

Small Worlds, Sour Grapes and Social Deprivation: the Computational Application of Peter Hedström's DBO Theory

Candidate number: 654934

Contents

1	Introduction	8
1.1	Approaches to the Social Sciences	8
1.2	The Use of Agent-Based Models in the Social Sciences	10
1.3	Criticism of Agent-Based Modelling	14
1.4	The DBO Model as an Agent-Based Model	16
2	Methodology: Parameters and Variables	18
2.1	The Parameters of the Computational Application of the DBO Model	18
2.2	The Outcomes of DBO Model Simulations	22
3	Processes Within Agents: Intra Individual-Mechanisms	26
3.1	The Basic Simulation	26
3.2	Non-Rational Intra-Individual Mechanisms	29
3.2.1	The Limits of Rationality	29
3.2.2	New Intra-Agent Mechanisms	38
3.3	Conclusion	41
4	Processes Between Agents: Social Networks and Thresholds	44
4.1	Network Models and Metrics	44
4.2	The Effects of Networks	53
4.2.1	Varying the Mean Degree	61
4.2.2	Varying Mean Shortest-Path Length	65
4.2.3	Varying Degree Distribution Skewness	67
4.3	The Aggregation Process: Thresholds	69
4.4	Conclusion	74
5	Conclusion	77

List of Figures

1	'Common sociological approaches defined on the basis of possible relationships between theory, substantive problems, data, and methods.'	9
2	The computational application of the DBO model	21
3	Pseudocode for a model with varying distributions of desire and belief	23
4	The relationship between action, desire, opportunity, deprivation and uninterest	25
5	The interaction process	27
6	The standard simulation	27
7	Typical distributions for the basic simulation	30
8	Attribute correlation for the basic simulation, before and after 20 interactions	31
9	Proportion acting as a function of interactions for various initial proportions acting	33
10	Proportion acting as a function of interactions for various initial proportions acting	34
11	Proportion acting as a function of interactions for various initial proportions acting and with different intra-agent mechanisms	35
12	The effect of wishful thinking on neighbours	36
13	Evolution deprivation for different types of actors	37
14	Evolution of uninterest for different types of actors	37
15	Action for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors	39
16	Deprivation for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors	40
17	Uninterest for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors	40
18	Carpe diem and wishful thinking versus come-what-may	41
19	Node attribute correlation for come-what may agents	42
20	The absence of transitive relationships in von Neumann neighbourhoods	45
21	A toroidal regular lattice	46

22	A ring lattice	47
23	A different ring lattice	47
24	A typical Erdős-Rényi graph	49
25	Degree distributions of the different network types	50
26	A typical Watts and Strogatz graph	52
27	A typical Barabási-Albert graph	54
28	The network formation processes of different network models	55
29	Toroidal lattice: eventual action and initial action	56
30	Ring lattice of degree 4: eventual action and initial action	57
31	Erdős-Rényi random graph: eventual and initial action	57
32	Watts and Strogatz network: eventual and initial action	58
33	Barabási-Albert network: eventual and initial action	58
34	Eventual and initial action in multiple network types	59
35	Barabási-Albert network, shown with points	60
36	Barabási-Albert network: distribution of outcomes for 50% initial action	61
37	Toroidal lattice: distribution of marginal outcomes	62
38	Ring lattice of degree 4: distribution of marginal outcomes	63
39	Ring lattice of degree 16: distribution of marginal outcomes	64
40	Ring lattice of degree 64: distribution of marginal outcomes	64
41	Mean shortest path lengths in the Watts and Strogatz model	65
42	Distribution of marginal outcomes in Watts and Strogatz networks with $\beta = 0.01, \beta = 0.02, \beta = 0.05$	66
43	Distribution of marginal outcomes in Watts and Strogatz networks with $\beta = 0.2, \beta = 0.4, \beta = 0.8$	67
44	Effect of the skewness of degree distribution on the variability of outcomes.	68
45	Effect of unconnected nodes on the variability of outcomes	69
46	The effects of thresholds on action in a toroidal lattice	72

47	The effects of thresholds on action in an Erdős-Rényi network	73
48	Decreasing the mean shortest path length and increasing the mean degree . . .	75

List of Tables

1	The standard simulation	28
2	Intra-agent mechanisms of rational, wishful thinking and sour grapes agents . .	32
3	Intra-agent mechanisms of rational, wishful thinking and carpe diem, and come- what-may agents	38
4	Comparison of empirical networks and network models	50

Nomenclature

ABM Agent-based model(ling)

ADM Action-decision mechanism: the process which determines whether DBO model agents will act, based on their mental states

DBA (state) Desire, belief and action (state)

DBO (state, model, theory) Desire, belief and opportunity

DBOMM Desire, belief and opportunity modification mechanism: the process which changes the initial societally-determined mental states of DBO model agents in a prescribed way

SIR Susceptible, infective, recovered: a biological-metaphor epidemiological model in which agents switch between three states

SIS Susceptible, infective, susceptible: a biological-metaphor epidemiological model in which agents switch between two states

1 Introduction

1.1 Approaches to the Social Sciences

The study of social behaviour is constituted of four interacting components: problems that need to be understood, data that bring the problems to light, methods that are used to acquire the data, and theories that explain the problem.¹ There are different approaches to sociology, each characterised by different relations of importance and temporal and practical priority between these components. The results that these approaches seek to produce are constrained by the way they make transitions between the theoretical and the empirical.

Peter Hedström describes four common approaches to sociology, each named after the component to which it attaches the most importance or priority. They are the *theory-driven*, *problem-driven*, *data-driven* and *methods-driven* approaches. Of these, the data-driven and problem-driven approaches might seem the most natural to social scientists. They seek to understand the social aspects of the human natural world, so where better to start than the object of study itself? But no researcher works in a vacuum: all aspects of their approach are informed by the work that has come before. The first social scientist may well have started by deriving a problem from the world and formulating a theory about it. But the second social scientist could choose to build on any part of the first's research, be it theory, problem, data, or method.

Agent-based modelling, the approach which is to be explored here, is an analytical method² that fits somewhere between the theory-driven and methods-driven approaches to sociology, or as Tim Liao would have it, it begins with a theoretical understanding of problems then seeks to understand the dynamics of the theoretical system.³ Agent-based modelling is methods-driven to the extent that its method determines which data can be gathered, so it lends itself more to examining certain types of theories more easily than others. However, within the large domain of problems that can be expressed as agent-based models (ABMs), particular theories determine

¹Hedström (2007, pp. 1–2)

²Gilbert (2008, p. 1)

³Tim F. Liao in Gilbert (2008, p. ix)

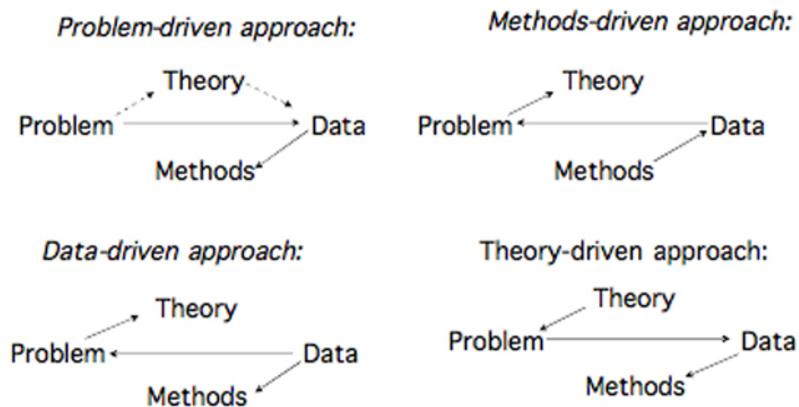


Figure 1: ‘Common sociological approaches defined on the basis of possible relationships between theory, substantive problems, data, and methods.’
Reproduced from *Actions and Networks: Sociology that really matters... to me*

which problems should be considered. In agent-based modelling – also known as generative social science⁴ – social processes (the problems) are explained by the system-level behaviour (the data) of interacting agents (the method) which act and interact on a local level following simple rules (the theory).

Agent-based models are necessarily methodologically individualistic, as they use interactions amongst agents rather than interaction amongst variables.⁵ As such, they fit into the multi-level approach described by Weber, whereby aggregate correlations must be backed up by intelligible patterns of individual action.⁶ ABMs are generative because they seek to explain ‘macroscopic regularities and organisations’,⁷ in terms of the ‘local interactions of heterogeneous autonomous agents’⁸, ‘growing’ the regularities from the bottom-up. Real societies are made up of autonomous agents, so the generativist creates an artificial society operating on simplified rules which represent some important characteristic of what is being modelled. If a computer simulation shows that the interacting agents do produce the requisite phenomena, then (given some provisos) the theory about the micro-level origin of the social processes is vindicated.

⁴The method is so important to this way of doing sociology that the whole approach takes its name

⁵Macy and Willer (2002, p. 148)

⁶Hedström (2005, p. 35)

⁷Epstein (2006a, p. 4)

⁸Epstein (2006a, p. 5)

Further (or prior) empirical work can be used to determine whether the agent-level processes occur in any real situation, but agent-based modelling is ‘much more concerned with theoretical development and explanation than with prediction’⁹, for which it is enough to show that the theoretical processes *can* explain the problem, it is not crucial to show that they *do*. When generative social science considers theories that might explain problems, the aim is not to ‘resolve such debates’, but to ‘contribute to them by suggesting alternative explanations, and by pointing out possible directions for empirical studies.’¹⁰, in order to ‘reveal new theoretical insights’¹¹.

ABMs model these theoretical mechanisms using ideal-typical actors rather than representations of any particular real actor. As such, they are part of the analytical-realist tradition: they focus only on what is relevant, but do so in a realistic way.¹² Properly-applied agent-based models are part of the procedure of ‘carefully dissecting [complex social processes] and then bringing into focus their most important constituent components [...] that seeks precise, abstract, realistic and action-based explanations for various social phenomena’.¹³ When ‘mechanism-based explanations are the most appropriate type of explanation for the social sciences’¹⁴, it is crucial to be able to test whether particular mechanisms offer plausible explanations.

1.2 The Use of Agent-Based Models in the Social Sciences

Epstein and Axtell identify¹⁵ three ingredients of all ABMs in the social sciences: agents, the ‘people’ of artificial societies, who have different internal states and behavioural rules; the environment, on which the agents operate and with which they interact; and the rules, both for the agents and their environment, which couple the agents to the environment, the environment to itself and agents to each other. The defining feature of these models is that ‘fundamental social structures and group behaviours emerge from the interaction of individual agents operating in ar-

⁹Gilbert (1997, p. 2.1) cited in Macy and Willer (2002, pp. 146–7)

¹⁰Watts and Dodds (2007, p. 445)

¹¹Macy and Flache (2009, p. 265)

¹²Hedström (2005, p. 3)

¹³Hedström (2005, p. 1)

¹⁴Hedström (2005, p. 2)

¹⁵Epstein and Axtell (1996, pp. 4–5)

tificial environments under rules that place only bounded demands on each agent's information and computational capacity'.¹⁶ Generative agent-based modelling is a computational method whereby experiments are carried out on these models. It is computational because the rules of the model are expressed as a program (carried out by hand or by computer), and uses experiments in which the human system can be isolated and influenced.¹⁷

This method of agent-based modelling is not dissimilar to the experiments carried out in natural sciences, except that the subjects of the experiments are models, not the natural phenomena themselves. Nigel Gilbert remarks that experimenting on models rather than the real world is not novel either: architects investigating the behaviour of buildings under high winds will typically use a scale model of the building in a wind tunnel: a representation of the real thing that has the same salient features and will behave in an analogous way.¹⁸ If the model is simple enough, the observer is placed at an Archimedian point above the society.

Willer and Macy note four key assumptions about the agents of ABMs: they are autonomous, interdependent (exert influence on each other), the rules they follow are simple (because the complexity of human behaviour is in the environment¹⁹) and they are adaptive and backward-looking (they do not calculate the most efficient action, but react to what has already happened).²⁰ The agents need not fit standard assumptions: they need be neither rational (they typically express bounded rationality) nor their populations homogeneous (in spatial or social locations or psychological characteristics), and models that use them can explain the dynamics of interactions rather than just their static equilibria.²¹

Because of this, agent-based modelling is most useful for considering theoretical questions involving agents (especially in situations in where it is otherwise difficult to relate hypotheses about individual behaviour to macroscopic regularities). The types of problems that generative sociology generally considers relate to the influences between agents and structures, diffusion and

¹⁶ Epstein and Axtell (1996, p. 6)

¹⁷ Gilbert (2008, p. 3)

¹⁸ Gilbert (2008, p. 3)

¹⁹ Simon (1998, p. 53)

²⁰ Macy and Willer (2002, p. 146)

²¹ Epstein and Axtell (1996, p. 2)

contagion processes, interactions between local and global phenomena, heterogeneity, bounded rationality, and social dynamics and equilibria.²² Specifically, research has addressed processes that require coordination (traffic slowing down on the opposite side of the road to accidents, the operation of daylight savings, the tragedy of the commons, congestion in resource consumption)²³, and flocks; emergent global patterns (the diffusion of information, emergence of norms, coordination of conventions, participation in collective action, revolutions, market crashes, fads, and feeding frenzies)²⁴; opinion dynamics, lock-in, consumer confidence, industrial networks, supply chain management and electricity markets²⁵; wealth distributions, firm size and growth rates, price distributions, spatial settlement patterns, economic classes, price equilibria in decentralised markets, trade networks, spatial unemployment patterns, excess volatility in returns to capital, military tactics, organisational behaviours, epidemics, traffic congestion patterns, cultural patterns alliance, stock market price time series, voting behaviours, cooperation in spatial games and demographic histories²⁶.

Examples of Agent-Based Models Most of the history of agent-based modelling is relatively recent. In part, this is because the simulations used in ABMs often (but not always) require the use of recently developed computational techniques and capabilities, such as object-oriented programming and parallel processing. Even so, the earliest common example of something that approaches agent-based modelling is found in the work of von Neumann in the 1940s.²⁷ Part of von Neumann's legacy in agent-based modelling is that, like the mathematical descendants of his cellular automata, most agent-based modelling is performed on computers. This allows repeated large-scale experiments to be carried out quickly, and the process is facilitated by those recently developed technologies which are particularly suited to design and verification of so-called *in silico* models. While, as Joshua Epstein observes, the use of computers to perform the

²²Epstein (2006a, p. xii-xviii)

²³Schelling (1971b)

²⁴Macy and Willer (2002, p. 148)

²⁵Gilbert (2008, pp. 6–14)

²⁶Epstein (2006a, pp. 7–8)

²⁷Gilbert (2008, p. ix)

simulations is not the point,²⁸ it is almost always a practical requirement.

For instance, Thomas Schelling's work on segregation and micromotives from the 1960s and 1970s²⁹ did not use computers; his simulations were carried out by moving of coins on a chessboard or marks on a piece of paper. Schelling's model of the process of segregation is one of the earliest efforts to carry agent-based simulation experiments in the social sciences. He models segregation in a society made up of people belonging to two different groups ('plusses and zeros'), initially randomly distributed in a representation of their neighbourhood. Each person has a preference over the composition of their local neighbourhood, and if these preferences are not currently satisfied, they move to the nearest point at which they are happy. Schelling finds that individual efforts to achieve a local majority will tend to result in super-majorities in the overall distribution of agents. If levels of tolerance are heterogeneous, then the only two stable equilibria are total segregation, where either plusses or zeros are absent (because otherwise individuals on the borders between the two groups will not be satisfied). Even if every agent is relatively tolerant and prefers a mixture of plusses and zeros rather than the total absence of the other group, the same results are produced. What's more, states of total segregation are stable, because no single agent can change the composition of the neighbourhood enough to affect the dispositions of others. Schelling's work already shows a few of the common features of agent-based models: the agents are simple, heterogeneous (in group membership), and have bounded rationality (they only consider the immediate consequences of their actions and are only aware of their immediate neighbours). The aggregate results obtained produced by uncoordinated individual action, were not intended by any individual and were not easily predictable from descriptions of the agents alone.

Modern computational ABMs only really began in the 1990s.³⁰ Epstein and Robert Axtell's Sugarscape is one such model³¹. It studies the interactions of simple animal-like agents with each other and with an environment, through the processes of resource acquisition, migration,

²⁸ Epstein (2006a, p. xiii)

²⁹ Schelling (1969); Schelling (1971a); Schelling (1971b)

³⁰ Gilbert (2008, p. xi)

³¹ Epstein and Axtell (1996)

reproduction, trade, evolution and war. They also find that ‘A wide range of important social, or collective phenomena, can be made to emerge from the spatio-temporal of autonomous agents operating on landscapes under simple local rules’.³² The process by which agents search for resources results in skewed wealth distributions and migration, the introduction of sex causes various population dynamics, culture segregates the groups into tribes, and combat results in a variety of conflicts.³³

1.3 Criticism of Agent-Based Modelling

According to Michael Macy, agent-based models have been criticised for being unrealistic, tautological and unreliable.³⁴ Because ABMs are *models*, they (and especially their agents) are necessarily simplified representations of aspects of the real world. This might make them unrealistic, but the simplicity of agents is probably not such a bad assumption to make. Macy and Willer note that the complexity in human behaviour is probably the result of complex environments: at least for some processes, we do follow simple rules.³⁵ Artificiality also allows us to deepen our understanding of particular aspects of behaviour while ignoring others; models that conformed fully to our understanding would not be easy to interpret. The agents of the Sugarscape model are doubtless unrealistic, but they still engage in realistic behaviour which is complex enough to be interesting.

Simulations, like mathematical models, are supposedly tell us nothing we do not already know. The solutions are built into the expression of the problem, so we are not likely to learn anything new or surprising from such models. In actual fact, agent-based models are exactly the opposite: while micro-level processes are described by the model, the macro-level patterns produced are not readily apparent from the formulation of the question. The results are present in the expression of the problem, but that does not make them obvious.

However, unlike mathematical models, agent-based models are numerical, so some claim that

³² Epstein and Axtell (1996, p. 153)

³³ Epstein and Axtell (1996, p. 153)

³⁴ Macy (2002, pp. 8–12)

³⁵ Simon (1998, p. 53) in Macy and Willer (2002, p. 146)

they can not be used to establish law-like regularities or generalisations. This problem is a consequence of the use of agents with adaptive rather than optimising strategies. Therefore, the only way to prevent this would be to forego one of the most significant benefits of agent-based models.³⁶ The use of agent-based models in such situations has been defended comprehensively: Axtell observes that they can have considerable advantages over equation-based models for certain types of problem,³⁷ and Epstein shows that all agent-based models can be systematically expressed as equations (although they are not as comprehensible in this form).³⁸ In fact, the computational nature of agent-based models gives them advantages over both ‘prose’ and mathematical models.³⁹ Unlike the former, they are complete, consistent and unambiguous (and must be so in order to be understood by the computer carrying out the simulation). Unlike most of the latter, they can account for heterogeneity and explain processes that are not at an equilibrium.

Epstein also points out some ‘fundamentally important’ epistemological issues with generative explanation⁴⁰. Of particular interest is the question of generative sufficiency versus explanatory necessity: a particular microspecification that generates the macroscopic regularity in which we are interested might do so in an absurd and non-explanatory way. When referring to a concrete phenomenon, the agent-interaction rule must provide a *plausible* explanation. Some irrelevant possibilities can be ruled out by close examination, empirical verification of the component assumptions, or the concerns of theoretical economy. However because of the difficulty of linking microspecification and macroscopic regularity other by than the use of ABMs, it might sometimes be difficult to determine which microspecification is the most plausible.

The Generative Standard of Explanation Despite these provisos, the generative approach to social science has proved useful and robust. Epstein observes that the generative approach might impose ‘certain changes in perspective [...] on the social sciences’⁴¹. Most fundamen-

³⁶Macy (2002, p. 12)

³⁷Axtell (2000)

³⁸Epstein (2006a, pp. 54–56)

³⁹Gilbert (2008, p. xi)

⁴⁰Epstein (2006a, pp. 53–67)

⁴¹Epstein (2006a, p. 8)

tally, it changes the standard of explanation: the generative standard of explanation for the social sciences is succinctly expressed by the “the bumper sticker reduction of the agent-based computational model”⁴²: ‘If you didn’t grow it, you didn’t explain it’⁴³. Generativists require that it can be shown that the phenomenon of interest ‘is effectively attainable under repeated application of agent-interaction rules [...] *effectively computable by agent society*’⁴⁴. However, generation need not be the only admissible form of explanation for the social sciences, because only ‘problems involving the formation or emergence of macroscopic regularities’⁴⁵ are under consideration. But for such problems, models of autonomous interacting agents are close enough to the real phenomena to be plausible, but abstract enough to be useful. A quote apocryphally attributed to Ernest Rutherford states that ‘All science is either physics or stamp collecting’⁴⁶. It might (somewhat uncharitably) be said that generativists claim that all sociology is either agent-based generation or pamphleteering.

1.4 The DBO Model as an Agent-Based Model

Hedström’s Desires, Beliefs and Opportunities (DBO) model, presented in *Dissecting the Social: On the Principles of Analytical Sociology*⁴⁷ is an agent-based model characterised by its explanation of agents’ psychology. In *Dissecting the Social*, computational applications of the DBO theory to agent-based generative sociological modelling are used to test theories about a number of social phenomena. The aim of the present discussion is to examine variations on the computational application of the DBO theory, by varying the parameters of the model. I intend to show the DBO theory can be used to model more social phenomena by doing so, and that some of the simplifying assumptions that were used can produce misleading results. Some validations of theories made on the basis of these simplified simulations would be mistaken, as only some

⁴²Macy and Flache (2009, p. 261)

⁴³Epstein (1999, p. 43)

⁴⁴Epstein (2006a, p. 8). Emphasis in original.

⁴⁵Epstein (2006a, p. 8)

⁴⁶Rutherford is also supposed to have said ‘The only possible interpretation of any research whatever in the “social sciences” is: some do, some don’t.’ On this point, I intend to disagree with him.

⁴⁷Hedström (2005)

of the results are robust across changes while others are artefacts of the assumptions⁴⁸.

One of the roles of agent-based modelling is ‘to subject certain core theories [...] to important types of stress’⁴⁹. This typically involves relaxing assumptions about micro-level behaviour, in order to see if the generated macro-level behaviour collapses. This has also been described as the ‘sensitivity test’: checking the effect on output of small changes in input.⁵⁰ The ‘certain core theories’ that I intend to stress are those expressed in the DBO model itself, which relate to ‘our intuitive understanding of social processes’,⁵¹. The method, as described in detail below, is to change characteristics of agents, environment and rules, to see whether previous macro-level results persist.

In the following section, I will discuss the DBO model in more detail, and outline which parameters are significant in its computational application. In Section 3, I will discuss some the of consequences of modifications of the rationality of the DBO model agents. Section 4 considers the results of changing the processes that occur between agents: how they are influenced by each other and the networks in which they interact. The concluding section remarks that there are some results that are indeed robust across changes of the model, but many of the more realistic assumptions produce different macro-level behaviour.

⁴⁸Axtell (2000, p. 5)

⁴⁹Epstein (2006b, p. 4)

⁵⁰Epstein (1999, pp. 52–53)

⁵¹Watts and Dodds (2007, p. 455)

2 Methodology: Parameters and Variables

2.1 The Parameters of the Computational Application of the DBO Model

According to Hedström, action theories should be psychologically and sociologically plausible (take into account the structure of social action), as simple as possible (trade off the complexity of agents towards that of the structures of interaction and the micro-macro link), and should explain action in meaningful intentional terms (with reference to the future state that actors seek to bring about).⁵²

At the micro-scale, the DBO model explains the actions of individual agents in terms of their psychological (and social) state, which is composed of desires, beliefs and opportunities (hence ‘DBO’). Because the micro-level theory is always about a particular voluntary action on the part of an agent, the ways in which the states of their ‘folk’ psychology are combined determine whether the agent acts or not. A rational agent will act if she desires a particular goal, believes that the action in question will help her to reach that goal, and has the opportunity to carry out the action. This makes the DBO both similar to and distinct from other models⁵³ of epidemiological processes. The macro-level phenomena are similar to those of other models: rate of spread, segregation of infected populations, influence of initial network and agent conditions on outcomes, and so on. But the DBO model is distinct from models that use a biological metaphor for contagion processes – like the SIR (susceptible, infective, recovered) and SIS (susceptible, infective, susceptible) models⁵⁴ – because each agent is endowed with a theory of mind composed of distinct parts. As a consequence, the model is more suited than others to show how *action* spreads in *social* networks.

