

# Parameterized convection, grid-scale clouds and resolution sensitivity in the Community Atmosphere Model

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## This paper describes...

Key Words: Climate models, physical parameterizations, resolution sensitivity

Received ...

#### 1. Introduction

An increasing number of Atmospheric General Circulation Models (AGCMs) are being developed to maximize efficiency on massively parallel systems, permitting regionally-refined highresolution, or even globally high-resolution weather ( $\Delta x \leq 5$ km) and climate ( $\Delta x \le 50$  km) simulations (Satoh *et al.* 2008; Skamarock et al. 2012; Zängl et al. 2014; Harris et al. 2016; Ullrich et al. 2017; Lauritzen et al. 2018). These models are built using unstructured meshes that allows for substantial grid flexibility, but this flexibility is limited by the need for physical parameterizations (physics) that behave consistently as the truncation scale of the model changes with different grid resolutions, referred to as scale-aware physics. The most common approach towards developing scale-aware physics is through the lens of limited area, large-eddy simulations (e.g., Plant and Craig 2008; Arakawa and Wu 2013; Song and Zhang 2018). By subsequently filtering large-eddy solutions to lower-resolution grids, a relationship between first- and higher-order moments (Germano 1992) may be understood and ultimately parameterized as a function of grid resolution. While this approach is likely necessary for developing scale-aware physics, it is not sufficient. The equations of motions have inherent scale dependencies, and the properties of dynamical modes are a function of native grid resolution (Orlanski 1981; Weisman et al. 1997; Pauluis and Garner 2006; Jeevanjee and Romps 2016). Scale-aware physics should also recognize these native grid dependencies.

The sensitivity of the Community Atmosphere Model (CAM; Neale et al. 2012), and its predecessor, the Community Climate Model (CCM) to resolution (resolution refers to horizontal resolution, hereafter) is well documented through convergence studies (Kiehl and Williamson 1991; Williamson et al. 1995; Williamson 2008; Rauscher et al. 2013; Zarzycki et al. 2014; Herrington and Reed 2017). CAM/CCM is a fully supported, well-funded climate model, but despite thirty years of continual model development, there are robust sensitivities to resolution that have persisted in all versions of the model. This study argues that a unifying cause, the inherent scale sensitivities of the underlying dynamical equations, can explain the robust responses to resolution that occur in CAM/CCM, since it is difficult to conceive that inevitable responses to native grid resolution could be ignored in the pursuit of scale-aware physics.

In CAM/CCM, the atmosphere progressively dries with increasing resolution, seen through a reduction in simulated total precipitable water (Kiehl and Williamson 1991; Williamson et al. 1995; Williamson 2008; Rauscher et al. 2013; Zarzycki et al. 2014; Herrington and Reed 2017), which typically, but not always (see Williamson et al. 1995; Zarzycki et al. 2014), coincides with a reduction in cloud cover. Kiehl and Williamson (1991) and Williamson et al. (1995) suggested that the drying of the atmosphere is due to greater magnitude resolved vertical velocities with increasing resolution, with greater subsiding motion increasing the export of dry air from the upper troposphere. This mechanism is consistent with an analysis of moisture budgets in CAM, version 4 (CAM4; Neale et al. 2010) across multiple resolutions (Yang et al. 2014; Herrington and Reed 2017).

It is well known that the magnitude of vertical velocities increase with resolution in atmospheric models. While the cause of this sensitivity has been established for large-eddy simulations (see Jeevanjee 2017, and references therein), only recently has the vertical velocity field in AGCMs and their sensitivity to resolution received attention (Donner et al. 2016; O'Brien et al. 2016), albeit with conflicting explanations (Rauscher et al. 2016; Herrington and Reed 2018). To generalize the relationship between vertical velocity and resolution, let  $\alpha$  refer to the ratio of  $W_0$ , the vertical velocity scale of some reference grid spacing  $\Delta x_0$ , to W, the vertical velocity scale for any  $\Delta x$ . A power law for  $\alpha^{-1}$  in  $\Delta x$  is then,

$$\alpha^{-1} = \frac{W}{W_0} = \left(\frac{\Delta x}{\Delta x_0}\right)^n,\tag{1}$$

where n is the power law exponent.

Rauscher et al. (2016) derive an estimate n=b-1 by combining a scale analysis of the continuity equation with a power law representation  $\Delta x^{2b}$  of the second-order structure function of the horizontal wind. Strictly speaking,  $\Delta x$  here refers to the distance between two points for which the velocity increment is computed in the structure function, but with this distance set to the model grid-spacing. Observations indicate that  $b=\frac{1}{3}$  for scales less than about 1000 km (Cho et al. 1999), which through the Weiner–Khinchin theorem  $-(2b+1)=-\frac{5}{3}$  is equal to the slope of the kinetic energy spectrum, and supported by observations of mesoscale flow (Nastrom and Gage 1985). Rauscher et al. (2016)

argue that the  $-\frac{5}{3}$  slope being common in both observations and models provides an emergent constraint for  $b=\frac{1}{3}$  and  $n=-\frac{2}{3}$ .

