ME 370B
Energy Systems II:
Modeling and Advanced Concepts

Chris Edwards
Winter 2023

More than you ever wanted to know about… Effectiveness, Capacity, and Pinch

Although you have all dealt with heat exchangers before, there are three concepts—capacity, effectiveness, and pinch—that you need to understand fully to deal with boilers and heat recovery steam generators (HRSGs)—and, in fact, any other form of heat-integrated device. To help you along with this, we have included a brief discussion of these issues below. Have a look if you are not entirely sure about what these terms mean and how they impact your analyses of energy systems.

The *effectiveness* of a heat exchanger is something that should be familiar to you from undergraduate courses in thermodynamics and/or heat transfer: It is the fraction of the maximum heat transfer permitted by the second law between two fluid streams. If you can figure out $\dot{Q}_{\max}$ between the steams—which is usually not impossible—then the actual heat transfer is obtained by multiplying $\dot{Q}_{\max}$ by the effectiveness ε

$$\dot{Q}_{\text{actual}} = \varepsilon \dot{Q}_{\max}$$

Since you will typically be given an effectiveness (by those who did the transport-level design of the heat exchanger), the key to integrating it into your thermodynamic analysis is to be able to calculate $\dot{Q}_{\max}$.

$\dot{Q}_{\max}$ is the largest rate of heat transfer between the two streams. This occurs when the streams are allowed to equilibrate in a lossless environment (in the energy, not exergy sense!). By this we mean that the streams exchange heat until a steady-state partitioning of the energy between the two streams is reached. The amount of heat moved in achieving that partitioning is $\dot{Q}_{\max}$.

Practically, this partitioning comes down to a question of the *capacity* of the two (or more) streams. Capacity is the product of mass flow rate and specific heat

$$C = \dot{m} c_p$$

A fluid stream has a high capacity if it has a large specific heat and/or a large mass flow rate. It has a high capacity in the sense that it will be able to absorb a large amount of heat without experiencing a significant change in its temperature. A stream with a low capacity has the opposite characteristic; a modest amount of heat transfer will cause a significant change in the temperature of the fluid.

Most heat exchangers in energy systems are of the counter-flow variety (at least on the macro scale). The objective is to pass thermal energy from a hot fluid to a cold one in the most *effective* (biggest transfer) way. Counterflow heat exchangers accomplish this by maintaining the largest possible temperature difference between the two fluids; the hot-side fluid's inlet state is paired with the cold-side fluid's outlet state (and vice versa) such that the largest driving potential for heat exchange always exists. In exchangers operating with equal capacity of the two fluids, this means that the temperature drop of the hot fluid will exactly match the temperature rise of the cold fluid. Under these conditions, the maximum heat transfer possible is that which brings the cold-fluid outlet temperature up to the value of the hot fluid at its inlet. Conversely, the hot-fluid outlet temperature is reduced to the cold-fluid inlet temperature and the world is full of wonderful symmetry.

The situation gets a little trickier when the two capacities are not the same. Thinking through the situation, it will probably not surprise you to find that the most heat that can be transferred corresponds to when the low-capacity fluid (whether hot or cold) is shifted in temperature such that its outlet temperature matches that of the high-capacity fluid at its inlet. In other words, since the high-capacity fluid is the 800-pound gorilla in the relationship, it is not too surprising that given enough time, space, copper fins, etc., the low-capacity fluid will be forced to accommodate to the constraint imposed by the high-capacity fluid. Since the only constraint that is really imposed on the high-capacity fluid is its inlet temperature, in a maximum-heat-transfer world, the low-capacity outlet temperature must match the high-capacity inlet value. (Note that if we tried to push the temperature of a low-capacity, low-temperature fluid up above the inlet temperature of a high-capacity, high-temperature fluid it would correspond to a case of heat transfer uphill against the temperature gradient—a not-too-subtle violation of the second law.)

So the key to finding $\dot{Q}_{\text{max}}$ is identifying the high- and low-capacity fluids. If you know the mass flow rates, and you know the heat capacities, this is straightforward. If you have fluids that have variable mass flow rates, or highly variable specific heats, or multiple fluids, or phase change (infinite specific heat) this gets trickier. All I can say about those cases is that if you can't figure out which is high and which is low, choose one and try it. The wonderful thing about thermo is that it has a marvelous way of letting you know when you have done something stupid (heat will flow up hill, entropy will be destroyed, the world as we know it will come to an end—just kidding). Try it one way and if you are wrong, the analysis will let you know.[1]

