1 Hyperball vs Hypercube

1.1 Volume of hyperball:

for n = 2k

$$V_{2k}(R) = \frac{\pi^{2k}}{k!} R^{2k} \tag{1}$$

for n = 2k + 1

$$V_{2k+1}(R) = \frac{2(k!)(4\pi)^k}{(2k+1)!}R^{2k+1}$$
(2)

1.2 Volume of hypercube:

for hyper cube with edge length of 'a':

$$V_n(a) = a^n (3)$$

1.3 Answer

a=2.0 and R=1.0 :

1.3.1 n = 2k

$$\frac{Hyperball \quad volume}{Hypercube \quad Volume} * 100\% = \frac{\pi^{2k}}{k!2^{2k}} * 100\% \tag{4}$$

1.3.2 n = 2k + 1

$$\frac{Hyperball \quad volume}{Hypercube \quad Volume} * 100\% = \frac{k!\pi^{2k}}{(2k+1)!} * 100\% \tag{5}$$