In large-eddy simulations, the sensitivity of vertical velocities to resolution is adequately explained by a scale analysis of the dynamical equations (Weisman et al. 1997; Pauluis and Garner 2006; Jeevanjee and Romps 2016). For hydrostatic scales relevant to AGCMs, a scale analysis of the Poisson equation gives  $W \propto$  $D^{-1}$ , where D is the horizontal scale of a buoyancy perturbation driving vertical motion (Herrington and Reed 2018). In CAM aqua-planet simulations, the largest source of buoyancy is from grid-scale cloud formation, whose horizontal extents are set by the effective resolution of the model (i.e., some multiple of  $\Delta x$ ), indicating n = -1 (Herrington and Reed 2018). Herrington and Reed (2017) has shown that the n = -1 scaling does not explain the behavior of CAM4 in a convergence experiment, but follow-up studies (Herrington and Reed 2018; Herrington et al. 2019) indicate that the inadequacy of the n = -1 scaling is not definitive, due to time-truncation errors associated with fixing the physics time-step ( $\Delta t_{phys}$ ) across resolutions in that study.

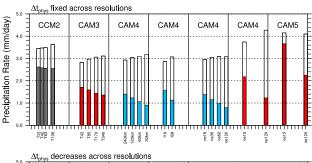
Another robust response of the CAM/CCM lineage to resolution is an increase in grid-scale precipitation rates at the expense of parameterized convective precipitation rates. Gridscale precipitation is formed from grid-scale clouds, but is often referred to in the literature as stratiform precipitaiton and stratiform clouds, and used hereafter. The resolution dependent partitioning between the two different precipitation routines is shown in Figure 1, which is a bar-graph of the climatological, global mean stratiform and convective precipitation rates in prior CAM/CCM convergence studies. The tendency for precipitation rates to shift from the convection scheme to the stratiform scheme with resolution has been documented in other models (Pope and Stratton 2002; Rauscher et al. 2016; Terai et al. 2018), but none have provided a satisfactory explanation for this sensitivity. The studies of Kiehl and Williamson (1991), Williamson et al. (1995) and Williamson (2013) indicate that the practice of reducing  $\Delta t_{phys}$  with resolution should by itself reduce the convective precipitation rates, however Figure 1 (top row) indicates that convergence studies with fixed  $\Delta t_{phys}$  still show a reduction in convective precipitation rates with resolution.

In this study, a convergence experiment using CAM, version 6 (CAM6; https://ncar.github.io/CAM/doc/ build/html/users\_guide/index.html) is carried out and analyzed in detail. It is shown that the resolution sensitivity of vertical velocities are well described with n = -1 in equation (1), provided  $\Delta t_{phys}$  is defined in a way that avoids large truncation errors across resolutions. The reduction in convective precipitation rates with resolution in CAM6 is shown to result from the greater magnitude subsiding motion, creating a more stable atmosphere in which the criterion for parameterized convection occurs less often. The increase in stratiform precipitation rates with resolution is shown to result more directly from the increase in vertical velocities, by increasing moisture fluxes through the cloud base level. The feedback of the resolved vertical motion on the physics indicates that the root cause of resolution sensitivity in CAM arises from the sensitivity of resolved dynamical modes to native grid resolution. Section 2 describes CAM6 and the details of the convergence experiment. Section 3 contains a thorough analysis of the CAM6 simulations and Section 4 provides some discussion and conclusions.

### 2. Methods

#### 2.1. Dynamical Core

This study uses the spectral-element dynamical core option of Community Atmosphere Model (CAM-SE; Dennis *et al.* 2012), coupled with a mass conserving, semi-Lagrangian advection



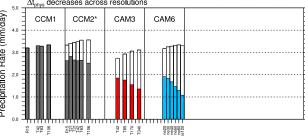


Figure 1. Bar-graph of the convective (solid) and stratiform (white) climatological precipitation rates in prior CAM/CCM convergence studies. Each window contains a single convergence study, with identical x-axis; the approximate grid resolution. Colors indicate the model configuration; January ensemble (black) and aqua-planet configurations with SST profiles QOBS (blue) and CNTL (red) after Neale and Hoskins (2000). Studies included in this figure are Kiehl and Williamson (1991) (CCM1), Williamson et al. (1995) (CCM2), Williamson (2008) (CAM3), Rauscher et al. (2013); Zarzycki et al. (2014); Herrington and Reed (2017) (CAM4), Zarzycki et al. (2014) (CAM5) and this study (CAM6). CCM2\* refers to the modified parameter experiment of Williamson et al. (1995), where parameters vary with resolution to reduce the dependence of cloud fraction on resolution.

method for accelerated multi-tracer transport (CSLAM; Lauritzen et al. 2017), and dry-mass vertical coordinate with comprehensive treatment of moisture and energy (Lauritzen et al. 2018). The dry dynamics are solved using the high-order, momentum, mass and energy conserving spectral element method (Taylor and Fournier 2010), with the elements defined by a cubed-sphere grid. The notation for the horizontal grid resolution is an 'ne' followed by the number of elements making up an edge of one cubed-sphere face, e.g., ne30. Hyper-viscous  $\nabla^4$  explicit numerical dissipation is applied to temperature, dry pressure thickness, rotational and divergent winds (Lauritzen et al. 2018). CSLAM tracer transport uses a finite volume grid constructed from the cubed-sphere of elements, and contains the same degrees of freedom as the dry dynamics.

