Investigating a Bouncing Basketball and the Effect of Air Loss on its Model.

SL IB Math Internal Assessment

By: Adam Sadek

<u>Table of Contents</u>

Cover page……..……………….…...………………...……………………..1

Table of Contents…………………..…………………...………………………...2

Introduction to the Sport……………..…...………………...……………………..3-4

Why I Chose This Exploration Topic……………………...………………...4-5

How I Will Go About It…………………………………...…………….....5-6

Important Measurements………………………………………...………………7

The Process of Graphing the Basketball's Bounce…………………...……………..7

Checking the psi of the Basketball………...………………………..…...………...8

Raw Data Collection………………………………………………...……..……...8

Graphing the Models…………………………………………………...………..9-10

Exploring the Mathematics Behind the Effect of Air Loss on the Bounce…….……..………...10

Finding the A and B Values to Determine the Functions….………..…...…...……..…………11

Joining the Values to Make the Functions…………………………………………………..12-14

Testing the Accuracy of the Calculated Piecewise Functions……………………………....15-16

Conclusion……………………………………………………………………………………..16-17

Works Cited………………………………………………………...………………...18

Introduction to the Sport

Basketball is a popular sport that requires a lot of precision, both from the players and from the equipment. Players have to shoot a ball into a small hoop ten feet high in order to score a point. This means that the players must be very precise with their shots, and the ball itself must be pumped just right, so that the air pressure in the ball does not affect the players' shot negatively. I have been playing basketball since I was five years old. I've played on my school team since 7th grade, and before that I was in different leagues outside of school. My favorite positions to play are point guard and center, even though they are very different positions. Growing up I would always play the point guard position, but then I got very tall and heavy, which made me a better center. The goal of the game is to score more points than your opponent, and each individual player plays both defense and offense, meaning it is their job to both try to stop their opponents from scoring, and score as much as they can. In basketball, you can shoot from anywhere on the court, however, obviously the further you are from your hoop, the harder it is to score.

One of the most important aspects about playing basketball is the ball itself. The ball that is used affects the game in many ways. If the ball is not perfectly round, then with every dribble the ball's direction will be unpredictable. If the ball is not pumped with enough air, the bounce may be too weak to dribble the ball with. If the grip on the ball is too weak, then it will be harder to dribble, shoot, pass, and catch the ball, as it will be more likely to slip. These are all vital factors when testing how good a ball is to play with, and the ideal ball takes all of these things into account.

FIBA (Federation International Basketball Association) has many rules in place regarding the ball used in games. For instance, FIBA writes that the ball shall "Be spherical, with black seams not exceeding 6,35 mm in width and, either of a single shade of orange or of the orange/light brown FIBA approved colour combination." FIBA also states that the ball must "Be inflated to an air pressure such that, when it is dropped onto the playing floor from a height of approximately 1,800 mm measured from the bottom of the ball, it will rebound to a height of between 1,200 mm and 1,400 mm, measured to the top of the ball." (Sports Balls, Balls). Likewise, the National Basketball Association suggests that the basketball have an air pressure between 7.5 - 8.5 psi (Carl Putman, SportsRec). There are also strict rules about the circumference and weight of the ball. Refer to figure 1 on the right for an image of the official FIBA basketball.

Figure 1: Official Fiba Basketball. Image taken from moltenusa.com

Why I Chose This Exploration Topic

Basketball has always been my favorite thing to spend my time doing. It is something that I can never get tired of, and something that I can always enjoy. Playing, watching and discussing basketball are all things I can spend multiple hours doing. Furthermore, there is a lot of math involved in basketball, and I believe that understanding the mathematics behind the sport is crucial to helping me improve as a basketball player. Achieving the objective of shooting a basketball, for example, involves the use of percentages and angles. Statistics is also a very large part of basketball, as it can help show which players are best at certain aspects of the sport, such as far range shots, passing, PER (player efficiency rating), etc. Another example would be the math involved in bouncing or dribbling the basketball, as this deals with height, angles, and the internal pressure within the ball. I will attempt to figure out and model the natural bounce of a basketball, because I believe it will help me become more skilled at handling and controlling the basketball.