Although the model is specified at the micro level, the aim of the DBO model is to explain phenomena that occur at the macro level. In fact, the phenomena of interest – characteristics of the *social* – cannot even be defined for an individual agent, only for a group, collectivity

⁵²Hedström (2005, pp. 35–36)

⁵³Newman (2003, pp. 40–43)

⁵⁴Newman (2003, pp. 40–43)

or a society as a whole.⁵⁵ Differences in the agents will produce different occurrences at the macro-level, but a study of the individual agents alone will not reveal how they will act when brought together into a community. In order to use the agents defined by the DBO model in an agent-based simulation, they need to be placed in a social context.

Consider the spread of a fad through a society, a (well-trodden) example of an epidemiological process. The DBO model is always concerned with a particular action, in this case the purchase of some paraphernalia required to take part in the fad, say, a yo-yo. In order to run the simulation, each agent in the society is considered in turn. Those other agents that will have an influence on the individual are determined by means of the *social network*, which shows which other agents are in close enough social proximity to influence the individual. The notion of ‘social proximity’ might represent friendship, or professional relationship, or whatever else is likely to affect the action under consideration. In the case of a fad, it is most likely that friends are those who mediate the spread.

The influential agents in social proximity to the focal agent are known as her *neighbours*, which collectively constitute the *neighbourhood*. The desires, beliefs, opportunities and action of the neighbours are combined by the *aggregation process* to make those of the *social aggregate actor*. The social aggregate actor is not really an actor of the model, but a *gestalt* that has the composite properties of the influential neighbours. The aggregate actor is a necessary fiction, as there are cases in which the individual neighbours do not exert influence in a uniform way, so that the influence of the neighbourhood on the focal agent is not just the average of individual influences. It could be the case that some neighbours matter more than others, there might be thresholds below which none has influence, and so on. In such cases, imagining that the focal agent is only influenced by a single actor with aggregate properties simplifies the process. It is possible that some of the friends are more respected and more influential than others, so their desire to buy a yo-yo, their belief that buying a yo-yo will allow them to participate in the fad, and their opportunity to buy yo-yos will count more than others.⁵⁶

⁵⁵Hedström (2005, p. 7)

⁵⁶It is possible that desire, belief and opportunity are affected differently by different people. So a person that people

The social aggregate actor's desire, belief and opportunity affect those of the individual in question by means of the *inter-agent mechanism*. Generally, the desires of the aggregate actor affect the desires of the individual, beliefs affect beliefs and opportunities affect opportunities, but other configurations are also possible. In cases where the agent is not privy to her neighbours' states of mind, she might try to guess them based on their actions: if the neighbours are acting, then (assuming they are rational), they will have positive desire, belief and opportunity. In this case, the action of the social aggregate directly affects the agent's DBO state. In our example, the agent will observe others with yo-yos and assume this means that they want(ed) to buy a yo-yo, believed doing so would allow them to take part in the fad, and know where to make the purchase. If she already believes and has opportunity, she might only base her desires on the actions of others.

For a rational agent, once the psychological state is determined, the next two steps are simple: the agent acts if and only if she desires to, has the belief that acting will be effective, and has the opportunity to act. Some agents are not rational, and will act without these conditions being fulfilled (or indeed, they may not act when these conditions are fulfilled), depending on their *DBO-modification mechanism* (DBOMM) and *action-decision mechanism* (ADM). In the example, if the desire to participate in the fad is overpowering, she may purchase a yo-yo without wanting to learn how to use it, so without the belief that the yo-yo will be useful. When the simulation has determined how one agent acts, it considers the next in turn until the round of interaction is complete (although the states of all agents are updated simultaneously). Figure 2 on the following page is a schematic of the structure of the whole process.

As is often the case with sociological models, there is a trade-off between the explanatory power and the comprehensiveness of the model. There are many ways in which the variables could be made more complicated, but wherever possible, I have tried to expand the model in the

want to resemble will have very influential desires, but one perceived as insightful will have more influential beliefs. This possibility can be accommodated either by having separate social networks for desire, belief and opportunity, or by appropriate aggregation processes. In the interests of simplicity, it is probably easier to consider the agent's neighbours being those that can have *any* sort of influence on her, and to deal with differences in influence between parts of the psychological state using the aggregation process.

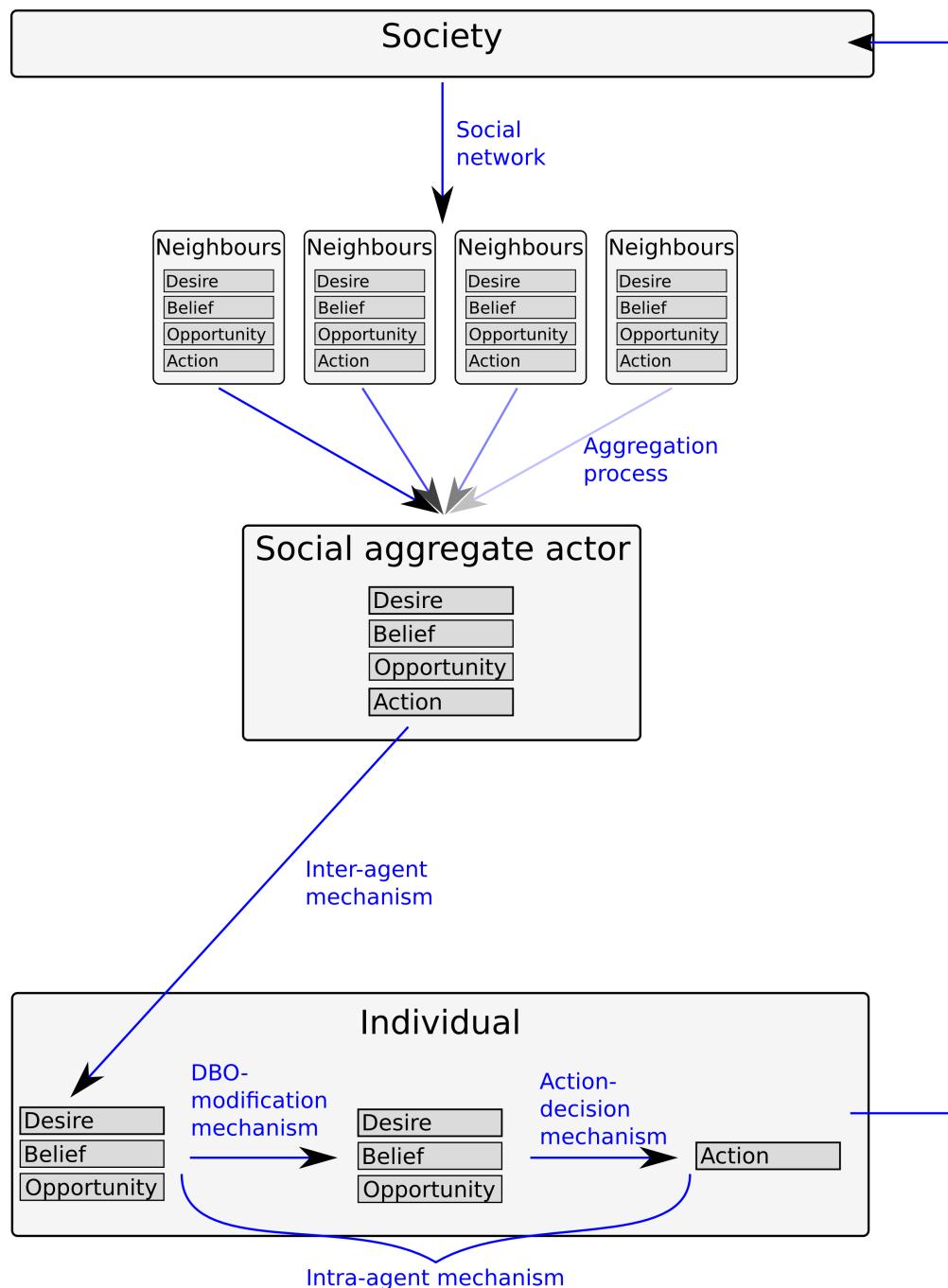


Figure 2: The computational application of the DBO model
Partially based on Figure 3.2 from *Dissecting the Social*

direction of greater verisimilitude. Whether the outcomes that are produced are themselves more realistic remains to be seen, and is beyond the present scope: I only seek to determine what the implications of different assumptions variables are on the outcome of simulations.

The models used for this particular application are written as a series of Python scripts. Python was chosen as a programming language for various reasons, including the ability to develop modular scripts with re-usable components and the availability of libraries for certain key mathematical functions. The scripts specifically written for this purpose are (with some significant double-counting) about 7000 lines long, containing about 27000 words⁵⁷. Each of the simulations in the following sections is generally run by one script over multiple iterations in order to span the required parameter space and to produce an average result from multiple simulations. The number of repetitions for the following simulations is often between 100 and 1000, but sometimes 10 or 20 for particularly computationally intensive simulations. A typical script for a particular simulation (re-written in pseudocode) is shown in Figure 3 on the next page.

2.2 The Outcomes of DBO Model Simulations

There are different ways of measuring the outcomes of DBO model simulations, some of which may be more important than others. As Hedström does, I will assume that all the agents have the opportunity to act. This assumption does not necessarily restrict the generality of the model, as the desire, belief and opportunity of the actors contribute to the agents' action in exactly the same way. Conclusions that are drawn about one of the parameters will be exactly equivalent to those for the others, while the interaction between two parameters allows a sufficient degree of control over the model, and results similar in kind.

Therefore, I will focus on measurements derived from the simplest social-level phenomena of the DBO model: the distributions of action, desire, and belief. For each agent, the DBA (desire, belief, action) state can be written as an ordered 3-tuple (or triplet) of the form $\langle D B A \rangle$, where D, B and A are replaced by 1 or 0 depending on whether each component the agent's

⁵⁷According to GNU coreutils' word counting program wc.

```

loop (500 times):
    for beta (between 0 and 1 in steps of 0.1):
        # Initialize the model
        Create the network of agents (grid, 50 by 50)
        for (each agent):
            Set DBO state of agent(DBO state from random distribution with mean beta)
    
    # Run 20 rounds of interaction for the model
    loop (20 rounds of interaction):
        for (each agent):
            # Find the social aggregate actor
            for (each neighbour in the neighbourhood of agent):
                Set DBO state of social aggregate actor (DBO state of neighbour, number of neighbours)
                
            # Inter-agent influence
            Set DBO state of agent (DBO state of social aggregate actor, inter-agent mechanism)
        
        for (each agent):
            # DBO-modification mechanism
            Set DBO state of agent (current DBO state, DBO-modification mechanism)
            # Action-decision mechanism
            Set acting state of agent (DBO state, action-decision mechanism of agent)

```

Figure 3: Pseudocode for a model with varying distributions of desire and belief

state is positive or negative. For example, the triplet $< 1 \ 0 \ 0 >$ corresponds to an agent who has desire, but not belief, and does not act. The triplet alone does not necessarily indicate what action-decision mechanism the agent is using. $< 1 \ 0 \ 0 >$ might be the state of a rational agent, but it might not. However, $< 1 \ 0 \ 1 >$ is definitely not the state of a rational agent, because such an agent would not act without belief.

The only possible DBA states of rational agents are those represented by the DBA-triplets $< 0 \ 0 \ 0 >$, $< 1 \ 0 \ 0 >$, $< 0 \ 1 \ 0 >$, and $< 1 \ 1 \ 1 >$. Because of this, simply measuring the proportion of agents who have, say, positive desire, will include those with both states $< 1 \ 0 \ 0 >$ and $< 1 \ 1 \ 1 >$ (see Figure 4 on the following page). The agents in these two states have the same desire, but they are otherwise quite dissimilar. It is more informative to measure the number of actors in each of the four DBA states. These states also have interesting interpretations: actors in state $< 0 \ 1 \ 0 >$ are deprived (desirous but not confident of the efficaciousness of action), actors in state $< 1 \ 0 \ 0 >$ are uninterested (knowing action will succeed but not wanting the outcome), those in state $< 1 \ 1 \ 1 >$ act, and those in state $< 0 \ 0 \ 0 >$ have no orientation to action at all. It is also of policy interest, as well as of theoretical interest, whether the proportions of people in a particular DBO state are stable, or how they evolve. On top of these basic measurements, more complex ones can be built, like the degree of segregation according to psychological state or the responsiveness of the measured outcomes to changes in the initial situation.

In the following sections, I will consider the effects of variations in intra-agent processes, social networks and aggregation processes on societies of DBO model agents. These parameters seem to be the most promising (and the most easily interpreted) candidates for modification. Intra-individual mechanisms were discussed in *Dissecting the Social*, where the use of different assumptions about agent psychology was found to yield different results. Thresholds like those that can be introduced into the aggregation process play an important role in other models, and social networks are so well-studied that it is easy to find realistic models with important and well-known characteristics. Therefore, it is worth examining what happens when modified –



Figure 4: The relationship between action, desire, opportunity, deprivation and uninterest

and arguably more realistic – versions of all of these parameters are used to apply the DBO model. The standard simulation (Section 3.1 on the next page) provides a baseline against which we can compare modified simulations.

3 Processes Within Agents: Intra Individual-Mechanisms

3.1 The Basic Simulation

Hedström uses agent-based simulation ‘to model an historical or temporal process’⁵⁸: starting from a random distribution of desires and beliefs, the agents are made to interact with their neighbours in the way prescribed by the model. Each agent in the simulation is examined in turn, and her desires and beliefs are affected by the actions of her neighbours following the inter-agent mechanism. In the basic simulation, the social network is a regular toroidal lattice: the agents are set out on a grid of which the top is connected to the bottom and the sides to each other, so that the grid forms a doughnut shape, or torus. Each agent’s neighbours are those directly to her left and right as well as those above and below her. The aggregation process is one of majority influence whereby the social aggregate actor’s DBO state is equal to the majority desire amongst her neighbours, and likewise for her beliefs (see Figure 5 on the following page). The inter-agent mechanism is simple and direct: the desires of neighbours determine the desires of the agent, beliefs determine beliefs and as a consequence the action of the neighbours determines the action of the focal actor (because all the agents have the same rational action-decision mechanism and no DBO-modification mechanism).

The society in the basic simulation is composed of 2500 agents. Before interaction occurs, a randomly selected 40% of them have positive desires, and 40% have positive beliefs. The number of agents acting, deprived and uninterested is measured during 20 interactions. Figure 6 on the next page and Table 1 on page 28 show the results.

These results confirm those found in *Dissecting the Social*, where the proportion of agents acting after 20 interactions was found to be .05, and the proportion of agents deprived was 0.17.⁵⁹ However, by showing how the actions of actors evolve at each interaction, it is also made apparent that the distributions settle very quickly: the situation after 20 interactions is very similar to that after 5 interactions; it might be surprising that with the sparse connections of a regular

⁵⁸ Hedström (2005, p. 78)

⁵⁹ Hedström (2005, p. 82), Table 4.2

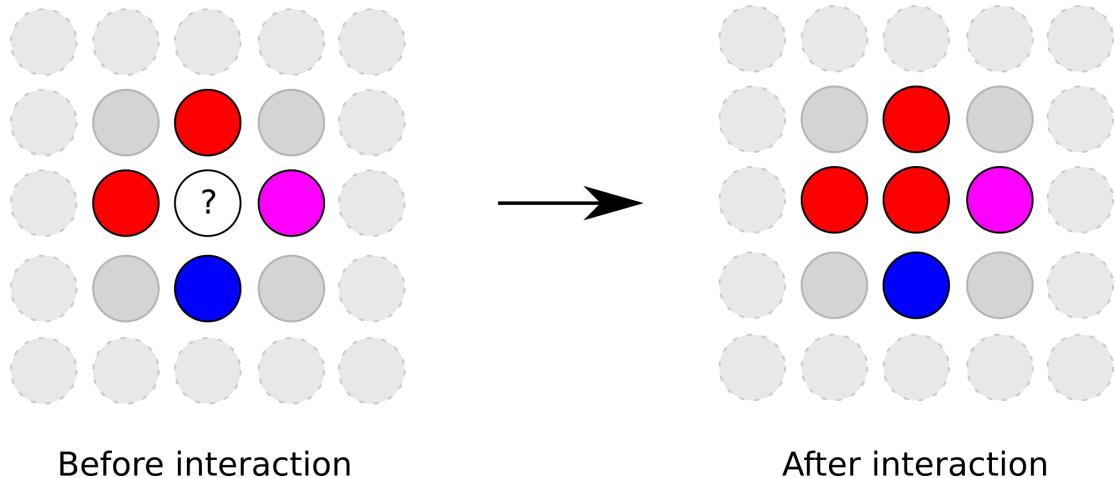


Figure 5: The interaction process

The agent under consideration is the one marked with a question mark. Her neighbours are the agents shown in colour: those shown in blue have positive belief, those in red positive desire, those in purple have both (because of which they act). Because a majority of the neighbours have positive desire, after interaction the agent does too. However, there is not a majority of agents with positive beliefs (the blue and purple agents), so the central agent does not have positive belief.

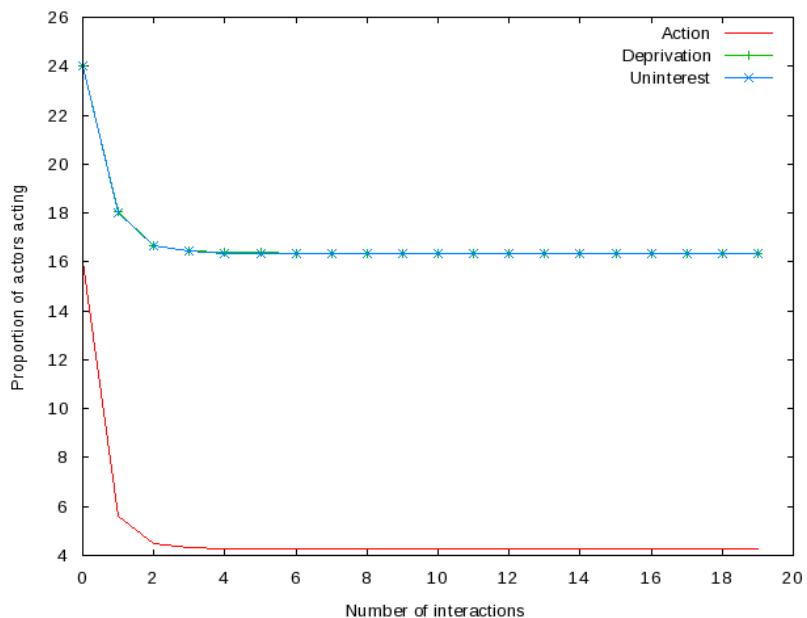


Figure 6: The standard simulation

Interaction number	Action	Deprivation	Uninterest
0	0.1600248	0.240204	0.2399776
1	0.056096	0.1807296	0.1802208
2	0.0448168	0.1666424	0.1664432
3	0.0433232	0.1644976	0.1643032
4	0.0428576	0.163912	0.1636648
5	0.042652	0.1637016	0.1634424
6	0.04258	0.1636272	0.1633408
7	0.0425552	0.1635888	0.1632816
8	0.0425456	0.1635736	0.1632544
9	0.0425384	0.1635672	0.1632488
10	0.042536	0.1635648	0.1632432
11	0.0425352	0.163564	0.1632408
12	0.0425352	0.163564	0.16324
13	0.0425352	0.163564	0.16324
14	0.0425352	0.163564	0.16324
15	0.0425352	0.163564	0.16324
16	0.0425352	0.163564	0.16324
17	0.0425352	0.163564	0.16324
18	0.0425352	0.163564	0.16324
19	0.0425352	0.163564	0.16324

Table 1: The standard simulation

lattice the society as a whole reaches a steady state after so few interactions. Desire and belief are treated the same, so the fraction of agents that are uninterested unsurprisingly follows an almost identical trajectory to the fraction of agents that are deprived.

Figure 7 on page 30 shows typical patterns in the society at different stages of the basic simulation. The blue circles represent agents with the belief that their actions will be effective, the red circles represent agents who desire to act, and the purple circles represent agents who have both desires and beliefs, and consequently do act. In the eventual situation, those agents with desire and belief have formed into relatively segregated groups. Where these groups overlap (for 4.25% of agents after 20 interactions), the agents act. The eventual macro-level stability is the result of meso-level islands of stability, which themselves result from random micro-level differences. The (uncoordinated) actions of thousands of individual actors are very efficient in producing stable results in the larger society, because each actor can reach a stable

state with respect to her neighbours (but the states reached are not uniform).

The segregation that occurs can also be expressed in terms of node attribute correlation, that is, the likelihood that a node with a particular DBO state will be connected to nodes with each DBO states. Figure 8 on page 31 shows the attribute correlation for the basic simulation before and after interaction. As before, the blue nodes have positive belief, red nodes have positive desire, grey nodes have neither, purple nodes have both, and consequently act. The size of the nodes in the figure shows the proportion of nodes of each type, and the width of the links between the nodes indicates the proportion of links between the types of nodes. The segregation in the final situation is evidenced by the relatively small proportions of connections between different node types, compared to the connections within the same node types. Even though the proportions of agents with desire, belief, and acting shrink, the proportion of links between agents with desire and other agents with desire, between agents with belief another agents with belief and between acting agents and other acting agents increases.

3.2 Non-Rational Intra-Individual Mechanisms

3.2.1 The Limits of Rationality

In order to establish whether the results found so far are robust, it is necessary to vary some of the parameters of the model. Of interest in this section is how the initial characteristics of individual actors affect their action. Each individual decides⁶⁰ whether to act or not based on her mental state⁶¹, but they need not all do so in the same way. Rational agents act if and only if their DBO state is $< 1 \ 1 \ 1 >$, but the DBO model does not require this; Hedström also considers the intra-individual mechanisms that he calls ‘wishful thinking’ and ‘sour grapes’. ⁶² An individual

⁶⁰The process by which actions are determined from mental states is not necessarily a conscious decision.

⁶¹Opportunities might be either mental or situational, but as noted above they are not used here for simplicity.