## 2.2. Physical Parameterizations

The physics are evaluated on the finite-volume CSLAM grid, and the tendencies mapped back to the spectral element grid. The coupled system, referred to as CAM-SE-CSLAM, conserves energy, mass and preserves linear correlations between two reactive species to within machine precision (Herrington et al. 2018). A coarser physics grid, containing  $\frac{5}{9}$  fewer degrees of freedom than the dynamical core grid is also available as part of the CAM-SE-CSLAM package (Herrington et al. 2019). This lower-resolution physics grid is used in this study, but only as a member of a perturbed parameter ensemble and not in the default convergence experiment. The dynamics time-step is subcycled within a longer physics time-step  $\Delta t_{phys}$ , and the temperature and momentum increments from the physics are divided by the number of subcycles and added to the dynamical core at the beginning of each subcycle. The full moisture increment from the physics is applied only at the start of the first subcycle to conserve tracer mass (ftype = 2 option in Lauritzen and Williamson 2019).

The simulations use the CAM6 physics package. The Cloud Layers Unified by Binormals (CLUBB Golaz *et al.* 2002;

Bogenschutz et al. 2013) is an assumed filtered density function (Germano 1992) high-order closure model that handles shallow convection, planetary boundary layer mixing and cloud macrophysics. The macrophysics are coupled with a two-moment bulk cloud microphysics scheme with prognostic precipitation (Gettelman et al. 2015), and microphysics are coupled with a three mode Modular Aerosol Model (Liu et al. 2012). The combined macrophysics/microphysics routines generate stratiform precipitation from stratiform clouds. Deep convection is parameterized using a quasi-equilibrium mass flux scheme (Zhang and McFarlane 1995) and an entraining plume model (referred to as the dilute convective available potential energy, or dilute CAPE hereafter; Raymond and Blyth 1992; Neale et al. 2008) is used as a convective trigger (convection occurs if dilute CAPE  $\geq 70 \text{ J/kg}$ ), and for closing the mass fluxes in the cloud ensemble. The deep convection scheme also parameterizes convective momentum transport (Richter and Rasch 2008).

#### 2.3. Experimental Design

The convergence experiment is performed in an aqua-planet configuration (Neale and Hoskins 2000; Medeiros et al. 2016), an all ocean planet with fixed, zonally symmetric sea surface temperatures modeled after present day Earth (QOBS in Neale and Hoskins 2000). The aqua-planets are in a perpetual equinox, and aerosols are largely absent from the simulations. Each simulation is ran for one simulated year. Six different horizontal grids are used in this study, which are provided in Table 1. In addition to the six simulations used in the convergence experiment, an ensemble of 24 simulations containing different model parameters (e.g., using the lower resolution physics grid) and across different resolutions are ran for one year in order to increase confidence in the assessment of resolution sensitivity in this study. All analyses exclude the first month of the simulations, and are computed on their native grids unless otherwise stated.

The horizontal hyper-viscosity operators  $\nu$  vary with resolution after Herrington *et al.* (2019), also provided in Table 1. The values of  $\nu$  are a factor 2.5 greater for divergence damping and are not shown.  $\Delta t_{phys}$  is chosen to scale with resolution, in proportion to the grid spacing,

$$\Delta t_{phys} = \Delta t_{phys,0} \times \frac{n_{e,0}}{n_e} \ s, \tag{2}$$

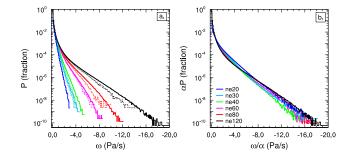
where  $\Delta t_{phys,0}$  is taken to be the standard 1800 s used in CAM-SE-CSLAM for the standard climate resolution,  $n_{e,0}=30$  (equivalent to an average equatorial grid spacing  $\Delta x=111.2$ km). This scaling was chosen to avoid large time-truncation errors in a rising moist bubble test (Appendix A in Herrington *et al.* 2019), and it is understood that this choice of  $\Delta t_{phys}$  will likely lead to greater resolution sensitivity (Williamson 2008). The convective time-scale in the deep convection scheme is fixed at 3600 s in all simulations.

# 3. Results

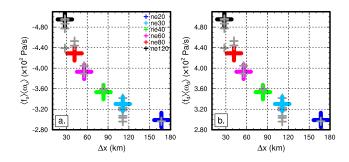
Table 1 provides some globally averaged, climatological metrics for the CAM6 convergence experiment, commonly published in CAM/CCM convergence studies. Total precipitable water, total cloud fraction and deep convective precipitation rate decreases, while stratiform precipitation increases, monotonically with resolution (also shown in Figure 1). Resolution sensitivity in CAM6 is similar to all prior versions of the model.

