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Construction of the E_7 Lie Algebra in the fundamental 56 representation

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Instructions for the User

The present NoteBook is devoted to the construction of the E7 Lie algebra in its fundamental representation that it is 56-dimensional. Evaluating this NoteBook one obtains a series of objects that are available for further calculations:

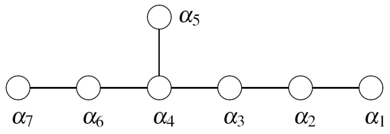
1. The object α is an array of 7 vectors with 7 components corresponding to the simple roots of E_7 as they are displayed in eq.(0.1) and eq. (0.2)
2. The object β is an array of 7 vectors with 7 components corresponding to the simple roots of $A_7 \subset E_7$ as they are displayed in eq.(0.4). A_7 is the Lie algebra of the subgroup SL(8,R) which is the so called electric subgroup. In the fundamental 56 representation of E_7 the matrices representing SL(8,R) are block-diagonal with two blocks 28×28.
3. The object **rho** is an array of 28 integer-valued 7-vectors. These are the 28 positive roots of A_7 expressed as integer valued linear combinations of the simple roots of E_7. Obviously these 28 roots are a subset of the 63 positive roots of E_7. These roots are enumerated in eq.(0.5)
4. The object **EP** is an array containing the 28 positive step operators of A_7 given as 56×56 matrices, namely within the fundamental representation of E_7. These step operators are enumerated in the same order as the corresponding roots are enumerated in **rho**.
5. The object **EM** is an array containing the 28 negative step operators of A_7 given as 56×56 matrices, namely within the fundamental representation of E_7. These step operators are enumerated in the same order as the corresponding roots are enumerated in **rho.** They are the transposed of the corresponding **EP**.
6. The object **racine** is an array of 63 integer-valued 7-vectors. These are the 63 positive roots of E_7 expressed as integer valued linear combinations of the simple roots of α . The roots are enumerated by height starting from the simple ones up to the highest one.
7. The object **rutte** is an array of 63 seven-vectors. These are the 63 positive roots of E_7 expressed in the same euclidian basis where the simple roots are those of eq.(0.2). They are enumerated in the same order as in the object **racine**.
8. The object **E7P** is an array that contains the positive step operators of E_7 written as 56×56 matrices and enumerated in the same order as the corresponding roots **rutte** (or **racine**)
9. The object **E7M** is an array that contains the negative step operators of E_7 written as 56×56 matrices and enumerated in the same order as the corresponding roots **rutte** (or **racine**)
10. The object **Cartani** is an array that contains the 7 Cartan generators of E_7 written as 56×56 matrices each corresponding to an axis of the euclidian basis in which the simple roots have been written in eq.(0.2).
11. The object **W** is an array that contains (given in the euclidian basis) the 7 fundamental weights of E_7.
12. The object **WW** is an array that contains (given in the euclidian basis) the 56 weights of the fundamental representation of E_7.
13. The weights can be represented as the maximal weight **vmax =** W_1 minus an integer valued linear combination of simple roots. The vectors expressing these linear combinations are encoded in the array **qqq** which corresponds to the weigths according to the formula:

$$W_i = W_{\max} - \sum_{j=1}^{7} \text{qqq}_{[[i,j]]}\, \alpha_j$$

Theoretical description

The E7 simple roots

The choice of the simple roots of E7 enumerated as in the Dynkin diagram shown below

E_7


(0.1)

is the following one

$$
\begin{aligned}
\alpha_1 &= \{1, -1, 0, 0, 0, 0, 0\} \\
\alpha_2 &= \{0, 1, -1, 0, 0, 0, 0\} \\
\alpha_3 &= \{0, 0, 1, -1, 0, 0, 0\} \\
\alpha_4 &= \{0, 0, 0, 1, -1, 0, 0\} \\
\alpha_5 &= \{0, 0, 0, 0, 1, -1, 0\} \\
\alpha_6 &= \{0, 0, 0, 0, 1, 1, 0\} \\
\alpha_7 &= \left\{-\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, -\frac{1}{2}, \frac{1}{\sqrt{2}}\right\}
\end{aligned}
\tag{0.2}
$$

Constructing the SL(8,R) subalgebra

SL(8,R) is the electric subgroup of the N=8 theory, since in application to supergravity it is the group that transforms electric gauge fields into electric gauge fields. From a purely mathematical viewpoint this subalgebra in the fundamental 56 representation is represented by matrices that are block diagonal as follows

$$\left(\begin{array}{c|c} A & 0 \\ \hline 0 & A^T \end{array}\right) \subset SL(8, R) \subset E_{7(7)} \tag{0.3}$$

The subalgebra SL(8,R) is regularly embedded since it shares with the bigger group the same Cartan subalgebra. The rank is the same and hence the simple roots of SL(8,R) can be expressed as linear combinations of the simple roots of $E_{7(7)}$.

