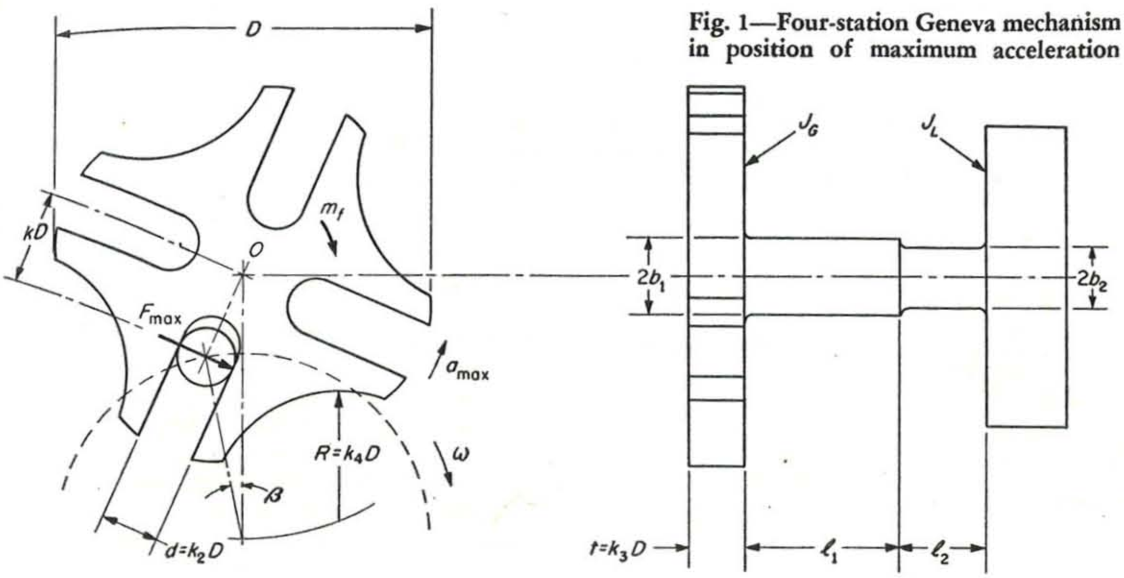

Fig. 1—Four-station Geneva mechanism in position of maximum acceleration

How to design Geneva Mechanisms

to minimize contact stress and torsional vibration

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INCREASED demands for high-speed Geneva wheel operation have focused attention on two undesirable characteristics inherent in this mechanism. These characteristics, found to some degree in many intermittent-motion mechanisms, are (1) high contact stress between the drive pin and the wheel slot and (2) vibratory motion of the driven inertia load. Both of these factors adversely affect performance and life.

Chief effect of high contact stress is to cause excessive drive-pin wear, with premature failure of the Geneva drive. On the other hand, vibratory motion often interferes with the operation of the mechanism or causes excessive wear on Geneva parts, such as the locking device.

This article outlines a simple method for minimizing drive-pin contact stress, and gives a procedure for reducing undesirable vibratory motion.

Minimizing Contact Stress

A four-station Geneva mechanism which intermittently rotates an inertia load J_L against a frictional retarding moment M_f is shown in *Fig.* 1. Lichtwitz has analyzed the motion of Geneva mechanisms and shows that, for a given number of stations, the angular motion of the Geneva wheel is a function of the angular position and velocity of the drive-pin shaft.* Angular motion of the Geneva wheel is also shown to be independent of its diameter D. The mechanism in *Fig.* 1 is shown in its position of maximum wheel acceleration, and the following discussion is concerned with the forces and stresses obtained in this instantaneous position.

