Natural deduction for predicate logic

And

$$\frac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge\mathrm{i} \qquad\qquad \frac{\phi \wedge \psi}{\phi}\ \wedge\mathrm{e}_1 \quad \frac{\phi \wedge \psi}{\psi}\ \wedge\mathrm{e}_2$$

Or

$$\frac{\phi}{\phi \vee \psi}\ \vee\mathrm{i}_1 \quad \frac{\psi}{\phi \vee \psi}\ \vee\mathrm{i}_2 \qquad\qquad \frac{\phi \vee \psi \quad \boxed{\begin{array}{c}\phi\\ \vdots\\ \chi\end{array}} \quad \boxed{\begin{array}{c}\psi\\ \vdots\\ \chi\end{array}}}{\chi}\ \vee\mathrm{e}$$

Implication

$$\frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \psi\end{array}}}{\phi \to \psi}\ {\to}\mathrm{i} \qquad\qquad \frac{\phi \quad \phi \to \psi}{\psi}\ {\to}\mathrm{e}$$

Negation

$$\frac{\boxed{\begin{array}{c}\phi\\ \vdots\\ \bot\end{array}}}{\neg\phi}\ \neg\mathrm{i} \qquad\qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg\mathrm{e}$$

Contradiction

no introduction

$$\frac{\bot}{\phi}\ \bot\mathrm{e}$$

Double negation

$$\frac{\phi}{\neg\neg\phi}\ \neg\neg\mathrm{i} \qquad\qquad \frac{\neg\neg\phi}{\phi}\ \neg\neg\mathrm{e}$$

Equality

$$\frac{}{t = t}\ {=}\mathrm{i} \qquad\qquad \frac{a = b \quad \phi[a/x]}{\phi[b/x]}\ {=}\mathrm{e}$$

Forall

$$\frac{\boxed{\begin{array}{ll}x_0 & \vdots\\ & \phi[x_0/x]\end{array}}}{\forall x \phi}\ \forall x\,\mathrm{i} \qquad\qquad \frac{\forall x \phi}{\phi[t/x]}\ \forall x\,\mathrm{e}$$

Exists

$$\frac{\phi[t/x]}{\exists x \phi}\ \exists x\,\mathrm{i} \qquad\qquad \frac{\exists x \phi \quad \boxed{\begin{array}{ll}x_0 & \phi[x_0/x]\\ & \vdots\\ & \chi\end{array}}}{\chi}\ \exists x\,\mathrm{e}$$

$$\frac{\phi \to \psi \quad \neg\psi}{\neg\phi}\ \mathrm{MT} \qquad\qquad \frac{\boxed{\begin{array}{c}\neg\phi\\ \vdots\\ \bot\end{array}}}{\phi}\ \mathrm{PBC} \qquad\qquad \frac{}{\phi \vee \neg\phi}\ \mathrm{LEM}$$

Linear-time temporal logic

$\mathcal{M} = (S, \rightarrow, L)$ a model and $\pi = s_1 \rightarrow \dots$ a path in $\mathcal{M}$. $i \geq 1, \pi^i = s_i \rightarrow s_{i+1} \rightarrow \dots$. $\mathcal{M}, s \models \phi$ if for every path π of $\mathcal{M}$ starting in s we have $\pi \models \phi$.

1. $\pi \models \top$
2. $\pi \not\models \bot$
3. $\pi \models p$ iff $p \in L(s_1)$
4. $\pi \models \neg\phi$ iff $\pi \not\models \phi$
5. $\pi \models \phi \wedge \psi$ iff $\pi \models \phi \wedge \pi \models \psi$
6. $\pi \models \phi \vee \psi$ iff $\pi \models \phi \vee \pi \models \psi$
7. $\pi \models \phi \rightarrow \psi$ iff $\pi \models \psi$ whenever $\pi \models \phi$
8. $\pi \models \mathrm{X}\,\phi$ iff $\pi^2 \models \phi$
9. $\pi \models \mathrm{G}\,\phi$ iff, for all $i \geq 1, \pi^i \models \phi$
10. $\pi \models \mathrm{F}\,\phi$ iff there is some $i \geq 1$ such that $\pi^i \models \phi$
11. $\pi \models \phi\ \mathrm{U}\ \psi$ iff there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \dots, i-1$ we have $\pi^j \models \phi$
12. $\pi \models \phi\ \mathrm{W}\ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \dots, i-1$ we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$
13. $\pi \models \phi\ \mathrm{R}\ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, \dots, i$ we have $\pi^j \models \psi$, or for all $k \geq 1$ we have $\pi^k \models \psi$

Computational tree logic

$\mathcal{M} = (S, \rightarrow, L)$ a model, $s \in S$, ϕ, ψ CTL formulas.

1. $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \bot$
2. $\mathcal{M}, s \models p$ iff $p \in L(s)$
3. $\mathcal{M}, s \models \neg\phi$ iff $\mathcal{M}, s \not\models \phi$
4. $\mathcal{M}, s \models \phi \wedge \psi$ iff $\mathcal{M}, s \models \phi \wedge \mathcal{M}, s \models \psi$
5. $\mathcal{M}, s \models \phi \vee \psi$ iff $\mathcal{M}, s \models \phi \vee \mathcal{M}, s \models \psi$
6. $\mathcal{M}, s \models \phi \rightarrow \psi$ iff $\mathcal{M}, s \models \psi$ whenever $\mathcal{M}, s \models \phi$
7. $\mathcal{M}, s \models \mathrm{AX}\phi$ iff for all s_1 such that $s \rightarrow s_1$ we have $\mathcal{M}, s_1 \models \phi$. 'In every next state.'
8. $\mathcal{M}, s \models \mathrm{EX}\phi$ iff for some s_1 such that $s \rightarrow s_1$ we have $\mathcal{M}, s_1 \models \phi$. 'In some next state.'
9. $\mathcal{M}, s \models \mathrm{AG}\phi$. For all paths starting in s ϕ holds globally.
10. $\mathcal{M}, s \models \mathrm{EG}\phi$. There exists a path starting in s where ϕ holds globally.
11. $\mathcal{M}, s \models \mathrm{AG}\phi$. For all paths starting in s there is some future state where ϕ holds.
12. $\mathcal{M}, s \models \mathrm{EF}\phi$. For all paths beginning in s there is some future state where ϕ holds.
13. $\mathcal{M}, s \models \mathrm{A}[\phi \mathrm{U} \psi]$. All paths starting in s satisfies $\phi \mathrm{U} \psi$.
14. $\mathcal{M}, s \models \mathrm{E}[\phi \mathrm{U} \psi]$. There exists some path starting in s where $\phi \mathrm{U} \psi$ holds.