Choice of the simple roots

The simple roots of SL $(8, R)$ are named β and are selected in the following way in terms of the simple roots of E_7 :

$$
\begin{aligned}
\beta_1 &= \alpha_2 + 2\,\alpha_3 + 3\,\alpha_4 + 2\,\alpha_5 + 2\,\alpha_6 + \alpha_7 \\
\beta_2 &= \alpha_1 \\
\beta_3 &= \alpha_2 \\
\beta_4 &= \alpha_3 \\
\beta_5 &= \alpha_4 \\
\beta_6 &= \alpha_6 \\
\beta_7 &= \alpha_7
\end{aligned}
\tag{0.4}
$$

Enumeration of all the roots of SL(8,R)

The complete set of positive roots of SL(8,R) is obviously made by 28 root vectors that we name ρ_a and we enumerate as follows:

$$\begin{array}{|l|}\hline \rho_1 \equiv \beta_2 \\ \rho_2 \equiv \beta_2+\beta_3 \\ \rho_3 \equiv \beta_2+\beta_3+\beta_4 \\ \rho_4 \equiv \beta_2+\beta_3+\beta_4+\beta_5 \\ \rho_5 \equiv \beta_2+\beta_3+\beta_4+\beta_5+\beta_6 \\ \rho_6 \equiv \beta_3 \\ \rho_7 \equiv \beta_3+\beta_4 \\ \rho_8 \equiv \beta_3+\beta_4+\beta_5 \\ \rho_9 \equiv \beta_3+\beta_4+\beta_5+\beta_6 \\ \rho_{10} \equiv \beta_4 \\ \rho_{11} \equiv \beta_4+\beta_5 \\ \rho_{12} \equiv \beta_4+\beta_5+\beta_6 \\ \rho_{13} \equiv \beta_5 \\ \rho_{14} \equiv \beta_5+\beta_6 \\ \rho_{15} \equiv \beta_6 \\ \rho_{16} \equiv \beta_1+\beta_2+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7 \\ \rho_{17} \equiv \beta_2+\beta_3+\beta_4+\beta_5+\beta_6+\beta_7 \\ \rho_{18} \equiv \beta_3+\beta_4+\beta_5+\beta_6+\beta_7 \\ \rho_{19} \equiv \beta_4+\beta_5+\beta_6+\beta_7 \\ \rho_{20} \equiv \beta_5+\beta_6+\beta_7 \\ \rho_{21} \equiv \beta_6+\beta_7 \\ \rho_{22} \equiv \beta_1 \\ \rho_{23} \equiv \beta_1+\beta_2 \\ \rho_{24} \equiv \beta_1+\beta_2+\beta_3 \\ \rho_{25} \equiv \beta_1+\beta_2+\beta_3+\beta_4 \\ \rho_{26} \equiv \beta_1+\beta_2+\beta_3+\beta_4+\beta_5 \\ \rho_{27} \equiv \beta_1+\beta_2+\beta_3+\beta_4+\beta_5+\beta_6 \\ \rho_{28} \equiv \beta_7 \\ \hline \end{array} \tag{0.5}$$

Mathematica Code

Loading inputs

In the next section we load the input of the simple root of $E_{7(7)}$ and we calculate the simple roots of SL(8,R)

Simple roots

In[1]:=

```
α = {
    {1, -1, 0, 0, 0, 0, 0},
     {0, 1, -1, 0, 0, 0, 0},
     {0, 0, 1, -1, 0, 0, 0},
     {0, 0, 0, 1, -1, 0, 0},
     {0, 0, 0, 0, 1, -1, 0},
    {0, 0, 0, 0, 1, 1, 0},
    {-1/2, -1/2, -1/2, -1/2, -1/2, -1/2, 1/Sqrt[2]}};
```