Since both the distance from the drive-pin axis to the drive-shaft axis and the distance from the drive-shaft axis to the Geneva-wheel axis are directly proportional to the wheel diameter D, the torque arm of the pin contact force with respect to the wheel axis is also directly proportional to the diameter. This torque arm is denoted by kD in *Fig.* 1, where k is a constant of proportionality. Because wheel acceleration is maximum in the position shown, the drive pin exerts a torque $F_{max}\ kD$ on the Geneva wheel. Summing

*Otto Lichtwitz—"Mechanisms for Intermittent Motion," MACHINE DESIGN, Dec. 1951, Pages 134-148.

torques about the wheel axis O and solving for F_{max},

$$F_{max} = \frac{(J_G + J_L)\, a_{max} + M_f}{kD} \qquad (1)$$

Since D is in the denominator of Equation 1, the pin force at the condition of maximum angular accelaration obviously depends on the Geneva wheel diameter. Also, D effectively exists in the numerator, since it is one of the factors that determines the value of Geneva wheel intertia J_G.

Drive pin contact stress is given by the well-known Hertz equation as

$$s_{max} = k_1 \sqrt{\frac{F_{max}}{td}} \qquad (2)$$

where

$$k_1 = 0.798 \sqrt{\frac{1}{\left(\frac{1 - \nu_1^2}{E_1}\right) + \left(\frac{1 - \nu_2^2}{E_2}\right)}} \qquad (3)$$

A study of Equation 2 shows that minimum contact stress is obtained when both the driving pin diameter d and the Geneva wheel thickness t are as large as practical. However, the Geneva wheel diameter D cannot be specified with certainty at this point. Equation 1, for example, shows that small wheel diameters cause high pin forces because D is in the denominator. Furthermore, large wheel diameters, because of high wheel inertia J_G, also cause high pin forces. Obviously there must be an optimum wheel diameter that minimizes the pin-slot contact stress.

In the derivation of optimum wheel diameter, proportional design techniques are used in the Geneva wheel layout. These techniques will recognize the desirability of having pin diameters as large as practicable. As for other principal dimensions, pin diameter d can be assumed to be directly proportional to wheel diameter. Denoting the constant of proportionality by k_2,

$$d = k_2 D \qquad (4)$$

Again from proportional design considerations, pin length—and therefore wheel thickness t—depend upon the diameter d of the drive pin. Longer pins are permissible with larger diameters. Because Equation 4 has already shown that the pin diameter is proportional to the wheel diameter, it follows that wheel thickness t can also be assumed proportional to wheel diameter. Denoting

Table 1—Geneva Design Constants

—Basic Geneva Properties—				——Suggested Design Constants——				
n	β	a_{max}/ω^2	a_v/ω^2	k	k_2	k_3	k_4	k_5
3	4° 46′	31.44	1.732	0.1546	0.16	0.16	0.70	0.00353
4	11° 24′	5.409	1.000	0.2384	0.15	0.15	0.35	0.00506
5	17° 34′	2.229	0.7265	0.2930	0.14	0.14	0.24	0.00516
6	22° 54′	1.350	0.5774	0.3311	0.14	0.14	0.17	0.00588
7	27° 33′	0.9284	0.4816	0.3592	0.14	0.14	0.13	0.00573
8	31° 38′	0.6998	0.4142	0.3807	0.13	0.13	0.10	0.00575
9	35° 16′	0.5591	0.3640	0.3977	0.12	0.12	0.08	0.00584
10	38° 30′	0.4648	0.3249	0.4112	0.10	0.10	0.08	0.00540

Nomenclature

a_{max} = Maximum angular acceleration of Geneva wheel, rad per sec^2

a_v = Angular acceleration of Geneva wheel at instant when drive pin leaves the wheel slot, rad per sec^2

b = Radius of Geneva wheel shaft, in.

c = Torsional stiffness of Geneva wheel shaft, lb-in. per rad

D = Diameter of Geneva wheel, in.

d = Diameter of Geneva drive pin, in.

E_1 = Modulus of elasticity of Geneva drive pin material, psi

E_2 = Modulus of elasticity of Geneva wheel slot material, psi

E_s = Shearing modulus of elasticity of Geneva wheel shaft material, psi

f_n = Natural frequency of free torsional vibrations for load inertia, cps

F_{max} = Force between the Geneva drive pin and wheel slot at instant when the angular acceleration of Geneva wheel is a maximum, lb

g = Acceleration due to gravity, 386 in. per sec^2

J_G = Mass moment of inertia of Geneva wheel about axis of rotation, lb-sec^2-in.