A basketball's bounce is largely dependent on the pressure within the ball. The less air within the ball, the less force the gases within the ball exert on the walls of the ball, and therefore the less force the ground exerts back on the ball when the ball makes contact with the ground. This lesser force that the floor exerts on the ball therefore results in a decreased bounce. Thus, being able to model how the ball's bounce changes as the air loss within the ball changes will allow me to form a greater level of comprehension around the nature of a basketball and how to style my game, my shot, my dribbling, etc. on the amount of air within the basketball. A lesser bounce makes it harder to dribble the ball, as more force must be put into each dribble to get the ball to return to its initial position. Less bounce also makes it so that the ball ricochets more softly off of the rim on a missed shot. This can be a crucial piece of information that can be used to a team's advantage when it comes to gathering rebounds from missed shots. Furthermore, the amount of air in the ball is also proportional to the mass of the ball. More air means a heavier ball, which can result in a better control on the ball when dribbling, passing and shooting. This is because a ball that is too light can become hard to predict its movements.

How I Will Go About It

To explore the effect of air loss on the model of a bouncing basketball, it helps to ensure that the height that the ball is dropped from is fixed for all of the trials. I chose to drop the ball from a height of 2 meters (±0.001 m) from the ground. This is a distance that is high enough for multiple bounces to be modelled before the basketball loses all energy and stops bouncing. Furthermore, it is also highly repeatable, as it is not too high that it becomes difficult to accurately drop the ball from that height.

It is very significant that the ball is dropped from free fall for every trial. If the ball is pushed down by a force or if the ball is thrown upwards initially before dropping down then it will be practically impossible to repeat these trials and form accurate models of the ball's bounces that represent the true effect of air loss. By dropping the ball from free fall, the only force acting on the ball is the force of gravity, which means that the only variable that could affect the ball's bounce model is the loss in air within the ball.

I want to find how drastic the difference in peak vertical heights is for the ball's bounces when the air pressure within the ball is changed. When graphed, there should be a visible change between the different air pressure trials, showing how with each bounce after being released from free fall, the heights that the ball is able to achieve vary significantly.

Variables:

- The material of the floor is fixed, which is important because the hardness of the floor's material will affect the energy return of the ball after making contact with the ground. The floor will be asphalt concrete.
- Size of the ball is fixed, as the same ball is used throughout the trial
- The height the ball is dropped at is fixed at 2 meters (±0.001 m)
- Air friction and wind are very insignificant and so will be disregarded
- The ball will be dropped with zero spin on it, as a spin on the ball will affect the direction the ball travels in and therefore will hinder its vertical height
- Gravity is fixed

When performing the investigation, it is crucial that when the ball is dropped, it does not travel in any direction horizontally. It must stay as vertical as possible. This is important because if the horizontal distance the ball travels is not controlled, then not all of the ball's energy will go into bouncing vertically up. This means that graphing the effect of air loss on the basketball's bounce will be undoable, as the vertical bounce will not accurately represent the balls total decrease in energy due to a lack of air pressure.

Important Measurements

Height at which ball is dropped into free fall: 2 meters (±0.001 m)

Horizontal Distance Travelled by ball: 0 meters

Time taken for ball to stop bouncing

Amount of times ball bounces before stopping

Air pressure within the ball (±0.5 psi)

The Process of Graphing the Basketball's Bounce

In order to graph the effect of air loss on a basketball's bounce, 5 different variations of air pressure will be tested on the ball, and each variation will be individually graphed. The challenge of this is that a bouncing ball is a partially inelastic collision, meaning that the energy within the ball is lost when it makes contact with the ground, and that is why after each bounce its maximum height decreases. This means that in order to graph each model, a piecewise function is necessary, as a sinusoidal wave will be modeled by the bounce. This is because the energy is dampening, so there will need to be a separate sin function for each bounce. This is because one sin function can not graph different amplitudes. Thus, a piecewise function will be used to graph the first three bounces of the ball at each different air pressure level. A piecewise function is a function in which more than one formula is used to define the output over different places of the domain (Lumenlearning, Piecewise-Defined Functions). The individual functions that will be used to graph the ball's bounce is:

$f(x) = |Asin(Bx)|$ A = Amplitude

B = $\frac{2\pi}{Period}$

Checking the psi of the Basketball

The psi of the basketball was changed and checked by using an air pump with a pressure gauge. Figure 2 below shows images of the basketball's psi before it was dropped.