## 3.1. Vertical Velocities and Resolution

The probability density function (PDF) of negative, or upward vertical pressure velocities  $\omega$  in the aqua-planets is provided in



**Figure 2.** Probability density distribution of the upward vertical pressure velocities  $\omega$  computed everywhere in the model from six-hourly output over the entirety of the year-long simulations. (a) Values on their native grid (solid) and values remapped to the ne20 grid (dotted), (b) values on their native grid, scaled to the ne120 resolution using a power law exponent n=-1 in equation 1.



**Figure 3.** Components of the climatological, global mean vertical pressure velocity, (a)  $\langle f_u \rangle \langle \omega_u \rangle$  and (b)  $\langle f_d \rangle \langle \omega_d \rangle$ . Grey crosses are for the 24 member perturbed parameter ensemble.

Figure 2a. The magnitude of upward  $\omega$  increases monotonically with resolution, with positive, or downward  $\omega$  behaving similarly (not shown). This monotonic increase in the magnitude of  $\omega$  is evident even after remapping all the model output to a common grid (ne20; dotted curves in Figure 2a).

The PDF's may be scaled to the highest-resolution resolution grid through  $P(\omega)_s = \alpha P(\omega/\alpha)$ , where  $\alpha$  is the scale factor from equation 1, and setting  $\Delta x_0$  to the ne120 grid-spacing. Figure 2b shows the scaled PDF's for power law exponent n=-1 in equation 1. The scaled PDF's all collapse onto the high-resolution reference, indicating that the power-law exponent n=-1 explains to first-order the variation in vertical velocity with resolution in the aqua-planet simulations.

Changes to the vertical velocity field can be further understood through decomposing the mass weighted vertical mean  $\langle \omega \rangle$  into upward and downward components,

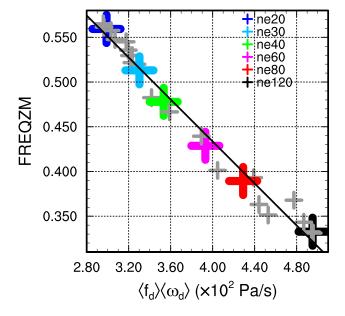
$$\langle \omega \rangle = \langle f_u \rangle \langle \omega_u \rangle + \langle f_d \rangle \langle \omega_d \rangle,$$
 (3)

where  $\langle f_x \rangle$  and  $\langle \omega_x \rangle$  refers to the vertical mass fraction  $\left(\frac{\int dp_x}{\int dp}\right)$  and the x component of the mass weighted vertical mean of  $\omega$   $\left(\frac{\int \omega_x dp_x}{\int dp_x}\right)$ , respectively, subscript u refers to upward motion and d, downward motion.

The global mean, climatological components  $\langle f_u \rangle \langle \omega_u \rangle$  and  $\langle f_d \rangle \langle \omega_d \rangle$  are provided in Figure 3a,b for the aqua-planet simulations. The magnitude of both  $\langle f_u \rangle \langle \omega_u \rangle$  and  $\langle f_d \rangle \langle \omega_d \rangle$  increase monotonically with resolution, and are equal and opposite, which is a requirement of mass conservation in the model and a convenient check of the calculation. While  $\langle f_d \rangle$  is about 25% larger than  $\langle f_u \rangle$  in all simulations, the vertical mass fractions vary by only few percent with resolution, and so the monotonic behavior of  $\langle f_x \rangle \langle \omega_x \rangle$  with resolution is primarily from variations in  $\langle \omega_x \rangle$  (not shown).

Variable ne20ne30ne40ne60ne80ne120 $\Delta x$  (km) 166.8 111.2 83.4 55.6 41.7 27.8  $1.5\times10^{15}$  $4.0\times10^{13}$  $1.5 \times 10^{13}$  $4.0 \times 10^{12}$  $4.0 \times 10^{14}$  $1.5\times10^{14}$  $\nu \left(m^4/s\right)$ 2700 1800 1350 900 675 450  $\Delta t_{phys}$  (s) Total Cloud Fraction 0.844 0.835 0.824 0.810 0.804 0.800 Total Precipitable Water (mm) 23.31 23.01 22.25 22.62 21.93 21.72 1.91 1.47 Convective Precipitation (mm/day) 1.83 1.68 1.29 1.08 Stratiform Precipitation (mm/day) 1.26 2.05 2.22

Table 1. Experimental design and global mean climatologies.  $\Delta x$  refers to the average equatorial grid-spacing.



**Figure 4.** Scatter plot of global mean, climatological  $\langle f_d \rangle \langle \omega_d \rangle$  and FREQZM, and the fitted linear regression which has a Pearson's R-value = 0.99, using all 27 simulations. Grey crosses are for the 24 member perturbed parameter ensemble runs

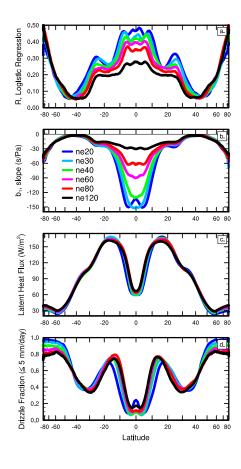
# 3.2. Vertical Velocities and Deep Convective Precipitation

The large increase in magnitude of the upward and downward vertical velocities with resolution may be expected to impact the behavior of other model components. Curiously, there is an excellent negative correlation (Pearson's R-value = 0.99, N = 27) between the global mean, climatological  $\langle f_d \rangle \langle \omega_d \rangle$  and a measure of the activity of the Zhang and McFarlane (1995) deep convection scheme (referred to as the *ZM scheme* hereafter), global mean, climatological FREQZM (Figure 4). At any grid-point and timestep, FREQZM is a binary variable: one if the ZM scheme is active, zero if it is not. Time mean FREQZM is therefore the fraction of the model time that the ZM scheme is triggered, i.e., dilute CAPE exceeds  $\geq 70$  J/kg. The regression indicates that model simulations with greater subsidence also have less convective activity.