Check of the Cartan Matrix of E7

In[2]:=

```
CarmatE7 = Table[α[[i]].α[[j]], {i, 1, 7}, {j, 1, 7}];
```

In[3]:=

```
MatrixForm[CarmatE7]
```

Out[3]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & -1 & 0 \\ 0 & 0 & 0 & -1 & 2 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

In[4]:=

```
β = Table[0, {i, 1, 7}, {j, 1, 7}];
β[[1]] = α[[2]] + 2 α[[3]] + 3 α[[4]] + 2 α[[5]] + 2 α[[6]] + α[[7]];
β[[2]] = α[[1]];
β[[3]] = α[[2]];
β[[4]] = α[[3]];
β[[5]] = α[[4]];
β[[6]] = α[[6]];
β[[7]] = α[[7]];
```

Step operators

The matrices of the step operators in this Dynkin basis were calculated in 1997 and are stored in a file that presently we load. They are arranged according to a certain filtration of the root space that singles out a nested succession of 6 ideals of the Borel subalgebra. The correspondence between the arrangement of the step operators according to the ideals and the enumeration of the roots is provided in an object named **dcode**

Loading step operators

Introducing the code for the roots

Check of the Cartan Matrix of SL(8,R)

In[16]:=

```
CarmatA7 = Table[β[[i]].β[[j]], {i, 1, 7}, {j, 1, 7}];
```

In[17]:=

```
MatrixForm[CarmatA7]
```

Out[17]//MatrixForm=

$$\begin{pmatrix} 2 & -1 & 0 & 0 & 0 & 0 & 0 \\ -1 & 2 & -1 & 0 & 0 & 0 & 0 \\ 0 & -1 & 2 & -1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 2 & -1 & 0 & 0 \\ 0 & 0 & 0 & -1 & 2 & -1 & 0 \\ 0 & 0 & 0 & 0 & -1 & 2 & -1 \\ 0 & 0 & 0 & 0 & 0 & -1 & 2 \end{pmatrix}$$

Identification of the roots of SL(8,R)

We start from writing the roots of SL(8,R) in the basis of simple roots of E_7

In[18]:=

```
bb1 = {0, 1, 2, 3, 2, 2, 1};
bb2 = {1, 0, 0, 0, 0, 0, 0};
bb3 = {0, 1, 0, 0, 0, 0, 0};
bb4 = {0, 0, 1, 0, 0, 0, 0};
bb5 = {0, 0, 0, 1, 0, 0, 0};
bb6 = {0, 0, 0, 0, 0, 1, 0};
bb7 = {0, 0, 0, 0, 0, 0, 1};
```

Then we write the enumeration of all the roots according to the labeling as in the main text of the book.

In[25]:=

```
rho = Table[0, {i, 1, 28}, {j, 1, 7}];
```

In[26]:=

```
rho[[1]] = bb2;
rho[[2]] = bb2 + bb3;
rho[[3]] = bb2 + bb3 + bb4;
rho[[4]] = bb2 + bb3 + bb4 + bb5;
rho[[5]] = bb2 + bb3 + bb4 + bb5 + bb6;
rho[[6]] = bb3;
rho[[7]] = bb3 + bb4;
rho[[8]] = bb3 + bb4 + bb5;
rho[[9]] = bb3 + bb4 + bb5 + bb6;
rho[[10]] = bb4;
rho[[11]] = bb4 + bb5;
rho[[12]] = bb4 + bb5 + bb6;
rho[[13]] = bb5;
rho[[14]] = bb5 + bb6;
rho[[15]] = bb6;
rho[[16]] = bb1 + bb2 + bb3 + bb4 + bb5 + bb6 + bb7;
rho[[17]] = bb2 + bb3 + bb4 + bb5 + bb6 + bb7;
rho[[18]] = bb3 + bb4 + bb5 + bb6 + bb7;
rho[[19]] = bb4 + bb5 + bb6 + bb7;
rho[[20]] = bb5 + bb6 + bb7;
rho[[21]] = bb6 + bb7;
rho[[22]] = bb1;
rho[[23]] = bb1 + bb2;
rho[[24]] = bb1 + bb2 + bb3;
rho[[25]] = bb1 + bb2 + bb3 + bb4;
rho[[26]] = bb1 + bb2 + bb3 + bb4 + bb5;
rho[[27]] = bb1 + bb2 + bb3 + bb4 + bb5 + bb6;
rho[[28]] = bb7;
```

Form of the SL(8) roots as euclidean vectors:

In[54]:=

$$\texttt{eurho = Table}\left[\sum_{j=1}^{7} \texttt{rho}_{[\![i,j]\!]} * \alpha_{[\![j]\!]},\ \{i,\ 1,\ 28\}\right];$$

coding the roots of SL(8)

In[55]:=
```
rhocode = Table[0, {i, 1, 28}, {j, 1, 2}];
```

In[56]:=
```
Do[Do[If[rho[[i]] == dcode[[j, 2]], rhocode[[i]] = dcode[[j, 1]], pippo = 0], {j, 1, 63}],
  {i, 1, 28}];
```

In[58]:=
```
Dimensions[step]
```

Out[58]=
```
{6}
```

Step operators of SL(8,R) within the 56 of E7

Here we define the final form of the step operators for the electric subalgebra. The step-operators associated with positive SL(8,R) roots are named EP, while those associated with negative SL(8,R) roots are named EM.

In[59]:=
```
EP = Table[0, {i, 1, 28}, {j, 1, 56}, {k, 1, 56}];
EM = Table[0, {i, 1, 28}, {j, 1, 56}, {k, 1, 56}];
```

In[61]:=
```
Do[ {EP[[i]] = step[[rhocode[[i,1]], rhocode[[i,2]]]];
   EM[[i]] = Transpose[step[[rhocode[[i,1]], rhocode[[i,2]]]]];}, {i, 1, 28}];
```

Storing the SL(8,R) generators

The SL(8,R) step operators associated with positive and negative roots are stored into the following files :

1) EP contains the generators associated with positive roots

2) EM contains the generators associated with positive roots

Matrix of scalar products for SL(8,R) roots

In[62]:=
```
scprd = Table[Simplify[eurho[[i]].eurho[[j]]], {i, 1, 28}, {j, 1, 28}];
```

In[63]:=

E7 roots

Algorithm to construct all the roots

In[64]:=

In this section we write an induction routine that constructs all the positive roots of E(7) as integer valued linear combination of the simple roots and obtaines them by induction on their height.

The result is organized in a tensor

racine *of* ***63*** *integer valued 7-vectors*

routine induczia and commands to run it

Running and displaying

Construction of the Cartan subalgebra

In[76]:=
```
CartE7 = Table[Simplify[E7P[[a]].E7M[[a]] - E7M[[a]].E7P[[a]]], {a, 1, 7}];
```

In[77]:=
```
Cartani = Simplify[Inverse[α].CartE7];
```

Verification of the Commutation relation of the Lie algebra

First we verify the relation

$$\left[E^{\alpha},\ E^{-\alpha}\right] = \alpha \cdot H \tag{1}$$

In[78]:=

```
giusto = 0; sbaglio = 0;
Do[{XPY = Simplify[E7P[[a]].E7M[[a]] - E7M[[a]].E7P[[a]] - rutte[[a]].Cartani],
   If[XPY == 0 * IdentityMatrix[56], giusto = giusto + 1, sbaglio = sbaglio + 1];}, {a,
   1, 63}];
Print[" giusto = ", giusto];
Print[" sbaglio = ", sbaglio];
```

```
 giusto = 63
 sbaglio = 0
```

Then we verify the relation

$$\left[H_i,\ E^{\alpha}\right] = \alpha_i\, E^{\alpha} \tag{2}$$

In[82]:=

```
giusto = 0; sbaglio = 0;
Do[Do[{XPY = Simplify[Cartani[[i]].E7P[[a]] - E7P[[a]].Cartani[[i]] - rutte[[a,i]] * E7P[[a]]],
    If[XPY == 0 * IdentityMatrix[56], giusto = giusto + 1, sbaglio = sbaglio + 1];}, {i,
    1, 7}], {a, 1, 63}];
Print[" giusto = ", giusto];
Print[" sbaglio = ", sbaglio];
```

```
 giusto = 441
 sbaglio = 0
```

Then we verify the relation

$$\left[H_i,\ E^{-\alpha}\right] = -\alpha_i\, E^{-\alpha} \tag{3}$$

In[86]:=

```
giusto = 0; sbaglio = 0;
Do[Do[{XPY = Simplify[Cartani[[i]].E7M[[a]] - E7M[[a]].Cartani[[i]] + rutte[[a,i]] * E7M[[a]]],
    If[XPY == 0 * IdentityMatrix[56], giusto = giusto + 1, sbaglio = sbaglio + 1];}, {i,
    1, 7}], {a, 1, 63}];
Print[" giusto = ", giusto];
Print[" sbaglio = ", sbaglio];
```

```
 giusto = 441
 sbaglio = 0
```