J_L = Mass moment of inertia of load about axis of rotation, lb-sec^2-in.

J_s = Polar moment of inertia of cross sectional area of Geneva wheel shaft, in.^{4}

k = Ratio of distance between Geneva center and drive pin to Geneva wheel diameter

k_1 to k_5 = Constants

l = Length of Geneva wheel shaft, in.

M_f = Retarding moment acting on Geneva wheel, lb-in.

n = Number of slots in Geneva wheel

R = Locking radius on Geneva wheel, in.

s_{max} = Contact compressive stress between Geneva drive pin and wheel slot at maximum angular acceleration of Geneva wheel, psi

T = Torque, lb-in.

t = Thickness of Geneva wheel, in.

W = Energy available for free torsional vibrations when Geneva drive pin leaves wheel slot, in.-lb

w = Specific weight of Geneva wheel material, lb per cu in.

β = Angle shown in *Fig.* 1 for Geneva mechanism in position of maximum angular acceleration, deg.

ν_1 = Poisson's ratio for Geneva drive pin material

ν_2 = Poisson's ratio for Geneva wheel slot material

ω = Angular velocity of Geneva drive shaft, rad per sec

θ = Elastic rotational deformation of Geneva wheel shaft when drive pin leaves wheel slot, rad this constant of proportionality by k_3,

$$t = k_3 D \qquad (5)$$

The proportional design assumption also provides a means for relating the Geneva inertia J_G to the wheel diameter D. From basic mechanics, the mass moment of inertia of the Geneva wheel is equal to its mass times its radius of gyration squared. The mass of the wheel is directly proportional to its specific weight and to its volume. Furthermore, under the proportional design assumption, the volume is proportional to D^3, and the square of the radius of gyration is proportional to D^2. Hence, from this line of reasoning the mass moment of inertia of a proportionally designed Geneva wheel can be expressed as

$$J_G = \frac{k_5 w D^5}{g} \qquad (6)$$

where k_5 is a constant of proportionality and w is the specific weight of the material. Incidentally, locking radius R, *Fig.* 1, is also related to the wheel diameter by the constant of proportionality k_4. Therefore

$$R = k_4 D \qquad (7)$$

When Equation 6 is substituted in Equation 1,

$$F_{max} = \frac{\left(\dfrac{k_5 w D^5}{g} + J_L \right) a_{max} + M_f}{k D}$$

This relationship, together with Equations 4 and 5, are substituted in Equation 2, and the expression

Example 1—Design for Minimum Stress

A four-station Geneva mechanism intermittently rotates a solid steel 4-in. diameter cylinder, ¼-in. thick. A constant frictional moment of 2 lb-in. opposes rotation of the cylinder and drive-shaft speed is 600 rpm.

First steps are to calculate the load inertia and the Geneva wheel acceleration. The moment of inertia of the load is

$$J_L = M_L \frac{R_L^2}{2} = \frac{\pi R_L^2 w}{4 g} \left(\frac{R_L^2}{2} \right)$$

$$= \frac{\pi (2)^2 (0.283) (2)^2}{4 (386) (2)}$$

$$= 0.0046 \text{ lb-sec}^2\text{-in.}$$

where M_L and R_L are the mass and radius of the load, respectively. From *Table* 1, maximum acceleration of the Geneva wheel is

$$a_{max} = 5.409\, \omega^2 = 5.409 \left[\frac{600\,(2\pi)}{60} \right]^2$$

$$= 21{,}300 \text{ rad per sec}^2$$

Contact stresses will now be determined for the optimum Geneva wheel and for wheels smaller and larger than optimum. In each case, the design proportions of *Table* 1 are used.

