Tutorial 10

April 2, 2020

Let $z_{1-\alpha}$ be the value such that $\mathbf{P}\left(Z \leq z_{1-\alpha}\right) = 1-\alpha$.

Let $t_{n-1,1-\alpha}$ be the value such that $\mathbf{P}\left(T_{n-1} \leq t_{n-1,1-\alpha}\right) = 1-\alpha$.

Question 1

A random sample of 110 lightning flashes in a certain region resulted in a sample average radar echo duration of 0.81 sec. and a standard deviation of 0.34 sec. Build a 99% confidence interval of the true average echo duration at that region.

From the question, we are given:

$\overline{x}$	s	n	α
0.81	0.34	110	0.01

Since $n > 40$, we construct a large sample interval for the true population mean:

$$\begin{aligned}\overline{x} \pm z_{1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} &= 0.81 \pm 2.576 \frac{0.34}{\sqrt{110}} \\ &= (0.7265,\, 0.8935)\end{aligned}$$

With 99% confidence, we conclude that the true average echo duration is between 0.7265 sec. and 0.8935 sec.

Question 2

In a sample of 1000 randomly selected consumers who had opportunities to send in a rebate claim form after purchasing a product, 250 of these people said they never did so. Calculate a 95% confidence interval of the true proportion of consumers who never apply for a rebate.

From the question, we are given:

$\widehat{p}$	n	α
$\frac{250}{1000} = 0.25$	1000	0.95

Since $n\widehat{p} = 250 \geq 10$ and $n(1-\widehat{p}) = 750 \geq 10$, we construct a large sample interval for the true population proportion:

$$\begin{aligned}\widehat{p} \pm z_{1-\frac{\alpha}{2}} \sqrt{\frac{\widehat{p}(1-\widehat{p})}{n}} &= 0.25 \pm 1.96 \sqrt{\frac{0.25 * 0.75}{1000}} \\ &= (0.2232,\, 0.2768)\end{aligned}$$

With 95% confidence, we conclude that the true proportion of customers who never apply for a rebate is between 0.2232 and 0.2768.

Question 3

The following are observations on degree of polymerization for paper specimens for which viscosity times concentration fell in a certain middle range:

418	421	421	422	425	427
431	434	437	439	446	447
448	453	454	463	465	

Assuming data are normally distributed, calculate a 95% confidence interval for the true average degree of polymerization. Does the interval suggest that 440 is a plausible value for the true average degree of polymerization? How about 450?

From the question, we are given: $\alpha = 0.05$. From the data it can be found that:

$$n = 17$$

$$\overline{x} = \frac{1}{n}\sum_{i=1}^{n} x_i = 438.2941$$

$$s = \sqrt{\frac{1}{n-1}\left(\sum_{i=1}^{n} x_i^2 - n\overline{x}^2\right)} = 15.1442$$

Assuming the data are normally distributed, since we are using s as a substitute for σ (unknown), we construct a t-interval for the true population mean:

$$\begin{aligned} \overline{x} \pm t_{n-1,1-\frac{\alpha}{2}} \frac{s}{\sqrt{n}} &= 438.2941 \pm 2.12 \frac{15.1442}{\sqrt{17}} \\ &= (430.5077,\ 446.0805) \end{aligned}$$

We conclude with 95% confidence that the true average degree of polymerization is between 430.5077 and 446.0805. As 440 is contained in this interval, it suggests that 440 is a plausible value for the true average degree of polymerization. As 450 is not contained in this interval, it suggests that 450 is *not* a plausible value for the true average degree of polymerization.

Question 4

The recommended daily dietary allowance for zinc among males older than 50 years is 15mg/day. A study on intake for a sample of 115 males ages 65-74 yielded a sample average zinc intake of 11.3mg/day and a standard deviation of 6.43mg/day. Does this survey indicate that the daily zinc intake for male population ages 65-74 falls below the recommended allowance?

