559

Modeling, Identification and Control of a Redundant Planar 2-DOF Parallel Manipulator

Yao-Xin Zhang, Shuang Cong*, Wei-Wei Shang, Ze-Xiang Li, and Shi-Long Jiang

Abstract: In this paper, the dynamic controller design problem of a redundant planar 2-dof parallel manipulator is studied. Using the Euler-Lagrange equation, we formulate the dynamic model of the parallel manipulator in the joint space and propose an augmented PD controller with forward dynamic compensation for the parallel manipulator. By formulating the controller in the joint space, we eliminate the complex computation of the Jacobian matrix of joint angles with end-effector coordinate. So with less computation, our controller is easier to implement, and a shorter sampling period can be achieved, which makes the controller more suitable for high-speed motion control. Furthermore, with the combination of static friction model and viscous friction model, the active joint friction of the parallel manipulator is studied and compensated in the controller. Based on the dynamic parameters of the parallel manipulator evaluated by direct measurement and identification, motion control experiments are implemented. With the experiments, the validity of the dynamic model is proved and the performance of the controller is evaluated. Experiment results show that, with forward dynamic compensation, the augmented PD controller can improve the tracking performance of the parallel manipulator over the simple PD controller.

Keywords: Augmented PD control, identification, parallel manipulator, PD control.

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1. INTRODUCTION

Parallel manipulators have the advantages of high speed and high precision in the theory of mechanisms. This has opened up great opportunities for parallel manipulators to be applied in many fields. But it is difficult to make use of parallel manipulators in real applications. One of the difficulties lies in the

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controller design of parallel manipulators. Due to inherent closed-loop constraints, the joints of parallel manipulators are tightly coupled and the dynamic characteristics are always highly nonlinear. With these difficulties, it is usually difficult to move the end-effector of a parallel manipulator along a trajectory accurately and quickly, and the controller design for a parallel manipulator is a work full of challenge, which has aroused the interests of researchers in recent years.

In literature, there are two basic controller design strategies for parallel manipulators: the kinematic control strategy and the dynamic control strategy. The kinematic control strategy is based on the assumption that the joints of parallel manipulators are all independent and the parallel manipulator can be decoupled into a group of single axis control systems. So the kinematic control method always results in a group of individual controllers, each for an active joint of the parallel manipulator. PID controller [1-4], NPID controller [5,6], fuzzy logic controller [7,8] and neural network controller [9] all belong to kinematic controllers. With the independent joint assumption, no priori-knowledge of parallel manipulator dynamics is needed in the kinematic controller design, so the complex computation of dynamics can be avoided and the controller design problem can be simplified greatly. This makes the great sense for the real-time control of parallel manipulators when powerful processors that can execute complex algorithms

rapidly are not available. But the performance of kinematic controllers is limited, for the neglect of joints coupling and the nonlinear dynamics of parallel manipulators. The synchronous control method [10] can be used to solve the problem of joints coupling and improve the accuracy of the trajectory tracking, but the nonlinear dynamics is still an unsolved problem for kinematic controllers, especially under high-speed motion control.

Unlike the kinematic control strategy, the dynamics of parallel manipulators is taken into account in the dynamic control strategy. So the nonlinear dynamics of parallel manipulators can be compensated, and the higher tracking performance can be achieved with dynamic controllers. Based on approximated linear dynamic models, dynamic controllers were proposed for parallel manipulators with improved tracking performance [11-13]. Although the controller is easy to implement by adopting the approximated linear model of parallel manipulators, the effect of the controller is limited in a small region of the configuration space for the inaccurate compensation of nonlinear dynamics. To solve this problem, full nonlinear dynamic models were adopted in the controller design in [14-16] and adaptive methods were also used to improve the tracking performance of parallel manipulators [4,17,18]. Furthermore, to gain a deep insight of the parallel manipulator dynamics, Liu developed an unified geometric approach for the modeling and the control of parallel manipulators [19], and Aghili proposed a more generalized method to solve the dynamic problem of parallel manipulators [20]. All these works have placed solid foundation for the further research of dynamic control strategy.

The control problem of parallel manipulators with redundant actuators is also of great interest for researchers. For redundant actuated parallel manipulators, infinite feasible solutions of control torques can be obtained to track a single desired trajectory. Among the feasible solutions, the best one may be picked out to minimize a predefined optimal function, so additional optimal control objective can be achieved during the trajectory tracking. In literature, redundant actuation is used to minimize the control torque [15,16], achieve optimal load distribution among actuators [21], improve the manipulability of parallel manipulators [22], obtain desired end-effector stiffness [23] and avoid the transmission backlash [24].

In this paper, the dynamic controller design problem of a redundant planar 2-dof parallel manipulator is studied. Using the Euler-Lagrange equation, we formulate the dynamic model of the parallel manipulator in the joint space and propose an augmented PD controller for the parallel manipulator. Compared with the controllers proposed by Kock [21] and Chen [16] which were formulated in the workspace, our controller is formulated in the joint

space, and the complex computation of the Jacobian matrix of the joint angles with end-effector coordinate is eliminated. With less computation, the controller proposed here is easier to implement, and a shorter sampling period can be achieved. This makes the controller more suitable for high-speed motion control. Furthermore, with the combination of static friction model and viscous friction model, the active joint friction of the parallel manipulator is compensated in the controller, while only the viscous friction of joints was considered in the previous controllers designed by Chen [16] and Yiu [4]. With trajectory tracking experiments, the validity of the dynamic model is proved and the performance of the controller is evaluated. Experiment results show that, with forward dynamic compensation, the augmented PD controller can improve the tracking performance of the parallel manipulator over simple PD controller.

