

$$p|_{\tau=0} = P_{start} + H = a$$

leading to the following coefficients:

$$g = -H \tag{13}$$

$$c = -3H \tag{14}$$

$$e = 3H \tag{15}$$

IV. Super Positioning

Movements like stacking cones on the top of each other or reaching for a shelf are not straight-line movements by nature. We used a technique similar to the one used by Flash and Henis [11] to achieve movements that are not straight-line movements or movements that are non-symmetrical by nature. New alterations can be made by superimposing two different trajectories to create the third one. Consider two dif-

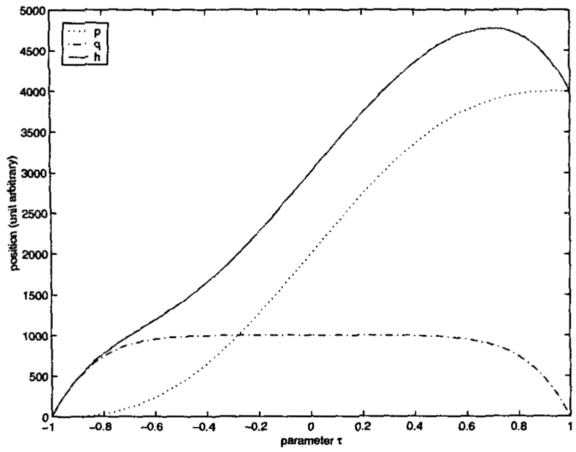

Fig. 6. Superimposed positions

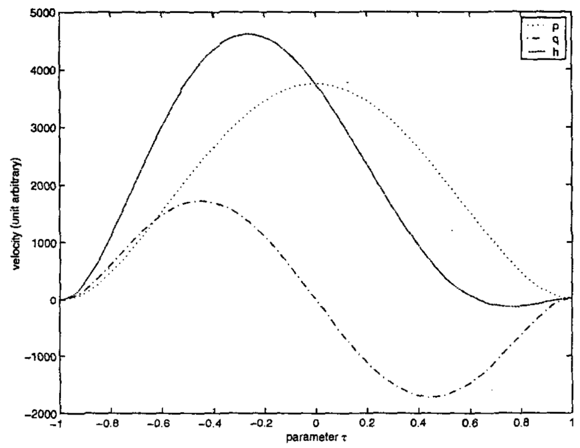

Fig. 7. Superimposed Velocities

ferent instance of the polynomials given below:

$$p = a + b\tau + d\tau^3 + f\tau^5 + h\tau^7 \tag{16}$$

$$q = a + c\tau^2 + e\tau^4 + g\tau^6 \tag{17}$$

By adding these two different polynomials vectorially, we can create a different trajectory (Figures 6 & 7):

$$h = p + q \tag{18}$$

The parameter τ for input polynomials (Equations 16 & 17) can be mapped to time differently therefore second trajectory do not need to begin at $\tau = -1$. To achieve a smooth resultant trajectory, we need to minimise jerk for sub-movements p and q separately.

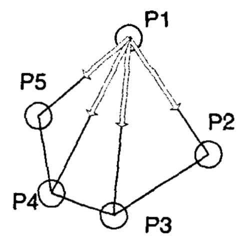

Fig. 8. Fork (target selection)

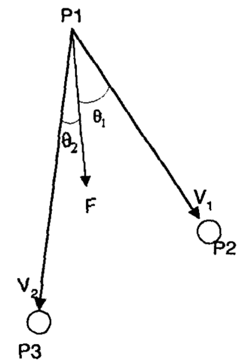

Fig. 9. vector dot products

V. Fork

One important aspect of this work is on machine mediated physiotherapies following a stroke, when a patient can receive intensive, challenging and daily exercise therapies. To promote the decision making process the patient is given a choice of targeted movements (Figure 8). The robot can then sense the intention of the patient via different ways. We used the force sensor mounted on the *HapticMaster*'s end-effector to sense the intention of the patient when he selects the targets. For this purpose, the Fork algorithm was developed and implemented (Figure 9). Fork is basically a target selection algorithm. Given the choice of the targets at P_2 and P_3, the user attempts to begin the movement toward one target. We can use vector dot products to detect the direction and amount of force exerted by the user toward the target. Vector dot product of two matrix, shows the projection of one vector on to the other, and by projecting the user's force vector, on to vectors $\vec{V_1}$ and $\vec{V_2}$, we can detect which target is selected by the user. We Know that:

$$\vec{V_1} = P_2 - P_1 \tag{19}$$

$$\vec{V_2} = P_3 - P_1 \tag{20}$$

If user is exerting a force $\vec{F}$ on the haptic interface, then Θ_1 and Θ_2 can be estimated as:

$$\cos\Theta_1 = \frac{\vec{F}.\vec{V_1}}{|\vec{F}||\vec{V_1}|} \tag{21}$$

$$\cos\Theta_2 = \frac{\vec{F}.\vec{V_2}}{|\vec{F}||\vec{V_2}|} \tag{22}$$

We also know that, if $\cos\Theta_1 \geq 0$, then $0 \leq \Theta_1 \leq \pi$, which means that both vectors are in the same side of the plane normal to the applied force. So the algorithm for detecting the target becomes:
Step 1 Calculate $\vec{V_1}$ and $\vec{V_2}$ using Equations 19 and 20
Step 2 Calculate $\cos\Theta_1$ and $\cos\Theta_2$ using Equations 21 and 22
Step 3 If $\cos\Theta_1 \geq 0$ OR $\cos\Theta_2 \geq 0$ AND $\vec{F} \geq$ a threshold value then:
Step 3.1 If $\cos\Theta_1 \geq \cos\Theta_2$ then $\Theta_2 \geq \Theta_1$ which means "Target P_2 is selected"
else If $\cos\Theta_2 \geq \cos\Theta_1$ then $\Theta_1 \geq \Theta_2$ which means "Target P_3 is selected"
step 3.2 initiate minimum jerk trajectory to appropriate target.
In order to find the target between more than two points, the algorithm becomes to find the maximum positive value for $\cos(q_i)$ where i is the index number assigned to targets on the fork junction.

