

# Lab 1 Notes

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The given Pareto distribution is

$$f(x) = \frac{\alpha 3^\alpha}{x^{\alpha+1}}, \quad x > 3, \quad \alpha > 1,$$

and zero otherwise.

(i) Find the cdf.

$$\begin{aligned} F(x) &= \int_{-\infty}^x \frac{\alpha 3^\alpha}{t^{\alpha+1}} dt \\ &= \int_3^x \alpha 3^\alpha t^{-(\alpha+1)} dt \\ &= \left. \frac{\alpha 3^\alpha t^{-(\alpha+1)+1}}{-(\alpha+1)+1} \right|_{t=3}^{t=x} \\ &= -3^\alpha t^{-\alpha} \Big|_{t=3}^{t=x} \\ &= -3^\alpha (x^{-\alpha} - 3^{-\alpha}) \\ &= -\frac{3^\alpha}{x^\alpha} + 1 \\ &= 1 - \left(\frac{3}{x}\right)^\alpha, \quad x > 3 \end{aligned}$$

and  $F(x) = 0$  for  $x \leq 3$ .

(ii) Find the quantile function.

Recall that the cdf was defined as

$$F(x) = \mathbf{P}(X \leq x) = p,$$

where  $x$  is the *quantile* and is given, and  $p$  is the probability that we need to often need to calculate. With the quantile function, the conditions are reversed:

$$Q(p) = x \quad \text{such that} \quad \mathbf{P}(X \leq x) = p,$$

where  $p$  is given is  $x$  is the value we seek. If  $F(x)$  is a one-to-one function, then  $Q(p)$  is simply the inverse of  $F(x)$ .

Using cdf from (i), we solve for  $x$ :

$$p = 1 - \left(\frac{3}{x}\right)^\alpha$$

$$\left(\frac{3}{x}\right)^\alpha = 1 - p$$

$$\frac{3}{x} = (1 - p)^{1/\alpha}$$

$$x = \frac{3}{(1 - p)^{1/\alpha}}$$

Therefore, our quantile function is:

$$Q(p) = \frac{3}{(1 - p)^{1/\alpha}}, \quad 0 < p \leq 1$$