Optimum Wheel: From Equation 9, the mass moment of inertia of the optimum Geneva wheel is

$$J_G = \frac{3}{2} \left[0.0046 + \frac{2}{21{,}300} \right]$$

$$= 0.00704 \text{ lb-sec}^2\text{-in.}$$

For a four-station Geneva wheel with the value of k_5 from *Table* 1, Equation 6 gives

$$J_G = 0.00506\, D^5 \left(\frac{0.283}{386} \right) = 0.00704$$

Therefore, $D = 4.53$ in.

The value of D and the constants from *Table* 1 are entered in Equations 4, 5, and 7. Results are $t = 0.15(4.53) = 0.68$-in., $d = 0.15(4.53) = 0.68$-in., and $R = 0.35(4.53) = 1.59$ in. Force torque arm $kD = 0.2384(4.53) = 1.08$ in.

From Equation 1,

$$F_{max} = \frac{(0.00704 + 0.0046)(21{,}300) + 2}{1.08}$$

$$= 232 \text{ lb}$$

For the steel drive pin and Geneva wheel, $E = 30{,}000{,}000$ psi and $\nu = 0.3$. Equation 3 therefore gives $k_1 = 3240$.

Finally from Equation 2, contact stress for the optimum Geneva design is

$$s_{max} = 3240 \sqrt{\frac{232}{(0.68)(0.68)}}$$

$$= 72{,}600 \text{ psi}$$

Wheel Too Small: Here, the Geneva wheel diameter is arbitrarily set at 2 in. Then: $D = 2.00$ in., $d = 0.30$-in., and $t = 0.30$-in. Inertia J_G is again calculated from Equation 6, using k_5 from *Table* 1. In this case, $J_G = 0.000119$ lb-sec²-in. From Equation 1 as before, $F_{max} = 215$ lb. From Equation 2 contact stress is $s_{max} = 158{,}200$ psi for the design where the Geneva wheel is too small. This contact stress is 218 per cent of the stress of 72,600 psi calculated for the optimum wheel.

This case shows the possible hazard of letting a desire for minimum inertia influence design of high-speed mechanisms.

Wheel Too Large: Assume that the Geneva wheel diameter is arbitrarily set at 6 in. When carried through as before, the analysis gives $F_{max} = 499$ lb, and $s_{max} = 80{,}400$ psi. This contact stress is 111 per cent of the optimum stress at 72,600 psi. Moreover, the pin force is decidedly larger than the optimum F_{max}, which in itself is a disadvantage.

for maximum contact stress becomes

$$s_{max} = \frac{k_1}{\sqrt{k\,k_2\,k_3}} \sqrt{\frac{\left(\dfrac{k_5\,w\,D^5}{g} + J_L\right) a_{max} + M_f}{D^3}} \quad (8)$$

To minimize the contact stress, a partial derivative of s_{max} with respect to D is taken in Equation 8 and equated to zero. This derivative expression, after simplification by use of Equation 6, can be rearranged to give

$$J_G = \frac{3}{2}\left(J_L + \frac{M_f}{a_{max}}\right) \quad (9)$$

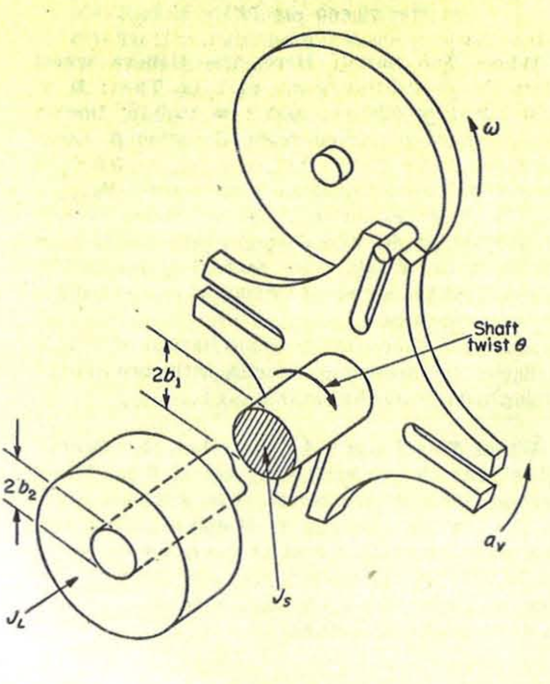
Equation 9 is the crux of the analysis to this point. It shows that minimum pin contact stress is obtained when the mass moment of inertia of the Geneva wheel has a particular value that depends on (1) inertia of the load, (2) frictional forces and (3) speed of operation. For most practical problems, the term M_f/a_{max} in Equation 9 will be negligible with respect to J_L and can be ignored.

