Algorithms
Dynamic Programming Group Project (75 pts)

This project requires you to first theoretically solve the dynamic programming problem below and then write a program that implements your solution. [1] **You are not allowed to use the internet or consult any references. The only people you can work with on this project are your group members.**

1. *Problem Description*:

   The owners of a gas station are faced with the following situation. They have an underground tank in which they store gas; the tank can hold up to L gallons overnight. Ordering gas is quite expensive, so they want to order relatively rarely. For each order, they need to pay a fixed price P for delivery in addition to the cost of the gas ordered (which we assume remains unchanged). However, it costs c to store a gallon of gas for each extra night, so ordering too much ahead increases the storage cost.

   They are planning to close for a week in the winter and they want their tank to be empty by the time they close. Luckily, based on years of experience, they have accurate projections for how much gas they will need each day until this point in time. Assume that there are n days left till they close and they need g_i gallons of gas for each of the days $i = 1, \ldots, n$. Assume that the tank is empty at the end of Day 0. Assume that any gas purchased and consumed on the same day does not need to be stored in the tank. Give an algorithm to decide on which days they should place orders and how much to order so as to minimize their total cost.

2. *Deliverables*: Please submit all of the items requested below in a single PDF file on Canvas for your group.

   (a) [40 pts] Theory: Devise a dynamic programming algorithm that optimizes the total cost. Demonstrate each of the following dynamic programming steps:

      i. Describe the main idea of your approach including how you break the problem into smaller recursive problems [20 pts].

      ii. Write pseudocode for your dynamic programming algorithm [10 pts].

      iii. Develop a traceback algorithm that returns the days on which to place orders and how much. [10 pts].

   (b) [10 pts] Theory: Derive the complexity of your algorithm in terms of n.

   (c) [15 pts] Implementation: Implement your algorithm and submit the print-out of your code. If you were unable to get your code to compile/run, please state this clearly.

   (d) [10 pts] Implementation: Demonstrate that your code works correctly by showing its results on the following instance.

   The first line contains n, L, P, c in order. The second line contains n integers g_i.

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15  6  17  4
4  3  1  4  2  2  4  3  1  6  6  1  5  5  5
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   Your output should be as follows: The first line contains C, the total cost of your plan. The second line should contain D, the number of days that gas would be purchased within this plan. The next D lines will contain in order starting from day 1: the day number and the amount of gas purchased on that day, separated by a space.

[1] The problem has been adapted from the text by Kleinberg and Tardos