⁶²After Elster (1979); Elster (1983), cited in Hedström (2005, p. 83)

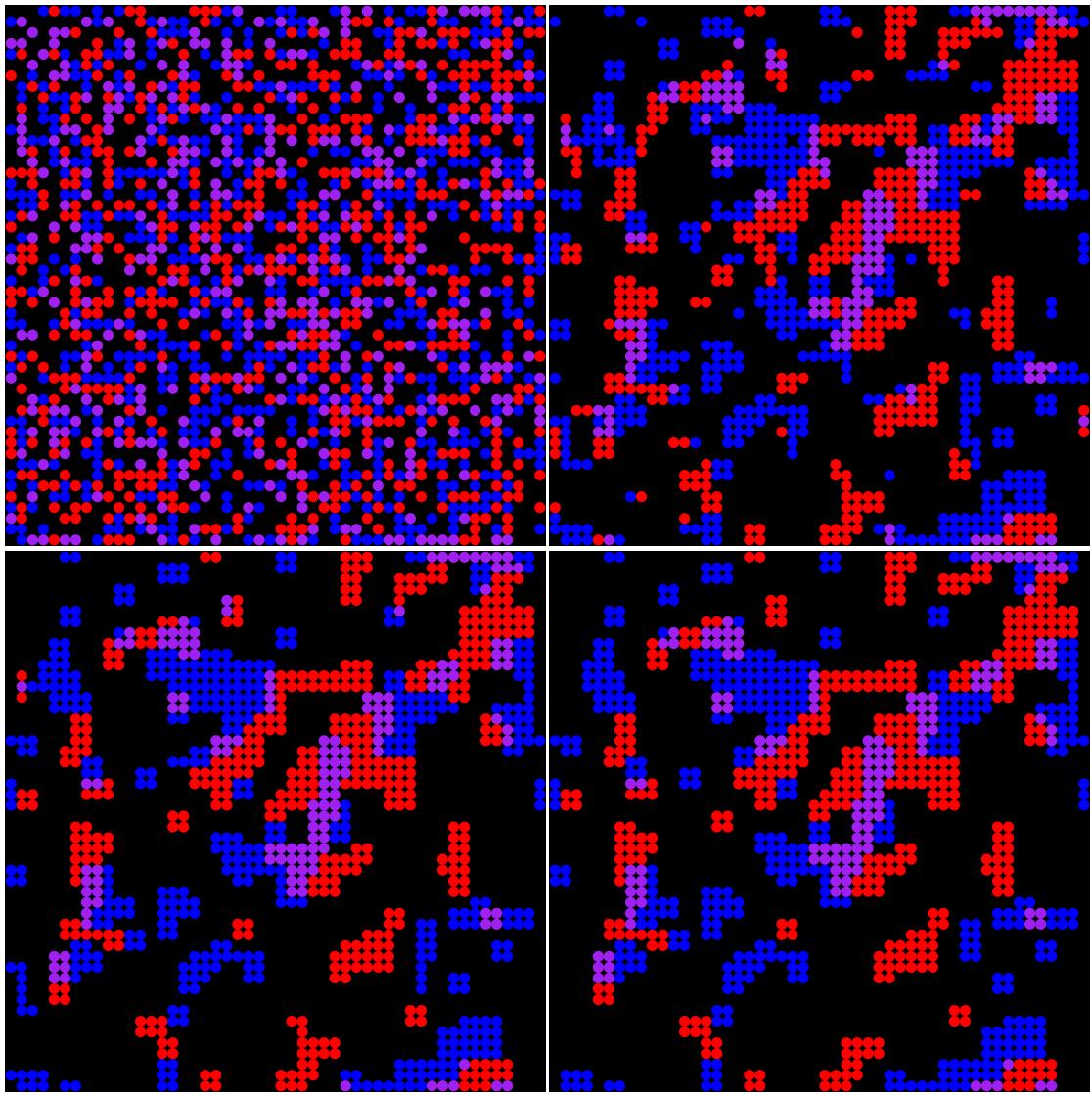


Figure 7: Typical distributions for the basic simulation
before interaction, after 1, 2 and 20 interactions

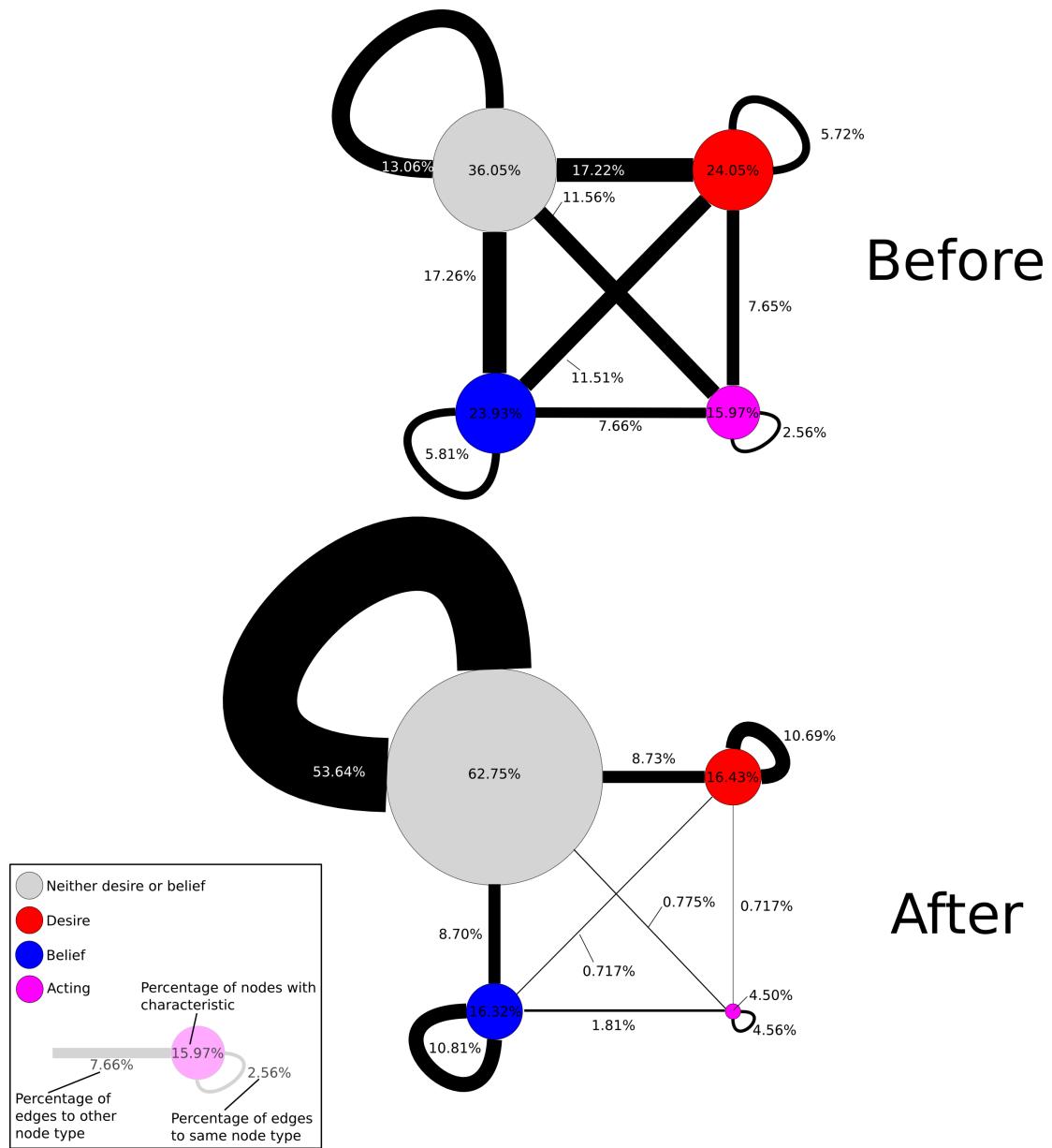


Figure 8: Attribute correlation for the basic simulation, before and after 20 interactions
Because of the counting algorithm, connections between nodes of the same type are counted twice. The results have been divided by two to produce this figure.

Initial				DBO modification →	Rational				Wishful Thinking				Sour Grapes			
D	B	O	A		D	B	O	A	D	B	O	A	D	B	O	A
1	1	1	1	modification →	1	1	1	1	1	1	1	1	1	1	1	1
1	1	0	0		1	1	0	0	1	1	0	0	1	1	0	0
1	0	1	0		1	0	1	0	1	1	1	1	1	0	0	0
1	0	0	0		1	0	0	0	1	1	0	0	0	0	0	0
0	1	1	0		0	1	1	0	0	1	1	0	0	1	1	0
0	1	0	0		0	1	0	0	0	1	0	0	0	1	0	0
0	0	1	0		0	0	1	0	0	0	1	0	0	0	1	0
0	0	0	0		0	0	0	0	0	0	0	0	0	0	0	0

Table 2: Intra-agent mechanisms of rational, wishful thinking and sour grapes agents
The lines in which opportunity is 0 are not under consideration here, because the agents are assumed to have the opportunity to act.

acting under the influence of wishful thinking thinks she can have everything she wants, whereas one acting under sour grapes does not want what she can not have. In other words, actors under wishful thinking will believe they can attain what they desire, and actors under sour grapes will not desire what they believe they can not attain. These dispositions are expressed in their DBO-modification mechanisms (DBOMMs). For wishful thinking agents, a DBA state of $<1\ 0\ 0>$ will be transformed into a DBA state of $<1\ 1\ 1>$, and for a sour grapes agent, a DBA state of $<1\ 0\ 0>$ will be transformed into the state $<0\ 0\ 0>$. The wishful thinking mechanism adds an extra DB(O)⁶³ state in which the agents act, while the sour grapes mechanism makes action less likely.

Even when the actors are all acting rationally, the initial distribution of their characteristics affects the macro-outcome. Figure 9 on the next page shows the typical evolution of societies with different initial proportions of acting agents. Broadly, if more than 25% of actors are acting initially, the proportion acting increases, otherwise, it decreases. This is because the proportions of actors with positive desires and beliefs increases (on average) if they are initially above 50%, because each agent takes on the state of the majority of its neighbours. Because rational actors require both positive desire and belief in order to act, and because desire and belief are distributed randomly and independently, when 50% of actors have positive belief and 50% have positive

⁶³DB(O) because opportunity is not used in these simulations.

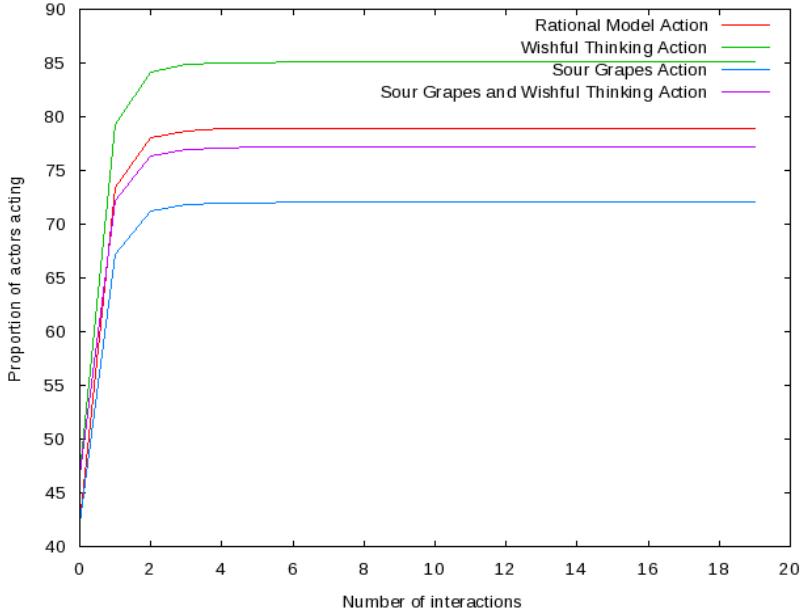


Figure 9: Proportion acting as a function of interactions for various initial proportions acting

desire, 25% will act. This is a consequence of action depending equally on desire and belief (and because, in these simulations, the proportions of agents with desire and belief are equal and the distribution independent). As a result, action does not vary linearly with desire and belief: the proportion acting will in fact be equal to the square of the proportion of desire or belief. If opportunity was included, the proportion acting would be equal to the cube of this value.

Although these results are relatively straightforward, there are some interesting observations to be made. Not only does the final level of action depend on the initial level, but the speed of the change does as well. The proportion of agents acting changes much faster at 70% initial action than it does at 40%, for example. However, all the simulations reach a stable level at around 5 interactions, which would seem to indicate that the society can absorb changes in initial state regardless of their magnitude.

The presence of even a small number of the aforementioned non-rational actors can have significant effects on the outcomes. When 20% of wishful thinking actors are added to the population, the final proportion acting increases to 85.01% from 78.94% with only rational actors. Con-

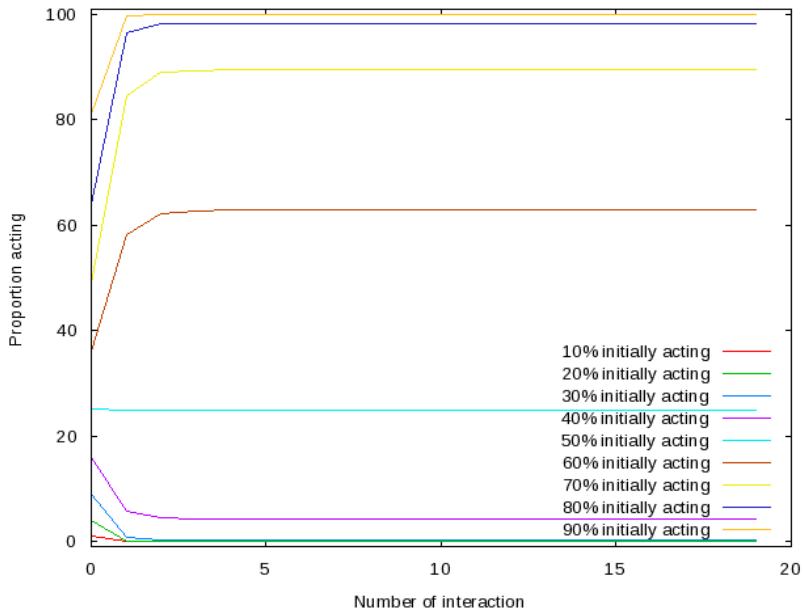


Figure 10: Proportion acting as a function of interactions for various initial proportions acting

versely, when 20% of actors act under sour grapes, the proportion acting decreases to 72.05%. Interestingly, the speed at which the society reaches a stable proportion acting is still the same: the society stable after about 5 interactions.

What's more, the presence of wishful thinking actors increases the magnitude of the changes that occur within the society. For every initial proportion acting, the change in the proportion acting over the course of interaction is greater (see Figure 11 on the following page). The presence of sour grapes actors broadly decreases the size of the changes, but when there are a number of both wishful thinking and sour grapes actors, the changes cancel each other out.

This might be expected, because wishful thinking and sour grapes have equal and opposite effects on the individual, respectively adding and removing a DB(O) state in which the agent acts. But at the level of the society, as Figure 9 on the previous page also shows, not all of the effects of wishful thinking and sour grapes cancel out. The presence of equal proportions of both types of non-rational actors in the population leads to a lower final proportion acting than if all the actors were rational. One reason for this is that the effects of an individual following one

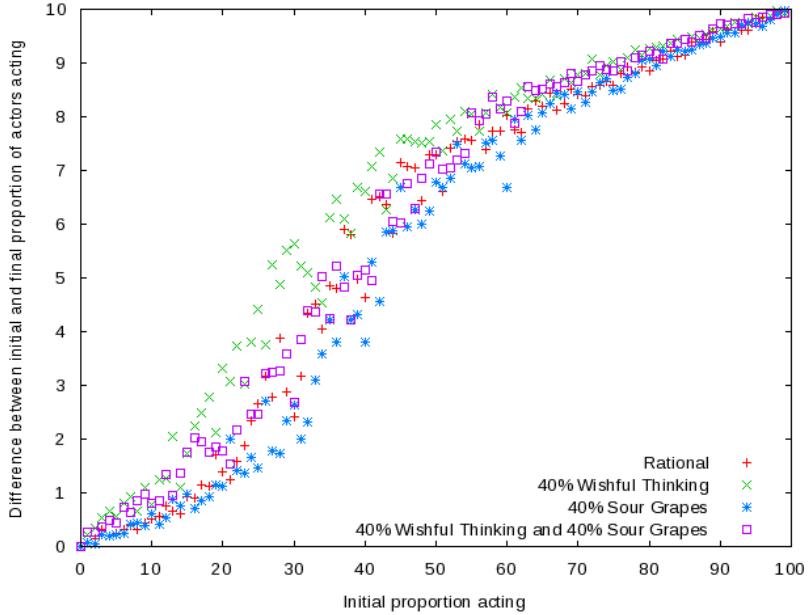


Figure 11: Proportion acting as a function of interactions for various initial proportions acting and with different intra-agent mechanisms

of these mechanisms does not end with the agent herself: because the agent's neighbours are affected by her desires and beliefs, anything that changes her desires and beliefs – even without changing her action – will affect her neighbours (see Figure 12 on the following page).

Whether agents act is not the only interesting measurable social outcome. As noted previously, those agents who want to act but do not believe that they will be successful are deprived, and those who believe that action would be effective but do not desire it are uninterested. Figures 13 on page 37 and 14 on page 37 show the effects: in the standard simulation, deprivation and uninterest decrease from 24.00% in the initial situation to 16.00% after 20 interactions. In all of the simulations with non-rational agents deprivation is substantially lower and uninterest is higher. In the case of deprivation, almost all of this is attributable to differences in the characteristics of the agents that are present right from the start: the vertical separation between the curves of Figure 13 on page 37 over the course of all the interactions remains almost the same. This is because both sour grapes and wishful thinking transform the DBA state that corresponds to deprivation ($<1\ 0\ 0>$) for each agent from the beginning. However, the models with wishful thinking

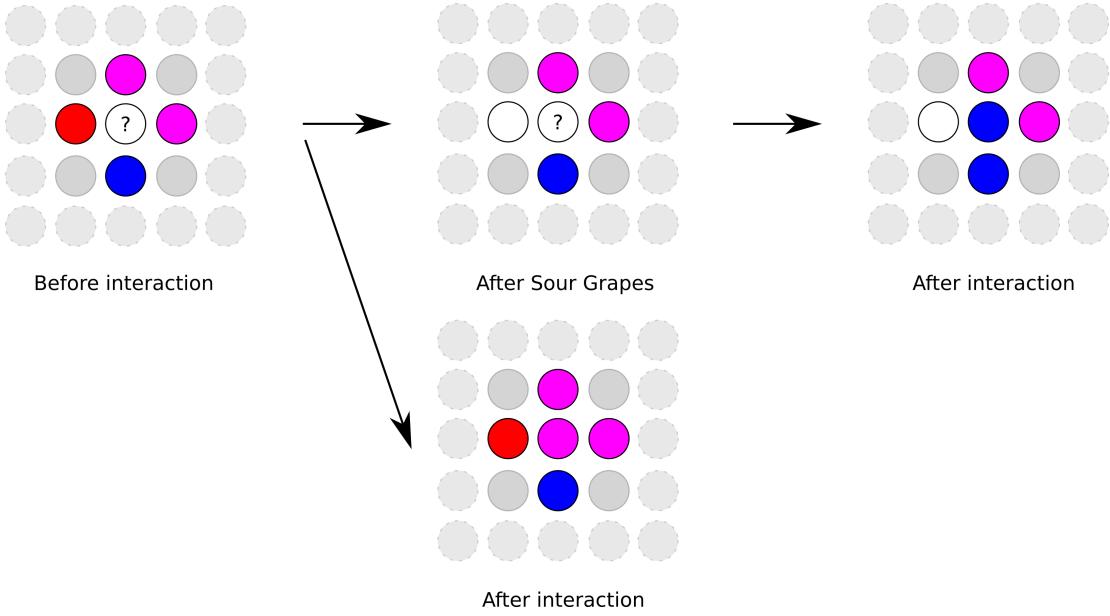


Figure 12: The effect of wishful thinking on neighbours

If the central agent’s neighbours had been acting rationally, the agent would act. Because they are acting under sour grapes, the agent is uninterested.

and sour grapes actors start with about the same proportion of disinterest, but end with different proportions. This is probably because wishful thinking agents transition to the state $< 1 \ 1 \ 1 >$ and stay there, while sour grapes agents that switch to the state $< 0 \ 0 \ 0 >$ are more likely to return to being deprived.

This seems to be confirmed by the effects on uninterest. Neither of the non-rational mechanisms affect uninterested agents directly, but they do increase the overall levels of uninterest in the population, relative to the simulation with only rational agents. In order to do this, they must make it more likely for agents to have the DBA state $< 0 \ 1 \ 0 >$. Wishful thinking agents only have a little effect, whereas sour grapes agents have more, and the combination of both has the greatest of all.

The effect of intra-agent mechanisms on social outcomes seems to be significant, so it is probably worth establishing the consequences the intra-agent mechanisms beyond the three considered so far.

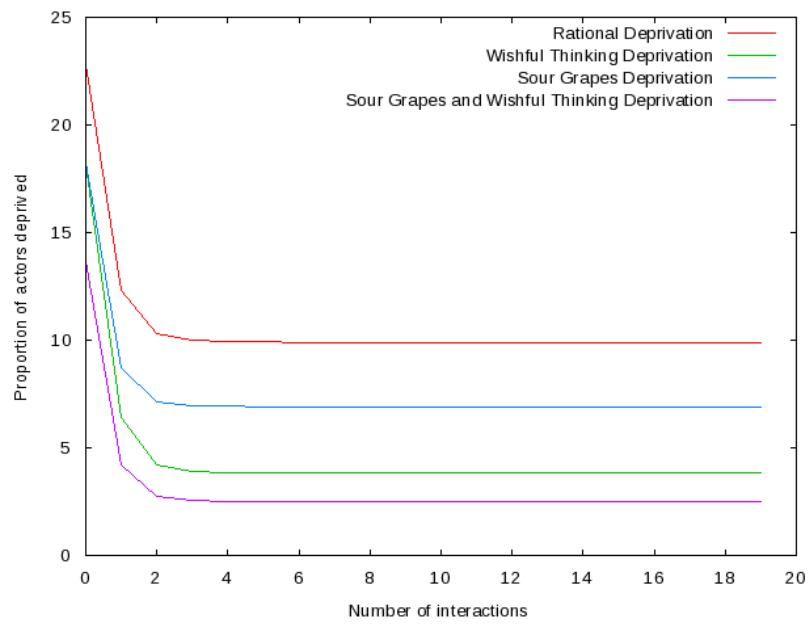


Figure 13: Evolution deprivation for different types of actors

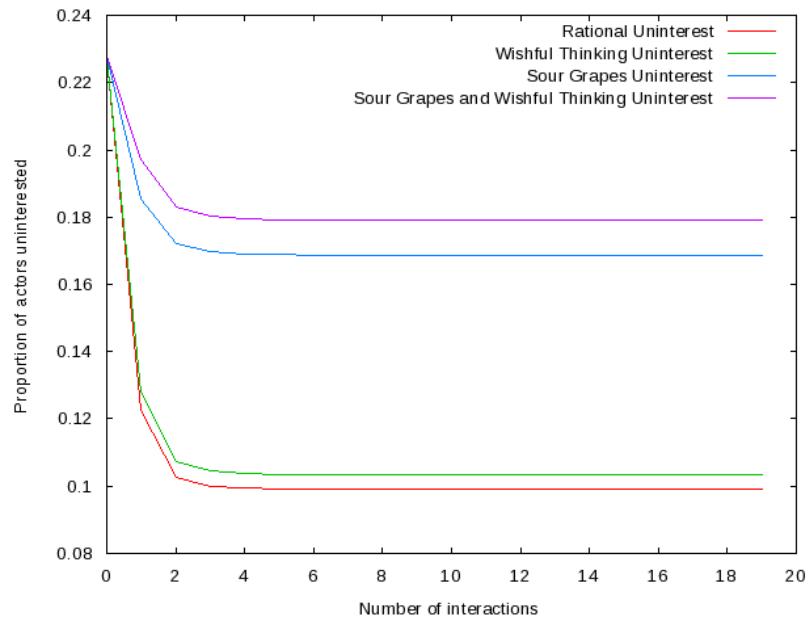


Figure 14: Evolution of uninterest for different types of actors

Initial				DBOMM →	Rational				Wishful Thinking + Carpe Diem				Come-What-May			
D	B	O	A		D	B	O	A	D	B	O	A	D	B	O	A
1	1	1	1	DBOMM →	1	1	1	1	1	1	1	1	1	1	1	1
1	0	1	0		1	0	1	0	1	1	1	1	1	1	1	1
0	1	1	0		0	1	1	0	1	1	1	1	1	1	1	1
0	0	1	0		0	0	1	0	0	0	1	0	1	1	1	0

Table 3: Intra-agent mechanisms of rational, wishful thinking and carpe diem, and come-what-may agents

3.2.2 New Intra-Agent Mechanisms

The most obvious way to introduce new intra-agent mechanisms is to increase the number of DB(O) states in which agents act. A rational agent acts when in one initial DBO state and a wishful thinking agent acts in two, so it would be interesting to see what happens when agents act in three or four cases. An actor who acts when she has a belief that her action will be effective, but without any particular desire for the outcome is taking advantage of any efficacious chance to act. Such an actor could be said to have a ‘*carpe diem*’ mentality. For such an actor the DBA state $<0\ 1\ 0>$ will always be transformed into the DBA state $<1\ 1\ 1>$. An actor under the influence of *both* wishful thinking and carpe diem will act in two additional cases compared to a rational actor.⁶⁴ Some actors might act in every circumstance, and they could be considered to have second-level preferences over action that override their first-order desires and beliefs. Regardless of the explanation, an actor who acts regardless of her desire and beliefs acts *come-what-may*. The effects of these DBOMMs on the initial DBO states of actors is shown in Table 3.

As might be expected, increasing the number of DB(O) states in which the agents act increases the proportion of agents acting, throughout their rounds of interaction. It is not surprising that actors who act ‘easily’ increase the society’s the level of action. When they act regardless of their mental states, the proportion acting reaches close to 100%. The come-what-may actors might seem far-fetched, but agents under the influence of both wishful thinking and carpe diem are not so difficult to imagine at all. The presence of 20% of this type of actor increases the action

⁶⁴This is not the same as in the previous simulations where some agents followed wishful thinking and some followed sour grapes, because there was no overlap.

