Mystery 1

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Scotland Yard has apprehended a person suspected of committing a heinous crime. They've stuck the suspect in a dark room. The door opens. Through the haze of cigar smoke, the suspect sees a detective walk in. He's carrying a mysterious black box. The suspect says: "I'm innocent, I swear!" The detective grunts: "We'll see about that."

In this mystery, you will infer the posterior probability of a discrete hypothesis – is the suspect guilty? – given evidence from a lie detector. The detective asks the suspect, "Did you commit the crime?" The machine beeps if it detects a lie. Unfortunately, the machine is old and unreliable. If the suspect is indeed lying, the machine only beeps 70% of the time (i.e. the false negative rate is 30%). If the suspect is telling the truth, the machine beeps 20% of the time (i.e. the false positive rate is 20%).

Part A

The suspect says "I didn't commit the crime!" a single time, and the machine doesn't beep. Assume you think he's innocent (say, the prior probability of guilt is 0.2). What's the posterior probability that he's guilty, given this test result? What about if you start off suspicious that he's guilty (say, the prior probability of guilt is 0.8)? Do this in MATLAB, but you can check your results with pencil and paper.

Part B

Now, the detective administers 10 tests in a row. Each time, the suspect says he's innocent, and the machine doesn't beep. Compute the posterior probability that he's guilty *at each stage of this process*, starting from before any tests were administered. In other words, compute `Prob(guilty | test results)` for no test results, 1 test result (with no beep), 2 test results (with no beeps), etc. (Hint: To compute the likelihood, look up a binomial distribution, and check out MATLAB's `binopdf` function. At this point, it will probably be helpful to write a separate function that computes the posterior for any given set of test results.)

Plot the posterior for each stage, assuming the innocent prior from Part A. Then, repeat the process assuming the suspicious prior. Describe how the posterior balances the prior and likelihood, as more and more evidence is accumulated. When there's not much evidence, how strong is the influence of the prior?

One way of describing the result is that, if you start off with a high prior suspicion that the suspect is guilty, it takes much more evidence to convince you that he's innocent. What kind of social-psychological phenomena does this idea capture?

Part C

It's not just the *amount* of evidence that matters; it's also the quality of that evidence. Suppose Scotland Yard buys a better lie detector, with a false negative rate of only 1%. Repeat Part B with this better lie detector, and compare the results. What effect does evidence quality have on the balance between the likelihood and prior?