Let μ represent the true average daily zinc intake among males between 65-74. From the question, we are given:

μ_0	n	$\overline{x}$	s
15	115	11.3	6.43

As the significance level is not given in the question, we will assume $\alpha = 0.05$. We want to test the claim that the average daily zinc intake among males between 65-74 **falls below** the recommended allowance. Therefore, our hypotheses are:

$$H_0 : \mu = 15, \quad H_A : \mu < 15$$

The value of the test statistic is:

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{11.3 - 15}{6.43/\sqrt{115}} = -6.171$$

Critical value method: Since this is a lower-tailed test, the critical value is:

$$-t_{n-1,1-\alpha} = -t_{114,0.95} = -1.658$$

and we reject the null hypothesis if $t < -t_{n-1,1-\alpha}$. Since $-6.171 < -1.658$, we reject the null hypothesis in favour of the alternative hypothesis. We conclude that at the 5% significance level, there is evidence to support the claim that the average daily zinc intake among males between 65-74 falls below the recommended allowance.

p-value method: Since this is a lower-tailed test, the p-value is the area to the left of t:

$$\mathbf{P}(T_{n-1} \leq t) = \mathbf{P}(T_{114} \leq -6.171) = 5.33 * 10^{-9}$$

and we reject the null hypothesis if the p-value is less than 0.05. Since $5.33 * 10^{-9} < 0.05$, we reject the null hypothesis in favour of the alternative hypothesis. We conclude that at the 5% significance level, there is evidence to support the claim that the average daily zinc intake among males between 65-74 falls below the recommended allowance.

Note: Your conclusion from both methods should be the same. If they are different, you've done something wrong!

Question 5

A manufacturer of nickel-hydrogen batteries randomly selects 100 nickel plates for test cells, cycles them a specified number of times, and determines that 14 of plates have blistered. Does this provide compelling evidence for concluding that more than 10% of all plates blister under such circumstances? State and test the appropriate hypotheses using a significance level $\alpha = 0.05$. In reaching your conclusion, what type of error might you have committed?

Let p represent the true proportion of nickel plates that blister after cycling. From the question, we are given:

p_0	$\widehat{p}$	n	α
0.10	$\frac{14}{100} = 0.14$	100	0.05

We want to test the claim that **more than** 10% of all plates blister under these circumstances. Therefore, our hypotheses are:

$$H_0 : p = 0.10, \quad H_A : p > 0.15$$

We should first check that we have satisfied the required conditions to perform a large-sample hypothesis test concerning a population proportion:

$$np_0 \;=\; 10 \;\geq\; 10, \quad n(1-p_0) \;=\; 90 \geq 10$$

As the required conditions are satisfied, we can proceed with a large-sample hypothesis test. The value of the test statistic is:

$$z \;=\; \frac{\widehat{p} - p_0}{\sqrt{\frac{p_0(1-p_0)}{n}}} \;=\; \frac{0.14 - 0.10}{\sqrt{\frac{0.10*0.90}{100}}} \;=\; 1.33$$

Critical value method: Since this is an upper-tailed test, the critical value is:

$$z_{1-\alpha} \;=\; z_{0.95} \;=\; 1.645$$

and we reject the null hypothesis if $z \;>\; z_{1-\alpha}$. Since $1.33 \not> 1.645$, we fail to reject the null hypothesis. We conclude that at the 5% significance level, there is insufficient evidence to support the claim that more than 10% of all plates blister after cycling.

***p*-value method:** Since this is an upper-tailed test, the p-value is the area to the right of z:

$$\mathbf{P}\,(Z > z) \;=\; \mathbf{P}\,(Z > 1.33) \;=\; 1 - \mathbf{P}\,(Z \leq 1.33) \;=\; 0.0918$$

and we reject the null hypothesis if the p-value is less than 0.05. Since $0.0918 \not< 0.05$, we fail to reject the null hypothesis. We conclude that at the 5% significance level, there is insufficient evidence to support the claim that more than 10% of all plates blister after cycling.

In reaching our conclusion, we may have committed a type II error. A type II error occurs when we fail to reject the null hypothesis when the null hypothesis is actually false.

In Question 4, we may have committed a type I error. A type I error occurs when we reject the null hypothesis when the null hypothesis is actually true.