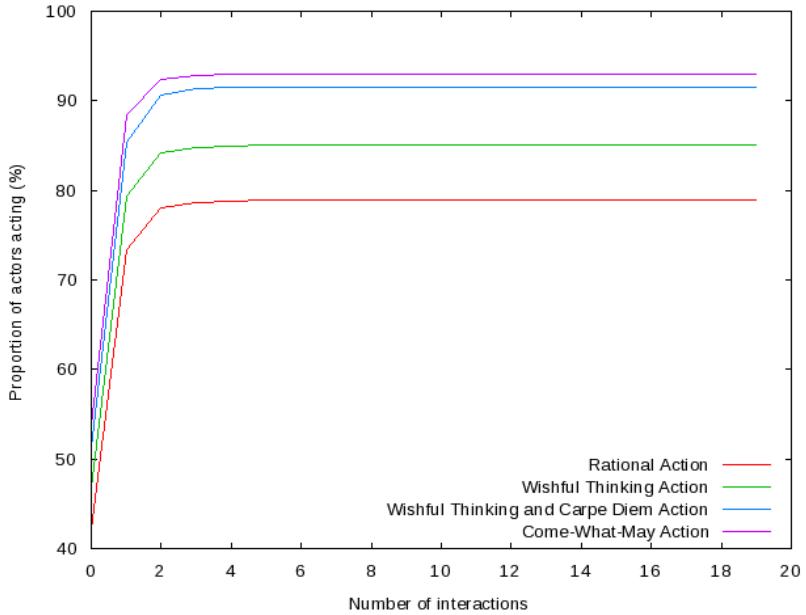


Figure 15: Action for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors

amongst all the agents acting from 79% to 92%, only slightly less than the effect of come-what-may actors.

Not only that, but the presence of these irrational agents decreases both the levels of deprivation and uninterest (Figure 16 on the next page and 17 on the following page): agents that act more easily are ‘beneficial’ for the society.

The difference between carpe diem/wishful thinking agents and come-what-may agents is in the way they react to the DBA state $< 0 \ 0 \ 0 >$. Because of this, more difference between the two mechanisms would be expected when there are low proportions of agents with desire and belief (that is, more agents in the state $< 0 \ 0 \ 0 >$). This is indeed what Figure 18 on page 41 shows: the 20% of come-what-may actors act from 0% of desire and belief, whereas the wishful thinking/carpe diem actors do not. However, as the initial proportion of desires and beliefs increases, the difference between the effects of the two mechanisms lessens.

The effect of come-what may agents can also be shown in a diagram of the node attribute correlations (Figure 19 on page 42). Compared to the basic simulation (Figure 8 on page 31)

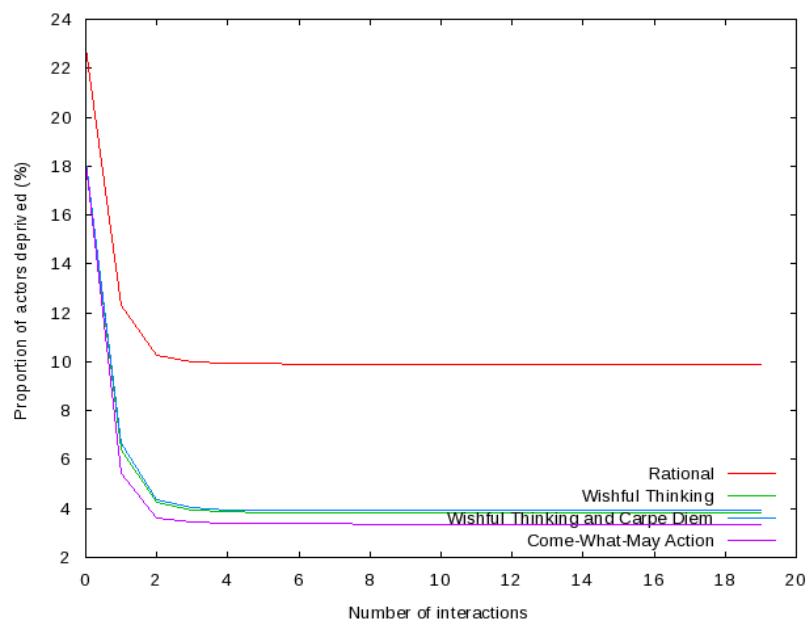


Figure 16: Deprivation for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors

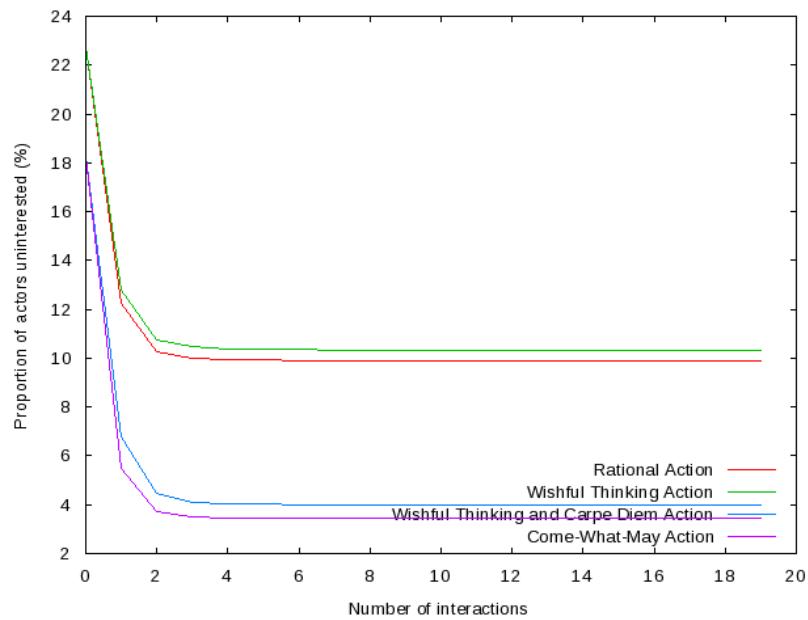


Figure 17: Uninterest for standard, wishful thinking, wishful thinking and carpe diem, and come-what-may actors

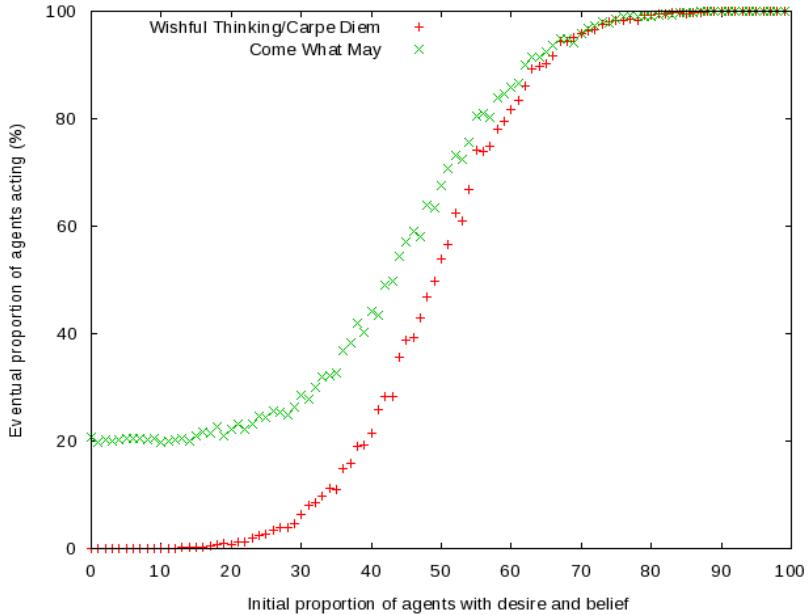


Figure 18: Carpe diem and wishful thinking versus come-what-may

even before interaction, there are significantly more agents acting (32% to 16%) and less with just desire or belief or neither. After interaction, there are even more agents acting, but there are also more with just desire or belief (compared to the basic simulation). However, it seems that the acting agents are less segregated than in the basic simulation: the proportion of acting-to-acting ties is lower than the proportion of acting agents, while the reverse was observed in the basic simulation. Close to 6% of the ties after interaction are between acting agents and those without desire or belief, compared to 0.775% in the basic simulation. Not only does the come-what-may mechanism make more agents act, but it ensures that a larger proportion of the population is in proximity to agents who are acting.

3.3 Conclusion

The ways in which a small number of non-rational agents decide to act can make a significant difference to a society. The DBO model can account for agents with different types of psychological dispositions, and because agents' actions are in part determined by the DBO states of

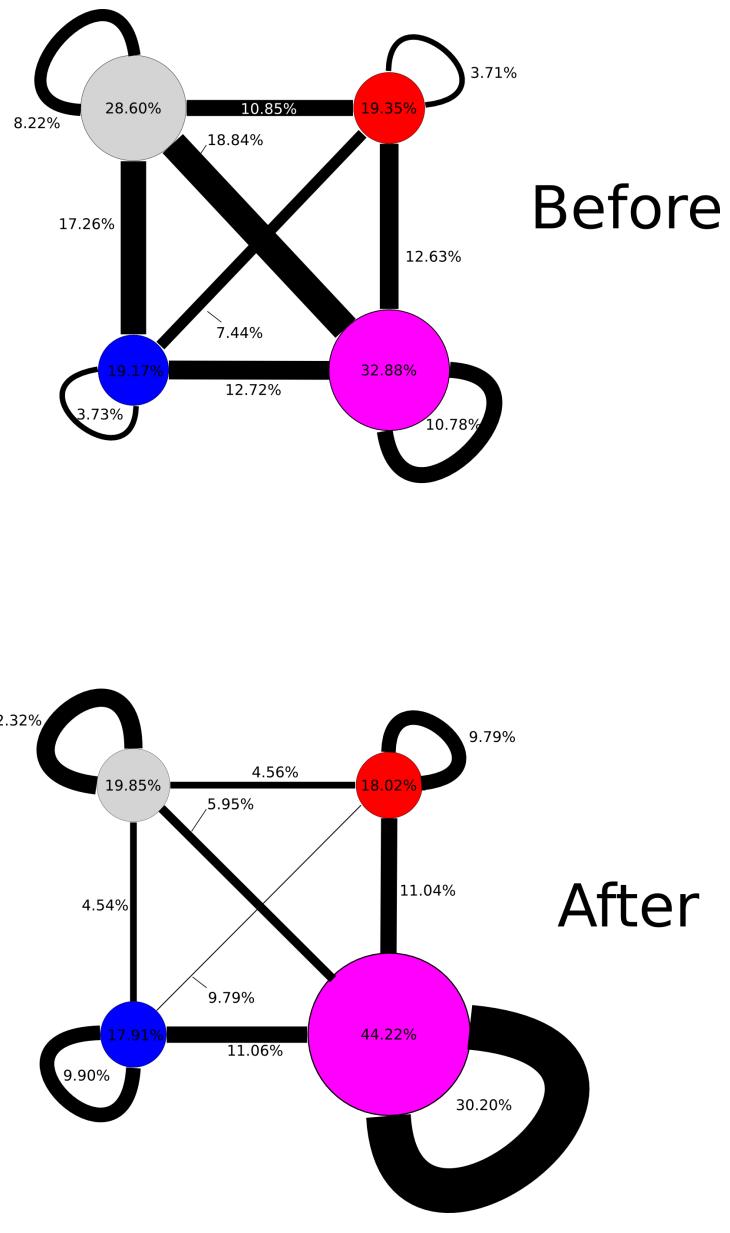


Figure 19: Node attribute correlation for come-what may agents

The model for this figure uses equivalent parameters to Figure 8 on page 31, but not to other figures in this section.

other agents, any irrationality on the part of an agent can have far-reaching consequences. A few actors who act more than would be rational can cause other rational actors to act, or at least make them not deprived from the belief that action will be effective. The *carpe diem* and *come-what-may* actors might be somewhat unrealistic, but there only a small number are needed to cause a snowball effect. However, these irrational agents are incomparable to rational agents: the graph of their eventual action against initial action still produces an S-curve. So, in very broad terms, the results of the basic simulation using the DBO model *are* robust, but in some cases the influence of non-rational agents will be important, especially where there is a low proportion of agents acting initially.

4 Processes Between Agents: Social Networks and Thresholds

4.1 Network Models and Metrics

The assumption that agents interact⁶⁵ on a toroidal lattice is convenient but not particularly realistic. The use of regular lattices is, in part, a heritage from the pioneers of agent-based modelling (including Schelling's chessboard and von Neumann's cellular automata)⁶⁶. Andreas Flache and Rainer Hegselmann, in their study of the effects of irregular grids on cellular models of social dynamics, point out that there are two *a priori* reasons for thinking that properties of their social models are *not* resistant to changes in the structure of the network.

Firstly, regular grids impose restrictions on the transitivity of relationships. In von Neumann neighbourhoods of degree 1 (as used in *Dissecting the Social*), relationships can not be transitive at all: if B is the neighbour of A and C is the neighbour of A, then B cannot be the neighbour of C. Because of this, no two nodes can be structurally equivalent (have the same sets of neighbours).⁶⁷ Not only are transitive relationships an important feature of real social networks, but they are a required component of some models of influence dynamics. Secondly, in regular rectangular grids, there is no variation in the number of neighbours of each agent; for von Neumann neighbourhoods of degree 1, each agent has 4 neighbours. Again, one of the characteristics of real world networks is variation in the number of neighbours each agent has, and there are theories of social processes that rely on this fact.

As Flache and Hegselmann note, regular grids need not be rectangular: they could be triangular, where each agent has 3 neighbours, or hexagonal, where each agent has 6.⁶⁸ In social networks, regular lattices can be made with arbitrary numbers of neighbours for each node (known

⁶⁵It would be more accurate to say that networks are composed of nodes rather than agents, connected by edges rather than by relationships. But because each node is an agent in these models, the terminology of agents and networks will be freely interchanged.

⁶⁶It is important to point out that the lattices in those models are spatial, not social, so there are restrictions there on which shapes can be used that do not apply to social networks. Macy and Willer (2002, p. 151)

⁶⁷Macy and Willer (2002, p. 151)

⁶⁸There is a further disanalogy between social and spatial networks. Regular tiling refers to plane tiling by regular polygons, and can only be done with rectangles, equilateral triangles and regular hexagons, so there can only be 4, 3 or 6 neighbours. (Weisstein (2010); Jablan (1995)) Social networks, which do not have to fit the plane, can have any number of neighbours greater than 1 (see below).

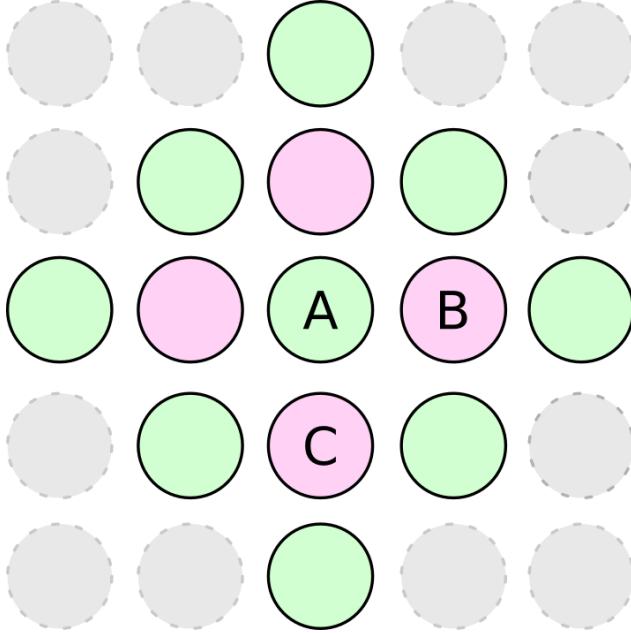


Figure 20: The absence of transitive relationships in von Neumann neighbourhoods
None of the neighbours of the neighbours of A (green) are the neighbours of A (which include B and C, shown in pink).

as the mean degree⁶⁹ of the lattice, which, for regular networks, is equal to the degree of every agent).

One regular lattice of particular interest is the ring lattice, in which each agent is connected to two (or any arbitrary greater number) agents to either side of them. Ring lattices are of particular interest because they are the regular lattices of the smallest possible degree (2)⁷⁰, and because they are used in the formation of other network types (see below).

Nevertheless, lattices of whatever degree are generally not good approximations of social space. In Schelling's segregation models, however, lines and regular grids are used to represent houses in a neighbourhood, where they are a reasonable approximation. There might be social situations, perhaps in organisational hierarchies or countries with strict family planning

⁶⁹The term 'degree', when referring to von Neumann neighbourhoods, has a slightly different meaning, it indicates the 'radius' of the neighbourhood on the grid: how many cells above, below and to the right and left of the target cell are included in the neighbourhood. If, in a network each agent is connected to her neighbours within the von Neumann neighbourhood of degree 1, each agent has degree 4.

⁷⁰Except for a network of two agents with a single link, where both have degree 1, or where unconnected nodes are allowed. The degree could also be 1 in graphs where ties have a direction.

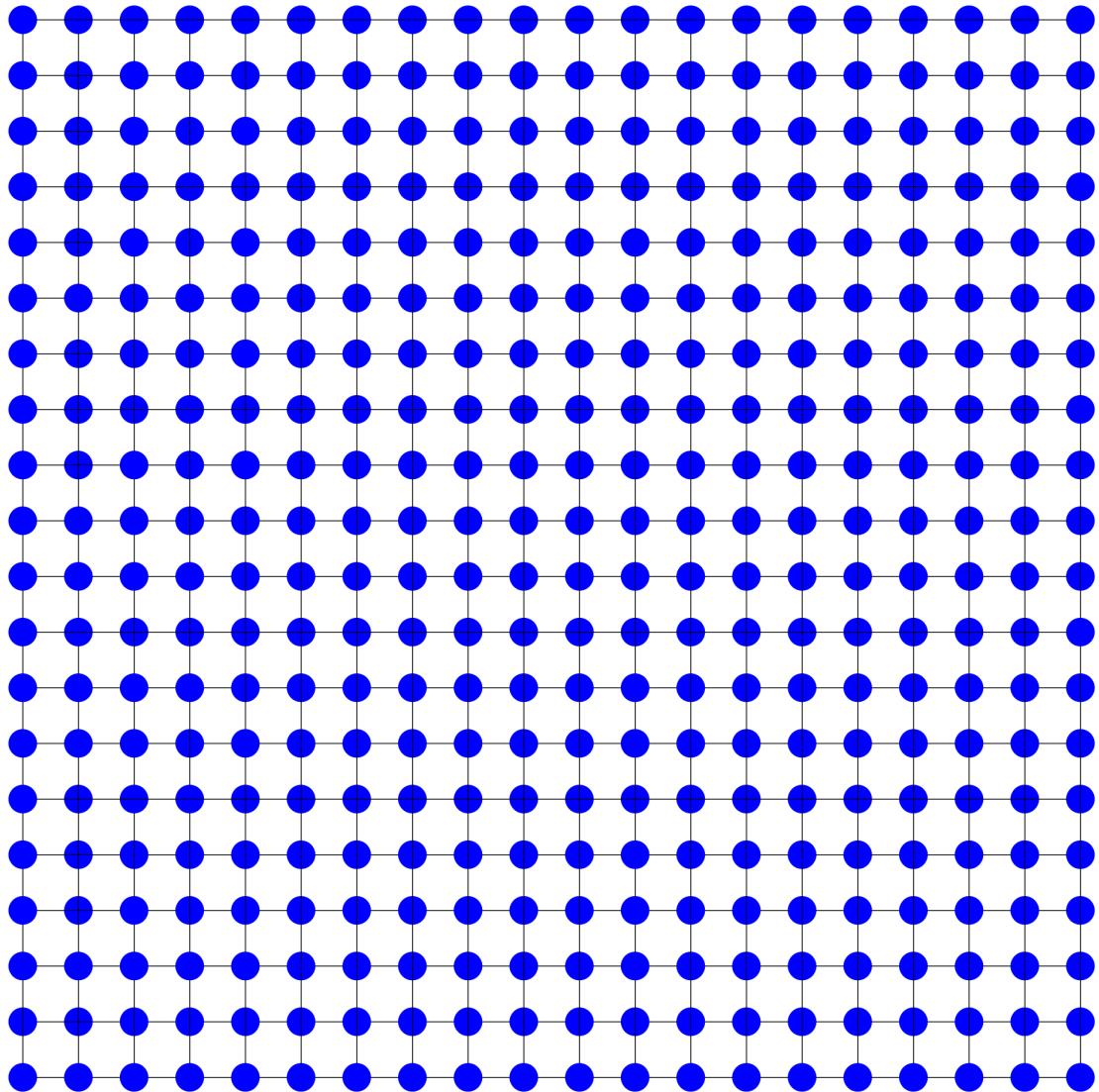


Figure 21: A toroidal regular lattice

2500 nodes. the nodes on the right are connected to the nodes on the left, and the nodes on the top to those on the bottom.

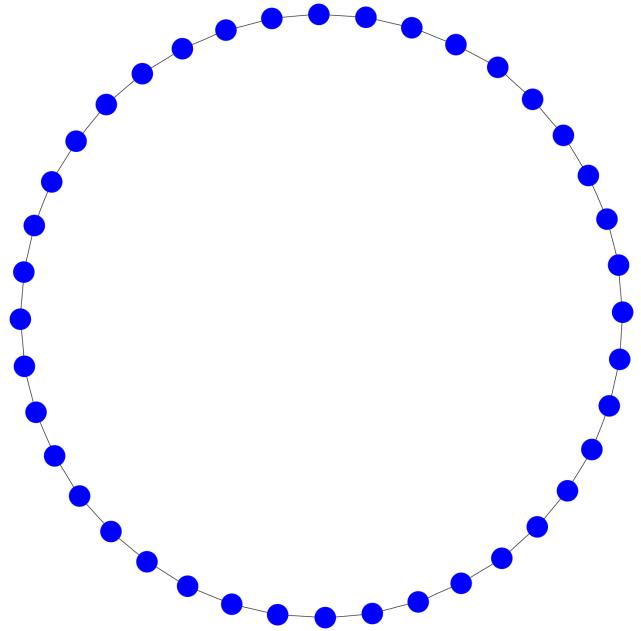


Figure 22: A ring lattice

20 nodes, each connected to two neighbours.

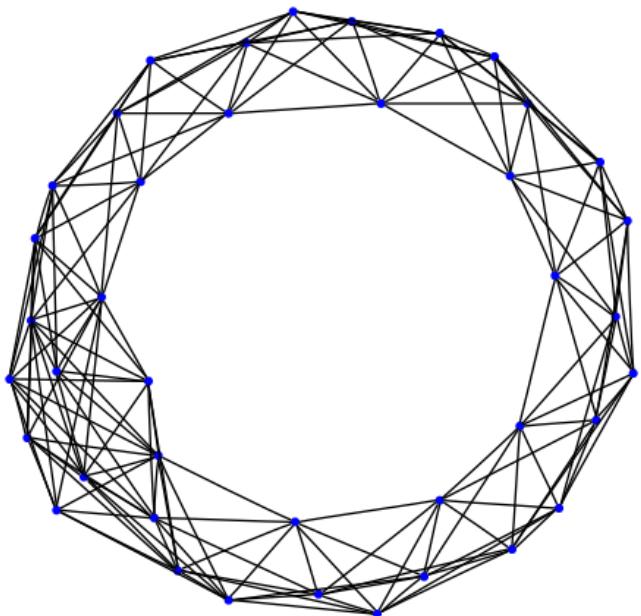


Figure 23: A different ring lattice

20 nodes, each connected to 8 neighbours.

laws, for which regular lattices might be of use. To remedy the problems of regular lattices, Flache and Hegselmann use irregular grids generated using a random process and find that, for their cellular automata models, the overall qualitative features of the models are not affected, but there are significant variations at the local level. Their process for generation of random spatial grids (using a Voronoi diagram) is analogous to Erdős and Rényi's G_{np} model for random social networks. In this model, first the number of nodes in the network is set, then they are connected, with the probability of any particular connection existing being set at some P^{71} .