Construction of the cocycle $N_{\alpha\beta}$

In[90]:=
```
Nalbet = Table[0, {alp, 1, 63}, {bet, 1, 63}];
Do[Do[{XYP = Simplify[E7P[[a]].E7P[[b]] - E7P[[b]].E7P[[a]]];
    If[XYP == 0 * IdentityMatrix[56], pippo = 0, {zorro = rutte[[a]] + rutte[[b]];
      Do[If[zorro == rutte[[k]], nappo = E7P[[k]], pippo = 0], {k, 1, 63}];
      If[XYP == nappo, Nalbet[[a, b]] = 1, pippo = 0];
      If[XYP == -nappo, Nalbet[[a, b]] = -1, pippo = 0];}];}, {b, 1, 63}], {a, 1, 63}];
```

Verification

In[92]:=
```
giusto = 0; sbaglio = 0;
Do[Do[{XPY = Simplify[E7P[[a]].E7P[[b]] - E7P[[b]].E7P[[a]]];
    If[XYP == 0 * IdentityMatrix[56], giusto = giusto + 1, {zorro = rutte[[a]] + rutte[[b]];
      Do[If[zorro == rutte[[k]], nappo = E7P[[k]], pippo = 0], {k, 1, 63}];
      If[XPY == Nalbet[[a, b]] * nappo, giusto = giusto + 1, sbaglio = sbaglio + 1];}]},
   {b, 1, 63}], {a, 1, 63}];
Print[" giusto = ", giusto];
Print[" sbaglio = ", sbaglio];
```

```
 giusto = 3969
 sbaglio = 0
```

Weights of the 56 representation

The next point is the construction of the weights of the fundamental 56 dimensional representation of E7

Construction of the fundamental weights

In[96]:=
```
W = Transpose[Inverse[α]];
```

In[110]:=
```
MatrixForm[W]
```

Out[110]//MatrixForm=

$$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 & \frac{1}{\sqrt{2}} \\ 1 & 1 & 0 & 0 & 0 & 0 & \sqrt{2} \\ 1 & 1 & 1 & 0 & 0 & 0 & \frac{3}{\sqrt{2}} \\ 1 & 1 & 1 & 1 & 0 & 0 & 2\sqrt{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & -\frac{1}{2} & \sqrt{2} \\ \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{1}{2} & \frac{3}{\sqrt{2}} \\ 0 & 0 & 0 & 0 & 0 & 0 & \sqrt{2} \end{pmatrix}$$

Theory of the q vectors identifying all the weights

Input of the q vectors

The following is a table containing 56 seven-component vectors that identify the weights of the 56 representation by giving the coefficient of the linear combination of simple roots one has to subtract from the fundamental weight W_1 in order to obtain the corresponding weight vector of fundamental representation