The paper is organized as follows. In Section 2, the structure of the manipulator is described and the kinematics is formulated. In Section 3, with the Euler-Lagrange equation, we formulate the dynamics of the parallel manipulator in the joint space. In Section 4, we formulate the active joint friction of the parallel manipulator with the combination of static friction and viscous friction. By least square method, we identify the unknown friction parameters and other unknown dynamic parameters of the parallel manipulator. In Section 5, based on the dynamic model in Section 3 and the friction model in Section 4, we propose a simple PD controller and an augmented PD controller with forward friction compensation for the parallel manipulator. With the two controllers, linear and circular trajectory tracking experiments are performed and experiment results are reported too. Section 6 concludes the paper with several important remarks.

2. MECHANISMS AND INVERSE KINEMATICS

The parallel manipulator to be controlled is Googol Tech. Ltd's GPM2002, which is a redundant planar 2-dof parallel manipulator. As shown in Fig. 1, the GPM2002 consists of 6 links denoted as $L_{ai}, L_{bi}, i = 1, 2, 3$, 3 active joints located at A_1, A_2, A_3 and 3 passive joints locate at B_1, B_2, B_3 respectively. The end-effector of the parallel manipulator coincides with O in Fig. 1.

According to Fig. 1, a reference frame is established in the workspace of the parallel manipulator and the unit of the frame is millimeter. With the reference frame, the zero positions of joint angles are all defined as the positive direction of the X axis of the frame, and the positive directions of angles are all the anticlockwise direction. The active joints of the
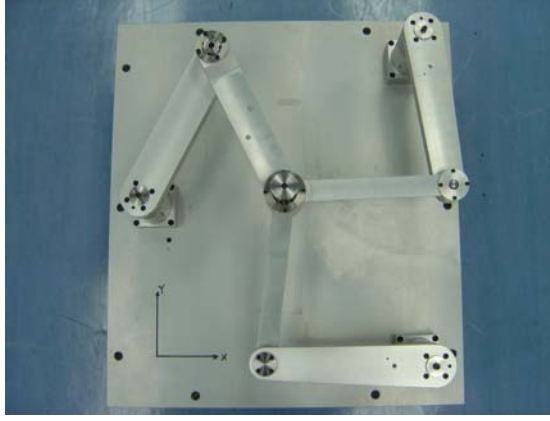
(a) Mechanisms of the GPM2002.
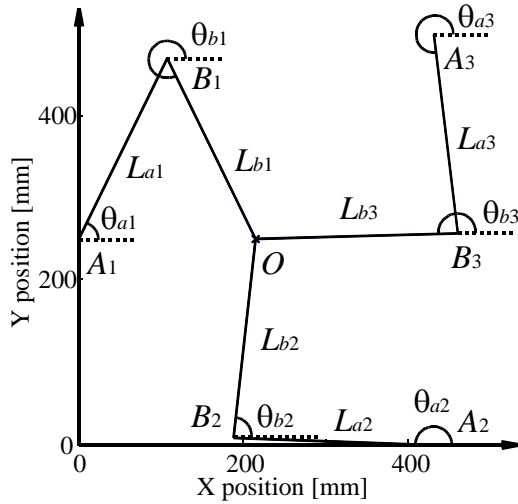
(b) Structure of the GPM2002.

Fig. 1. Redundant planar 2-dof parallel manipulator GPM2002.

GPM2002 are actuated by AC servo motors. All of the motors are embedded with internal absolute encoders as joint sensors, and controlled by motion control board GT-400-PCI-SV from Googol Tech. Ltd.

For parallel manipulators, the desired trajectories to be tracked are always described by end-effector coordinates, while the only available feedback signals are active joint angles. So the tracking error of the manipulator can't be calculated directly. To solve this problem, inverse kinematics of the GPM2002 can be employed and the desired positions of joints corresponding to the desired position of the end-effector are obtained through inverse kinematic transformation. Then the tracking error can be calculated by subtracting real joint angles from desired joint angles.

For parallel manipulators GPM2002, the lengths of 6 links are same and all equal to 244mm. Denote the lengths of links by symbol l, and the coordinates of active joint positions A_1, A_2, and A_3 by symbols

(x_{ai}, y_{ai}), $i = 1, 2, 3$ which equal to (0,250), (433,0), and (433,500), respectively. Then from the end-effector coordinate (x, y), active joint angles $\theta_{ai}, i = 1, 2, 3$ can be calculated through following equations:

$$\theta_{ai} = arctg\, 2(x - x_{ai}, y - y_{ai}) + \arccos\left(\frac{d_i}{2l}\right), \quad (1)$$
$$i = 1, 2, 3.$$

In (1), symbols d_i, $i = 1, 2, 3$ are defined as follows:

$$d_i = \sqrt{(x - x_{ai})^2 + (y - y_{ai})^2}, \, i = 1, 2, 3. \quad (2)$$

Then the passive joint angles θ_{bi}, $i = 1, 2, 3$ can be obtained through following equations:

$$\theta_{bi} = arctg\, 2(x - x_{bi}, y - y_{bi}) \in (-\pi, \pi], \quad (3)$$
$$i = 1, 2, 3.$$

In (3), symbols x_{bi}, y_{bi}, $i = 1, 2, 3$ refer to the coordinates of passive joint locations in the reference frame, which can be calculated through following equations:

$$\begin{cases} x_{bi} = x_{ai} + l\cos\theta_{ai} \\ y_{bi} = y_{ai} + l\sin\theta_{ai}, \quad i = 1, 2, 3. \end{cases} \quad (4)$$

With (1) and (3), the angles of active joints and the passive joints corresponding to the end-effector coordinate (x, y) can be calculated, so the tracking error of the parallel manipulator can be expressed by the tracking errors of joints.