While random networks like those generated by the Erdős-Rényi model are better approximations of some social networks, they are still not particularly realistic. If a process relies on people making eye-contact in the street, then the network of interaction might be close to random. But for the networks of friendship over which influence is most likely to take place, there are better models, as noted in Table 4 on page 50. The table shows some of the measurable characteristics of real-world networks compared to those of different models. The significance of these metrics shown merits some explanation. The toroidal lattice model seems to perform relatively well if only the mean degree (average number of neighbours) is considered (and, indeed, it should be possible to produce a model with an arbitrary mean degree, using appropriate neighbourhoods). But the mean degree of a network is easily varied within other models as well. Therefore, in order to compare the consequences of the other parameters, in the comparison of networks models in the table, the mean degree was held constant at ≈ 4 .⁷²

The Watts and Strogatz β model attempts to model small-world networks, a common type of real world network. The metrics of Table 4 on page 50 are particularly characteristic of small-world networks: they have a low mean-shortest path length, low mean degree and high clustering coefficient. In order to generate networks with these properties, the Watts and Strogatz model starts with a ring lattice, then randomly deletes and replaces a certain proportion of the connections.⁷³ This process might be that hypothesised to occur in real-world friendship networks:

⁷¹Erdős and Rényi (1959); Erdős and Rényi (1960)

⁷²In some models, like the toroidal lattice, the mean degree is exactly 4. In others, the mean degree cannot be controlled directly.

⁷³Watts and Strogatz (1998, pp. 66–69)

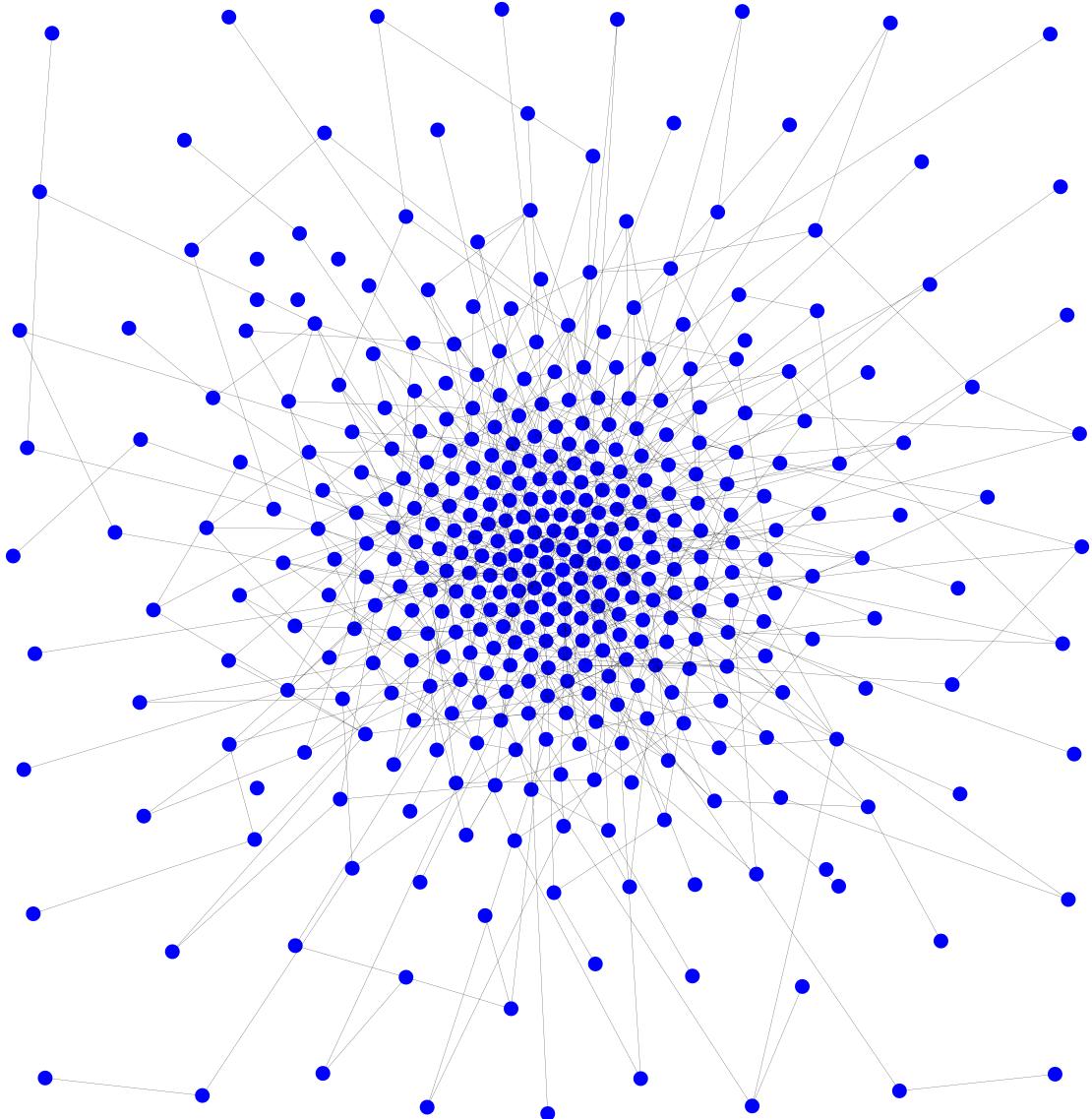


Figure 24: A typical Erdős-Rényi graph

400 nodes, $P = 0.005$

Network	n	m	z	l	C
Film Actors	449913	25516482	113.43	3.48	0.78
Company Directors	7673	55392	14.44	4.60	0.88
Toroidal Lattice	400	1600	4.00	10.03	0.00
Ring Lattice	400	1604	4.01	50.25	0.00
Erdős-Rényi	400	1569	3.92	4.36	0.02
Watts and Strogatz β	400	1604	4.01	5.24	0.31
Barabási-Albert	400	1692	4.23	2.14	0.20

Table 4: Comparison of empirical networks and network models

The characteristics measured are the number of nodes n , the number of edges m , the mean degree z , the mean shortest-path length l and the clustering coefficient C . The first two lines are derived from empirical data (from Newman (2003, p. 10), Table II), and the last five from simulations of the relevant models. Model parameters used: Ring Lattice: $N = 400, k = 4$; Erdős-Rényi: $N = 400, P = 0.005$; Watts and Strogatz: $N = 400, k_0 = 4, \beta = 0.2$; Barabási-Albert: $N = 400, N_0 = 50, k_0 = 4$.

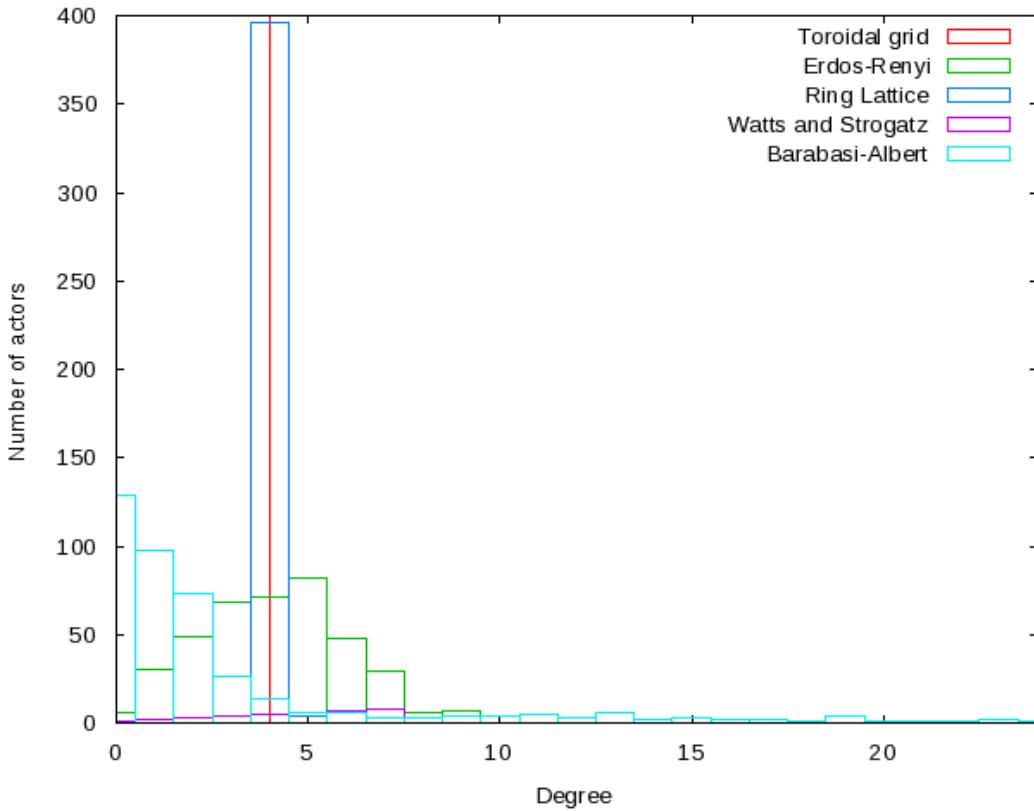


Figure 25: Degree distributions of the different network types

everyone is born into a regular network of people they have not chosen, but they will (randomly) break and re-form connections over time. The ring lattice is one extreme case of the Watts and Strogatz model, in which no rewiring has taken place, while an Erdős-Rényi network is the other extreme case, where all the initial connections have been randomly rewired. Networks in between the two extremes have characteristically low mean shortest-path length compared to that of a random network built on the same nodes.

A relatively high clustering coefficient is also evidence of real-world network formation processes. The clustering coefficient indicates the extent to which, in a group of three linked nodes between which there are two ties, the third tie is likely to be present. In other words, it indicates the transitivity of the friendship relation: the extent to which the friend of a friend is also likely to be a friend. Formally, the clustering coefficient for a single node is defined as

$$C_i = \frac{n_i}{k_i(k_i-1)/2}$$

where k_i is the number of neighbours the node has and n_i is the number of existing links between them. $k_i(k_i-1)/2$ is the total number of possible connections between k_i nodes.⁷⁴ The clustering coefficient for the graph is the average of the coefficients for all the nodes.

The Barabási-Albert model⁷⁵ is a preferential attachment model: broadly, it starts with a random network and those individuals that have more neighbours will get more as the model progresses. The main advantage of this model over the Watts and Strogatz model is that it has a more realistic degree distribution, for a similar mean degree. In the Watts and Strogatz model, nearly all of the nodes have the same degree, but in real-world networks, the degree distribution is right-skewed (and often follows a power-law). That is, there is a very large number of nodes with one neighbour, but many fewer with a larger number of neighbours. In other words, there are a few network hubs, and many quasi-isolates. The degree distributions of the lattices and

⁷⁴There are multiple definitions of clustering coefficients. Here, the version in Watts and Strogatz (1998, p. 441) is used.

⁷⁵Barabási and Albert (1999)

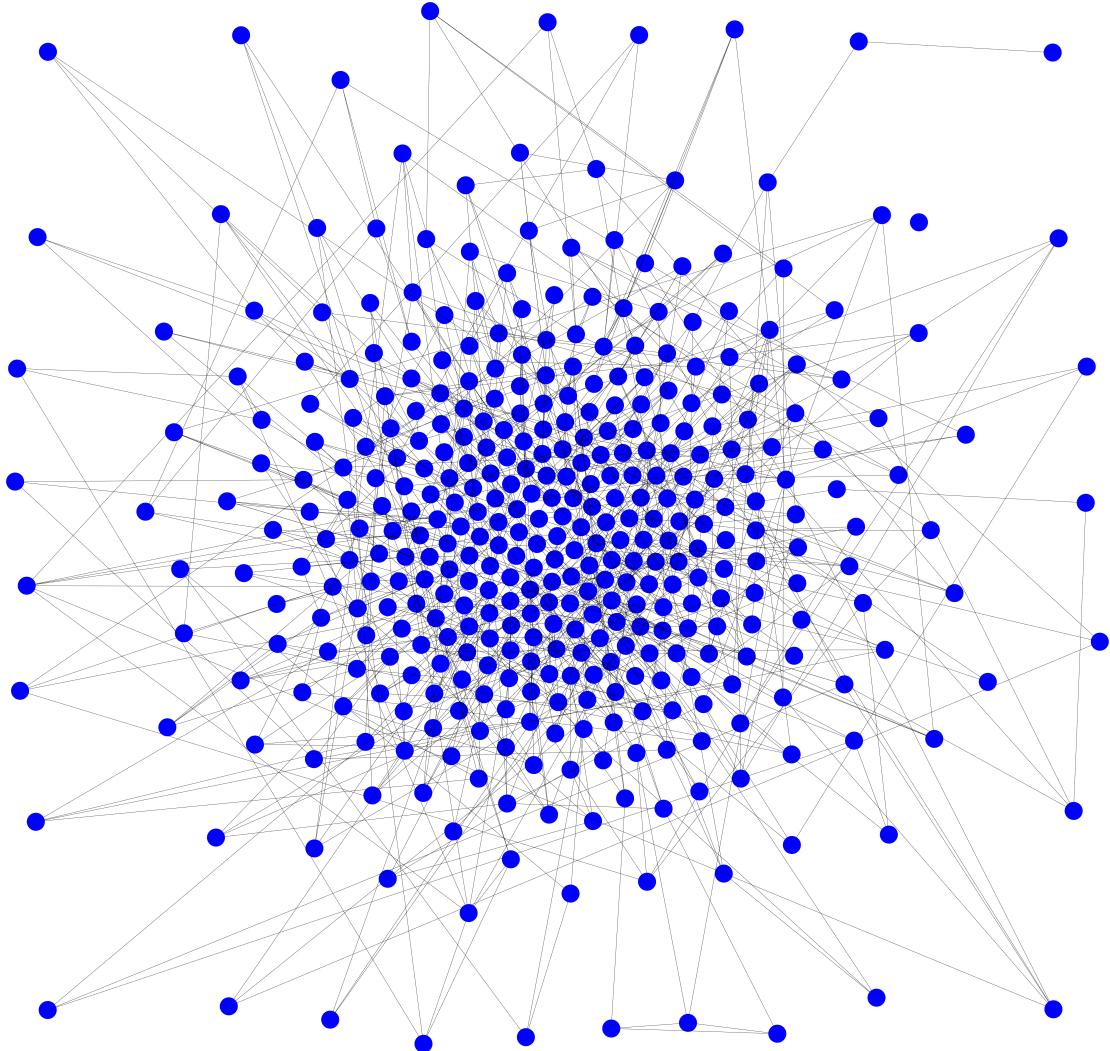


Figure 26: A typical Watts and Strogatz graph
400 nodes, initial lattice degree: $k_0 = 4$, rewiring probability $\beta = 0.2$

of the Watts and Strogatz models are quite similar: almost all nodes in such networks have the same degree.

All of these models have been used to approximate real social networks, with varying degrees of success. For some particular processes, each might work well, but for the small-world networks that characterise friendships and other important social relations, the most realistic seems to be the Barabási-Albert model. It is worth noting that there will be some variation in the metrics between networks in different situations: film actors have more films in common than company directors do companies. This is why comparative terms are used to evaluate how realistic the network models are: small-world networks have ‘low’ mean degrees and ‘high’ clustering coefficients.

It might seem therefore that the Barabási-Albert model should be used at the exclusion of all others, but I do not intend to do this for three reasons. Firstly, as noted, different networks might be better models for different processes. Secondly, there are different networks metrics of interest, and comparing more than one model allows us to determine the relative importance of these different network characteristics for creating the relevant outcomes. Finally, creating Barabási-Albert networks (especially large networks over many iterations) is very computationally intensive compared to other models.⁷⁶

4.2 The Effects of Networks

In order to compare the effects of these different networks, the initial proportion of agents acting was varied, as previously. The following graphs (Figures 29 on page 56, 30 on page 57, 31 on page 57, 32 on page 58, 33 on page 58 and the comparative Figure 34 on page 59) show the initial proportions of desire and belief, compared to the proportion acting after 20 interactions for the different types of networks.⁷⁷ These data are more informative here than the proportion

⁷⁶Although it is beyond the current scope to prove this, it appears that the time taken to generate a network using the Watts and Strogatz model grows linearly with the number of nodes, while the time taken by the Barabási-Albert model grows exponentially. Although the size of a scale-free (small-world) network should not affect its characteristics, in practice it is necessary to use networks of several hundred nodes in order to ensure consistency between the simulations.

⁷⁷Hedström (2005, p. 86), Figure 4.5

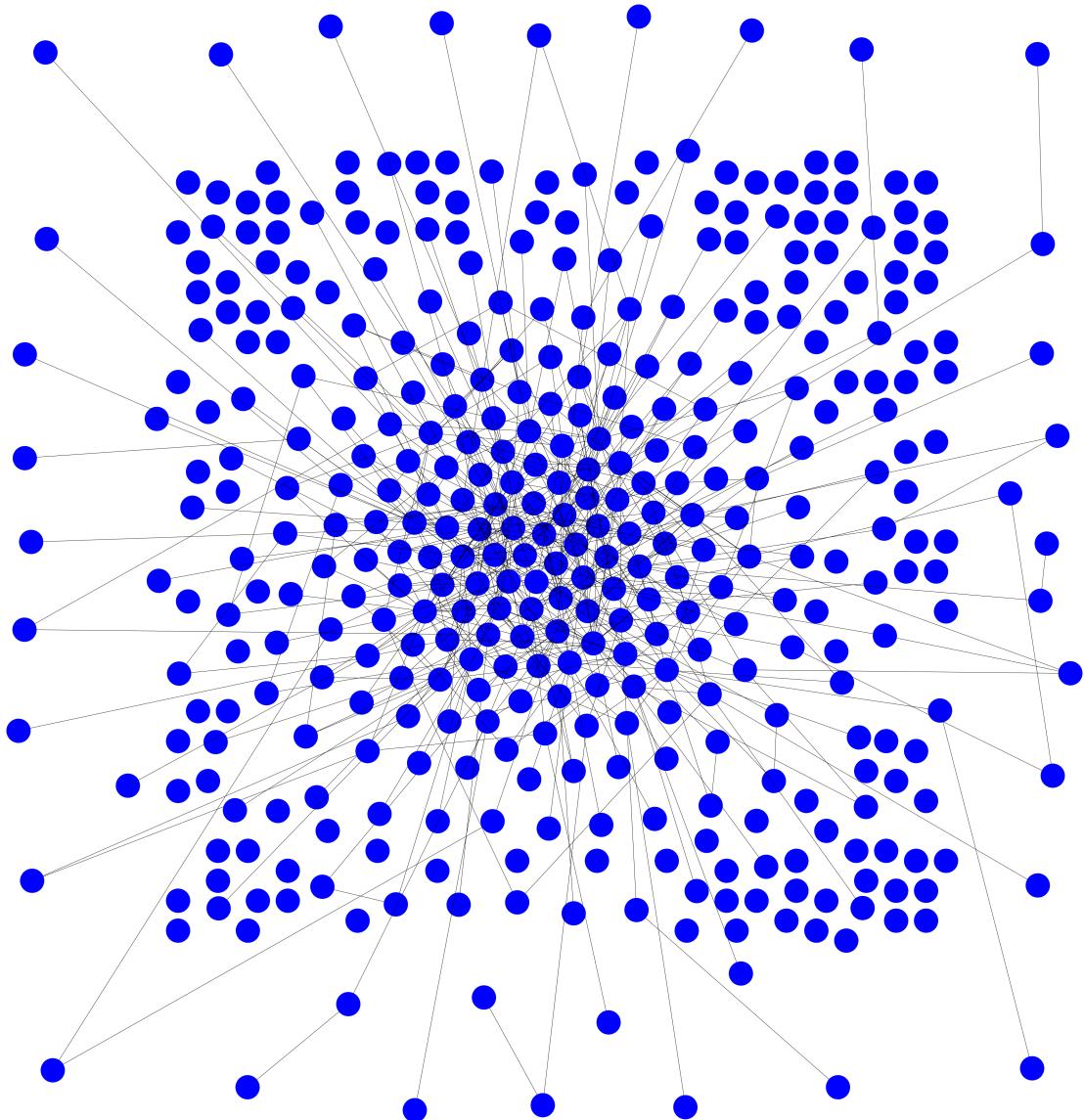
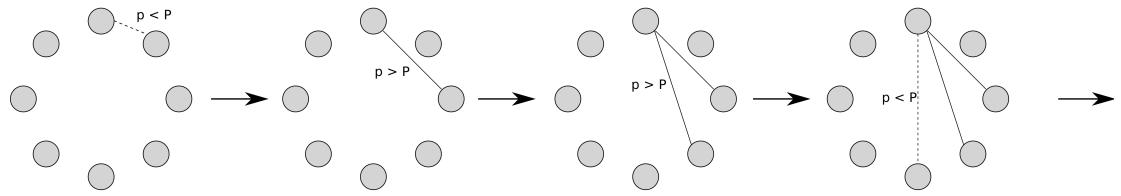
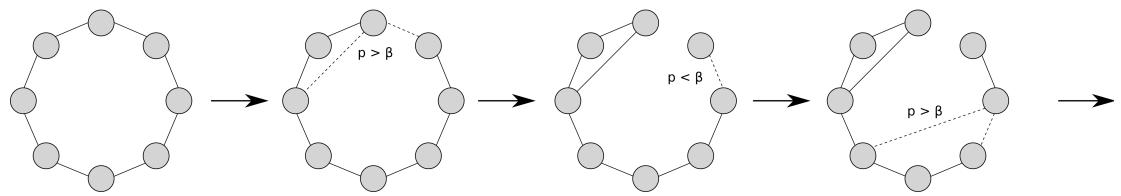


Figure 27: A typical Barabási-Albert graph
Number of nodes $N = 400$, initial number of nodes $N_0 = 50$, initial mean degree $k_0 = 10$

Erdős-Renyi $G(n,p)$



Watts and Strogatz β



Barabási-Albert

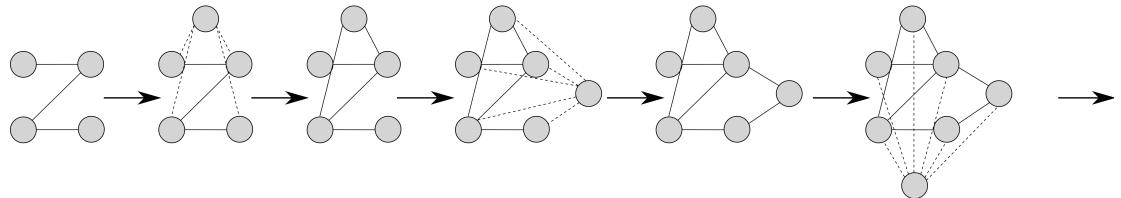


Figure 28: The network formation processes of different network models

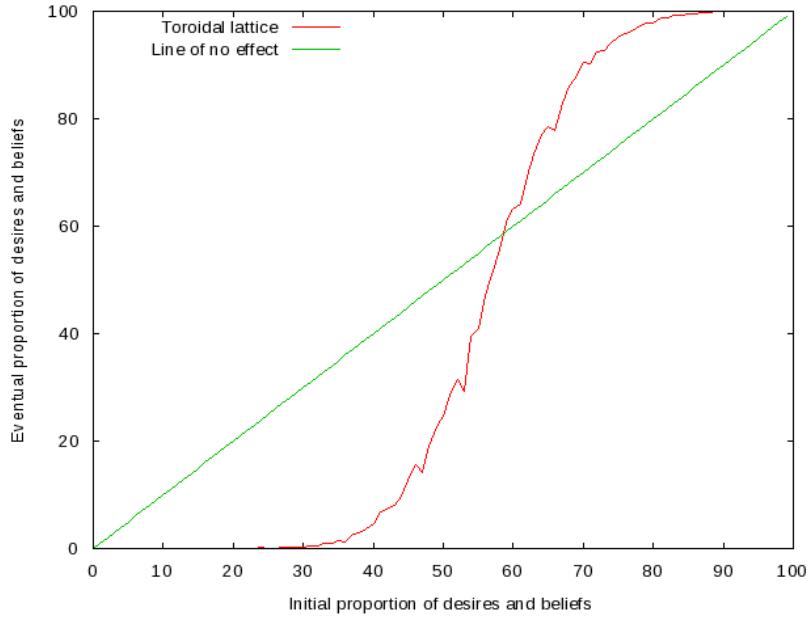


Figure 29: Toroidal lattice: eventual action and initial action

Proportion acting after 20 interactions as a function of the initial proportion of desires and beliefs. This figure is replicated from the figure 4.5 in *Dissecting the Social* (p. 86)

of agents acting shown over multiple interactions, because they show the ‘responsiveness’ of the final outcome to changes in the distribution of agent characteristics in the initial situation. On each graph, the ‘line of no effect’ shows where the proportion acting at the end of the simulation is identical to the proportion acting at the beginning.