In[97]:=

```
qqq = {{2, 3, 4, 5, 3, 3, 1}, {2, 2, 2, 2, 1, 1, 1}, {1, 2, 2, 2, 1, 1, 1}, {1, 1, 2, 2, 1, 1, 1},
   {1, 1, 1, 2, 1, 1, 1}, {1, 1, 1, 1, 1, 1, 1}, {2, 3, 3, 3, 1, 2, 1}, {2, 2, 3, 3, 1, 2, 1},
   {2, 2, 2, 3, 1, 2, 1}, {2, 2, 2, 2, 1, 2, 1}, {1, 2, 2, 2, 1, 2, 1}, {1, 1, 2, 2, 1, 2, 1},
   {1, 1, 1, 2, 1, 2, 1}, {1, 2, 2, 3, 1, 2, 1}, {1, 2, 3, 3, 1, 2, 1}, {1, 1, 2, 3, 1, 2, 1},
   {2, 2, 2, 2, 1, 1, 0}, {1, 2, 2, 2, 1, 1, 0}, {1, 1, 2, 2, 1, 1, 0}, {1, 1, 1, 2, 1, 1, 0},
   {1, 1, 1, 1, 1, 1, 0}, {1, 1, 1, 1, 1, 0, 0}, {3, 4, 5, 6, 3, 4, 2}, {2, 4, 5, 6, 3, 4, 2},
   {2, 3, 5, 6, 3, 4, 2}, {2, 3, 4, 6, 3, 4, 2}, {2, 3, 4, 5, 3, 4, 2}, {2, 3, 4, 5, 3, 3, 2},
   {1, 1, 1, 1, 0, 1, 1}, {1, 2, 3, 4, 2, 3, 1}, {2, 2, 3, 4, 2, 3, 1}, {2, 3, 3, 4, 2, 3, 1},
   {2, 3, 4, 4, 2, 3, 1}, {2, 3, 4, 5, 2, 3, 1}, {1, 1, 2, 3, 2, 2, 1}, {1, 2, 2, 3, 2, 2, 1},
   {1, 2, 3, 3, 2, 2, 1}, {1, 2, 3, 4, 2, 2, 1}, {2, 2, 3, 4, 2, 2, 1}, {2, 3, 3, 4, 2, 2, 1},
   {2, 3, 4, 4, 2, 2, 1}, {2, 2, 3, 3, 2, 2, 1}, {2, 2, 2, 3, 2, 2, 1}, {2, 3, 3, 3, 2, 2, 1},
   {1, 2, 3, 4, 2, 3, 2}, {2, 2, 3, 4, 2, 3, 2}, {2, 3, 3, 4, 2, 3, 2}, {2, 3, 4, 4, 2, 3, 2},
   {2, 3, 4, 5, 2, 3, 2}, {2, 3, 4, 5, 2, 4, 2}, {0, 0, 0, 0, 0, 0, 0}, {1, 0, 0, 0, 0, 0, 0},
   {1, 1, 0, 0, 0, 0, 0}, {1, 1, 1, 0, 0, 0, 0}, {1, 1, 1, 1, 0, 0, 0}, {1, 1, 1, 1, 0, 1, 0}};
```

explicit calculation of the weights

```
wmax = W[[1]];
WW = Simplify[Table[wmax - Sum[qqq[[i,j]] * α[[j]], {j, 1, 7}], {i, 1, 56}]];
```

$$WW = \text{Simplify}\left[\text{Table}\left[wmax - \sum_{j=1}^{7} qqq_{[[i,j]]} * \alpha_{[[j]]},\ \{i, 1, 56\}\right]\right];$$

Construction of the Cartan generators of the SU(8) maximal compact subalgebra

In this section we recall the explicit embedding of the SU(8) subgroup into $E_{7\,(7)}$ and we construct the Cartan generators of the SU(8) subalgebra.

Defining the SU(8) Cartan generators inside E(7)

The 7 Cartan generators of SU(8), obtained from the step operators of $E_{7\,(7)}$ are named in the programme **CH[[i]]** which is a seven vector of 56 x 56 matrices.

In[100]:=

```
CH1 = step[[2, 1]] - Transpose[step[[2, 1]]];
CH2 = step[[2, 2]] - Transpose[step[[2, 2]]];
CH3 = step[[4, 1]] - Transpose[step[[4, 1]]];
CH4 = step[[4, 2]] - Transpose[step[[4, 2]]];
CH5 = step[[6, 1]] - Transpose[step[[6, 1]]];
CH6 = step[[6, 2]] - Transpose[step[[6, 2]]];
CH7 = step[[6, 11]] - Transpose[step[[6, 11]]];
CH = {CH1, CH2, CH3, CH4, CH5, CH6, CH7};
```

The basis of Cartan duals to the simple roots

The basis we choose is by definition such that the following is a true equation:

$$H_{a_i} = [\ E^{a_i}, E^{-a_i}]$$

where a_i are the simple roots of SU(8). The explicit form of the SU(8) simple roots in the basis of the CH(i) was calculated in 1997 and it is recalled here. The Cartan generators H_{a_i} in the Dynkin basis are stored in a vector named rotdyCH[[i]]

In[108]:=

```
aa = {{-1, -1, 1, 1, 0, 0, 0},
       {0, 0, -1, -1, 1, 1, 0},
       {1, 1, 1, 1, 0, 0, 0},
       {-1, 0, -1, 0, -1, 0, -1},
      {1, -1, 0, 0, 1, -1, 0},
     {0, 0, 1, -1, -1, 1, 0},
     {-1, 1, 0, 0, 1, -1, 0}};
```

$$\text{rotdyCH} = \text{Table}\left[\sum_{j=1}^{7} \text{aa}_{[\![i,j]\!]} * \text{CH}_{[\![j]\!]},\ \{i, 1, 7\}\right];$$