Figure 2: Psi of the basketball before each trial

Air Pressure (±0.5 psi)	1	4	7	10	13
Images					

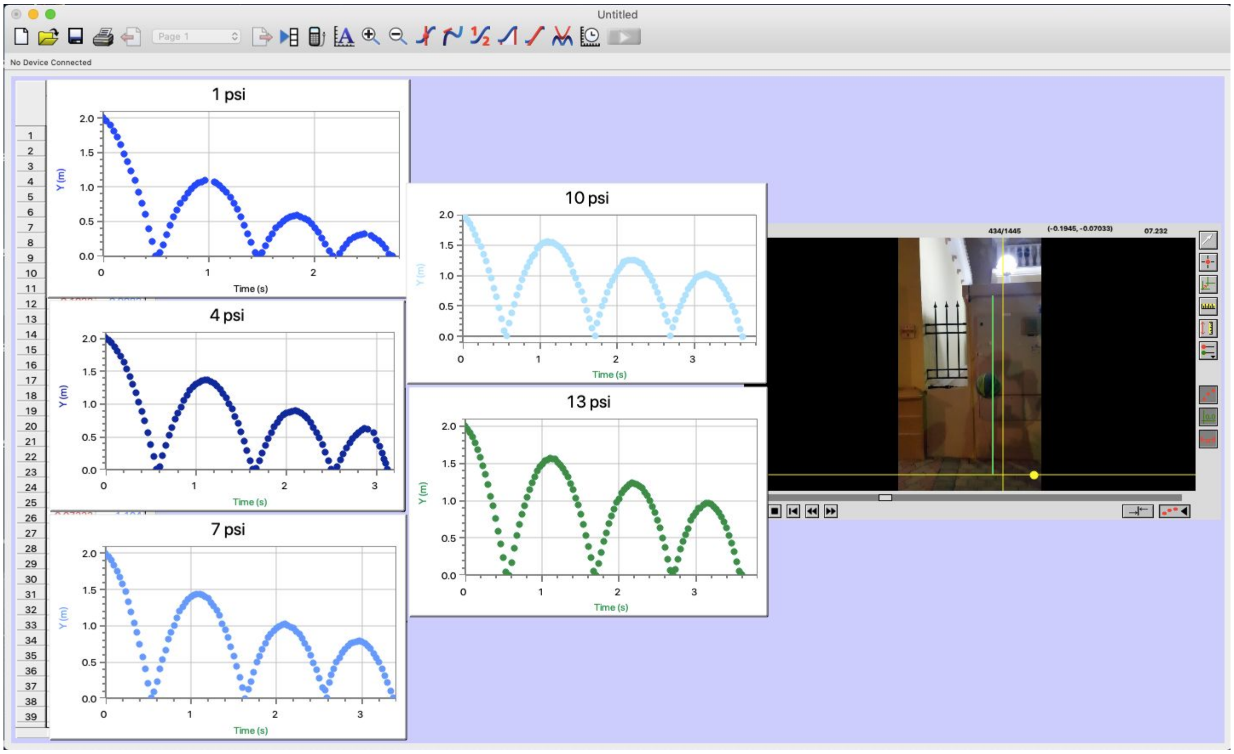
Raw Data Collection

Figure 3: Raw Data Showing Vertical Distance (±0.001 m) Covered for First Three Bounces Depending on Air Pressure (±0.5 psi) Within the Ball

	Maximum Vertical Distance (±0.001 m)		
Air Pressure (±0.5 psi) Within the Ball	**Bounce 1**	**Bounce 2**	**Bounce 3**
1	1.094	0.594	0.319
4	1.362	0.901	0.624
7	1.440	1.021	0.793
10	1.531	1.234	0.969
13	1.555	1.250	1.023