3. DYNAMIC MODEL BASED ON THE EULER-LAGRANGE EQUATION

The parallel manipulator GPM2002 consists of 3 serial kinematic chains, and each of kinematic chains is a planar 2-dof serial manipulator. The motion of the parallel manipulator equals to the motion of the 3 serial kinematic chains under the closed-loop constraints, so the dynamics of the parallel manipulator can be formulated by combining the dynamics of 3 serial kinematic chains under the constraint forces.

3.1. Dynamic model of serial kinematic chain

Cutting the parallel manipulator at the end-effector O, one can divide the GPM2002 into 3 serial kinematic chains, each of which is a planar 2-dof serial manipulator consisting of 2 links and 2 joints.

The structure of serial kinematic chain is shown in Fig. 2. According to Fig. 2, the links are denoted by symbols L_a and L_b, and the joint angles are
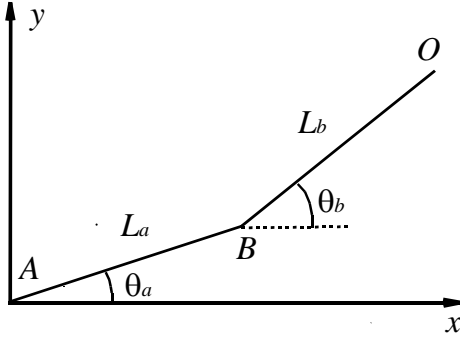
Fig.2. Structure of serial kinematic chain.

denoted by θ_a and θ_b. The base of serial kinematic chain coincides with A and the end-effector coincides with O. The lengths of 2 links are same, which is denoted by l.

The links of the kinematic chain are supposed to be ideal rigid bodies. Because the kinematic chain moves in the horizontal plane, the effect of the gravity can be ignored. So the mechanical energy KE_a and KE_b of links equal to their kinetic energy, and can be expressed as following equations:

$$\begin{cases} KE_a = \dfrac{1}{2} J_a \dot{\theta}_a^2 + \dfrac{1}{2} m_a \left(\dot{x}_{ca}^2 + \dot{y}_{ca}^2 \right) \\ KE_b = \dfrac{1}{2} J_b \dot{\theta}_b^2 + \dfrac{1}{2} m_b \left(\dot{x}_{cb}^2 + \dot{y}_{cb}^2 \right), \end{cases} \quad (5)$$

where symbols m_a and m_b refer to link masses, J_a and J_b refer to the moments of inertia of links relative to mass center, (x_{ca}, y_{ca}) and (x_{cb}, y_{cb}) refer to the mass center coordinates of links.

Denote the distances between the mass center and the joint of links by r_a and r_b respectively, one can express the coordinates of mass centers as following equations:

$$\begin{cases} x_{ca} = r_a \cos \theta_a \\ y_{ca} = r_a \sin \theta_a \\ x_{cb} = l \cos \theta_a + r_b \cos \theta_b \\ y_{cb} = l \sin \theta_a + r_b \sin \theta_b. \end{cases} \quad (6)$$

Substitute (6) into (5), one can have:

$$\begin{aligned} KE_a &= \frac{1}{2} J_a \dot{\theta}_a^2 + \frac{1}{2} m_a r_a^2 \dot{\theta}_a^2, \\ KE_b &= \frac{1}{2} J_b \dot{\theta}_b^2 + \frac{1}{2} m_b \left(l^2 \dot{\theta}_a^2 + r_b^2 \dot{\theta}_b^2 \right) \\ &\quad + m_b l r_b \cos \left(\theta_a - \theta_b \right) \dot{\theta}_a \dot{\theta}_b. \end{aligned} \quad (7)$$

And the total mechanical energy of the kinematic chain can be formulated as follows:

$$KE = \frac{1}{2} \alpha \dot{\theta}_a^2 + \frac{1}{2} \beta \dot{\theta}_b^2 + \gamma \cos \left(\theta_a - \theta_b \right) \dot{\theta}_a \dot{\theta}_b, \quad (8)$$

where symbols α, β, γ are defined as follows:

$$\begin{cases} \alpha = J_a + m_a r_a^2 + m_b l^2 \\ \beta = J_b + m_b r_b^2 \\ \gamma = m_b l r_b. \end{cases} \quad (9)$$

Let $\theta = [\theta_a, \theta_b]^T$ be the vector of joint angles, $\tau = [\tau_a \ \tau_b]^T$ be the vector of joint torques, $f = [f_a \ f_b]^T$ be the vector of joint frictions. Then the Euler-Lagrange equation of the kinematic chain can be expressed as the following equation:

$$\frac{d}{dt} \left(\frac{\partial L}{\partial \dot{\theta}} \right) - \frac{\partial L}{\partial \theta} = \tau - f. \quad (10)$$

Expands (10), one can get the dynamics of the kinematic chain as following equation:

$$M \ddot{\theta} + C \dot{\theta} = \tau - f. \quad (11)$$

In (11), symbols M, C are defined as follows:

$$M = \begin{bmatrix} \alpha & \gamma \cos \left(\theta_a - \theta_b \right) \\ \gamma \cos \left(\theta_a - \theta_b \right) & \beta \end{bmatrix}, \quad (12)$$

$$C = \begin{bmatrix} 0 & \gamma \sin \left(\theta_a - \theta_b \right) \dot{\theta}_b \\ -\gamma \sin \left(\theta_a - \theta_b \right) \dot{\theta}_a & 0 \end{bmatrix}. \quad (13)$$

3.2. Dynamic model of the parallel manipulator

Similar with (9), constant coefficients can be defined for each serial kinematic chains

$$\begin{cases} \alpha_i = J_{ai} + m_{ai} r_{ai}^2 + m_{bi} l^2 \\ \beta_i = J_{bi} + m_{bi} r_{bi}^2 \\ \gamma_i = m_{bi} l r_{bi}, \quad\quad\quad i = 1, 2, 3, \end{cases} \quad (14)$$

where m_{ai}, m_{bi} refer to the masses of links, J_{ai}, J_{bi} refer to the moments of inertia of the links relative to the mass centers, r_{ai}, r_{bi} refer to the distances between mass centers and joints of links, and l refers to the length of links.

