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Construction of the G2 Lie algebra

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Instructions

Group Theory of $G_{2,2}$

In an orthonormal basis the $g_{2\,(2)}$ positive roots α (rutte) read:

$\alpha_1 = (1, 0);$

$\alpha_2 = \left(-\frac{3}{2}, \frac{\sqrt{3}}{2}\right);$

$\alpha_3 = \alpha_1 + \alpha_2 = \left(-\frac{1}{2}, \frac{\sqrt{3}}{2}\right);$

$\alpha_4 = 2\,\alpha_1 + \alpha_2 = \left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right);$

$\alpha_5 = 3\,\alpha_1 + \alpha_2 = \left(\frac{3}{2}, \frac{\sqrt{3}}{2}\right);$

$\alpha_6 = 3\,\alpha_1 + 2\,\alpha_2 = \left(0, \sqrt{3}\right);$

Instructions for the user

After running this NoteBook you have at your disposal the explicit form ot the $\mathfrak{g}_2$ Lie algebra in the fundamental 7-dimensional representation and in the maximally split real section where all the generators are completely real.

The available objects are the following ones

1. The array **rutte** contains the six positive roots enumerated as above
2. The array **Steppig2** contains the step operators associated with the positive roots and enumerated in the same order as in **rutte.**
3. The array **MSteppig2** contains the step operators associated with the negative roots enumerated in the same order as their opposites in **rutte**.
4. The array **Cartanig2** contains the Cartan generators in an orthonormal basis.
5. The array **TTsolv** contains the 8 generators of the Borel solvable Lie algebra enumerated as follows $=\{\mathcal{H}_1,\mathcal{H}_2,\mathcal{E}^{\alpha_1},...,\mathcal{E}^{\alpha_6}\}$

Construction of the fundamental

representation in the standard Weyl basis