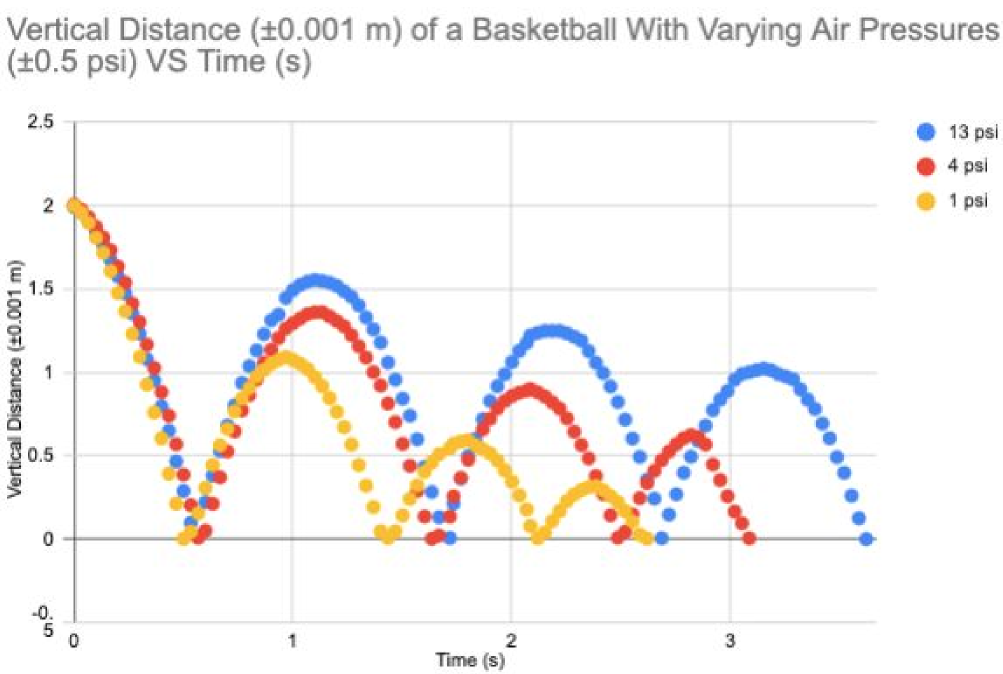
Graphing the Models

Figure 4: Logger Pro Analysis of Video with the psi Variations' Respective Graphs



These graphs were created by first filming the drop tests at each varying pressure level. A measuring tape was present in the background to serve as a scale, as can be seen on the right side of the image (yellow line). After taking all of the test drops, the videos were analyzed using logger pro, and all of the data points were plotted.

Figure 5: Scatter Plot Graph of the Basketball's Bounce Height (±0.001 m) for the First Three Bounces For 3 of the 5 Air Pressure Variations (±0.5 psi) VS Time (s)



Logger pro had the different psi variations spaced out on 5 separate graphs. So, I took all of the data that logger pro found from the video and put it into a google sheets spreadsheet (as shown in the image on the left). A scatterplot containing only three of the psi variations (1, 4, and 13) was created because putting all of the variations made the graph too cluttered and more difficult to visualize and understand.

Exploring the Mathematics Behind the Effect of Air Loss on the Bounce

In order to associate a piecewise function to each of the air loss models, the changing amplitudes and periods must be found. Each graph will be made up of 3 sin functions, as there will be one for each bounce (3 bounces). In order to determine the sin functions, the A and B values must be determined.

Finding the A and B Values to Determine the Functions

Figure 6: Unique Amplitudes and Periods of Each Bounce for the Basketballs Depending on pressure Level Within the Ball.

	Bounce 1		**Bounce 2**		**Bounce 3**	
Air Pressure (±0.5 psi) Within the Ball	**A Value (Amplitude)**	**B Value ($\frac{2\pi}{Period}$)**	**A Value (Amplitude)**	**B Value ($\frac{2\pi}{Period}$)**	**A Value (Amplitude)**	**B Value ($\frac{2\pi}{Period}$)**
1	1.094	3.25	0.594	4.48	0.319	5.54
4	1.362	2.94	0.901	3.62	0.624	5.09
7	1.440	2.85	1.021	3.30	0.793	4.01
10	1.531	2.77	1.234	3.14	0.969	3.49
13	1.555	2.73	1.250	3.10	1.023	3.36

The A value is the amplitude, which in this case, is the maximum distance travelled vertically upwards by the basketball. The B value is, as previously mentioned, $\frac{2\pi}{Period}$ which means the amount of cycles the function can make in an interval from 0 to 2π. Now, with the information provided in figure 5, the sin functions for each bounce can be derived.

B-Value Sample Calculation:

Points of contact with the ground are at 0.567s and 1.718s for the first bounce.