Then with (11), the dynamics of serial kinematic chains can be established. Combine the dynamics of the 3 serial chains and consider the constraint forces due to the closed-loop constraints, the dynamic model of the parallel manipulator can be formulated in the joint space as follows:

$$M\ddot{\theta} + C\dot{\theta} = \tau - f + A^T\lambda, \tag{15}$$

where $\theta = [\theta_{a1}, \theta_{a2}, \theta_{a3}, \theta_{b1}, \theta_{b2}, \theta_{b3}]^T$ refers to the vector of joint positions, $\tau = [\tau_{a1}, \tau_{a2}, \tau_{a3}, \tau_{b1}, \tau_{b2}, \tau_{b3}]^T$ refers to the vector of input torques, $f = [f_{a1}, f_{a2}, f_{a3}, f_{b1}, f_{b2}, f_{b3}]^T$ refers to the vector of joint frictions and $A^T\lambda$ refers to the vector of constraint forces.

The inertia matrix M and coriolis matrix C in (15) are defined as follows:

$$M = \begin{bmatrix} \alpha_1 & 0 & 0 & \gamma_1 c_{ab1} & 0 & 0 \\ 0 & \alpha_2 & 0 & 0 & \gamma_2 c_{ab2} & 0 \\ 0 & 0 & \alpha_3 & 0 & 0 & \gamma_3 c_{ab3} \\ \gamma_1 c_{ab1} & 0 & 0 & \beta_1 & 0 & 0 \\ 0 & \gamma_2 c_{ab2} & 0 & 0 & \beta_2 & 0 \\ 0 & 0 & \gamma_3 c_{ab3} & 0 & 0 & \beta_3 \end{bmatrix}, \tag{16}$$

$$C = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ -\gamma_1 s_{ab1}\dot{\theta}_{a1} & 0 & 0 \\ 0 & -\gamma_2 s_{ab2}\dot{\theta}_{a2} & 0 \\ 0 & 0 & -\gamma_3 s_{ab3}\dot{\theta}_{a3} \end{bmatrix}$$

$$\begin{matrix} \gamma_1 s_{ab1}\dot{\theta}_{b1} & 0 & 0 \\ 0 & \gamma_2 s_{ab2}\dot{\theta}_{b2} & 0 \\ 0 & 0 & \gamma_3 s_{ab3}\dot{\theta}_{b3} \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \Bigg]. \tag{17}$$

In (16) and (17), symbols $c_{abi}, i = 1,2,3$ are defined as $c_{abi} = \cos(\theta_{ai} - \theta_{bi}), i = 1,2,3$, and symbols $s_{abi}, i = 1,2,3$ are defined as $s_{abi} = \sin(\theta_{ai} - \theta_{bi}), i = 1,2,3$.

As shown in (15), the dynamics of the GPM2002 is formulated in the joint space and redundant variables are involved in the formulation, so the constraint force $A^T\lambda$ is included explicitly to guarantee the closed-loop constraints of the parallel manipulator are satisfied at every instant. Here matrix A is the differential of the closed-loop constrained equations, and multiplier λ denotes the magnitude of constraint forces.

Adopting the symbols defined in Section 2, one can formulate the closed-loop constraints of the GPM2002 as following equation:

$$H(\theta) = \begin{bmatrix} x_{a1} + lc_{a1} + lc_{b1} - x_{a2} - lc_{a2} - lc_{b2} \\ y_{a1} + ls_{a1} + ls_{b1} - y_{a2} - ls_{a2} - ls_{b2} \\ x_{a1} + lc_{a1} + lc_{b1} - x_{a3} - lc_{a3} - lc_{b3} \\ y_{a1} + ls_{a1} + ls_{b1} - y_{a3} - ls_{a3} - ls_{b3} \end{bmatrix} = 0. \tag{18}$$

In (18), symbols $s_{ai}, c_{ai}, i = 1,2,3$ are defined as $s_{ai} = \sin(\theta_{ai}), c_{ai} = \cos(\theta_{ai}), i = 1,2,3$, and symbols $s_{bi}, c_{bi}, i = 1,2,3$ are defined as $s_{bi} = \sin(\theta_{bi}), c_{bi} = \cos(\theta_{bi}), i = 1,2,3$.

Differentiating (18), one can have:

$$\frac{dH(\theta)}{dt} = \frac{\partial H(\theta)}{\partial\theta}\dot{\theta} = A(\theta)\dot{\theta} = 0. \tag{19}$$

Then matrix A can be formulated as follows:

$$A = \begin{bmatrix} -ls_{a1} & ls_{a2} & 0 & -ls_{b1} & ls_{b2} & 0 \\ lc_{a1} & -lc_{a2} & 0 & lc_{b1} & -lc_{b2} & 0 \\ -ls_{a1} & 0 & ls_{a3} & -ls_{b1} & 0 & ls_{b3} \\ lc_{a1} & 0 & -lc_{a3} & lc_{b1} & 0 & -lc_{b3} \end{bmatrix}. \tag{20}$$

With (15), (16), (17), and (20), the dynamics of the GPM2002 is established in the joint space.

