Natural deduction for predicate logic

And

$$\frac{\phi \quad \psi}{\phi \wedge \psi}\ \wedge\mathrm{i} \qquad \frac{\phi \wedge \psi}{\phi}\ \wedge\mathrm{e}_1 \qquad \frac{\phi \wedge \psi}{\psi}\ \wedge\mathrm{e}_2$$

Or

$$\frac{\phi}{\phi \vee \psi}\ \vee\mathrm{i}_1 \qquad \frac{\psi}{\phi \vee \psi}\ \vee\mathrm{i}_2 \qquad \frac{\phi \vee \psi \qquad \begin{array}{|c|}\hline \phi \\ \vdots \\ \chi \\ \hline \end{array} \qquad \begin{array}{|c|}\hline \psi \\ \vdots \\ \chi \\ \hline \end{array}}{\chi}\ \vee\mathrm{e}$$

Implication

$$\frac{\begin{array}{|c|}\hline \phi \\ \vdots \\ \psi \\ \hline \end{array}}{\phi \to \psi}\ \to\mathrm{i} \qquad \frac{\phi \quad \phi \to \psi}{\psi}\ \to\mathrm{e}$$

Negation

$$\frac{\begin{array}{|c|}\hline \phi \\ \vdots \\ \bot \\ \hline \end{array}}{\neg\phi}\ \neg\mathrm{i} \qquad \frac{\phi \quad \neg\phi}{\bot}\ \neg\mathrm{e}$$

Contradiction

no introduction

$$\frac{\bot}{\phi}\ \bot\mathrm{e}$$

Double negation

$$\frac{\phi}{\neg\neg\phi}\ \neg\neg\mathrm{i} \qquad \frac{\neg\neg\phi}{\phi}\ \neg\neg\mathrm{e}$$

Equality

$$\frac{}{t = t}\ =\mathrm{i} \qquad \frac{a = b \quad \phi[a/x]}{\phi[b/x]}\ =\mathrm{e}$$

Forall

$$\frac{\begin{array}{|l|}\hline x_0 \qquad \vdots \\ \qquad\quad \phi[x_0/x] \\ \hline \end{array}}{\forall x \phi}\ \forall x\,\mathrm{i} \qquad \frac{\forall x \phi}{\phi[t/x]}\ \forall x\,\mathrm{e}$$

Exists

$$\frac{\phi[t/x]}{\exists x \phi}\ \exists x\,\mathrm{i} \qquad \frac{\exists x \phi \qquad \begin{array}{|l|}\hline x_0 \quad \phi[x_0/x] \\ \qquad\quad \vdots \\ \qquad\quad \chi \\ \hline \end{array}}{\chi}\ \exists x\,\mathrm{e}$$

$$\frac{\phi \to \psi \quad \neg\psi}{\neg\phi}\ \mathrm{MT} \qquad \frac{\begin{array}{|c|}\hline \neg\phi \\ \vdots \\ \bot \\ \hline \end{array}}{\psi}\ \mathrm{PBC} \qquad \frac{}{\phi \vee \neg\phi}\ \mathrm{LEM}$$