To further understand this relationship, a logistic regression between  $\langle f_d \rangle \langle \omega_d \rangle$  and FREQZM is performed for each grid column within each of the simulations. Logistic regression uses an iterative method to fit a continuous variable predictor, x to a binary predictand p using the exponential (Wilks 2011),

$$p = \frac{exp[b_0 + b_1 x]}{1 + exp[b_0 + b_1 x]},\tag{4}$$

where  $b_0$  and  $b_1$  are the shape parameters of the exponential. The predictor is the instantaneous  $\langle f_d \rangle \langle \omega_d \rangle$  of a grid column, and the predictand the binary FREQZM. The assumption is then that subsidence is the independent variable, which the authors believe is reasonable since the environment of subsiding regions is generally more stable than its surroundings, and the ZM scheme is modulated by a stability calculation, dilute CAPE. Grid column



**Figure 5.** Zonal mean (a) R-values and (b) sensitivity parameter in the logistic regression, (c) climatological surface latent heat fluxes and (d) drizzle fraction.

regressions that are statistically significant at the 95% level using a log-likelihood test (Wilks 2011) are retained for analysis. Since the aqua-planets have zonally symmetric boundary conditions, there is a zonally varying structure in the goodness of fit (R-value) and parameter  $b_1$  (hereafter referred to as the sensitivity parameter; Figure 5a,b).

The zonal mean R-values indicate the greatest goodness of fit in the  $\pm 10^\circ$  latitude band, hereafter referred to as the deep tropics. In this region, the sensitivity parameter is large and negative (Figure 5b), consistent with the idea that subsiding motion stabilizes the environment and actively depresses dilute CAPE and the activity of the ZM scheme in the simulations. The sensitivity parameter becomes less negative in the deep tropics with resolution, likely due to the greater magnitude  $\langle f_d \rangle \langle \omega_d \rangle$  with resolution, which requires a lower sensitivity parameter to predict the binary FREQZM. The R-values generally decrease with resolution indicating that there is degradation in the relationship with resolution.

Table 2 shows the fractional contribution of the deep tropics to the climatological, global mean change in convective precipitation with resolution. This is a reflection of changes in the partitioning of the Intertropical Convergence Zone (ITCZ) from convective to stratiform precipitation with resolution. The table indicates that a majority (60-70%) of the reduction in convective precipitation

Table 2. Fractional contribution of latitude bands  $\pm 10^{\circ}$  and  $\pm 15^{\circ}$  to changes in global mean precipitation with resolution. The grid headers refer to differences with respect to the next lowest grid resolution, e.g., ne30 = ne30 - ne20, ne40 = ne40 - ne30, etc... All differences are computed after remapping the data to the ne20 grid.

Variable	ne30	ne40	ne60	ne80	ne120
$\pm 10^{\circ}$ (17.6% of global area)					
Convective Precipitation	-0.58	0.62	0.66	0.72	0.70
Stratiform Precipitation	0.55	0.63	0.69	0.67	0.41
$\pm 15^{\circ}$ (25.8% of global area)					
Convective Precipitation	0.22	0.75	0.73	0.79	0.72
Stratiform Precipitation	0.46	0.64	0.71	0.70	0.49

with resolution is from changes within the deep tropics (except in going from ne20 to ne30, where convective precipitation rates increase due to a wide double-ITCZ in the ne20 run that spans well outside of  $\pm 10^\circ$  latitude, but which is contained within  $\pm 10^\circ$  latitude in the ne30 run). Expanding the latitude boundaries marginally to  $\pm 15^\circ$ , roughly 75% of the changes in convective precipitation with resolution occurs in this region (again, ignoring ne30-ne20; Table 2). Taken together, the region with the largest change in convective precipitation with resolution is also the region where the logistic regression indicates that subsiding motion is most skillful at depressing the activity of the convection scheme.

To estimate the dilute CAPE values associated with subsiding motion in the deep tropics, temperature and moisture profiles are conditionally sampled depending on whether  $\langle\omega\rangle$  is positive or negative, indicating predominantly subsiding or ascending grid columns. The time mean temperature and moisture profiles of subsiding and ascending regions are then used to compute the dilute CAPE used in the ZM scheme, offline. Figure 6a shows the dilute CAPE values associated with mean conditions for ascending, descending and all grid columns in the deep tropics, with resolution. Ascending regions are associated with larger values of dilute CAPE (> 180 J/kg) relative to subsiding regions (< 110 J/kg), and the dilute CAPE value computed for mean conditions over the entire deep tropics decreases monotonically with resolution, consistent with the reduction in convective activity with resolution.