4. IDENTIFICATION OF DYNAMIC PARAMETERS

For the GPM2002, the dynamic parameters of links such as masses m_{ai}, m_{bi}, moments of inertia relative to the mass centers J_{ai}, J_{bi}, distances between mass centers and joints r_{ai}, r_{bi} and length l, can be measured directly. But the constraint forces and the joint frictions in (15) are still unknown. In this section, the unknown constraint force $A^T\lambda$ is eliminated by projection method, and the active joint friction is modeled with the combination of static friction model and viscous friction model. By formulating the dynamics of the GPM2002 as a group of linear equations about dynamic parameters, the unknown friction parameters and other unknown dynamic parameters of the GPM2002 are identified with least squared method.

4.1. Elimination of constraint forces

The constraint force magnitude λ in (15) is unknown, and it is difficult to be measured directly. Fortunately this unknown term can be eliminated by projection method [19].

With (15), the magnitude of constraint force can be calculated through following equations:

$$\lambda = \left(AM^{-1}A^T\right)^{-1}AM^{-1}\left(M\ddot{\theta} + C\dot{\theta} - (\tau - f)\right). \tag{21}$$

Substituting (21) into (15), one can obtain following equations:

$$\left(I - A^T \left(A M^{-1} A^T \right)^{-1} A M^{-1} \right) \left(M \ddot{\theta} + C \dot{\theta} - (\tau - f) \right) = 0.$$

(22)

With (22), one can eliminate constraint forces by projecting the dynamic model onto the image space of matrix $P = I - A^T \left(A M^{-1} A^T \right)^{-1} A M^{-1}$. The parameters of P consist of the dimension parameters of the parallel manipulator and the inertia parameters of links, all of which can be measured directly. So the matrix P can be calculated directly and constraint forces can be eliminated with (22).

4.2. Model of joint frictions

In (22), joint friction vector f is still an unknown term. Here, static friction model and viscous friction model are adopted to formulate the joint frictions. Furthermore, the zero drift of the control board that results a nonzero output torque for motors is considered too. For the GPM2002, the passive joints are well lubricated, so the frictions of passive joints can be ignored and only the frictions of active joints are considered. The frictions of active joints can be formulated as following equations:

$$f_i = d_i + sign\left(\dot{\theta}_{ai} \right) f_{si} + f_{vi} \dot{\theta}_{ai}, \quad i = 1, 2, 3. \tag{23}$$

Here, symbols d_i, $i = 1, 2, 3$ refer to the zero drifts of control board, symbols f_{si}, $i = 1, 2, 3$ refer to the static frictions, and symbols f_{vi}, $i = 1, 2, 3$ refer to viscous friction coefficients.

4.3. Formulation of parameter identification

In (22) and (23), the dynamic parameters of links can be measured directly and only the friction parameters have to be identified. Besides, the unit of output torque of the motors has to be transformed to the unit of numerical control value, so one more unknown unit transformation coefficient has to be identified. Based on (22), following equation can be got:

$$P \left(M \ddot{\theta} + C \dot{\theta} + f - \tau \right) = P \cdot D \cdot k + P \cdot f - P \cdot \tau = 0.$$

(24)

In (24), symbol k refers to the unknown unit transformation coefficient, while symbol D refers to the sum of the inertia term and the coriolis term of the manipulator. Let $P = \left[P_{ij} \right]_{6 \times 6}$ and $D = \left[D_j \right]_{6 \times 1}$, one can formulate (24) as a group of linear equations about the unknown dynamic parameters.

$$W_i p = t_i, \quad i = 1, \cdots, 6. \tag{25}$$

In (25), symbols W_i, t_i, p are defined as follows:

$$p = \begin{bmatrix} k & f_{v1} & f_{v2} & f_{v3} & d_1 + f_{s1} & d_1 - f_{s1} \\ d_2 + f_{s2} & d_2 - f_{s2} & d_3 + f_{s3} & d_3 - f_{s3} \end{bmatrix}^T, \tag{26}$$

$$W_i = \begin{bmatrix} \sum_{j=1}^{6} P_{ij} D_j & P_{i1} \dot{\theta}_{a1} & P_{i2} \dot{\theta}_{a2} & P_{i3} \dot{\theta}_{a3} \\ u_{i1} & l_{i1} & u_{i2} & l_{i2} & u_{i3} & l_{i3} \end{bmatrix}, \quad i = 1, \cdots, 6, \tag{27}$$

$$t_i = \sum_{j=1}^{3} P_{ij} \tau_{aj}, \quad i = 1, \cdots, 6. \tag{28}$$

For simplicity, parameter combinations $d_i + f_{si}$ and $d_i - f_{si}$ are viewed as unknown parameters to be identified as shown in (26). In (27), the coefficients $u_{ik}, l_{ik}, k = 1, 2, 3$ are determined by following rules: $u_{ik} = P_{ik}$, $l_{ik} = 0$ when $\dot{\theta}_{ak} > 0$ and $u_{ik} = 0$, $l_{ik} = P_{ik}$ when $\dot{\theta}_{ak} < 0$. In (28), only the input torque of active joints is considered, for the input torque of passive joints always equals zero.

There are 10 parameters to be identified in (25) and 2 independent equations can be got for each sampling point. So a group of linear equation about the unknown parameters can be got with the sampling data of a continuous trajectory. By solving the linear equations with least squared method, the unknown parameters can be identified.

4.4. Parameter identification experiment

The dynamic parameters of links can be measured directly, as shown in Table 1.

To identify the friction parameters and the unknown dynamic parameters of the GPM2002, the end-effector of the parallel manipulator is driven to track a circular trajectory. Based on (25), a group of linear equations about the unknown parameters can be obtained with the sampling data of trajectory, and the

Table 1. Dynamic parameters of the links.

