Does Location and Amount of Weight Affect Distance a Paper Airplane Flies?

Adam Rockett, Cody Zimmerman, Eva Beyer

June 2019

Introduction

For our experiment, we wanted to test whether or not weight location and amount of weight affected the distance a paper airplane could fly. Our experiment is a two factor factorial design where we had three levels for each of our two factors, which were the amount of paper clips on the plane during flight, and the location of those paper clips on the plane. Our levels for the amount of paper clips were one, three, and five paper clips and were put on the very front of the paper airplane, the back of the wings, and the body in the back of the airplane.

We collected our data on the lawn of the Communications Facility on Wednesday, May 29th, where we had a tape measure on the ground and threw the paper airplanes at each level three times. We tried to do our experiment relatively quickly so the wind wouldn't change direction suddenly and kept the same person throwing the airplane to keep the experiment consistent. However, there is always some human error involved no matter what. The data we collected is in Figure 1 given below.

	Number of Paper Clips		
Location of Paper Clips	1	3	5
Front	126, 132, 147	128, 129, 153	114, 130, 129
Back Body	147, 121, 121	122, 175, 122	228, 156, 215
Back Wings	193, 153, 147	210, 126, 131	102, 89, 140

Figure 1: Table of Data Collected.

Analysis of Data

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Analysis of Variance Table

Response: Distance
                             Df  Sum Sq Mean Sq F value   Pr(>F)
number.factor                 2    14.3     7.1  0.0098 0.990275
location.factor               2  2667.6  1333.8  1.8245 0.189901
number.factor:location.factor  4 13900.6  3475.1  4.7535 0.008554 **
Residuals                     18 13159.3   731.1
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Signif. codes:  0 ‘***’ 0.001 ‘**’ 0.01 ‘*’ 0.05 ‘.’ 0.1 ‘ ’ 1
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Figure 2: ANOVA Table for Data Collected.

From the ANOVA table seen in Figure 2, we can see that the only significant factor from our data is the interaction term for the two factors. We can also

tell that there is significant interaction from the plot given in Figure 3 since the lines for the different factor levels are not parallel. Therefore, we will keep all of the factors in the model when doing further analysis of the data.

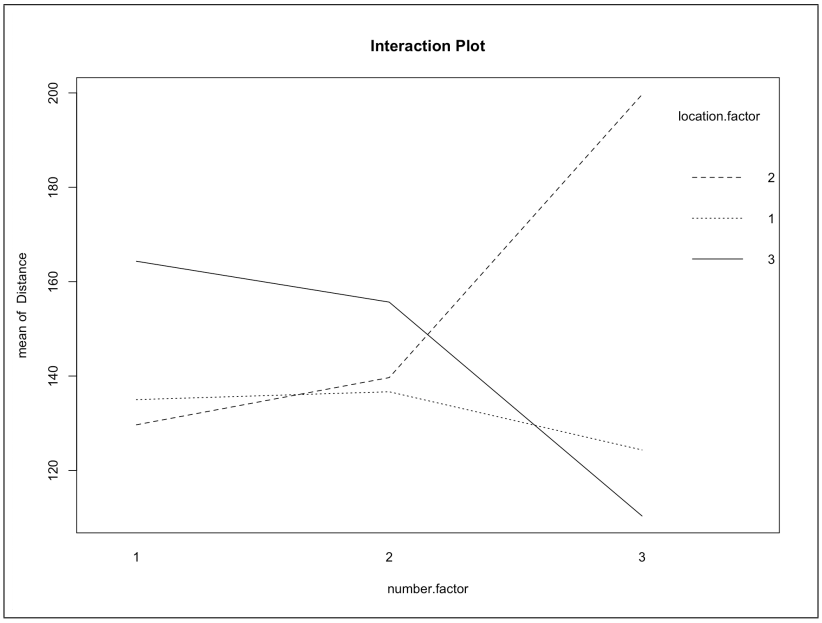

Figure 3: Interaction Plot.

Since our experiment has a two-factorial design, we will do a Tukey and a Fisher's LSD pairwise comparison test to see if there is a difference in mean flight distance at the different levels for our location factor.

Now, for the Tukey pairwise comparison with $FWER = 0.05$, we will use the following test statistic,

$$\begin{aligned} q_{FWER,a,edf}\sqrt{\frac{MSE}{bn}} &= q_{0.05,3,18}\sqrt{\frac{731.1}{9}} \\ &= (3.609304)(9.012954) \\ &= 32.53049 \end{aligned}$$

Then, finding the treatment means for all of the different levels of the location

factor, we get,

$$\begin{aligned}\bar{y}_{1\cdot} &= 143\\ \bar{y}_{2\cdot} &= 144\\ \bar{y}_{3\cdot} &= 144.778.\end{aligned}$$

Then taking all of the possible combination differences, we find

$$\begin{aligned}\bar{y}_{1\cdot} - \bar{y}_{2\cdot} &= -1\\ \bar{y}_{1\cdot} - \bar{y}_{3\cdot} &= -1.778\\ \bar{y}_{2\cdot} - \bar{y}_{3\cdot} &= -0.7778.\end{aligned}$$

Since the absolute values of all of these differences are less than the critical value from above, we can conclude that there are no significant differences in the mean flight distance for all of the different levels of the location factor.