The space-time weights associated with ascending and descending grid columns in the deep tropics vary drastically with resolution (Figure 6b). The subsiding (ascending) space-time weights change from 0.32 (0.68) at ne20, monotonically increasing (decreasing) with resolution to 0.51 (0.49) in ne120. The increasing occurrence of stable, subsiding grid columns with resolution results in a reduction in dilute CAPE for the entire deep tropics, verified by the similar values derived through taking the weighted sum of the ascending/descending CAPE values (grey crosses in Figure 6a).

Figure 7 shows the time mean temperature and specific humidity profiles of subsiding grid cells in the deep tropics, expressed as anomalies from the mean temperature and specific humidity of the entire deep tropics. The mean profiles of subsiding regions have an anomalous warming layer in the 600-800 hPa layer and an anomalous moisture deficit throughout the entire column. This warming and drying patterns is consistent with the effects of subsidence, whose motion adiabatically warms the environment while simultaneously advecting drier conditions aloft downward. Both warming and drying the environment oppose the growth of dilute CAPE through reducing parcel buoyancy; warming the environment relative to the temperature of rising air parcels reduces parcel buoyancy (Zhang 2002), and mixing drier environmental air into rising air parcels reduces the moisture available to warm parcels through latent heating (Raymond and Blyth 1992).

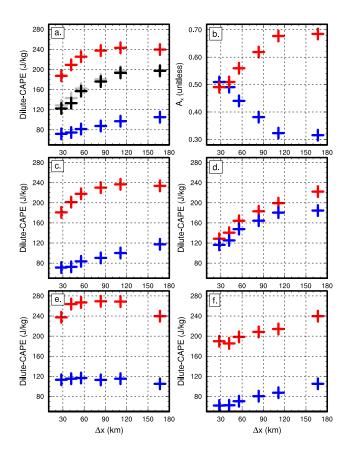
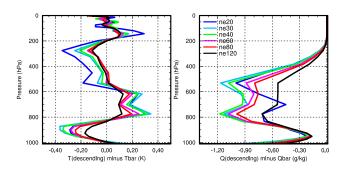


Figure 6. (a) Dilute CAPE computed from time mean temperature and moisture profiles of ascending (red), subsiding (blue) and all grid columns (black) in the deep tropics  $(\pm 10^{\circ})$  latitude), and (b) space-time weights of ascending (red) and descending (blue) grid columns in the deep tropics. (c) Dilute CAPE computed for ascending/descending grid columns, but using the mean temperature profile for the entire deep tropics, (d) Same as (c) but using the mean moisture profile for the entire deep tropics. (e) Same as (c) but using the fixing the temperature to the ne20 profiles, and (f) same as (d) but fixing the moisture to the ne20 profiles. Grey crosses in (a) are dilute CAPE derived from the sum of the products of space-time weights with the dilute CAPE values of ascending/descending grid columns.



**Figure 7.** Time mean (a) temperature and (b) specific humidity profiles of subsiding grid cells in the deep tropics ( $\pm 10^{\circ}$  latitude) in the convergence experiment, presented as anomalies from the mean temperature and specific humidity of the entire deep tropics in each simulation.

An experiment is carried out to unravel the contributions of warming and drying to dilute CAPE values of subsiding grid columns. Through recomputing dilute CAPE using the mean specific humidity of subsiding regions in the deep tropics, but setting the temperature profile to the mean profile for the entire deep tropics, one can isolate the role of moisture in subsiding regions on the dilute CAPE calculation. Doing the reverse then isolates the role of temperature in subsiding regions on dilute CAPE.

Figure 6c shows the influence of moisture on the dilute CAPE of subsiding/ascending regions in the deep tropics, and Figure 6d the influence of temperature. To first order, the

large spread in dilute CAPE between subsiding and ascending regions is due to the difference in moisture in subsiding and ascending regions, while temperature differences between the two regimes has a smaller, second order influence on the spread of dilute CAPE. A large spread in dilute CAPE between ascending/descending regions is important for maintaining a strong relationship between subsidence and the activity of the ZM scheme. If the model operated more similar to the fixed moisture experiment (Figure 6d), then subsidence would be associated with larger values of dilute CAPE, and less effective at depressing dilute CAPE below the threshold for convection.

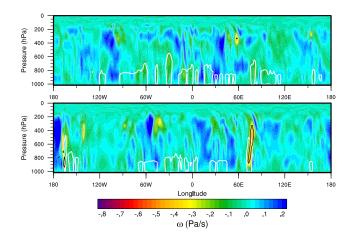
The large spread in dilute CAPE of subsiding/ascending regions multiplied by the large change in space-time fraction of subsiding/ascending are responsible for about half of the change in the dilute CAPE computed from the mean temperature and moisture profiles over the entire deep tropics. There is a systematic offset of dilute CAPE values in both subsiding/ascending regions with resolution (Figure 6a) that accounts for another half of the resolution sensitivity. To understand whether moisture or temperature changes with resolution are responsible for this behavior, a similar experiment is performed in which dilute CAPE is recomputed at all resolutions, but fixing the moisture profiles to the lowest resolution ne20 profile, and then again through fixing the temperature profile to the ne20profile. Figure 6e,d shows the influence of the change of resolution on moisture, and the change due to temperature, and indicates that temperature changes with resolution are primarily responsible for the other half of the decrease in dilute CAPE in deep tropics with resolution.