For all of the networks, the number of agents acting after interaction increases with the number of agents acting before, as would be expected. The results are also similar to the S-curves that Hedström found: growth is slow for low initial proportions acting, very rapid around 50%, and saturation is reached soon after.⁷⁸ The process by which this occurs is relatively easy to explain: at low levels of action very few agents will have a majority of neighbours with desire or belief, so there will be little spread of these states, and most agents will be inactive. Those few that do act will be in isolated stable islands surrounded by a ‘sea of defection’⁷⁹ (or, more accurately

⁷⁸Hedström graphs the final proportion acting against the initial proportion acting, which would be the equivalent of taking the square root of all the data-points in these graphs.

⁷⁹Flache and Hegselmann (2001), Section 3.2

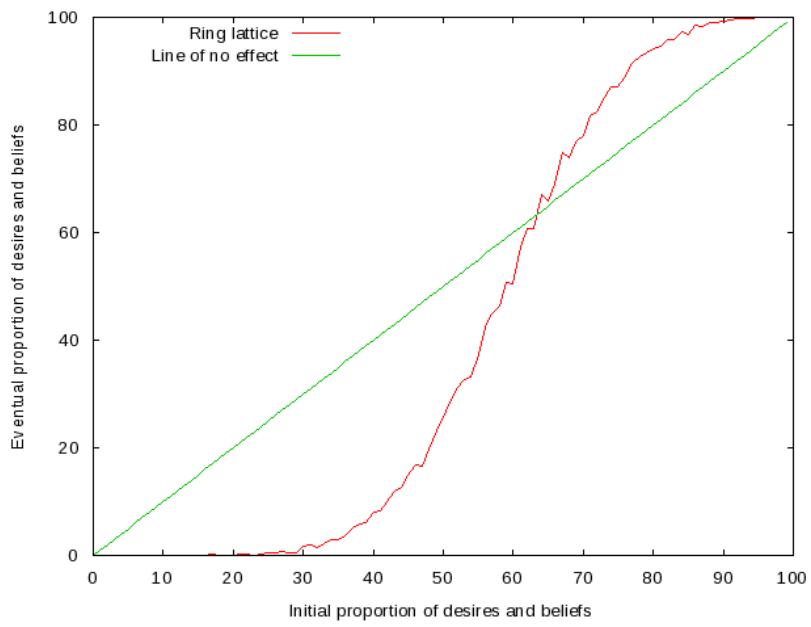


Figure 30: Ring lattice of degree 4: eventual action and initial action
 Proportion acting after 20 interactions as a function of the initial proportion acting.

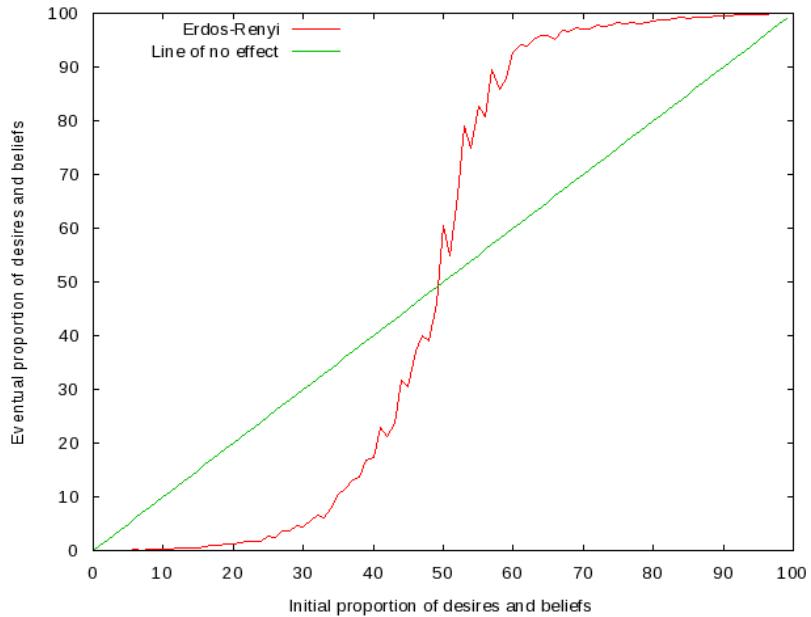


Figure 31: Erdős-Rényi random graph: eventual and initial action
 Proportion acting after 20 interactions as a function of the initial proportion acting.

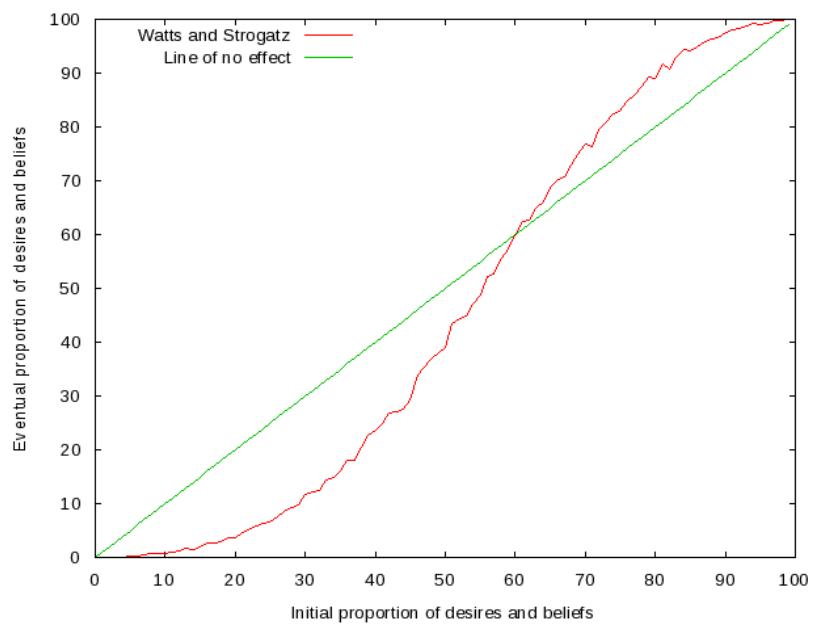


Figure 32: Watts and Strogatz network: eventual and initial action
Proportion acting after 20 interactions as a function of the initial proportion acting.

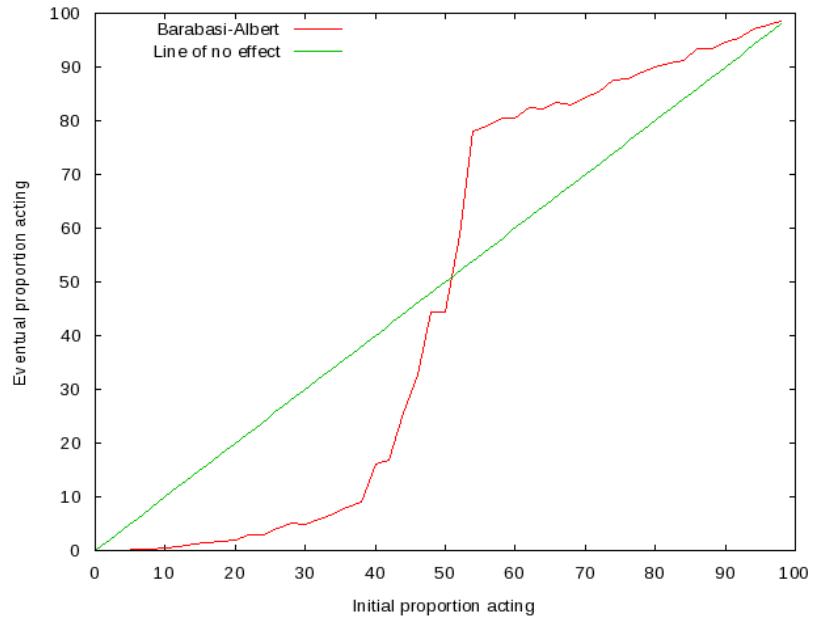


Figure 33: Barabási-Albert network: eventual and initial action
Proportion acting after 20 interactions as a function of the initial proportion acting.

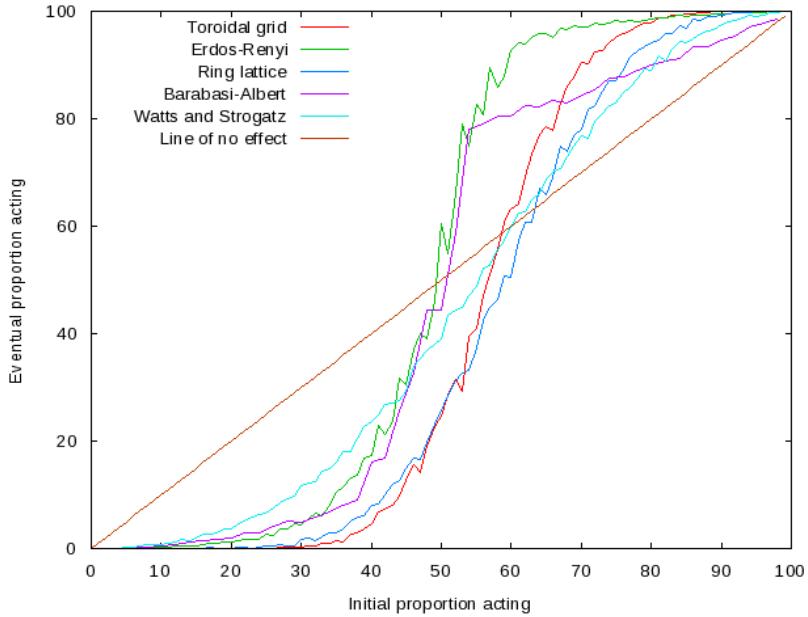


Figure 34: Eventual and initial action in multiple network types
Proportion acting after 20 interactions as a function of the initial proportion acting.

in this case, a sea of inaction). The islands of action will grow larger for a time as the initial proportion of acting agents increases, and the addition of any more acting agents will cause a disproportionately large change in the final proportion acting until the curve crosses the line of no effect. Up until this point, agents who happen to be acting in proximity to islands of action will be able to continue acting, and the islands will grow larger. Eventually, as the proportion acting approaches 100%, saturation is reached and the curve levels off.

Although all of the curves display these similarities, there are still some differences between the different networks. In the simulations with Erdős-Rényi networks, the curve crosses the line of no effect quite early, at 50% compared to 60% for the others. This would indicate that saturation begins earlier in Erdős-Rényi networks, but whether this is significant, and why it occurs, is not immediately clear. It might be because the number of nodes in an Erdős-Rényi network was not controlled directly, but probabilistically, resulting in some random variation. If, as a result, the mean degree is higher than for other networks, this might be causing saturation to occur faster (see Section 4.2.1 on page 61), which it appears to do: the Erdős-Rényi network

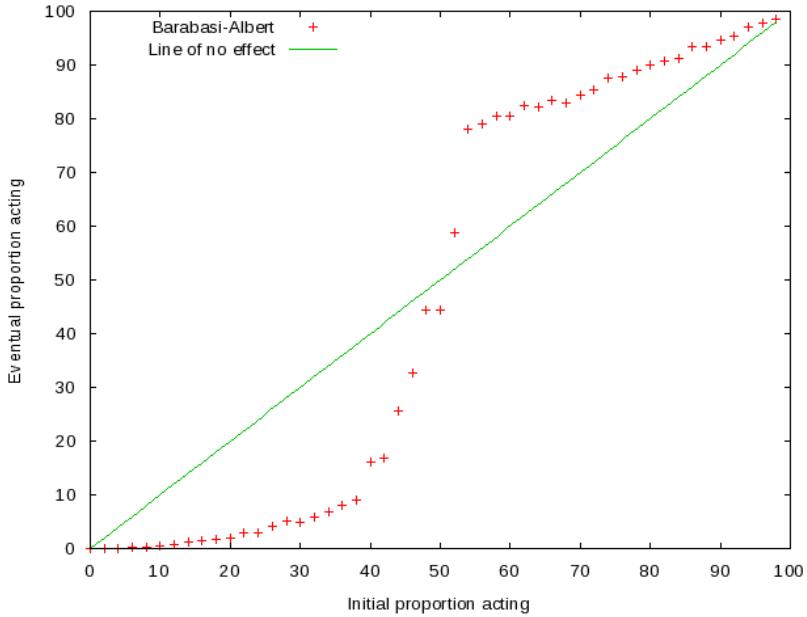


Figure 35: Barabási-Albert network, shown with points

overtakes the Watts and Strogatz network at 20%, and reaches 80% action before any others, at just over 50% initial action. The network type with the lowest mean degree, the ring lattice, reaches the line of no effect last of all, which would seem to confirm this possibility.

Moreover, in the Barabási-Albert networks, there is a discontinuity in the graph at around 50% of initially positive desires and beliefs (which is visible in Figure 35, where data points are graphed without the lines connecting them). In situations where about half of agents act initially, there is a lot of variation between individual simulations (Figure 36 on the next page). When the saturation of desire and belief reaches 50% (and 25% of agents are acting), the majority of the neighbours of many agents will have positive desires or beliefs, causing these agents to desire or believe as well. With 50% of desires or beliefs, the distribution of outcomes is spread widely: the outcome of the simulation can be greatly affected by small differences in the initial circumstances (both the shape of the network and the initial random distribution of mental states).

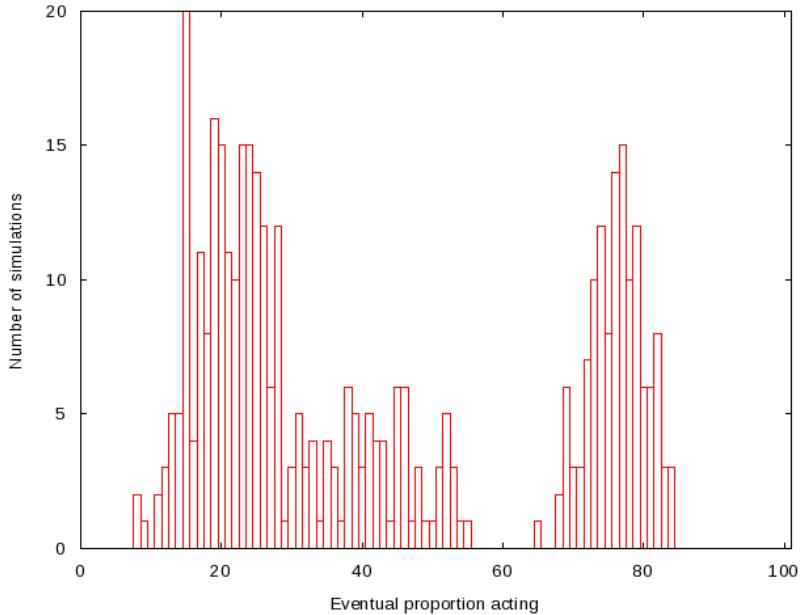


Figure 36: Barabási-Albert network: distribution of outcomes for 50% initial action

But why does this occur for the Barabási-Albert network in particular? Compared to the other networks, it has a low mean degree, a low mean shortest-path length⁸⁰, and a right-skewed degree distribution⁸¹. Some of these network characteristics might then be important in creating social instability under certain sensitive distributions of agent characteristics. This result that represents a significant departure from the toroidal lattice model. Therefore, the effects of the mean degree, mean shortest-path length and skewness of the degree distribution skewness bear further investigation.

4.2.1 Varying the Mean Degree

The higher the mean degree of the network relative to the number of nodes in the network, the more other nodes each has the chance to influence. At the same time, the amount of influence

⁸⁰The Barabási-Albert network shown above also has a high clustering coefficient. Ideally, the effects of this parameter would also be studied, but there is no easy way of isolating this variable.

⁸¹The low mean degree might be in part the result of this implementation of the model, because the mean degree of the initial random network is only determined approximately. However, this is not an important factor in large networks, because most of the edges in such networks will be connected using preferential attachment, not randomly.

that a node has over each other decreases. That is, each individual is better connected, but her influence over any particular person is diluted by that of others. Nevertheless, the increase in degree seems to increase the ability of societies to become homogeneous: the outcomes (in terms of percentages desiring and believing) are more extreme. If the mean degree was 0, then each actor's behaviour would be totally independent of any other's. When each actor is connected to every other, then the overall outcome will be the average of the distribution in the society; any global distribution above 50% will tend towards 100% action and anything below will fall to 0%.

However, when the degree of the networks is low, local disturbances in the distributions of desires and beliefs remain contained. Therefore, there is a relatively wide spread of outcomes around a mean of 25% of actors acting, which is observable in the toroidal lattice (Figure 37) and the ring lattice of degree 4 (Figure 38 on the next page).

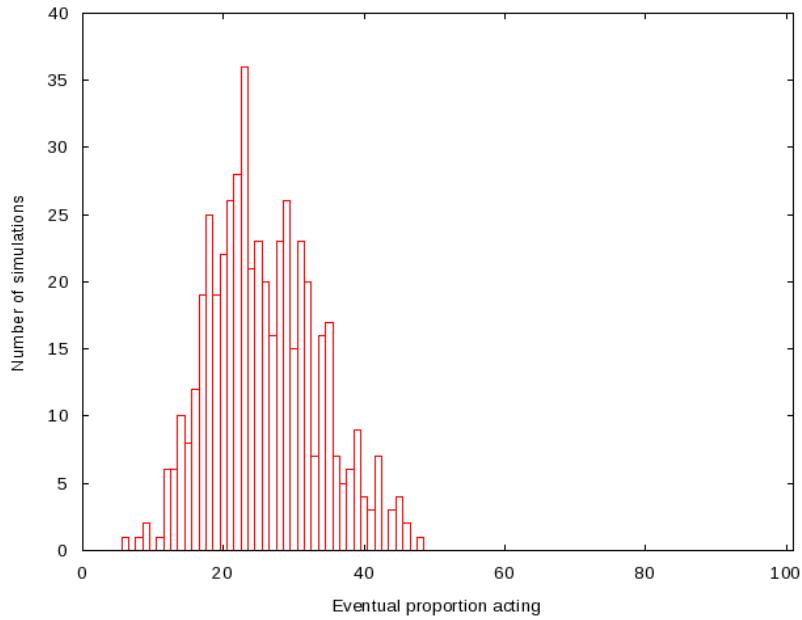


Figure 37: Toroidal lattice: distribution of marginal outcomes

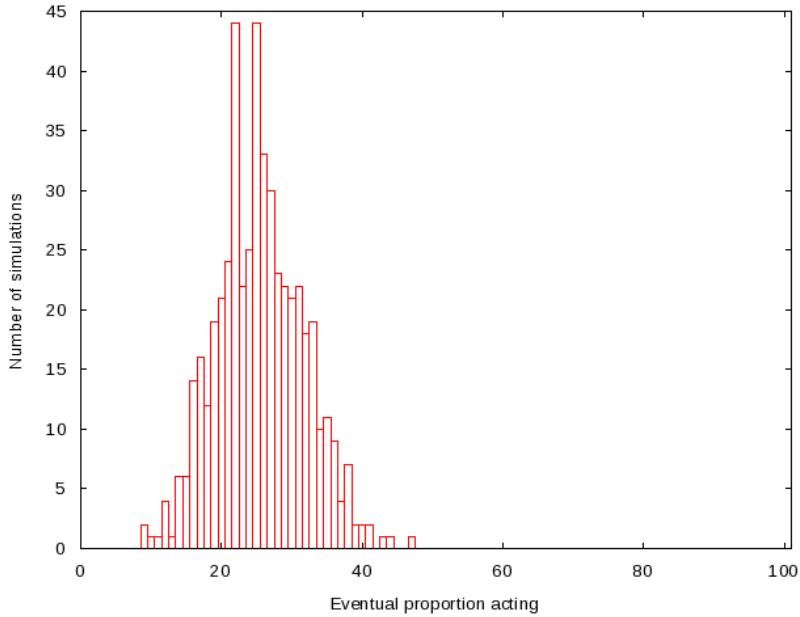


Figure 38: Ring lattice of degree 4: distribution of marginal outcomes

In order to vary the mean degree of networks, the following simulations use ring lattices. When the mean degree of the network increases, local imbalances are averaged out over larger numbers of agents. When the degree of a ring lattice of 400 nodes is 16 (Figure 39 on the following page), this change is already observable: most of the outcomes result in either none or all actors acting. There is substantial overlap between the neighbourhoods of all agents, so the action of each depends on many other agents. The few remaining intermediate outcomes represent situations in which stable islands have formed and remained due to insufficient interconnection.

When the mean degree is 64 (Figure 40 on the next page), there are no intermediate outcomes: everyone acts in 22% of the simulations and no-one does in the others. In this case, each agent has in her neighbourhood about one sixth of the total population, nearly every agent will conform to the average. Therefore, the discontinuity observed in the Barabási-Albert network's characteristics cannot be the result of this network type's typically low mean degree: the instability of marginal situations actually increases with the mean degree of the network.

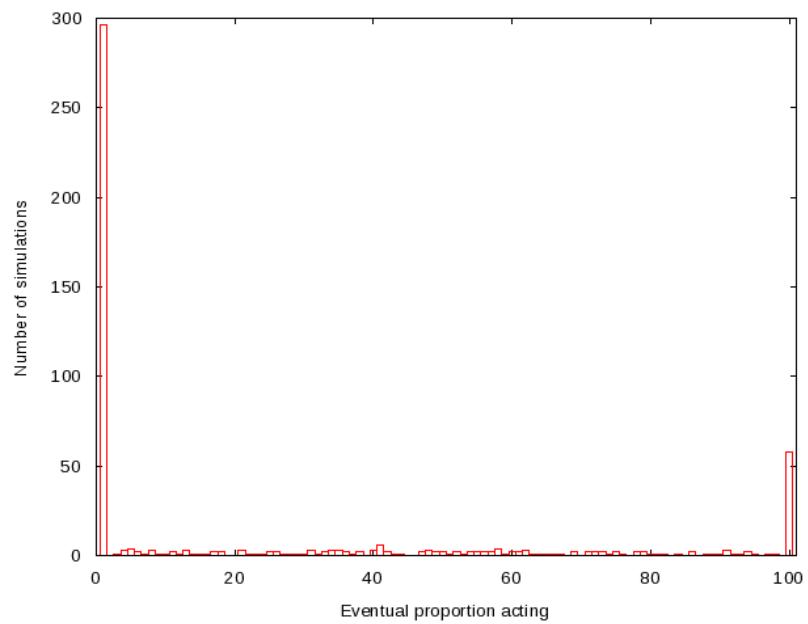


Figure 39: Ring lattice of degree 16: distribution of marginal outcomes

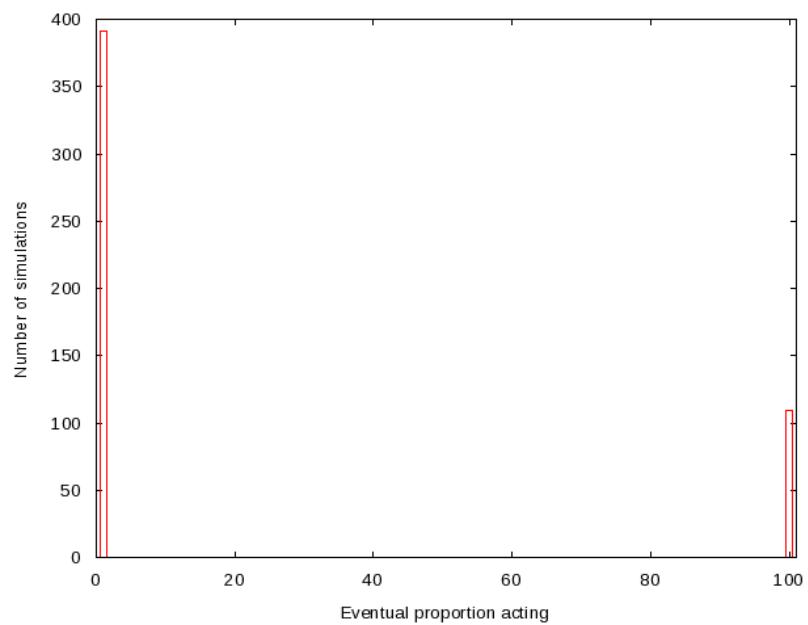


Figure 40: Ring lattice of degree 64: distribution of marginal outcomes

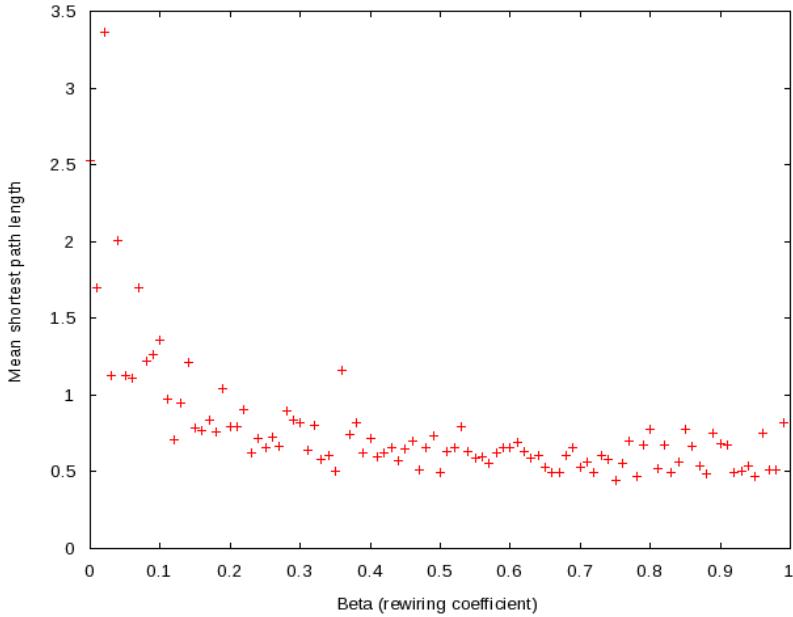


Figure 41: Mean shortest path lengths in the Watts and Strogatz model

4.2.2 Varying Mean Shortest-Path Length

If the mean degree of Barabási-Albert networks does not explain the instability in outcomes, it might be the networks' low mean shortest-path length. The mean shortest-path length of networks generated by the Watts and Strogatz model varies inversely with the β parameter (Figure 41), so this model can be used to determine the effects of mean shortest-path length on outcomes of the DBO model. When the mean shortest-path length is decreased, nodes are reachable from each other in a smaller number of 'hops', so it might be that states are transmitted more easily.