$2 \times (1.718 - 0.567) = 2.302$

$\frac{2\pi}{2.302} = 2.729 \approx 2.73$

Joining the Values to Make the Functions

Figure 7: The Sin Functions for Each Bounce of the Basketball Depending on the Air Pressure Within the Ball.

Air Pressure (±0.5 psi) Within the Ball	Bounce 1	Bounce 2	Bounce 3
1	$f(x) = \lvert 1.094sin(3.25x) \rvert$	$f(x) = \lvert 0.594sin(4.48x) \rvert$	$f(x) = \lvert 0.319sin(5.54x) \rvert$
4	$f(x) = \lvert 1.362sin(2.94x) \rvert$	$f(x) = \lvert 0.901sin(3.62x) \rvert$	$f(x) = \lvert 0.624sin(5.09x) \rvert$
7	$f(x) = \lvert 1.440sin(2.85x) \rvert$	$f(x) = \lvert 1.021sin(3.30x) \rvert$	$f(x) = \lvert 0.793sin(4.01x) \rvert$
10	$f(x) = \lvert 1.531sin(2.77x) \rvert$	$f(x) = \lvert 1.234sin(3.14x) \rvert$	$f(x) = \lvert 0.969sin(3.49x) \rvert$
13	$f(x) = \lvert 1.555sin(2.73x) \rvert$	$f(x) = \lvert 1.250sin(3.10x) \rvert$	$f(x) = \lvert 1.023sin(3.36x) \rvert$

The B value was calculated by multiplying the time taken by the ball in between contact with the ground by 2, and then dividing 2π by that number. This is because the time taken between contact with the ground is exactly half of a full sinusoidal cycle, thus, it is half the period. And according to the equation B= $\frac{2\pi}{period}$, that time must be multiplied by 2.

Trends include that the A value increase as air pressure increase, and decreases as bounce number increases within trials. Also, B value decreases as air pressure increases, and increases as bounce number increases. The B and A values are opposite to each other. When one is increasing, the other is decreasing. In fact, the two have a strong negative correlation of -0.947. This is significant, because the B-Value is inversely proportional to the period. This means that as the amplitude decreases, so does the period. This can actually be seen in the graphs as well. This also makes logical sense, because the only force on the basketball is the force of gravity. As the ball loses energy within the system, it can no longer sustain the same vertical distance, so it travels less. By travelling a lesser distance, it is able to reach its amplitude quicker, and therefore with a smaller period. It is also important to recognize that there is a trend regarding change in amplitudes and the change in B-values between the different psi variations. The change in amplitudes and B-values can be seen in Figure 8.

