Renal/Urinary Equations

Week 1

Functions of the Kidney and Physiology of Glomerular Filtration

Net filtration pressure

$$\text{NFP} = K_f \left(P_{\text{GC}} - P_{\text{BS}} - \pi_{\text{GC}}\right)$$

Symbol	Meaning	Units	Description
NFP	Net Filtration Pressure	mmHg	Overall pressure driving fluid out of glomerular capillaries into Bowman's space
K_f	Filtration coefficient	mL/min/mmHg	Reflects permeability × surface area of the glomerular filtration barrier
P_{GC}	Glomerular capillary hydrostatic pressure	mmHg	Pushes fluid out of the capillaries into Bowman's space
P_{BS}	Bowman's space hydrostatic pressure	mmHg	Pushes fluid back into the capillaries (opposes filtration)
π_{GC}	Glomerular capillary oncotic pressure	mmHg	Osmotic pull of plasma proteins back into the capillaries (opposes filtration)

Glomerular filtration rate

$$C_x = \frac{U_x \cdot V}{P_x}$$

Symbol	Meaning	Units	Description
C_x	Clearance of substance x	mL/min	Volume of plasma completely cleared of substance x per unit time
U_x	Urine concentration of substance x	mg/mL (or mEq/L)	Concentration of substance x in the urine
V	Urine flow rate	mL/min	Rate at which urine is produced
P_x	Plasma concentration of substance x	mg/mL (or mEq/L)	Concentration of substance x in the plasma

Renal Circulation & its Relationship to Glomerular Tubular Function

Renal plasma flow, renal blood flow, and filtration fraction

$$\text{RPF} = \text{Clearance}_{\text{PAH}} = \frac{U_{\text{PAH}} \cdot V}{P_{\text{PAH}}}$$

$$\text{RBF} = \frac{\text{RPF}}{1 - \text{Hematocrit}}$$

$$\text{FF} = \frac{\text{GFR}}{\text{RPF}}$$

Symbol	Meaning	Units	Description
RPF	Renal Plasma Flow	mL/min	Volume of plasma delivered to the kidneys per unit time; measured using PAH clearance since PAH is both filtered and secreted (nearly 100% cleared from plasma).
U_{PAH}	Urine concentration of PAH	mg/mL (or mEq/L)	Amount of para-aminohippuric acid in urine.
P_{PAH}	Plasma concentration of PAH	mg/mL (or mEq/L)	Amount of para-aminohippuric acid in plasma.
V	Urine flow rate	mL/min	Rate of urine production.
RBF	Renal Blood Flow	mL/min	Total blood flow through the kidneys, including both plasma and cells.
Hematocrit	Fraction of blood volume occupied by RBCs	Unitless (decimal)	Used to convert between plasma flow and total blood flow.
FF	Filtration Fraction	Unitless	Fraction of plasma that is filtered through the glomerulus.
GFR	Glomerular Filtration Rate	mL/min	Volume of plasma filtered into Bowman’s space per unit time.

Solute & Water Transport Along the Nephron

$$\text{Serum osmolality} = 2 \times [Na^+]_{(\text{mEq/L})} + \frac{\text{Glucose}_{(\text{mg/dL})}}{18} + \frac{\text{BUN}_{(\text{mg/dL})}}{2.8}$$

$$\text{Serum tonicity (normal glucose)} \approx 2 \times [Na^+]$$

$$\text{Serum tonicity (hyperglycemia)} \approx 2 \times [Na^+] + \frac{\text{Glucose}_{(\text{mg/dL})}}{18}$$

Symbol	Meaning	Units	Description
Serum osmolality	Total calculated serum osmolality	mOsm/kg	Reflects total concentration of solutes (effective + ineffective) in plasma, influencing overall osmotic balance.
Serum tonicity	Effective osmolality	mOsm/kg	Measures osmoles that influence water movement between ICF and ECF (mainly Na^+ and glucose).
Na	Sodium concentration	mEq/L	Primary determinant of extracellular fluid osmolality; doubled because sodium is balanced by accompanying anions (Cl^-, HCO_3^-).
Glucose	Serum glucose concentration	mg/dL	Adds to effective osmolality; becomes significant during hyperglycemia.
BUN	Blood Urea Nitrogen	mg/dL	Contributes to total osmolality but is ineffective (freely crosses cell membranes, so it doesn't cause water shifts).
18	Conversion factor for glucose	—	Converts glucose from mg/dL to mmol/L (divide by 18).
2.8	Conversion factor for BUN	—	Converts BUN from mg/dL to mmol/L (divide by 2.8).

$$\text{Daily osmotic excretion} = U_{\text{osm}} \times U_{\text{vol}}$$

$$\text{FE}_{\text{Na}} = \frac{\text{Na}^+_{\text{excreted}}}{\text{Na}^+_{\text{filtered}}} = \frac{V \times U_{\text{Na}}}{\text{GFR} \times P_{\text{Na}}} = \frac{P_{\text{Cr}} \times U_{\text{Na}}}{U_{\text{Cr}} \times P_{\text{Na}}} \times 100$$

Symbol	Meaning	Units	Description
FeNa	Fractional excretion of sodium	%	Fraction of filtered sodium that is excreted in urine; helps differentiate prerenal vs intrinsic renal failure.
V	Urine flow rate	mL/min	Rate of urine formation.
U_{Na}	Urine sodium concentration	mEq/L	Amount of sodium in urine.
P_{Na}	Plasma sodium concentration	mEq/L	Amount of sodium in plasma.
U_{Cr}	Urine creatinine concentration	mg/dL	Used to normalize excretion to filtration.
P_{Cr}	Plasma creatinine concentration	mg/dL	Used as a filtration reference (since creatinine is freely filtered).
Na^+ excreted	Sodium excretion rate	mEq/min	Amount of sodium removed via urine per unit time.
Na^+ filtered	Sodium filtered load	mEq/min	Amount of sodium entering Bowman's space per unit time.
GFR	Glomerular Filtration Rate	mL/min	Volume of plasma filtered into Bowman's space per unit time.

Body Fluids/Mechanisms of Urine Concentration & Dilution

Free water clearance

$$\dot{V} = C_{\text{osm}} + C_{H_2O}$$

$$C_{\text{osm}} = \frac{U_{\text{osm}} \times \dot{V}}{P_{\text{osm}}}$$

Symbol	Meaning	Units	Description
V	Urine flow rate	mL/min	The total volume of urine produced per unit time.
C_{H2O}	Free water clearance	mL/min	The volume of plasma cleared of solute-free water (pure H_2O) per unit time.
C_{osm}	Osmolar clearance	mL/min	The volume of plasma cleared of all solutes per unit time.
U_{osm}	Urine osmolality	mOsm/L	Total solute concentration in urine
P_{osm}	Plasma osmolality	mOsm/L	Total solute concentration in plasma