Accurate layouts were made by the author for Geneva wheels that have three to ten stations, using a set of proportions that seemed to conform well with good practice. The proportionality constants for these designs are given in *Table* 1.

Use of these suggested constants will simplify the design procedure. The calculated value for J_G from Equation 9 can be placed in Equation 6 and the desired wheel diameter D found at once by extracting k_5 from *Table* 1. However, a designer can choose different proportions, if he wishes and then work out the design by use of Equation 9.

How these foregoing ideas may be applied in actual design is demonstrated by *Example* 1.

Fig. 2—Geneva mechanism at instant drive pin is leaving slot



Minimizing Vibration

Lichtwitz has shown in his analysis that the Geneva wheel is decelerating at the instant the drive pin leaves the wheel slot. Deceleration a_v is given in *Table* 1. Because the Geneva wheel shaft, (*Fig.* 2), is decelerating the inertia load J_L, it transmits a torque at the instant the drive pin leaves the wheel slot. The magnitude of this shaft torque, neglecting frictional forces, is given by

$$T = J_L\,a_v \quad (10)$$

When a shaft of length l transmits a reasonable torque, it undergoes elastic rotational deformation, and the angle of deformation is

$$\theta = \frac{T\,l}{J_s\,E_s}$$

Therefore, from Equation 10, the elastic rotational deformation at the instant the pin leaves the slot is

$$\theta = \frac{J_L\,a_v\,l}{J_s\,E_s} \quad (11)$$

Polar moment of inertia J_s is

$$J_s = \frac{\pi b^4}{2} \quad (12)$$

where b is the shaft radius.

The shaft twist θ (*Fig.* 2) at the beginning of the rest period represents stored potential energy which is capable of initiating torsional vibrations of the inertia load. These vibrations persist during the rest period of the Geneva wheel and have an initial amplitude given by Equation 11.

For many applications, vibrations of even small magnitude cannot be tolerated. Large vibrations, on the other hand, often cause exorbitant wear on parts such as the Geneva locking device. Equations 10, 11 and 12 reveal a means for reducing the amplitude of such free torsional vibrations.

For example, from Equation 10, it is obvious that shaft torque can be reduced by decreasing the inertia of the load. Decreasing the load inertia J_L is also desirable from a pin-stress standpoint.

From Equation 11, the length of the Geneva wheel shaft l should also be as small as possible, and the shaft should be a stiff material with large shearing modulus E_s.

Finally, from Equation 12, the polar moment of inertia of the shaft J_s should be as large as practical, implying the use of a large-diameter shaft.

For a shaft made of a single material, but consisting of various diameter cross sections, the torsional deflection equation corresponding to

Equation 11 is

$$\theta = \frac{J_L \alpha_o}{E_s}\left(\frac{2}{\pi}\right)\left(\frac{l_1}{b_1^4}+\frac{l_2}{b_2^4}+\frac{l_3}{b_3^4}+\ldots\right) \quad (13)$$

Considering that the Geneva wheel is clamped during the rest period, the natural frequency for free torsional vibrations of the load inertia J_L is

$$f_n = \frac{1}{2\pi}\sqrt{\frac{c}{J_L}} \quad (14)$$

The torsional stiffness c of the shaft is found from Equation 13:

$$c = \frac{T}{\theta} = \frac{\left(\frac{\pi}{2}\right)E_s}{\left(\frac{l_1}{b_1^4}+\frac{l_2}{b_2^4}+\frac{l_3}{b_3^4}+\ldots\right)} \quad (15)$$

From basic mechanics, the energy of free torsional vibrations is

$$W = \frac{c\,\theta^2}{2} \quad (16)$$

Equations 11 through 16 enable a designer to calculate the amplitude, frequency and energy of the load vibrations and demonstrate that, through careful selection of components, the magnitude of vibrational motion can be held to an acceptable value.