Figure 8: Change in Amplitude and B-Value

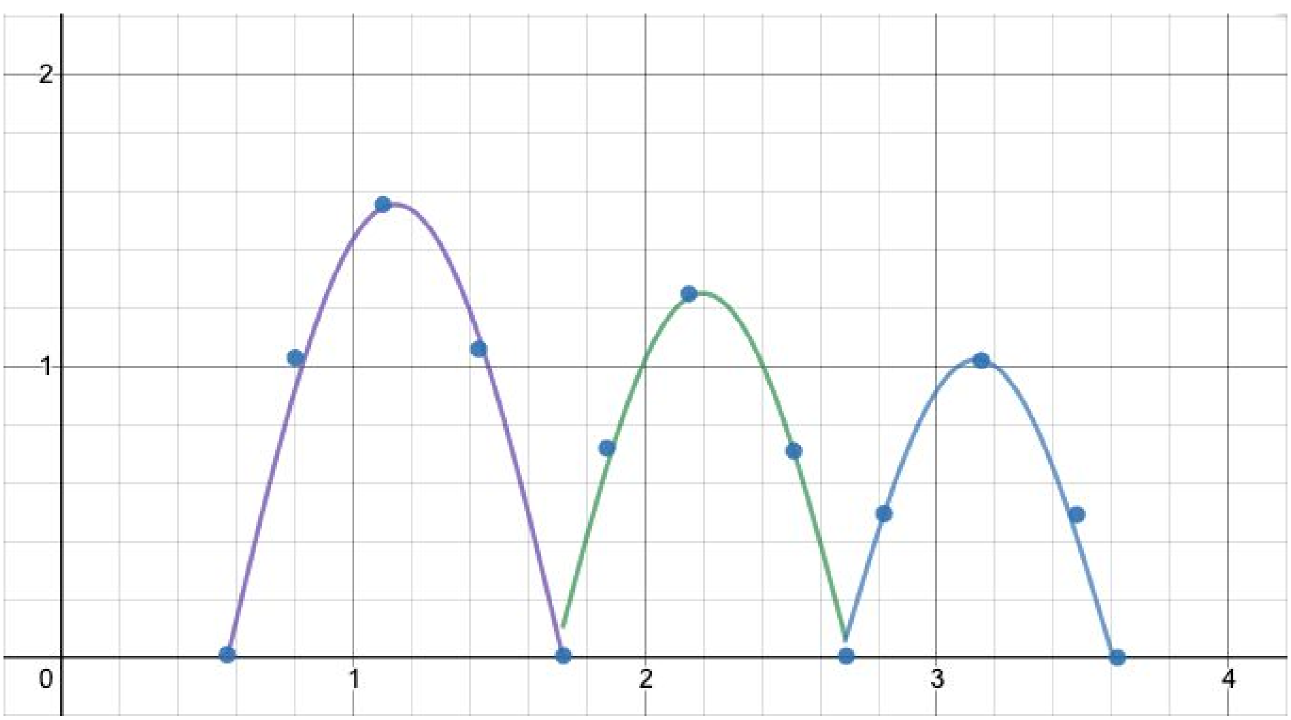
Air Pressure (±0.5 psi) Within the Ball	**Change in Amplitude from Bounce 1 to Bounce 2**	**Change in Amplitude from Bounce 2 to Bounce 3**	**Change in B-Value from Bounce 1 to Bounce 2 (Absolute Value)**	**Change in B-Value from Bounce 1 to Bounce 2 (Absolute Value)**
1	0.5	0.275	1.23	1.06
4	0.461	0.277	0.68	1.47
7	0.419	0.228	0.45	0.71
10	0.297	0.265	0.37	0.35
13	0.305	0.227	0.37	0.26

A higher B-value, means a decreased period. This is because they are inversely proportional (B= $\frac{2\pi}{Period}$). A decreased period, in this case, translates to a smaller amount of time spent in the air by the ball. As figure 8 shows, the change in B-values between bounces decreases with air pressure. Thus, the change in time spent in the air by the ball increases with air pressure. Analyzing these piecewise functions provides evidence to the idea that air pressure affects the amount of time spent in the air by the ball. Specifically, increasing air pressure results in an increase in the change in time spent in the air by the ball. This means that with more air pressure, there is a more significant difference in the time spent in the air between the bounces, and with less air pressure, the time spent in the air becomes more similar.

The A-value is the amplitude. In figure 8, the change in Amplitude from the first bounce and the second bounce is much greater than the change in amplitude from the second bounce and the third bounce. This means that the change in amplitude decreases the more bounces there are, regardless of air pressure. In other words, the nature of a basketball's bounce is that as the ball continues to bounce the difference in height achieved by the ball decreases drastically. This makes sense too, as this will allow for the ball to eventually lose all energy and stop bouncing entirely. This is irrespective of air pressure.

Testing the Accuracy of the Calculated Piecewise Functions

Figure 9: Graphing the Points From Video Analysis with the Piecewise Function for the Basketball's Bounce with a pressure of 13 psi.



The purple, green, and blue lines are functions for bounces 1, 2, and 3 respectively, of the basketball's bounce with a pressure of 13 psi. The first function is $f(x) = |1.555sin(2.73x)|$ .The second function is $f(x) = |1.250sin(3.10x)|$. The third function is $f(x) = |1.023sin(3.36x)|$. These functions were put into desmos.com and their respective domains were put in. The domains were simply the times at which the ball made contact with the ground. The graphs were also horizontally shifted to the right, all by unique amounts. The first function was shifted to account for the initial drop of the ball, the second function was shifted to account for the initial drop and the first bounce, and the third function was shifted to account for the initial drop and the first two bounces. The blue dots placed on the graph are actual points plotted by logger pro's program when I did the video analysis. This graph shows that the piecewise function lines up pretty well with the actual model, demonstrating a high level of accuracy within the functions that were created.