However, it seems that the spread of the distributions of outcomes does not change considerably with the mean shortest-path length, although for very low values of the β parameter (Figure 43 on page 67), the mean of the distribution does vary. The mean of the distribution probably changes because changes in the mean shortest-path length are greatest for smaller values of β (see Figure 41). The stability of the spread of the distribution (especially with higher β parameters (Figure 43 on page 67) probably occurs because: agents of the DBO model are only affected

by their immediate neighbours (those to whom the path length is 1), so unless the decrease in the mean path length also increases the mean degree, it will have little effect on the agents.

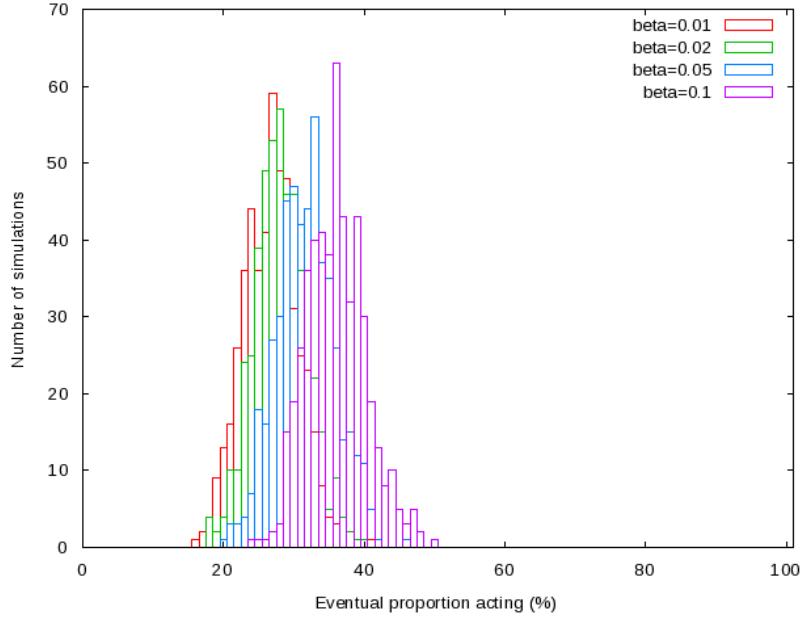


Figure 42: Distribution of marginal outcomes in Watts and Strogatz networks with $\beta = 0.01, \beta = 0.02, \beta = 0.05$

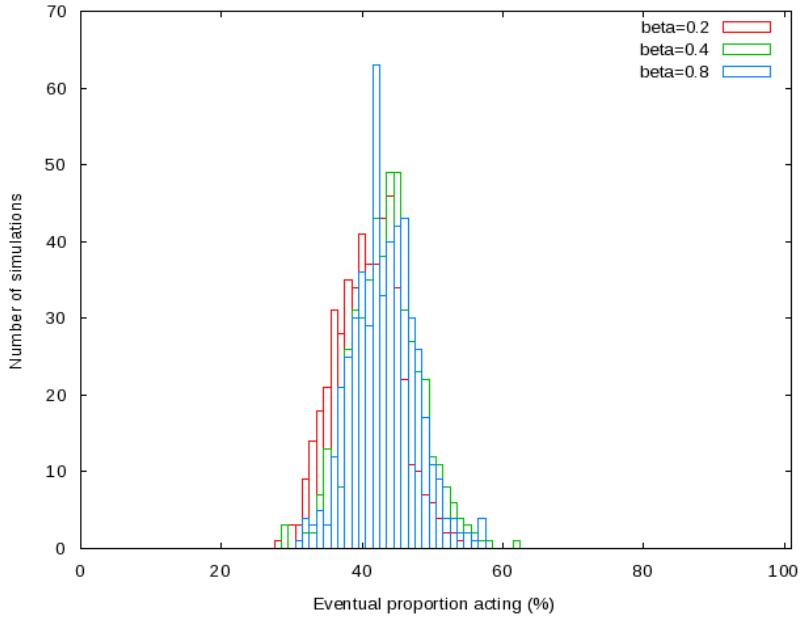


Figure 43: Distribution of marginal outcomes in Watts and Strogatz networks with $\beta = 0.2, \beta = 0.4, \beta = 0.8$

4.2.3 Varying Degree Distribution Skewness

While the mean degree of the network does have an effect on the levels of action, it does not do so in a way that might explain the instability of simulations in the Barabási-Albert network. But this might be because the mean degree statistic is not telling the whole story. As noted previously, Barabási-Albert networks also have a characteristic right-skewed degree distribution, so the mean degree of the network will be significantly affected by low-degree nodes (some of which are not connected to the rest of the network at all). The skewness of the degree distribution can be affected by the N_0 parameter of the model, which controls the size of the initial random network relative to the size of the whole network. The smaller the initial random network, the more nodes connected using preferential attachment, so the more skewed the degree distribution.

However, as the skewness of the degree distribution increases, the spread of the outcome distribution actually decreases. That is, the more nodes with a low degree, the less likely the outcomes will tend towards the extremes. However, this occurrence can be explained by the

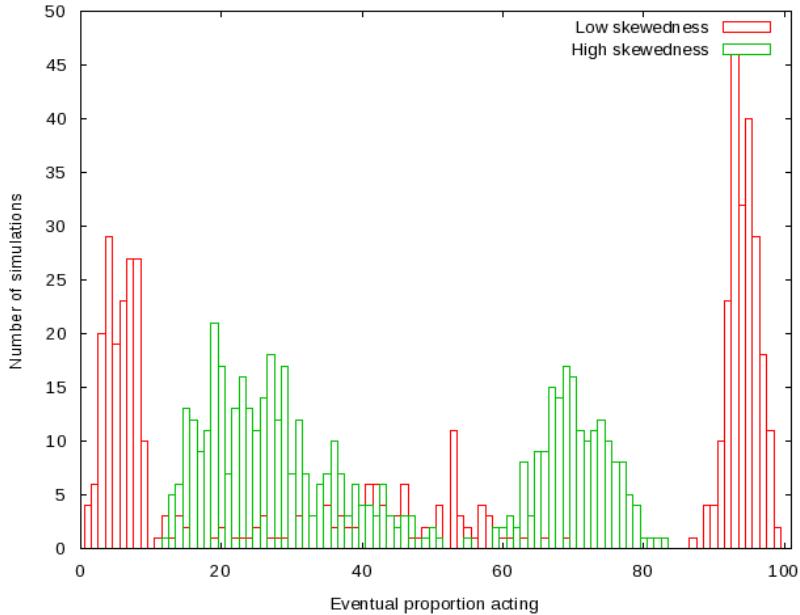


Figure 44: Effect of the skewness of degree distribution on the variability of outcomes.
Model parameters $N = 100$, $m_0 = 90$, $k = 4$ and $N = 100$, $m_0 = 10$, $k = 4$.

peculiar behaviour of nodes on the very extreme of the degree distribution. Normally, as the degree of the node decreases, fewer others are required to make it act. Nodes with lower degrees will be more sensitive to changes in their environment. But nodes at the very edge of the degree distribution, with no neighbours at all, will be totally unaffected by changes in their environment and will remain in the initial random state they started in. Such nodes even out the average of whatever other changes might occur in the wider society.

In networks with very right-skewed degree distributions there will be many such nodes, and the distribution of outcomes will be shifted away from the extremes. When unconnected nodes are removed, the number of simulations with intermediate eventual proportions acting decreases in both networks with both high and low skewness. (Figure 45 on the following page)

Even when unconnected nodes are removed, the distributions of outcomes still tend more to the extremes with low skewness than with high skewness. Therefore, it seems that the characteristics that distinguish Barabási-Albert networks from the other network models (low mean shortest-path length, low mean degree and right-skewed degree distribution) cannot explain why

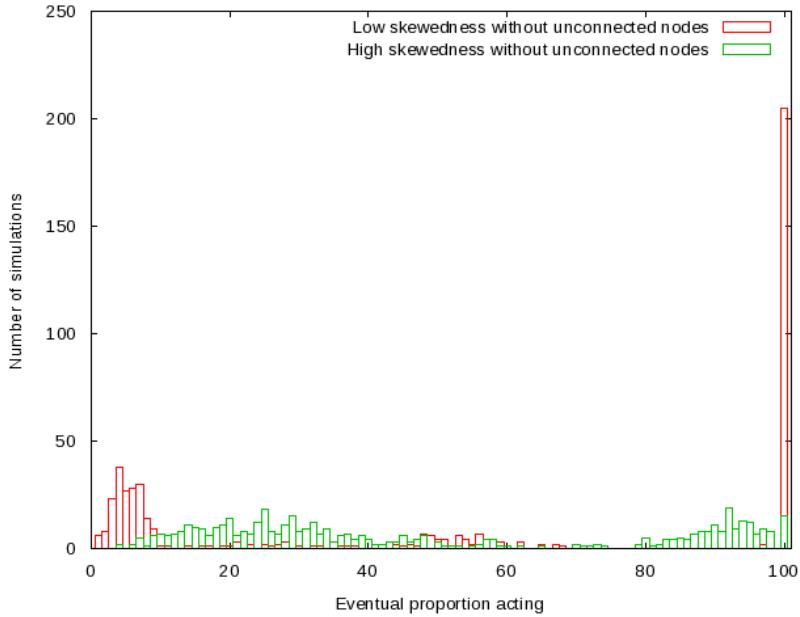


Figure 45: Effect of unconnected nodes on the variability of outcomes

simulations run on these networks show such high variability in outcomes. The mean degree of the network does not seem to affect the variability in outcomes at all, while mean shortest-path lengths and skewness of the degree distribution affect the outcomes in the opposite way to what would explain the particularities of the Barabási-Albert model.

It is possible that some other distinguishing characteristic of the Barabási-Albert model is responsible for the results, or it might be some combination of the factors already considered. It might also be that the tests were problematic: while one of the network parameters was varied others changed as well, producing an opposite effect. What is clear is that the characteristics of networks do have significant effects on societies, and small-world networks in particular produce outcomes very different to those of regular lattices.

4.3 The Aggregation Process: Thresholds

Even these more realistic network models do not capture some important characteristics of networks in the real world. The ties between nodes are all non-directed (reciprocal) and unweighted

(of the same strength), while real networks need not be. For some processes tie weight and direction do not need to be taken into account, there are surely others where they do matter. What's more, particular agents might on the whole be more susceptible to influence than others so that, regardless of the strength of the ties to her neighbours, their DBO state will change for less than majority influence. Agents who are more likely to jump on the bandwagon are essential to explanations of some forms of collective behaviour: they are the radicals in revolutions and the instigators in hostile crowds.

Indeed, as Granovetter's study of threshold models of collective behaviour⁸² points out, there are some situations of collective behaviour which can not be simply accounted for assuming that they are produced by average institutionalised norms or values, or simple deviance from such norms. Where collective results seem 'paradoxical' in light of the average properties of individuals, threshold models can have a role to play. Threshold models allow the strength of norms and preferences to vary between agents, so they have varying responses to influence. Instances of 'strange' collective action do not rely on strange behaviour of any individual but on the strange dynamics of their situations. In Coleman's explanation of the behaviour of hostile crowds for instance, variations in the influenceability of agents are required to explain differences in the volatility of the groups. Coleman's hypothesis is that in such situations, the more heterogeneous the normative constraints, the greater the chance of expressive action.⁸³

Varying thresholds can be incorporated into the DBO model by changing part of the process that occurs between agents, the aggregation process. As in Granovetter's study, rational agents make binary decisions and similarly, the chances of any individual acting depend on the number of others acting. However, the dependence of individuals' action on that of their neighbours is not, as in Granovetter's study, because of costs and benefits depending on the number of others in the crowd, but because of contagion processes in desires and beliefs.

Figure 46 on page 72 shows the proportion of agents acting for different proportions of agents with different levels of thresholds in the basic simulation. Unsurprisingly, the highest level of

⁸²Granovetter (1978)

⁸³Coleman (1994, p. 239)

action is for 100% of agents with thresholds at 0%, where all the agents with even one acting neighbour (or one with belief and another with desire) act⁸⁴. The highest proportion of agents acting is 40%, which is the same as the initial random proportion acting. In other words, in these simulations, the amount of action falls unless there are very low thresholds to preserve it. As thresholds decrease and the number of agents with the modified threshold increases, the overall level of action increases.

There are differences between the results in the basic simulation's regular lattice and in Erdős-Rényi networks (Figure 47 on page 73). In the toroidal lattice (Figure 46 on the next page), there are marked discontinuities in the levels of action where thresholds are at 25%, 50% and 75%. This is, of course, an artefact of the network structure, because each node only has four neighbours, so agents with thresholds between those levels will be influenced in the same way. Even though this does not occur in Erdős-Rényi networks, there is a similar ledge at 50% and fainter ledges every 7% to 8% above and below that. This might be explained because agents in Erdős-Rényi networks can still only have a discrete number of neighbours (which is less than the total number of agents). As a consequence, there are situations – especially obvious in networks with low degrees – when a small change in thresholds will have a large effect on social-level outcomes.

Being influenced by a minority opinion might be a plausible model for agents who have second-order preferences about acting. In the current model, the bias is only towards action, that is, if an agent has a minority of neighbours with positive desire or belief, their own state will be influenced, but not if there is a minority of neighbours with *negative* desires and beliefs. It is possible that an agent would consistently switch to the minority DBO state, which would be evidence of another type of second-order desire, for demarcation or eccentricity.

⁸⁴Agents with 0% thresholds and no acting neighbours might be expected to act as well, but this does not occur in these simulations because the proportion of agents must be strictly greater than the threshold (in order to allow agents not to be influenced at all when there is a tie).

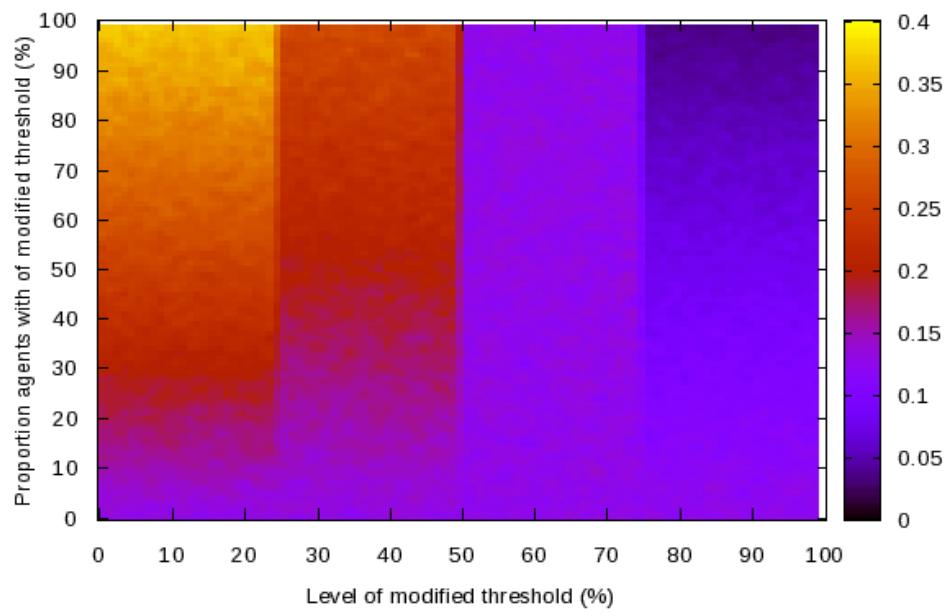
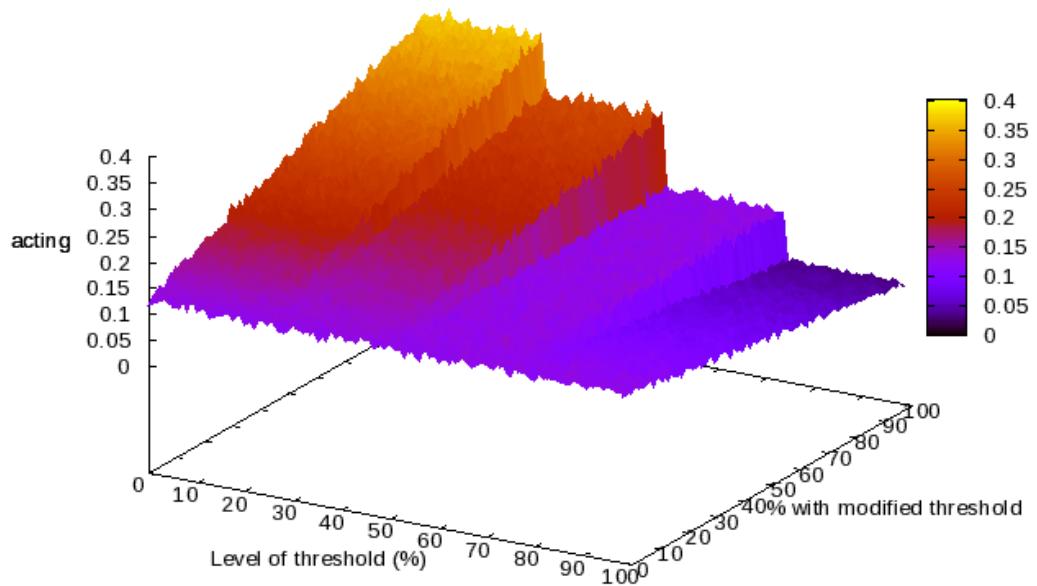


Figure 46: The effects of thresholds on action in a toroidal lattice

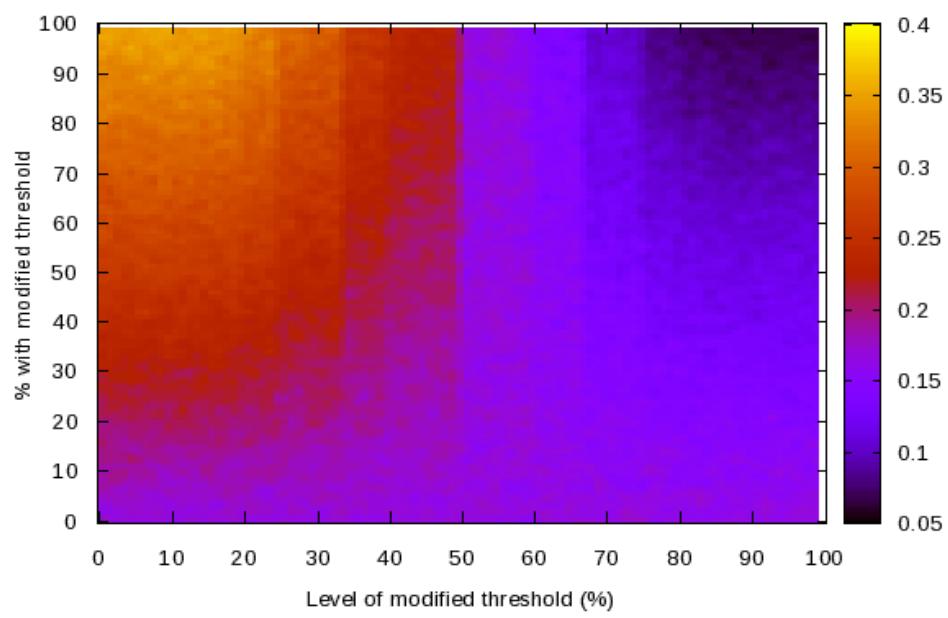
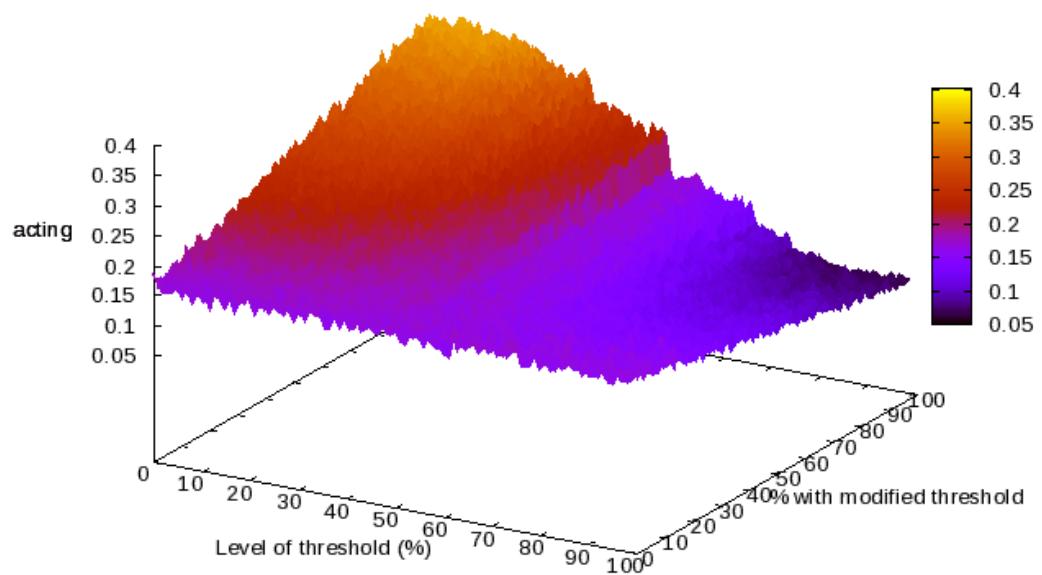


Figure 47: The effects of thresholds on action in an Erdős-Rényi network

4.4 Conclusion

As might have been expected, the factors that shape the processes between agents affect the results of their interactions. In different models that attempt to approximate real-world networks, the proportion of agents acting initially is positively related⁸⁵ to the proportion of agents still acting after interaction has occurred. In general, the relationship describes an S-curve, but there are important differences between the different types of networks. Some reach total action for lower initial proportions than others and the outcomes of simulations in some network types are more sensitive to small changes in the initial situation than in others. In certain circumstances, the results of interaction are similar across network types, but in others they are indeed ‘dramatically different’, such that ‘even if we knew everything there was to know about the properties of the actor (including their action logics), we would be a long way from knowing the social wholes they would be likely to bring about’.⁸⁶

Increases in the mean degree of networks cause more extreme social outcomes, by making the co-ordination of action across the networks much easier, so the outcomes of marginal initial situations will depend on small differences between them. A high mean degree relative to network size allows networks to approximate what Watts⁸⁷ calls a ‘mean-field network’: a network where each actor is connected to every other. In other words, the social aggregate actor for each agent is the average of all the other agents, so that the agents are aware of a large proportion of the others in their society, and can act accordingly.