	Mass (Kg)	Length (m)	Distance between the mass center and the joint (m)	Moment of inertia relative to the mass center (Kg*m²)
L_{a1}	1.2525	0.2440	0.1156	0.0124
L_{a2}	1.3663	0.2440	0.0657	0.0122
L_{a3}	1.3663	0.2440	0.0657	0.0122
L_{b1}	1.0771	0.2440	0.1621	0.0098
L_{b2}	0.4132	0.2440	0.1096	0.0036
L_{b3}	0.4132	0.2440	0.1096	0.0036

Table 2. Results of the identification experiment.

	Active joint 1	Active joint 2	Active joint 3
f_v	1425.5	2363.1	1947.0
$d + f_s$	625.4	392.6	417.8
$d - f_s$	-923.6	-567.8	-651.6
k	1481.6		

unknown parameters can be identified with least squared method. In the identification experiment, the velocity of joints is got by the numerical differentiation of joint positions, and the acceleration of joints is got by numerical differentiation of joint velocity. Since the acceleration of joints is calculated by two-time numerical differentiations, the joint accelerations are quite noisy. A low-pass filter is adopted to filter the velocity signal before the calculation of acceleration.

Based on the parameter values in Table 1, the unknown parameters of the GPM2002 are identified with MATLAB program and results are shown in Table 2.

5. CONTROLLER DESIGN AND MOTION CONTROL EXPERIMENTS

With the dynamic formulation (22) and (23), a simple PD controller and an augmented PD controller are proposed in this section. With the controllers, motion control experiments are performed and experiment results are reported in this section too.

5.1. Simple PD controller

Denote the desired trajectory in the joint space by symbol $\theta_d = \begin{bmatrix} \hat{\theta}_{a1} & \hat{\theta}_{a2} & \hat{\theta}_{a3} & \hat{\theta}_{b1} & \hat{\theta}_{b2} & \hat{\theta}_{b3} \end{bmatrix}^T$, where $\hat{\theta}_{ai}, i = 1, 2, 3$ and $\hat{\theta}_{bi}, i = 1, 2, 3$ refer to desired active joint positions and desired passive joint positions respectively, and denote the actual trajectory by symbol $\theta = \begin{bmatrix} \theta_{a1} & \theta_{a2} & \theta_{a3} & \theta_{b1} & \theta_{b2} & \theta_{b3} \end{bmatrix}^T$ where $\theta_{ai}, i = 1, 2, 3$ and $\hat{\theta}_{bi}, i = 1, 2, 3$ refer to actual active joint positions and actual passive joint positions respectively.

The structure of the simple PD controller is shown in Fig. 3.

Command torque of three motors can be calculated by following equations:

$$\tau_i = K_p e_i + K_d \dot{e}_i, \quad i = 1, 2, 3. \tag{29}$$

In (29), K_p refers to the proportion gain of the PD controller, K_d refers to the derivative gain of the PD
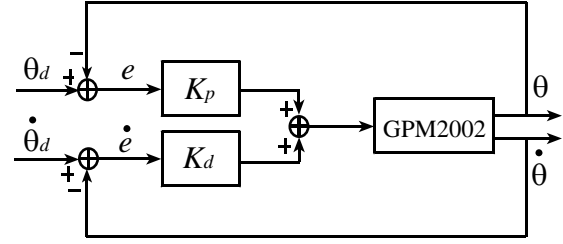

Fig. 3. Structure of simple PD controller.

controller, and e_i refers to the tracking error of active joints which can be formulates as follows:

$$e_i = \hat{\theta}_{ai} - \theta_{ai}, \quad i = 1, 2, 3. \tag{30}$$

5.2. Augmented PD controller

The structure of augmented PD controller is shown in Fig. 4.

For the GPM2002, the dynamics including the friction at active joints dominates the dynamics of the parallel manipulator, so we ignore the dynamics including the friction at passive joints. Then computation of the command torque of motors can be simplified further, which can be calculated by following equations:

$$\tau_i = K_p e_i + K_d \dot{e}_i + k D_i + f_i, \quad i = 1, 2, 3, \tag{31}$$

where K_p refers to the proportion gain of the PD controller, K_d refers to derivative gain and k refers to unit transformation coefficients.

In (31), symbol e_i refers to the tracking error of active joint, D_i refers to the compensation of dynamics and f_i refers to the compensation of frictions.

$$D_i = \sum_{j=1}^{3} \left(M_{ij} \ddot{\theta}_{aj} + C_{ij} \dot{\theta}_{aj} \right) \\ + \sum_{j=1}^{3} \left(M_{ij+3} \ddot{\theta}_{bj} + C_{ij+3} \dot{\theta}_{bj} \right), \quad i = 1, 2, 3, \tag{32}$$

$$f_i = d_i + sign\left(\dot{\hat{\theta}}_{ai} \right) f_{si} + f_{vi} \dot{\hat{\theta}}_{ai}, \quad i = 1, 2, 3. \tag{33}$$
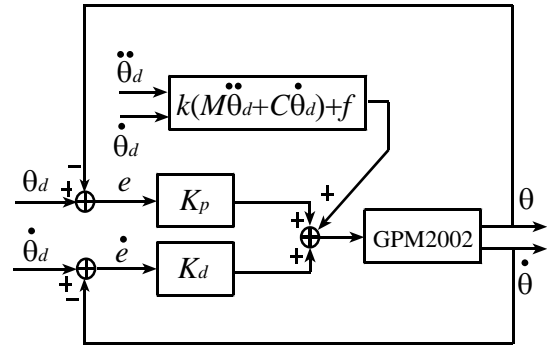

Fig. 4. Structure of augmented PD controller.

Here symbols d_i, $i = 1,2,3$ refer to the zero drifts of the motion control board, f_{si}, $i = 1,2,3$ refer to the static frictions of active joints, f_{vi}, $i =1,2,3$ refer to the viscous friction coefficient of active joints.

