# THE UNIVERSITY OF CALGARY

# DEPARTMENT OF GEOMATICS ENGINEERING

**ENGO 623: INERTIAL SURVEYING AND INS/GPS INTEGRATION**

**Winter semester 2023**

Developing an INS Navigation Module – Due date: Monday April 17, 2020

## Objectives

Design and implement the INS mechanization in the local level frame and show step by step implementation details.

**Deliverables**

1. A document detailing
    a. the step by step implementation of the mechanization module.
    b. Plots of the final latitude, longitude and height and comments on the results
    c. Plots of the position (in meters), velocity (m/sec) and attitude errors (degrees)
2. Source code of the module.

The information for the dataset attached is provided below. The simulated data mimics a situation in which an aligned IMU was left running without any motion for a period of time.

The data was simulated using the following parameters:

| | | |
|---|---|---|
| IMU data rate | 64 | Hz |
| Gyro bias (calibration residual error + random constant) | 0.1 | deg/hr |
| gyro white noise (ARW) | 0.01 | $deg/hr^{1/2}$ |
| Gyro bias instability (1st-order Gauss Markov) | 0.015 | deg/hr |

| | | |
|---|---|---|
| Gyro correlation time | 1 | hr |
| Accelerometer bias (calibration residual error + random constant) | 3 | μg |
| Accelerometer white noise (VRW) | 0.003 | $m/s/hr^{1/2}$ |
| Accelerometer bias instability ($1^{st}$-order Gauss Markov) | 50 | μg |
| Accelerometer correlation time | 1 | hr |

The initial conditions are as follows:

| | | |
|---|---|---|
| Initial latitude | 51.07995352 | deg |
| Initial longitude | -114.13371127 | deg |
| Initial height | 1118.502 | m |

The data format is as follows:

| | |
|---|---|
| Time | sec |
| Gyro x | rad/s |
| Gyro y | rad/s |
| Gyro z | rad/s |
| Accelerometer x | $m/s^2$ |
| Accelerometer y | $m/s^2$ |
| Accelerometer z | $m/s^2$ |

Where x is right (sideways), y is forward and z is pointing up.

Needed Information:

- ☐ Earth Fixed Frame related parameters: WGS84
  - Earth is not round and the semi-major axis a=6378137;
  - Eccentricity squared is $e^2$=6.69438$e^{-3}$;
  - Earth rotation rate in rad/s is $w_e$ = 7.292115147$e^{-5}$

- ☐ Gravity is a function of height and latitude
  - a1 = 9.7803267715; a2 = 0.0052790414;
  - a3 = 0.0000232718; a4 = -0.000003087691089;
  - a5 = 0.000000004397731;
  - a6 = 0.000000000000721;
  - Where $\varphi$ = latitude and h = height

$$g = a1 \cdot (1 + a2 \cdot \sin^2\varphi + a3 \cdot \sin^4\varphi) + (a4 + a5 \cdot \sin^2\varphi) \cdot h + a6 \cdot h^2$$

- ☐ Radius of curvature in the prime vertical (N) and meridian (M) directions

$$M = \frac{a\left(1-e^2\right)}{\left(1-e^2\sin^2\varphi\right)^{\frac{3}{2}}} \qquad N = \frac{a}{\left(1-e^2\sin^2\varphi\right)^{\frac{1}{2}}}$$

## *1. Notes about alignment*

Roll, pitch and azimuth are defined as follows:

- Roll to right is positive
- Pitch upwards is positive
- Azimuth is from North direction clockwise

Using the above definitions:

1. The roll, pitch and azimuth in ENU frame (the one used in the lecture notes) can be calculated as:

$$r = -sign(f_z)\sin^{-1}\left(\frac{f_x}{g}\right)$$

$$p = sign(f_z)\sin^{-1}\left(\frac{f_y}{g}\right)$$

$$A = \tan^{-1}\left(-\frac{\omega_x^b}{\omega_y^b}\right)$$

2. The roll, pitch and azimuth in NED frame (the one used in Eun Shin MSc thesis) can be calculated as:

$$r = sign(f_z)\sin^{-1}\left(\frac{f_y}{g}\right)$$

$$p = -sign(f_z)\sin^{-1}\left(\frac{f_x}{g}\right)$$

$$A = \tan^{-1}\left(\frac{\omega_y^b}{\omega_x^b}\right)$$

## *2. Scale Factor*

SF is a unit matrix for this dataset.

## 3. DCM to Quaternions

$$M = \begin{bmatrix} m11 & m12 & m13 \\ m21 & m22 & m23 \\ m31 & m32 & m33 \end{bmatrix} --> Q = \begin{bmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{bmatrix}$$

For computation compute $q_0$ to $q_3$ first and then take the biggest component to compute the rest. $q_0$ is the scalar.

$$q_0 = \frac{\sqrt{m11 + m22 + m33 + 1}}{2} \Rightarrow q_1 = \frac{m23 - m32}{4q_0}, q_2 = \frac{m31 - m13}{4q_0}, q_3 = \frac{m12 - m21}{4q_0}$$

$$q_1 = \frac{\sqrt{m11 - m22 - m33 + 1}}{2} \Rightarrow q_0 = \frac{m23 - m32}{4q_1}, q_2 = \frac{m12 + m21}{4q_1}, q_3 = \frac{m31 + m13}{4q_1}$$

$$q_2 = \frac{\sqrt{-m11 + m22 - m33 + 1}}{2} \Rightarrow q_0 = \frac{m31 - m13}{4q_2}, q_1 = \frac{m12 + m21}{4q_2}, q_3 = \frac{m23 + m32}{4q_2}$$

$$q_3 = \frac{\sqrt{-m11 - m22 + m33 + 1}}{2} \Rightarrow q_0 = \frac{m12 - m21}{4q_3}, q_1 = \frac{m31 + m13}{4q_3}, q_2 = \frac{m23 + m32}{4q_3}$$

## 4. Data File to be Used

File name is “project_data.bin”.