To check and see if this is actually the case, we will also perform Fisher's LSD pairwise comparisons, at $\alpha = 0.05$, on the locations factor to confirm the test above. For this comparison, we will have

$$\begin{aligned}t_{\alpha/2,edf}\sqrt{\frac{2MSE}{bn}} &= t_{0.025,18}\sqrt{\frac{2(731.1}{9}}\\ &= (2.100922)(12.74624)\\ &= 26.778.\end{aligned}$$

Once again, none of the difference between the factor levels are greater than this critical value, so there is no significant difference in mean flight distance between the three different levels of location. A graphical representation of this is given in Figure 4 below.

Based on the data we collected and the interaction plot given above, the best combination of our two factors would be the third level of the weight, which was having five paper clips on the plane, and the second level of location, which was having the paper clips on the back of the body of the plane.This combination had a mean distance traveled of 199.667, which flew 35.33 inches further than the next highest treatment-factor combination.

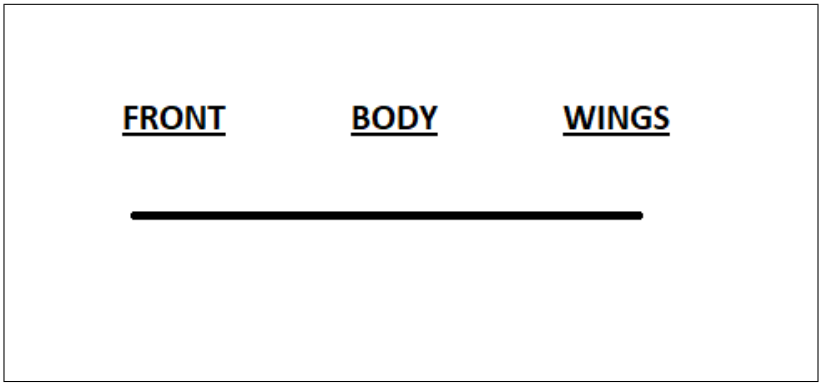

Figure 4: Graphical Representation of Significant Differences of Location Factor.

Residuals Analysis

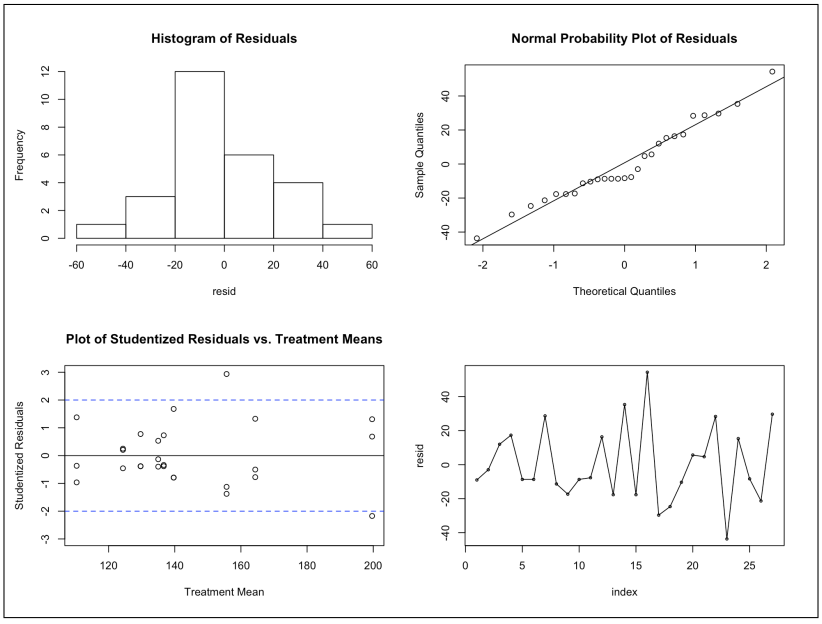

Figure 5: Residual Plots.

Looking at the histogram of the residuals and the normal probability plot from Figure 5, nothing seems to be violating the assumption of normality. We also ran an Anderson-Darling Test on the residuals, which returned a p-value of 0.1462, which is large enough for us to no reject the null hypothesis of normality. Also, the plot of studentized residuals vs. treatment means appear to be randomly

scattered throughout, implying constant variance throughout. Therefore, we can conclude that none of the assumptions for the model are violated.

Conclusion

The distance traveled by the planes may have been influenced by a few outside factors including the human error of consistently throwing the planes (i.e. initial angle, speed, height of plane) and any wind from collecting the data outdoors. If we were to redo the experiment, we would prefer an indoor location, as well as some type of automated plane launcher in order to stabilize the trajectory and force at which the plane is thrown. Nevertheless it is clear from the interaction plot that with more weight on the body of the plane, the average distance the plane travels will increase. It was interesting to learn that varying the location of the weight as well as the weight itself impacted the travel distance.