5.3. Trajectory tracking experiment results

   With the controllers proposed above, motion control experiments are implemented. In the experiments, the end-effector of the GPM2002 is driven to track a linear trajectory and a circular trajectory both for 2 times, once with lower velocity and once with higher velocity. The control system runs on a Pentium III CPU at 333MHz with the sampling period 2ms. In the controller, the unit of feedback signal is *rad* for joint position and *rad/*2ms for joint velocity. The value of the proportion gain K_p of simple PD controller is 800000, and derivative gain K_d is 5000000. For the augmented PD controller, the same proportion gain and derivative gain with simple PD controller are adopted.

   Furthermore, root-square mean error (*RSME*) is adopted to evaluate the performance of the simple PD controller and the augmented PD controller [16].

$$RSME = \sqrt{\frac{1}{N}\sum_{i=1}^{N}\left(\left(\hat{x}_i - x_i\right)^2 + \left(\hat{y}_i - y_i\right)^2\right)} \qquad (34)$$

In (34), symbols $\hat{x}_i, \hat{y}_i$ refer to the coordinates of the ith interpolation point of desired trajectory and x_i, y_i refer to the coordinates of corresponding point of real trajectory.

   The starting point of linear trajectory is (150,300) and the ending point is (300,150). The velocity profile of desired linear trajectory is a T-curve. First the end-effector of the GPM2002 is driven to track the line with maximum velocity 0.2m/s and acceleration 4m/s^2, then with maximum velocity 0.5m/s and acceleration 10m/s^2. Tracking errors of the end-effector are shown in Figs. 5 and 6, while the *RSME*s of the linear trajectory tracking experiments are shown in Table 3.
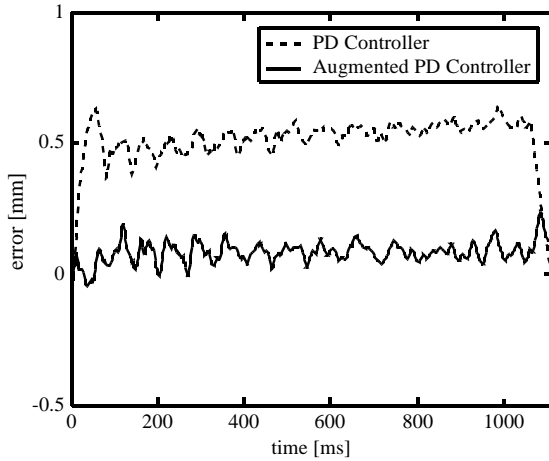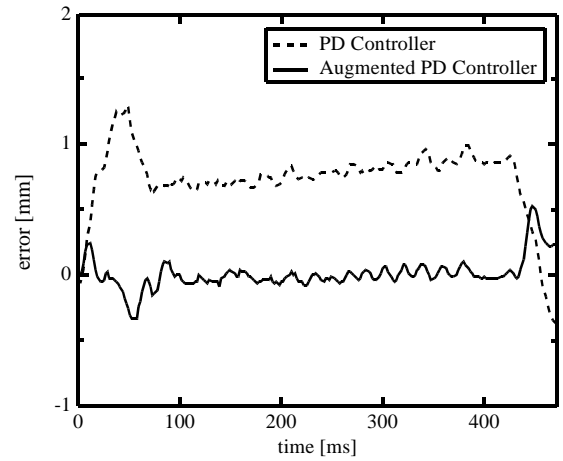

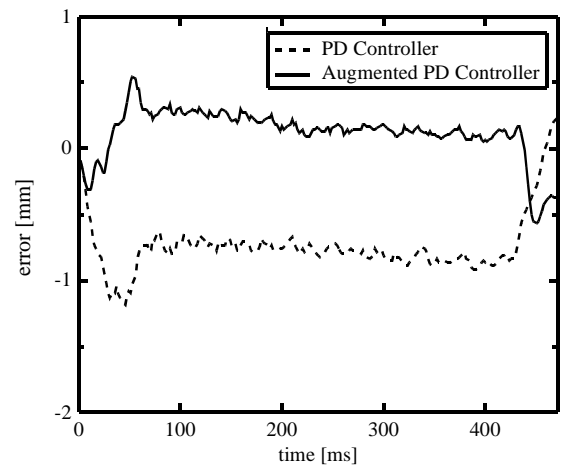(a) Tracking error in X direction.
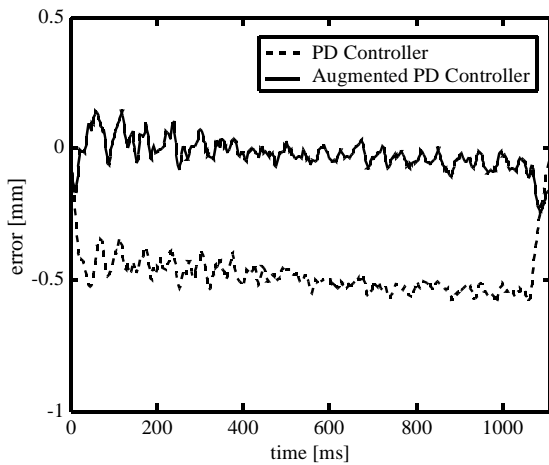
(b) Tracking error in Y direction.

Fig. 5. Linear trajectory tracking error of the end-effector with maximum velocity 0.2m/s.
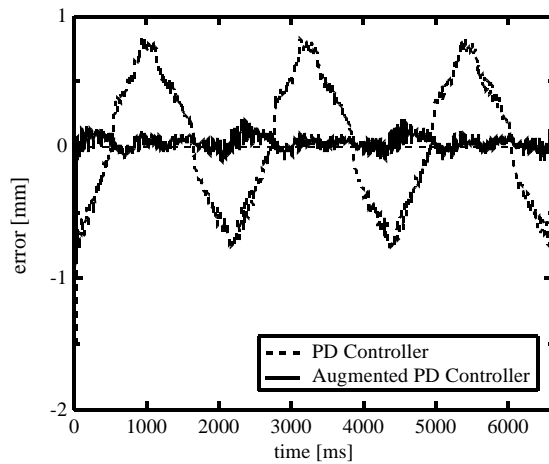

(a) Tracking error in X direction.