# 3.3. Vertical Velocities and Shallow Convective Precipitation

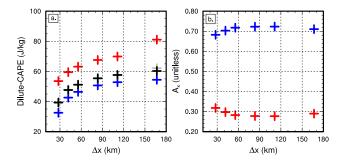
AGCMs are known to suffer from a drizzle bias, producing too much light rain relative to observations (Dai 2006). Figure 5d shows the fraction of ZM precipitation  $\leq 5$  mm/day in the simulations, which stubbornly persists at 70% in the region between  $10^{\circ}-20^{\circ}$  latitude irrespective of resolution. Focusing in on this drizzle region, Figure 8 shows two snapshots of  $\omega$  in the longitude-pressure plane at  $\sim 18^{\circ}$  latitude in the ne30 simulations, overlain by an isoline delineating where the ZM mass fluxes are quite active. The ZM mass fluxes typically only extend up to about the 800 hPa level, which often occurs with abundant subsiding motion aloft.

In drizzling regions, there are opposing influences on dilute CAPE, a global maximum in latent heat fluxes (Figure 5c), influencing the thermodynamic state of boundary layer parcels and increasing dilute CAPE from below (Zhang 2002), and subsidence, which opposes dilute CAPE from above. It seems plausible that these two opposing influences on stability translates into small values of dilute CAPE, but still large enough to trigger the ZM scheme. The ZM scheme, properly a deep convection scheme, is operating as a shallow convection scheme in the  $10^{\circ}-20^{\circ}$  latitude region, and produces drizzle.

Figure 9 shows the dilute CAPE values computed from mean temperature and moisture of subsiding and ascending regions in the  $\pm (10^{\circ} - 30^{\circ})$  latitude bands, hereafter referred to as the subtropics. The spread in dilute CAPE between ascending and descending grid columns is much smaller than for the deep tropics (Figure 6a), and the dilute CAPE computed from the mean temperature and moisture over the entire subtropics varies much less with resolution compared with the deep tropics ( $\sim$  20 J/kg across all resolutions, compared with  $\sim$  80 J/kg across resolutions in the deep tropics). Also unlike the deep tropics, the reduction in dilute CAPE with resolution is not from an increase in occurrence of subsiding grid columns, since the spacetime weights of ascending/descending motion are more-or-less invariant with resolution (Figure 9b). The decline in dilute CAPE



**Figure 8.** (a,b) Two snapshots of  $\omega$  for a longitude-pressure transect at  $\sim 18^\circ$  latitude in the ne30 simulation, overlain by the  $0.0075~{\rm kg/m^2/s}$  contour of the ZM mass flux (white) delineating the region where the ZM scheme is active, and the 15 K/day contour of the total physics tendencies (black), indicating stratiform cloud formation.



**Figure 9.** (a) Dilute CAPE computed from time mean temperature and moisture profiles of ascending (red), subsiding (blue) and all grid columns (black) in the subtropics ( $\pm$  ( $10^{\circ}-30^{\circ}$ ) latitude bands), and (b) space-time weights of ascending (red) and descending (blue) grid columns in the subtropics. Grey crosses in (a) are dilute CAPE derived from the sum of the products of space-time weights with the dilute CAPE values of ascending/descending grid columns.

with resolution in the subtropics is a result of a greater magnitude of downward motion (not shown), but this sensitivity is not sufficient to suppress the occurrence of drizzling in the subtropics.

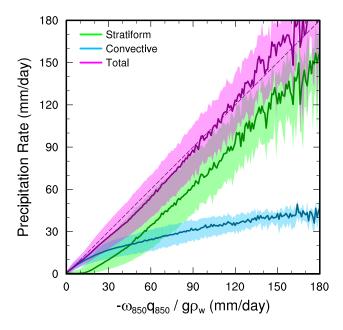
# 3.4. Vertical Velocities and Stratiform Precipitation

In contrast to the impact of vertical motion on the ZM scheme, the stratiform scheme is more intuitively connected to vertical velocities. Rauscher *et al.* (2016) proposed that total precipitation rates in models (R, sum of convective and stratiform precipitation rates) are determined by the upward moisture flux through cloud base,

$$R \approx -\frac{1}{q\rho_w}\omega^+ q^+ \tag{5}$$

where  $\omega^+$  and  $q^+$  are the upward  $\omega$  and specific humidity q at cloud base, respectively, and  $\rho_w$  is the density of rainwater.