Fork algorithm was used in *'ActiveAssisted'* and *'Active'* therapy modes. In these modes, the patient is in later stages of recovery after stroke and is able to initiate a movement but often not able to finish the movement on his own, or lacks the movement coordination. For this purpose, we used Fork algorithm to sense movement initiations and to start assisting patient toward his selected targets.

VI. Time mapping

One advantage of using a parameter such as τ in our polynomial implementation is that, we can scale this parameter to time using a linear scaling or quadratic scaling and dependent on scaling, different movements are achievable. Equation 23 can be used to scale time parameter t to τ linearly with respect to $t_{Start} = 0$ and $t_{End} = duration$.

$$1 + \left(\frac{2}{t_{end} - t_{start}}\right) * (t - t_{end}) \tag{23}$$

Other time mappings are possible, for example we can use a quadratic mapping:

$$\tau = at^2 + bt + c \tag{24}$$

where:

$$0 \leq t \leq duration \tag{25}$$

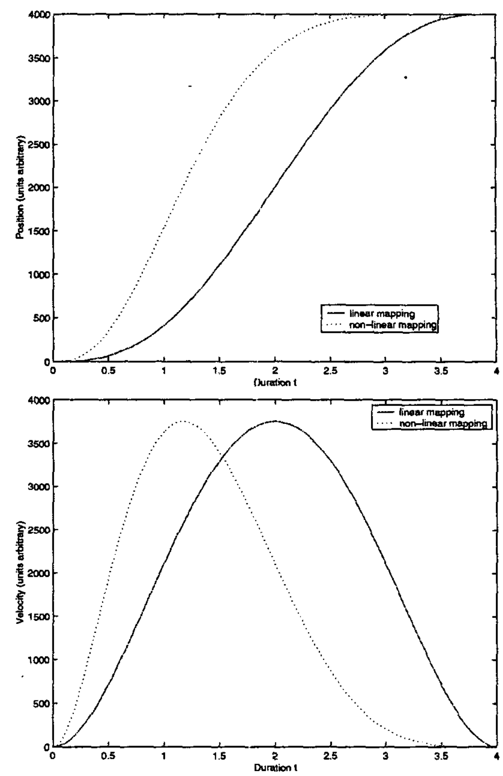

Fig. 10. time mapped positions and velocities

the coefficients of this quadratic can easily be found as:

$$c = -1 \tag{26}$$

$$a = \left(\frac{2}{d^2} - \frac{b}{d}\right) \tag{27}$$

Figures 10 shows positions and velocities, when τ is mapped linearly and compares it with the positions obtained with a non-linear mapping ($b = 1$). This allows easy implementation of non-symmetric non-minimum jerk movements.

VII. Application and implementation

Using the minimum jerk polynomials on Gentle/s robot (Figure 4), three different therapy mode were defined. During the session, patient's arm is attached to robot arm using an orthosis. *'Panic'* consideration was made as having an easy to detach mechanism as well as a mechanical breakaway. This will allow us to detach the patient from the robot as soon as a panic situation arises.

Our first therapy mode is *Passive* mode. In this mode, patient remain passive while the robot moves his arm on a predefined trajectory. This mode is mostly used for the patients that are in their early stages after stroke, as they lack the ability to move their arms. The trajectory was implemented using minimum jerk polynomials. Duration of the movement and system's assistive strength(spring and damper parameters) are set by the physiotherapist, according to the needs. When the robot reaches to the end point, it can reverse the movement back to the start point, or continue moving to the next position defined in the target list.

In our second therapy mode, $Active - Assisted$ mode, patients are able to initiate the movement but often not able to finish the movement correctly, or completely. In this mode, we used fork algorithm to sense patient's intention toward his selected target. Upon initiating the movement, robot helps the patient to finish the movement, based on the defined trajectory.

Our last therapy mode is *Active* mode, in this mode, patient's can move their arm toward the target but they lack the ability to coordinate their arm. Similar to previous mode, patient initiates the movement, but robot stays passive, until user deviates from the predefined trajectory. In this case, robot encourages the patient to return to the trajectory. Unlimited time is given to patient to complete the movement by using a different time mapping[8].

Patient can use these three different therapy modes in conjunction with the aid of visual cues. Exercises can be performed in three different environment, $'Real'$,$'virtual'$ and $'Augmented'$ (mixed-reality) environments. [9], [10].

VIII. Conclusion

In this paper we have introduced our new mathematical approach to trajectory control. Polynomials were used in conjunction with Minimum jerk theory in order to create smooth trajectories. We have introduced different approaches to polynomials to create more human-like movements. These approaches are useful to create non-symmetric non-minimum jerk movement as well as symmetric minimum jerk movements. We have also demonstrated that, using the polynomials we can control the position, velocity and acceleration as well as the duration of the movement and smoothness. Our algorithm for target selection and time mapping was also presented.

We have successfully implemented our mathematical approach in two experimental working systems for stroke rehabilitation. This allows us to create movement trajectories that encourage the so called 'Errorless Learning', in several modes where the patient must take an active part in movement initiation and execution. Further research is under going in the area of target selection to predict the position of the target and also in the area of obstacle avoidance.

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