(b) Tracking error in Y direction.

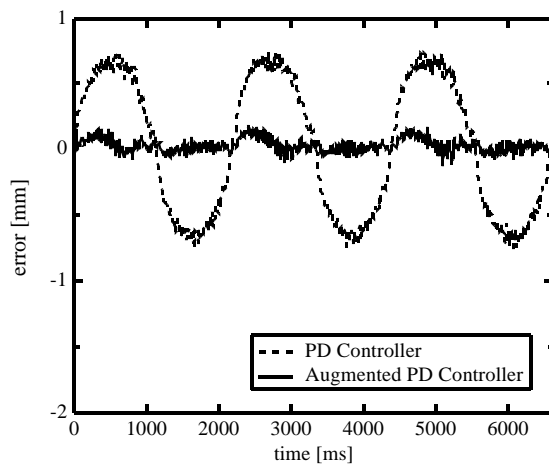Fig. 6. Linear trajectory tracking error of the end-effector with maximum velocity 0.5m/s.

Table 3. *RSME* of linear trajectory tracking experiments.

	RSME(mm)	
	PD controller	Augmented PD controller
End-effector velocity 0.2m/s	0.7140	0.1111
End-effector velocity 0.5m/s	1.1011	0.2560

The starting point of circular trajectory is (216.5,180) and center is (216.5,250). The end-effector of the GPM2002 is driven to track the circle for two times, first with constant velocity 0.2m/s and then with constant velocity 0.5m/s. Tracking errors of the end-effector are shown in Figs. 7 and 8, while the *RSME*s of the circular trajectory tracking experiments are shown in Table 4.
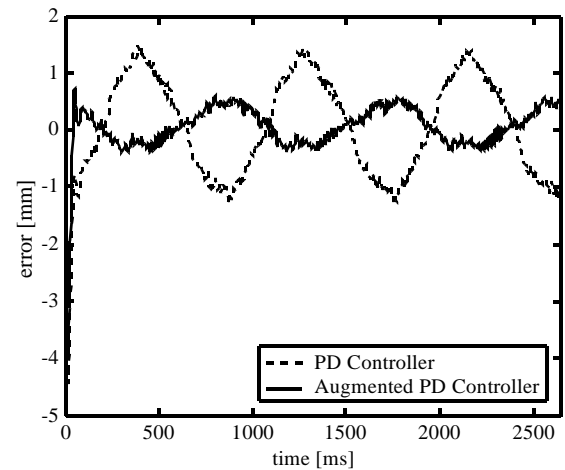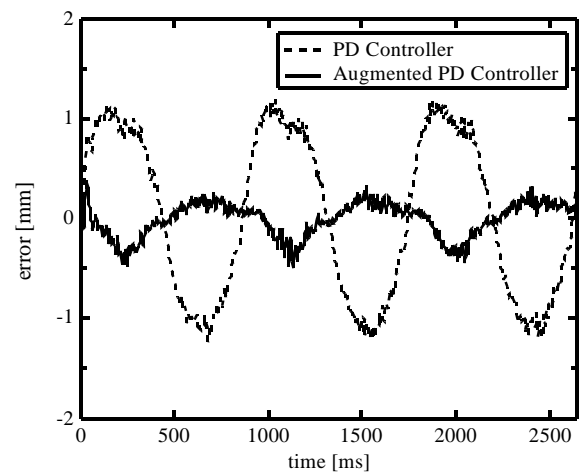

(a) Tracking error in X direction.

(b) Tracking error in Y direction.

Fig. 7. Circular trajectory tracking error of the end-effector with velocity 0.2m/s.

(a) Tracking error in X direction.

(b) Tracking error in Y direction.

Fig. 8. Circular trajectory tracking error of the end-effector with velocity 0.5m/s.

From the experiment results, one can see that the tracking error grows with the velocity of the end-effector. But, with the compensation of parallel manipulator dynamics and the active joint frictions, the augmented PD controller can achieve more accurate trajectory tracking results than the simple PD controller under the same trajectory.

Table 4. *RSME* of circular trajectory tracking experiments.

	RSME(mm)	
	PD controller	Augmented PD controller
End-effector velocity 0.2m/s	0.7210	0.1029
End-effector velocity 0.5m/s	1.2189	0.4525

6. CONCLUSIONS

In this paper, we proposed an augmented PD controller for a redundant planar 2-dof parallel manipulator. By formulating the controller in the joint space, we eliminated the complex computation of the Jacobian matrix of joint angles with end-effector coordinate. So with less computation, our controller is easier to implement, and a shorter sampling period can be achieved, which makes the controller more suitable for high-speed motion control. Furthermore, with the combination of static friction model and viscous friction model, the active joint friction of the parallel manipulator was studied. With the compensation of active joint friction, more accurate forward dynamic compensation was achieved. To evaluate the dynamic parameters of the parallel manipulator, we formulated the dynamic model as a group of linear equations and identified the unknown parameters with least squared method. With linear trajectory tracking experiments and circular trajectory tracking experiments, the validity of dynamic model was proved and the performance of the controller was evaluated. Experiment results showed that, based on the forward dynamic compensation, better tracking performance could be achieved with the augmented PD controller proposed over simple PD controller.

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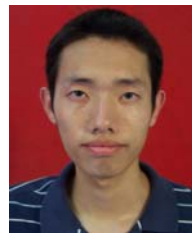
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