Assuming that you have identified which fluid is which, $\dot{Q}_{\text{max}}$ can be found directly

$$\dot{Q}_{\text{max}} = \dot{m}_{\text{lc}}\left[h_{\text{lc}}\left(T_{\text{hc, inlet}}, P_{\text{lc, outlet}}\right) - h_{\text{lc}}\left(T_{\text{lc, inlet}}, P_{\text{lc, inlet}}\right)\right]$$

where the subscripts *lc* and *hc* stand for the low-capacity and high-capacity fluids, respectively. From this expression it is apparent that although it is the high-capacity fluid that sets the temperature end-point for the low-capacity fluid, it is the *capacity* of the low-capacity fluid that

[1] In many cases we will consider in this class, the task is less one of matching two specified streams for heat exchange (as in process heat integration) than it is of *mining* a thermal resource for its available energy. In this case, the resource is always taken to be the high-capacity stream and the stream used to "mine it" is adjusted (usually via flow rate) so as to meet an effectiveness specification. Note that it would make no sense to change which stream one takes as dominant in such an analysis since the mining stream can always be adjusted up in flow rate until it becomes dominant. At that point, however, the objective of the transfer is no longer to capture the available energy of the resource stream since the increased capacity of the mining stream will now result in a reduced capture of available energy (i.e., a lower outlet temperature).

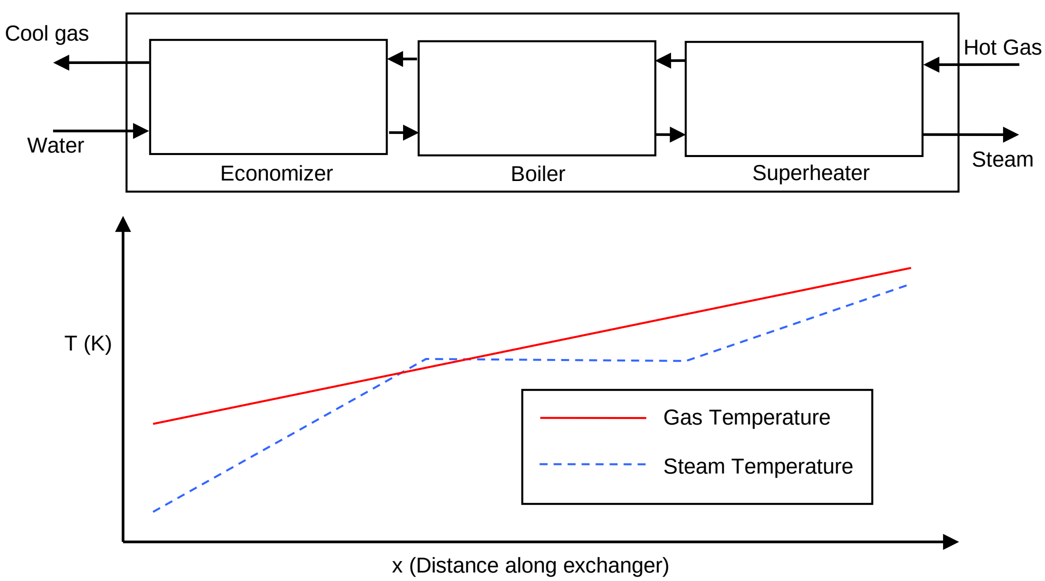
sets the overall amount of heat that can be transferred between the two streams. (As such we speak of the low-capacity stream as being the *controlling* stream of the heat exchanger.)

With $\dot{Q}_{\max}$ determined and the effectiveness known, the actual heat transfer for each side can be found and the outlet states of each fluid can be determined by solving

$$\begin{aligned}\dot{Q}_{actual} &= \varepsilon \dot{m}_{\text{lc}} \left[h_{\text{lc}} \left(T_{\text{hc, inlet}}, P_{\text{lc, outlet}} \right) - h_{\text{lc}} \left(T_{\text{lc, inlet}}, P_{\text{lc, inlet}} \right) \right] \\ &= \dot{m}_{\text{lc}} \left[h_{\text{lc}} \left(T_{\text{lc, outlet}}, P_{\text{lc, outlet}} \right) - h_{\text{lc}} \left(T_{\text{lc, inlet}}, P_{\text{lc, inlet}} \right) \right] \\ &= \dot{m}_{\text{hc}} \left[h_{\text{hc}} \left(T_{\text{hc, inlet}}, P_{\text{hc, inlet}} \right) - h_{\text{hc}} \left(T_{\text{hc, outlet}}, P_{\text{hc, outlet}} \right) \right]\end{aligned}$$

for the fluid temperatures (either explicitly or implicitly).