However, decreasing the mean shortest-path length does not increase the coordination of the simulated societies. It seems reasonable that such networks do not approximate mean-field networks: while a low mean shortest-path length means that all nodes are within a small number of connections from each other, it does not increase the number of influential neighbours that the agent has (Figure 37 on page 62).

Degree distribution does play a role in determining the distribution of outcomes, but it is not

⁸⁵Hedström (2005, p. 86)

⁸⁶Hedström (2005, pp. 86–7)

⁸⁷Watts (1999b), Chapter 7: Global Computation in Cellular Automata

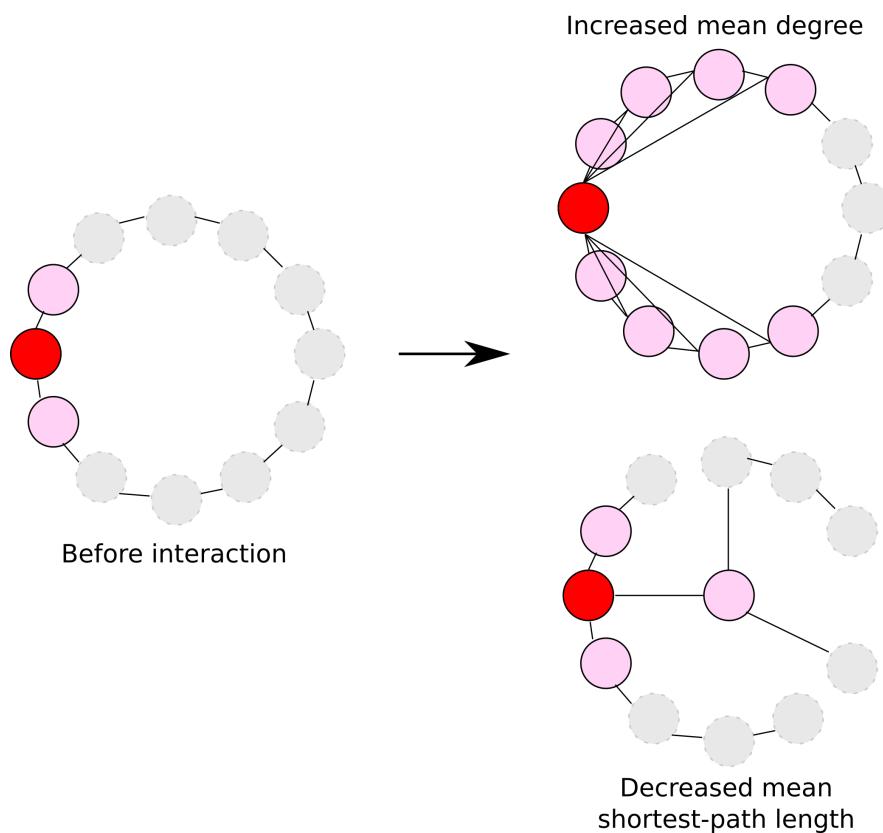


Figure 48: Decreasing the mean shortest path length and increasing the mean degree
The red node's neighbours are shown in pink.

entirely clear what role this is. Because of the particularities of the Barabási-Albert model, it is not possible to adjust the degree distribution without affecting the number of agents that are entirely unconnected to the network. These agents have a significant effect on outcomes because their state is not affected by any other actors, only by the initial random allocation.

When the thresholds required for action are changed, the level of action changes too. However, the change is not continuous: the mean degree of the network interacts with the threshold level, sometimes producing large changes in action for small changes in thresholds.

While the characteristics of networks do affect the outcomes of simulations, it is not always obvious which characteristics are important or in what ways they bring about the observed results – although some possibilities have been suggested. In part, this is because it is difficult to make parameters vary independently of each other in network models. As a consequence, even knowing the basic facts about the ways in which actors interact with and are connected to each other is not enough: neither the number of influential neighbours each actor has, nor the distribution thereof, nor the mean shortest-path length between any two actors can account for all of the differences in social outcomes between DBO simulations. In particular, the simulations of interaction in Barabási-Albert networks have outcomes very sensitive to small differences between initial situations when there is an initial level of action close to 50%, which is of particularly noteworthy because Barabási-Albert networks are particularly good analogues for real small-world networks.

5 Conclusion

Computational simulations that use agents based on the DBO theory can account for a wide range of behaviour. When assumptions about the ways agents act and interact are made more realistic, outcomes can be created that differ greatly from those obtained with strict rationality and rigid networks. There only need be a few agents that are somewhat irrational for macro-level outcomes to change significantly. Similarly, if the networks that connect agents have certain properties, the macro-level outcomes also differ. In particular, there seem to be some characteristics of small-world networks that make the outcomes of interaction very sensitive to changes in the initial conditions. Thresholds in the aggregation process combine with networks to produce discontinuities in the sensitivity of outcomes to changes.

Despite the difference from the initial situation, similarities persist across all of the simulations. Perhaps most notably, the time the simulations take to reach stability has been constant at around 5 interactions. It seems likely that the time to stability is some fundamental unvarying consequence of the basic structure of the DBO model. In Epstein and Axtell's Sugarscape, where agents move around and interact based on more than three parameters, the macro-level properties of the societies sometimes oscillate over time, and do not reach stability for thousands of interactions.⁸⁸ Perhaps if the social networks used in the DBO model were allowed to vary over time, rather than being fixed, similar disturbances in the outcomes would be observed.

Such a process might be implemented in a similar way to Epstein's norm-following agents that are modelled as 'lazy statisticians' in *Learning to Be Thoughtless*⁸⁹. These agents want to make an informed decision, but they also want to expend as little effort as possible. Therefore, they only consult the minimal number of their neighbours that (they think) will give them a reliable sample of the society. In the DBO model, this might mean a varying neighbourhood, so that agents either choose to consult no-one, or just their friends, or friends of friends, and so on.

There are other modifications that could also be carried out. For instance, at present the state

⁸⁸ Epstein and Axtell (1996, pp. 66–67)

⁸⁹ Epstein (2006a), Chapter 10

of each actor at time T only depends on the state of the society at time $T-1$. This prevents the model from representing any processes that require agents to respond and adapt to repeated interaction. All the agents are also updated at the same time (synchronously), whereas it might be more realistic if some reacted before others (asynchronously).

Another direct consequence of the design of the model is that the simulations have some limitations. In particular the same outcome can be explained by many different mechanisms, depending on the measurements available. Epstein notes that ‘in principle, there may be competing microspecifications with equal generative sufficiency, none of which can be ruled out [...] easily. The mapping from the set of microspecifications to the macroscopic explanandum might be many-to-one. In that case, further work is required to adjudicate among the competitors.’⁹⁰ The simplicity that makes the DBO model adaptable to different situations also makes it vulnerable to certain ambiguities. Most significantly, there are situations in which multiple mechanisms can create the same macro-level outcome: different combinations of DBO-modification mechanism and action-decision mechanism can be contrived that produce exactly the same result for individual agents, but not for society as a whole. However, many such mechanisms will not be psychologically plausible and can be disregarded.

Such problems will be common to all agent-based models, and they can be accounted for as long as they are known. In empirical applications extra measurements can be made, in theoretical uses the hypotheses can be adapted. Despite these possible limitations, the DBO model has so far shown itself to be quite versatile in producing varying outcomes for different parameters. While there are still variations to be explored, the model is a useful tool in the generativist’s arsenal for the purpose of explaining the actions and interactions of psychologically-plausible rational (and not-so-rational) agents. The agents of the DBO model might be simple but their societies can be as complex as required.

In sociology, ‘all explanations [...] provide plausible causal accounts for why events happen, why something changes over time, or why events co-vary in time or space’⁹¹. Within the

⁹⁰ Epstein (2006b, p. 1589)

⁹¹ Hedström (2005, p. 13)

mechanism-based tradition, ‘A mechanism should [...] be seen as an empirical commitment on the part of the theorist as to how a process would unfold if the assumptions upon which it rests were well founded’⁹² and ‘an appropriate explanation consists in detailing the constellation of entities and activities that regularly bring about the type of outcome to be explained’⁹³, while the mechanisms remain ‘propositions about particular aspects of a causal totality, with no claim that the tendency in question is the dominant one’⁹⁴. Testing such mechanisms appears to be a fruitful approach to sociology, which benefits in clarity and explanatory power from focussing on actors, their interactions, and the way they bring about social phenomena.

Within the analytical tradition, the DBO theory and the computational models based on it can be adapted to test different hypotheses about individual behaviour, yielding results that are diverse and non-trivial. The link between the individual and the social is not necessarily straightforward, as small changes in social structure can have large effects.⁹⁵ Clarity and precision in such theories can make them seem detached, but they need not ignore the real world.⁹⁶ Putting DBO agents in realistic societies allows the development of specific aggregate correlations between the components of mechanisms and their consequences, which are backed up by intelligible patterns of individual action.⁹⁷

⁹²Hedström (2005, p. 31)

⁹³Hedström (2005, p. 9)

⁹⁴Hedström (2005, p. 32)

⁹⁵Hedström (2005, pp. 145–6)

⁹⁶Hedström (2005, p. 4)

⁹⁷Hedström (2005, p. 35)

References

- Albert, Réka and Barabási, Albert-László**, Statistical mechanics of complex networks. *Review of Modern Physics*, 74 2002 URL: http://prola.aps.org/abstract/RMP/v74/i1/p47_1 .
- Axtell, Robert**, Why Agents? On the Varied Motivations for Agent Computing in the Social Sciences - Brookings Institution. Center on Social and Economic Dynamics, Working Paper No. 17 November 2000 URL: http://www.brookings.edu/reports/2000/11technology_axtell.aspx .
- Barabási, Albert-László and Albert, Réka**, Emergence of Scaling in Random Networks. *Science*, 286 1999:5439, pp. 509–512 URL: <http://www.jstor.org/stable/2899318> , ISSN 00368075.
- Boudon, Raymond**, The ‘Cognitivist Model’: A Generalized ‘Rational-Choice Model’. *Rationality and Society*, 8 1996:2, pp. 123–150 URL: <http://rss.sagepub.com/cgi/content/abstract/8/2/123> .
- Coleman, James Samuel**, Foundations of Social Theory. Cambridge, Massachusetts: Belknap Press of Harvard University Press, 1994.
- Crossley, Nick**, Small-World Networks, Complex Systems and Sociology. *Sociology*, 42 2008:2, pp. 261–277 URL: <http://soc.sagepub.com/cgi/content/abstract/42/2/261> .
- Elster, Jon**, Ulysses and the Sirens: Studies in Rationality and Irrationality. Cambridge: Cambridge University Press, 1979.

Elster, Jon, Sour Grapes: Studies in the Subversion of Rationality. Cambridge: Cambridge University Press, 1983.

Epstein, Joshua M., Agent-based computational models and generative social science. Complexity, 4 1999:5, pp. 41–60 URL: [http://dx.doi.org/10.1002/\(SICI\)1099-0526\(199905/06\)4:5<41::AID-CPLX9>3.0.CO;2-F](http://dx.doi.org/10.1002/(SICI)1099-0526(199905/06)4:5<41::AID-CPLX9>3.0.CO;2-F).

Epstein, Joshua M., Generative social science : studies in agent-based computational modeling. Princeton University Press, 2006.

Epstein, Joshua M.; Idem, editor, Chap. 34 In Remarks on the Foundations of Agent-Based Generative Social Science. Volume 2, Elsevier, 2006 URL: <http://www.sciencedirect.com/science/article/B7P5C-4JR414P-T/2/48ba7cb4bc73777204311257c55e6aa6>, pp. 1585–1604, ISBN 1574–0021.

Epstein, Joshua M. and Axtell, Robert, Growing Artificial Societies: Social Science from the Bottom Up. Brookings Institution Press/The MIT Press, 1996.

Erdős, P. and Rényi, A., On Random Graphs. Publicationes Mathematicae, 6 1959, pp. 290–297.

Erdős, P. and Rényi, A., On the evolution of random graphs. Publications of the Mathematical Institute of the Hungarian Academy of Sciences, 5 1960, pp. 17–61.

Flache, Andreas and Hegselmann, Rainer, Do Irregular Grids make a Difference? Relaxing the Spatial Regularity Assumption in Cellular Models of Social Dynamics. Journal of Artificial Societies and Social Simulation, 4 2001:4 URL: <http://www.soc.surrey.ac.uk/JASSS/4/4/6.html>.

Gilbert, Nigel, A simulation of the structure of academic science. Sociol. Res. Online, 2 1997:2, pp. 3.1–3.21.

Gilbert, Nigel; Liao, Tim F., editor, Agent-Based Models. USA: SAGE Publications, 2008, Quantitative Approaches in the Social Sciences.

Goldthorpe, John H., Rational Action Theory for Sociology. *The British Journal of Sociology*, 49 1998:2, pp. 167–192 URL: <http://www.jstor.org/stable/591308> , ISSN 00071315.

Granovetter, Mark, Threshold Models of Collective Behaviour. *American Journal of Sociology*, 83 May 1978:6, p. 1420.

Hardin, Russell, Collective action as an agreeable n-prisoners' dilemma. *Behavioral Science*, 16 1971:5, pp. 472–481.

Heckathorn, Douglas D., The Dynamics and Dilemmas of Collective Action. *American Sociological Review*, 61 1996:2, pp. 250–277 URL: <http://www.jstor.org/stable/2096334> , ISSN 00031224.

Hedström, Peter, Kolm, Ann-Sofie and Åberg, Yvonne, Social Interactions and Unemployment. Uppsala University, Department of Economics, 2003 – Technical report URL: http://ideas.repec.org/p/hhs/uunewp/2003_018.html .

Hedström, Peter, Dissecting the social : on the principles of analytical sociology. Cambridge University Press, 2005, p. 177, ISBN 9780521792295.

Hedström, Peter, Actions and Networks: Sociology that really matters... to me. *Sociologica*, 1 2007 URL: <http://www.sociologica.mulino.it/doi/10.2383/24187> .

Huckfeldt, Robert, Political Environments, Cohesive Social Groups, and the Communication of Public Opinion. *American Journal of Political Science*, 39 1995:4, pp. 1025–1054 URL: <http://www.jstor.org/stable/2111668> , ISSN 00925853.

Jablan, Slavik V., Chap. 1.2 In Transformations and Symmetry Groups. Mathematical Institute of the Serbian Academy of Science and Arts, 1995 URL: <http://emis.library.cornell.edu/monographs/jablan/chap12.htm> .

Kollock, Peter, Social Dilemmas: The Anatomy of Cooperation. *Annual Review of Sociology*, 24 1998, pp. 183–214 URL: <http://www.jstor.org/stable/223479> , ISSN 03600572.

Lazer, David, SOCIAL SCIENCE: Computational Social Science. *Science*, 323 February 2009:5915, pp. 721–723 URL: <http://www.sciencemag.org> .

Macy, Michael W., Social Order and Emergent Rationality. in: **Sica, A, editor:** What is Social Theory: The Philosophical Debates. Blackwell, 1998.

Macy, Michael W.; Smelser, N. and Baltes, P., editors, Chap. 2.2.150 In Social Simulation. Elsevier, 2002.

Macy, Michael W. and Flache, Andreas; Hedström, Peter and Bearman, Peter, editors, Chap. 11 In Social Dynamics from the Bottom Up: Agent-Based Models of Social Interaction. Oxford University Press, 2009.

Macy, Michael W. and Willer, Robert, From Factors to Actors: Computational Sociology and Agent-Based Modeling. *Annual Review of Sociology*, 28 2002, pp. 143–166 URL: <http://www.jstor.org/stable/3069238> , ISSN 03600572.

Marwell, Gerald and Oliver, Pamela, The critical mass in collective action : a micro-social theory. Cambridge University Press, 1993, p. 206.

Newman, M. E. J, The structure and function of complex networks. *SIAM Review*, 45 2003, pp. 167–256 URL: <http://arxiv.org/abs/cond-mat/0303516> .

Nicosia, V., Exploiting social networks dynamics for P2P resource organisation. *Lecture Notes in Computer Science*, 4263/2006 2006, pp. 726–734 URL: <http://arxiv.org/abs/cs/0702127> .

Olson, Mancur, The logic of collective action : public goods and the theory of groups. Harvard University Press, 1965, p. 186.

Opp, Karl-Dieter, Peter Hedstrom: Dissecting the Social. On the Principles of Analytical Sociology. *Eur Sociol Rev*, 23 2007:1, pp. 115–122 URL: <http://esr.oxfordjournals.org>.

Peeters, Hans, Review: Theoretical Approaches: Peter Hedstrom, Dissecting the Social: On the Principles of Analytical Sociology. Cambridge: Cambridge University Press, 2005, 177 pp., ISBN 0521796679, pound15.99. *International Sociology*, 22 2007:5, pp. 606–609 URL: <http://iss.sagepub.com>.

Pickel, Andreas, Dissecting the Social: On The Principles of Analytical Sociology. *Canadian Journal of Sociology Online*, September-October 2006 2006 URL: <http://www.cjsonline.ca/reviews/dissectsocial.html>.

Raub, Werner, Structural-Individualist Social Theory: Advances and New Challenges Comments on Hedström, Buskens, and Flache. 2006.

Schelling, Thomas C., Models of Segregation. *American Economic Review, Papers and Proceedings*, 59 1969:2, pp. 488–93.

Schelling, Thomas C., Dynamic Models of Segregation. *Journal of Mathematical Sociology* 1 1971:143-86.

Schelling, Thomas C., On the Ecology of Micromotives. *The Public Interest*, 25 1971:Fall, pp. 61–98.

Scott, John, Social network analysis : a handbook. Sage, 2000, p. 208.

Siegel, David A., Social Networks and Collective Action. *American Journal of Political Science*, 53 2009:1, pp. 122–138.

Simon, H., The Sciences of the Artificial. Cambridge, MA: MIT Press, 1998.

Steyvers, Mark and Tenenbaum, Joshua B., The Large-Scale Structure of Semantic Networks: Statistical Analyses and a Model of Semantic Growth. *Cognitive Science*, 29

2005:1, pp. 41–78–41–78 URL: http://www.leaonline.com/doi/abs/10.1207/s15516709cog2901_3.

Takahashi, Nobuyuki, The Emergence of Generalized Exchange. *The American Journal of Sociology*, 105 2000:4, pp. 1105–1134 URL: <http://www.jstor.org/stable/3003889>, ISSN 00029602.

Taylor, Michael; Idem, editor, *The possibility of cooperation*. Cambridge University Press, 1987, p. 205, ISBN 8200183866.

Troitzsch, Klaus G., Theory and Practice of Multi-Agent Methodology in Microsimulation: Presentation of a new JAVA-Based Tool. 2007 URL: www.uni-koblenz.de/~kgt/Pub/Barcelona.pd.

Watts, Duncan J., Networks, Dynamics, and the Small-World Phenomenon. *The American Journal of Sociology*, 105 1999:2, pp. 493–527 URL: <http://www.jstor.org/stable/2991086>, ISSN 00029602.

Watts, Duncan J., *Small worlds : the dynamics of networks between order and randomness*. Princeton University Press, 1999, p. 262.

Watts, Duncan J. and Dodds, Peter Sheridan, Influentials, Networks, and Public Opinion Formation. *The Journal of Consumer Research*, 34 2007:4, pp. 441–458 URL: <http://www.jstor.org/stable/4498509>, ISSN 00935301.

Watts, Duncan J. and Strogatz, Steven H., Collective dynamics of ‘small-world’ networks. *Nature*, 393 1998:6684, pp. 440–442, ISSN 0028–0836.

Weisstein, Eric W., Regular Tessellation. 2010 URL: <http://mathworld.wolfram.com/RegularTessellation.html>.

Wikipedia contributors, Average path length. 2008 URL: http://en.wikipedia.org/w/index.php?title=Average_path_length&oldid=192788785.

Wikipedia contributors, Affinity group. 2009 URL: http://en.wikipedia.org/w/index.php?title=Affinity_group&oldid=279640049 .

Wikipedia contributors, Affinity group. 2009 URL: http://en.wikipedia.org/w/index.php?title=Affinity_group&oldid=279640049 .

Wikipedia contributors, BA model. 2009 URL: http://en.wikipedia.org/w/index.php?title=BA_model&oldid=274153527 .

Wikipedia contributors, Clustering coefficient. 2009 URL: http://en.wikipedia.org/w/index.php?title=Clustering_coefficient&oldid=287832691 .

Wikipedia contributors, Degree (graph theory). 2009 URL: [http://en.wikipedia.org/w/index.php?title=Degree_\(graph_theory\)&oldid=286952496](http://en.wikipedia.org/w/index.php?title=Degree_(graph_theory)&oldid=286952496) .

Wikipedia contributors, Distance (graph theory). 2009 URL: [http://en.wikipedia.org/w/index.php?title=Distance_\(graph_theory\)&oldid=272217377](http://en.wikipedia.org/w/index.php?title=Distance_(graph_theory)&oldid=272217377) .

Wikipedia contributors, Erdős–Rényi model. 2009 URL: http://en.wikipedia.org/w/index.php?title=Erd%C5%91s%E2%80%93R%C3%A9nyi_model&oldid=285506371 .

Wikipedia contributors, Fat tail. 2009 URL: http://en.wikipedia.org/w/index.php?title=Fat_tail&oldid=280918854 .

Wikipedia contributors, Graph (mathematics). 2009 URL: [http://en.wikipedia.org/w/index.php?title=Graph_\(mathematics\)&oldid=282043558](http://en.wikipedia.org/w/index.php?title=Graph_(mathematics)&oldid=282043558) .

Wikipedia contributors, Nash equilibrium. 2009 URL: http://en.wikipedia.org/w/index.php?title=Nash_equilibrium&oldid=285673229 .

Wikipedia contributors, Power law. 2009 URL: http://en.wikipedia.org/w/index.php?title=Power_law&oldid=285223774 .

Wikipedia contributors, Preferential attachment. 2009 URL: http://en.wikipedia.org/w/index.php?title=Preferential_attachment&oldid=284797989 .

Wikipedia contributors, Random graph. 2009 URL: http://en.wikipedia.org/w/index.php?title=Random_graph&oldid=286203303 .

Wikipedia contributors, Scale invariance. 2009 URL: http://en.wikipedia.org/w/index.php?title=Scale_invariance&oldid=274399921 .

Wikipedia contributors, Small-world network. 2009 URL: http://en.wikipedia.org/w/index.php?title=Small-world_network&oldid=287709988 .

Wikipedia contributors, Watts and Strogatz model. 2009 URL: http://en.wikipedia.org/w/index.php?title=Watts_and_Strogatz_model&oldid=287049153 .

Acknowledgements

- Simulations written in Python from the Python Software Foundation (www.python.org), using the python-graph (<http://code.google.com/p/python-graph/>) and pydot (<http://dkbza.org/pydot.html>) libraries for graph-related functions.
- Typeset in X_ELATEX (<http://scripts.sil.org/xetex>) (a derivative of LATEX) using LyX (<http://www.lyx.org/>) on Fedora GNU/Linux from the Fedora Project and sponsored by Red Hat, Inc. (<http://fedoraproject.org/>).
- KOMA-script typesetting style by Frank Neukam, Markus Kohm, and Axel Kielhorn (<http://www.ctan.org/tex-archive/macros/latex/contrib/koma-script/>).
- Bibliography management using Zotero from the Center for History and New Media (<http://www.zotero.org/>), BibTEX by Alexander Feder (<http://www.bibtex.org/>) and KBibTEX by Thomas Fischer (<http://www.unix-ag.uni-kl.de/~fischer/kbibtex/>).

- Graph images generated with the ‘neato’ global-energy minimisation layout algorithm by Emden Gansner and Yehuda Koren from Graphviz (<http://www.graphviz.org/>).
- Data plotting by Gnuplot (<http://www.gnuplot.info/>).
- Some figures drawn with Inkscape (<http://www.inkscape.org/>).