NATURAL DEDUCTION FOR PREDICATE LOGIC

AND
$$\frac{\phi \quad \psi}{\phi \land \psi} \land i \qquad \frac{\phi \land \psi}{\phi} \land e_1 \quad \frac{\phi \land \psi}{\psi} \land e_2$$

$$OR \qquad \frac{\phi}{\phi \lor \psi} \lor i_1 \quad \frac{\psi}{\phi \lor \psi} \lor i_2 \qquad \frac{\phi \lor \psi}{\chi} \quad \frac{\psi}{\chi} \quad \frac{\psi}{\chi} \lor e$$

$$IMPLICATION \qquad \frac{\phi}{\phi \to \psi} \to e$$

$$NEGATION \qquad \frac{\phi}{\phi \to \psi} \to e$$

$$CONTRADICTION \qquad \frac{\phi}{\neg \phi} \to e$$

$$DOUBLE NEGATION \qquad \frac{\phi}{\neg \neg \phi} \to e$$

$$EQUALITY \qquad t = t = i \qquad \frac{a = b \quad \phi[a/x]}{\phi[b/x]} = e$$

$$FORALL \qquad \frac{\psi}{\forall x \phi} \quad \forall x i \qquad \frac{\forall x \phi}{\phi[t/x]} \quad \forall x e$$

$$EXISTS \qquad \frac{\phi[t/x]}{\exists x \phi} \exists x i \qquad \frac{x_0 \quad \psi[x_0/x]}{\chi} \quad \exists x$$

$$\frac{\phi \to \psi \quad \neg \psi}{\neg \phi} \text{ MT} \qquad \frac{\phi \lor \neg \phi}{\downarrow} \text{ LEM}$$

Linear-time temporal logic

 $\mathcal{M} = (S, \to, L)$ a model and $\pi = s_1 \to \ldots$ a path in \mathcal{M} . $i \geq 1, \pi^i = s_i \to s_{i+1} \to \ldots$ $\mathcal{M}, s \models \phi$ if for every path π of \mathcal{M} starting in s we have $\pi \models \phi$.

- 1. $\pi \models \top$
- 2. $\pi \not\models \bot$
- 3. $\pi \models p \text{ iff } p \in L(s_1)$
- 4. $\pi \models \neg \phi \text{ iff } \pi \not\models \phi$
- 5. $\pi \models \phi \land \psi$ iff $\pi \models \phi \land \pi \models \psi$
- 6. $\pi \models \phi \lor \psi$ iff $\pi \models \phi \lor \pi \models \psi$
- 7. $\pi \models \phi \rightarrow \psi$ iff $\pi \models \psi$ whenever $\pi \models \phi$
- 8. $\pi \models X \phi \text{ iff } \pi^2 \models \phi$
- 9. $\pi \models G \phi$ iff, for all $i \geq 1, \pi^i \models \phi$
- 10. $\pi \models F \phi$ iff there is some $i \ge 1$ such that $\pi^i \models \phi$
- 11. $\pi \models \phi \cup \psi$ iff there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$
- 12. $\pi \models \phi \ W \ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \psi$ and for all $j = 1, \ldots, i-1$ we have $\pi^j \models \phi$; or for all $k \geq 1$ we have $\pi^k \models \phi$
- 13. $\pi \models \phi \ \mathbb{R} \ \psi$ iff either there is some $i \geq 1$ such that $\pi^i \models \phi$ and for all $j = 1, \ldots, i$ we have $\pi^j \models \psi$, or for all $k \geq 1$ we have $\pi^k \models \psi$

Computational tree logic

 $\mathcal{M} = (S, \to, L)$ a model, $s \in S$, ϕ, ψ CTL formulas.

- 1. $\mathcal{M}, s \models \top$ and $\mathcal{M}, s \not\models \bot$
- 2. $\mathcal{M}, s \models p \text{ iff } p \in L(s)$
- 3. $\mathcal{M}, s \models \neg \phi \text{ iff } \mathcal{M}, s \not\models \phi$
- 4. $\mathcal{M}, s \models \phi \land \psi \text{ iff } \mathcal{M}, s \models \phi \land \mathcal{M}, s \models \psi$
- 5. $\mathcal{M}, s \models \phi \lor \psi \text{ iff } \mathcal{M}, s \models \phi \lor \mathcal{M}, s \models \psi$
- 6. $\mathcal{M}, s \models \phi \rightarrow \psi \text{ iff } \mathcal{M}, s \models \psi \text{ whenever } \mathcal{M}, s \models \phi$
- 7. $\mathcal{M}, s \models AX\phi$ iff for all s_1 such that $s \to s_1$ we have $\mathcal{M}, s_1 \models \phi$. 'In every next state.'
- 8. $\mathcal{M}, s \models \mathrm{EX}\phi$ iff for some s_1 such that $s \to s_1$ we have $\mathcal{M}, s_1 \models \phi$. 'In some next state.'
- 9. $\mathcal{M}, s \models AG\phi$. For all paths starting in $s \phi$ holds globally.
- 10. $\mathcal{M}, s \models \mathrm{EG}\phi$. There exists a path starting in s where ϕ holds globally.
- 11. $\mathcal{M}, s \models AF\phi$. For all paths starting in s there is some future state where ϕ holds.
- 12. $\mathcal{M}, s \models \text{EF}\phi$. For all paths beginning in s there is some future state where ϕ holds.
- 13. $\mathcal{M}, s \models A[\phi U \psi]$. All paths starting in s satisfies $\phi U \psi$.
- 14. $\mathcal{M}, s \models \mathrm{E}[\phi \mathrm{U}\psi]$. There exists some path starting in s where $\phi \mathrm{U}\psi$ holds.