Conclusion

In conclusion, I have found that the air loss in a basketball has a significant effect on the ball's model. It is clear that as the ball loses air pressure within it, it is unable to bounce nearly as high for all of its bounces, and it spends less time in the air in between bounces. This is supported by figure 5, as it is very clear to see that the basketball bounced much higher and for much longer when it had a pressure of 13 psi vs when it had a pressure of 1 psi. The 13 psi basketball took 3.62 seconds to bounce three times, while the 1 psi basketball only took 2.74 seconds. Furthermore, as figure 6 and 7 show, the maximum vertical distances for the first and third bounce for the 13 psi basketball were 1.555 and 1.023 meters respectively. However, the maximum vertical distances for the first and third bounce for the 1 psi basketball were 1.094 and 0.594 meters respectively. This also proves that the pressure psi affects the bounce and the time spent in the air. In fact, the correlation coefficient for the psi and the maximum vertical distance for the initial bounce is 0.92, which suggests as well that there is a clear relationship between the two variables.

Another important note is the accuracy of this experiment. According to figure 9, the sin functions that were made were very accurate to the actual models that were made by logger pro's video analysis. The points match up pretty well, with only a very small amount of error/distance between the points and the piecewise function. However, although the accuracy matched well with the logger pro models, there were some places where inaccuracy could have taken place. For example, the camera may not have been perfectly angled directly perpendicular to the ground, thus resulting in a tilt that lead to some places seeming further or closer than they actually are. This would have messed with the scale that was used in logger pro to determine the distance travelled by the ball. Another point of weakness that could have caused inaccuracies is the horizontal distance travelled by the ball. Ideally, the ball would only move strictly upwards and strictly downwards. This is unfortunately very difficult to achieve and almost impossible to manage, especially when dealing with multiple trials such as in this case. This is because if the ball travels more horizontally in one trial than in another, then the maximum vertical distance travelled by the ball will decrease more than it should have in that trial, which will result in skewed data that does not represent the true nature of the ball's bounce.

The overall significance of this experiment is that it can be used to further the understanding of a basketball player behind the math involved in playing the sport. If a basketball player genuinely understands the correlation between the air pressure within the ball and the time it takes for the ball to bounce and how high it potentially can bounce, then they will have a competitive advantage, as they will be able to make more informed decisions and predictions including sharper passes, where to hit the backboard during a shot, or where the ball will be directed to after hitting the rim or the backboard. A possible extension to this idea is investigating how the type of material the floor is made out of affects the ball's bounce height, and why all NBA courts are made out of the same highly polished maple wood.

Works Cited

"Basketball and Math." *Math in Basketball – How Math Is Used in Sports – Cool Math*, Knowledge Adventure®, 2018, www.mathblaster.com/coolmath/articles/basketball-and-math.

Balls, Sports. "FIBA Has Detailed Rules." *Balls.com - Index of Balls Used in Sporting Games and Events*, 23 June 2013, balls.com/rules/fiba-has-detailed-rules.html.

Desmos Team. "Desmos Graph." *Desmos Graphing Calculator*, Desmos, Inc., 2019, www.desmos.com/calculator.

Floris. "Model/Formula for Bouncing Ball." *Physics Stack Exchange*, Stack Exchange Inc., Apr. 2016, physics.stackexchange.com/questions/256468/model-formula-for-bouncing-ball.

Gonterman, Margo. "Exploring Asin(Bx+C)+D." *Exploring y=Asin(Bx+C)+D*, The University of Georgia, jwilson.coe.uga.edu/EMAT6680Fa09/Gonterman/Gonterman1/Gonterman1.html.

Kelvin, Angelica, and Iris Latorre. "Mathematics Improves Basketball." *JJay College of Criminal Justice*, JJay College of Criminal Justice. https://www.jjay.cuny.edu/sites/default/files/contentgroups/sasp/poster_gallery/poster48.pdf

LumenLearning. "College Algebra." *Piecewise-Defined Functions*, Pressbook, courses.lumenlearning.com/waymakercollegealgebra/chapter/piecewise-defined-functions/.

Putman, Carl. "Basketball Official Size & Weight." *SportsRec*, Leaf Group Ltd., 11 Dec. 2018, www.sportsrec.com/4271855/basketball-official-size-weight.

The Editors of Encyclopaedia Britannica. "Kinematics." *Encyclopædia Britannica*, Encyclopædia Britannica, Inc., 21 June 2017, www.britannica.com/science/kinematics.