Through approximating the cloud base as the 850 hPa level, equation 5 was found to be a good approximation to total precipitation rates in a regional model (Rauscher et al. 2016) and in the CAM-SE AGCM with CAM5 physics (O'Brien et al. 2016). Figure 10 shows the median convective, stratiform and total precipitation rates conditioned on the 850 hPa moisture flux using 1 mm/day bins, in the CAM6 ne30 simulation. The figure shows that the moisture flux is a good approximation to the median total precipitation rate, and that the increase in precipitation rates with moisture flux is primarily from the stratiform precipitation scheme.



**Figure 10.** Precipitation rates vs. upward moisture flux at the 850 hPa level. Solid lines refer to median stratiform (green), convective (blue) and total (magenta) precipitation rates conditional on bins of the moisture flux, and shaded regions refer to the conditional interquartile ranges.

The increase in stratiform precipitation rates with resolution is likely due to the increase in magnitude of  $\omega$  with resolution. To unravel the contributions of  $\omega$  and q at the 850 hPa level,  $\omega_{850}$  and  $q_{850}$  to the increase in climatological stratiform precipitation with resolution, the area averaged, climatological stratiform precipitation rate  $\overline{R_s}$  can be decomposed into a double sum of the product of the time mean magnitude  $M_s$  and time mean spatial frequency  $f_s$ , over  $\omega_{850}$  and  $q_{850}$  space,

$$\overline{R_s} = \sum_{i} \sum_{j} f_s \left(\omega_i, q_j\right) M_s \left(\omega_i, q_j\right) \tag{6}$$

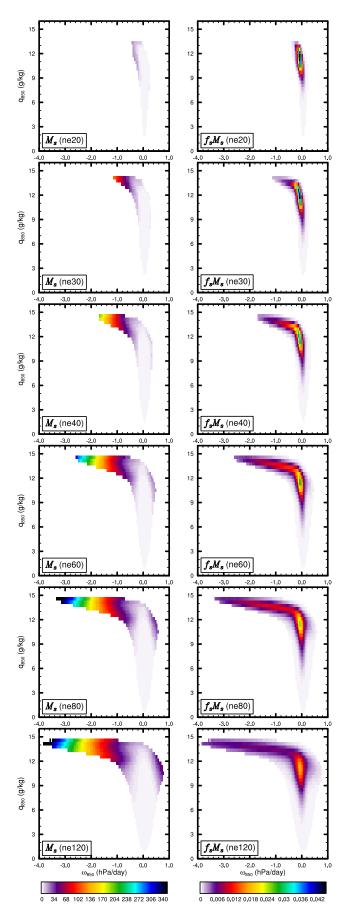
after Terai *et al.* (2018), and the subscript 850 is dropped from  $\omega$  and q for brevity.

Table 2 indicates that the  $\pm 15^{\circ}$  latitude band accounts for most of the change in global mean stratiform precipitation with resolution, and so  $\overline{R_s}$  is defined as the mean over the  $\pm 15^{\circ}$ latitude region. Figure 11 is a plot of the terms  $M_s\left(\omega_i,q_i\right)$  and  $f_s(\omega_i, q_i) M_s(\omega_i, q_i)$  for all resolutions, in the  $\pm 15^{\circ}$  latitude region. The plots are computed using 6-hourly instantaneous output of  $\omega_i$  and  $q_j$ , with 0.05 Pa/s and 0.4 g/kg bins. Values in  $(\omega_i, q_j)$  space are only shown for bins with  $f_s \ge$  $1 \times 10^{-5}$ , a reasonable cut-off to a bins' contribution to  $\overline{R_s}$ . The  $M_s$  plots show that larger magnitude  $\omega_{850}$  correspond to larger magnitude time-mean stratiform precipitation rates, and these larger precipitation rates are associated with larger spatial frequencies  $f_s(\omega_i, q_j)$  at higher resolutions. These large magnitude stratiform precipitation rates increasingly contribute to the increase in  $\overline{R_s}$  with resolution, as seen by the expansion of  $f_s\left(\omega_i,q_j\right)M_s\left(\omega_i,q_j\right)$  to include larger magnitude  $\omega_{850}$ . Changes to the time mean spatial frequency of the  $q_{850}$  field contributes comparatively much less to changes in stratiform precipitation.

## 4. Conclusions

#### Acknowledgement

Funding support for this work was in part provided by the U.S. Department of Energy Office of Science (DE-SC0019459) and the National Science Foundation (AGS1648629). Computing and



**Figure 11.** Decomposition of the climatological stratiform precipitation rates, averaged over the  $\pm 15^{\circ}$  latitude band into  $\omega_{850}$  and  $q_{850}$  environmental conditions. Left column shows the time mean magnitude term  $M\left(\omega_{i},q_{j}\right)$  and the right column is the magnitude term multiplied by the space-time frequency term  $f\left(\omega_{i},q_{j}\right)\times M\left(\omega_{i},q_{j}\right)$ . Integrals over  $f\times M$  gives the climatological, area averaged stratiform precipitation rate. Panel labels denote the grid resolution of the model run.

data storage resources, including the Cheyenne supercomputer (doi:10.5065/D6RX99HX), were provided by the Computational and Information Systems Laboratory (CISL) at the National Center for Atmospheric Research (NCAR). Herrington would like to thank Dr. Sultan Hameed for his assistance with the statistical analysis used in this study.

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