Note that this same approach applies to counterflow exchangers with multiple (more than two) fluid streams. The key to untangling this situation is to determine which *flow direction* has the higher capacity. The flow direction with the higher capacity will determine the other flow direction's outlet temperature. The capacities of the low-capacity-flow-direction streams will then control the maximum amount of heat transfer. (Did that make any sense?) I mention this because we will encounter it in a few weeks when we tackle a project on gas liquefaction.[2]



Using *effectiveness* and understanding the *capacity* of the fluid streams is great for making sure that a heat exchanger doesn't violate the first or second law at the inlets and outlets, but it says nothing about what goes on inside. Imagine a boiler in which the temperature of the hot gas

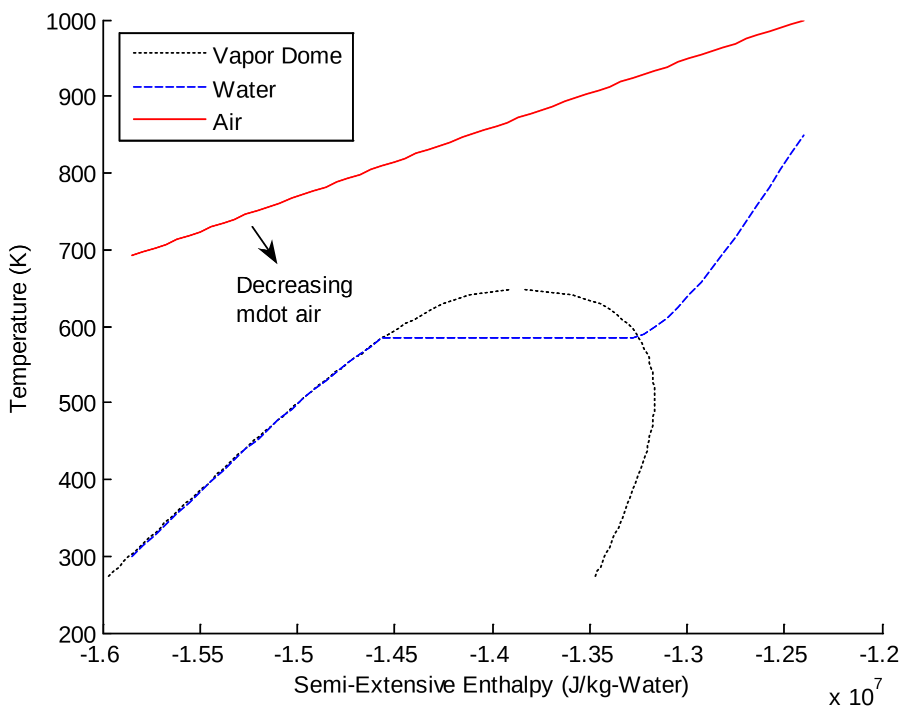
---

[2] In lieu of effectiveness, we also speak some times of the ΔT between the outlet of the low-capacity stream and the inlet of the high-capacity stream. For systems with well-known configurations (e.g., a boiler or HRSG) knowing that the state of the art is to provide sufficient fin/tube area for these to approach within, say, 10 or 20 K is often more convenient than making a specification in terms of effectiveness.

must steadily decline as it transfers heat to the boiling fluid. The boiling fluid will enter a section where its temperature is constant (saturation). A sketch of temperature vs. position in the heat exchanger might look as shown above.

The interesting thing to note is that the temperature of the boiling fluid drops off so quickly in the economizer section that, from the perspective of one who knows only the END states of the two streams, the first and second laws are obeyed. But in part of the boiler, heat is attempting to flow up a temperature gradient! *Pinch* or *pinch-point* analysis fills in the "middle" part of understanding the heat-exchange process, and it ensures that you aren't attempting to make heat flow uphill at any point.

The key to showing that the exchanger is well behaved is a semi-extensive *T-h* diagram. Rather than look at temperature as a function of distance, look at the temperature of each of the streams as a function of enthalpy per unit mass of total flow (this can be normalized to your choice of mass flow rate in the system). If we offset the enthalpies so that one of the endpoints of each *T-h* curve lines up with the corresponding point for the other fluid, we know that the other endpoint must also line up since the heat transferred from one stream was absorbed by the other (assuming no losses to the outside world). Furthermore, all points along the *h*-axis represent real points in the heat exchanger; any movement along the *h*-axis represents a change in enthalpy and that change must be reflected by its mirror-image in the other fluid. (Yep, conservation of energy!) Also note that the mapping from temperature-distance to *T-h* is not necessarily linear.



The pinch temperature specifies how close these two curves are allowed to approach each other internally. (They're never allowed to cross!) It is your job as the engineer to watch the slope and position of these curves as the flow rates change to ensure that the pinch specifications of the exchanger are always met.