Application of these analytical methods is demonstrated in *Example* 2.

Summary of Geneva Design

For a Geneva mechanism having a given number of stations, the following design steps are recommended:

1. Using Equations 10 through 16, select a Geneva wheel shaft that will give satisfactory vibratory characteristics. Equation 13 is the most important equation in this group and should be used at the start of the analysis for choosing an acceptable shaft size.
2. Calculate the mass moment of inertia of the Geneva wheel shaft and, if appreciable, include it with the load inertia, J_L.
3. Use Equation 9 to calculate the optimum moment of inertia for the Geneval wheel, J_G.
4. Using *Table* 1 and Equation 6, calculate the optimum diameter D of the Geneva wheel. Determine the optimum Geneva wheel physical dimensions by using Equations 4, 5 and 7.
5. The pin stress at the condition of maximum angular acceleration may then be calculated by Equation 8.

Example 2—Design for Acceptable Vibration

Design data and calculated values from *Example* 1 are used in this example. Also, a constant-diameter shaft connects the Geneva wheel with the load. A shaft design that gives exorbitant vibratory motion will be considered first.

Exorbitant Vibratory Motion: Here, the Geneva-wheel shaft diameter $2b$ is chosen as ⅜-in. Such a small diameter might be selected with the idea of keeping inertias low. The shaft is steel and is 5 in. long.

From Equation 10, shaft torque is

$$J_L \alpha_o = 0.0046\ [1.000]\left[\frac{600\ (2\pi)}{60}\right]^2$$
$$= 18.15 \text{ lb-in.}$$

From Equation 12,

$$J_s = \frac{\pi b^4}{2} = \frac{\pi}{2}\left(\frac{3}{16}\right)^4$$
$$= 0.00194\text{-in.}^4$$

Hence, from Equation 11,

$$\theta = \frac{18.15\ (5)}{0.00194\ (12{,}000{,}000)}$$
$$= 0.00390\text{-rad}$$

Therefore, at the periphery of the 4-in. diameter load cylinder, the amplitude of free vibrations will be $2\theta = 2(0.0039) = 0.0078$-in., or 0.0156-in. total vibratory motion. This vibrational motion is exorbitant.

Acceptable Vibratory Motion: Assume that the wheel shaft diameter $2b$ is increased to 1 in. and that the shaft length l is reduced to 3 in. Then, from Equation 12, $J_s = 0.0982$-inch^4. Because the initial shaft torque is still 18.15 lb-in., Equation 11 gives

$$\theta = \frac{18.15\ (3)}{0.0982\ (12{,}000{,}000)}$$
$$= 0.0000462\text{-rad}$$

Therefore, at the periphery of the load cylinder the amplitude of free vibrations has been reduced to $2(\theta) = 2(0.0000462) \approx 0.0001$-in., and the total vibratory motion is now less than 0.0002-in.

Reduction of vibration, by a factor of 85 to 1, has been achieved with comparatively little increase in inertia.

The mass moment of inertia of the 1-in. by 3 in. shaft is only 0.000216 lb-sec²-in., which is less than 5 per cent of the load inertia of 0.0046 lb-sec²-in.

A comparison of the vibratory characteristics is given below for the two designs. Frequencies and energies shown were obtained directly from Equations 14, 15 and 16.

Shaft Diam. (in.)	Shaft Length (in.)	Double Amplitude (in.)	Frequency (cps)	Vibration Energy (in.-lb)
⅜	5	0.0156	159	0.0354
1	3	0.0002	1470	0.00042