Energy-Aware Ergodic Search: Continuous Long-Term Exploration for Multiagent Systems

Adam Seewald[1], Marvin Chancán[1], Cameron J. Lerch[1], Hector Castillo[1], Aaron M. Dollar[1], and Ian Abraham[1]

***Abstract*—**

***Index Terms*—Motion and Path Planning; Energy and Environment-Aware Autonomation.**

I. Introduction

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II. Problem Formulation

This work addresses the problem of exploring a bounded space with multiple agents, continuously, and proportionally to a spatial distribution. In the remainder of the text, we will use the term "continuously" to indicate that there is at least one agent that is exploring the space at all times.

Canonical ergodic search [1] does not deal with continuous exploration. It derives an agent's control so that its trajectory maximizes an ergodic metric defined in the spectral domain [2].

Problem II.1 (Ergodic search)**.** Consider a bounded space $\mathcal{Q} \subset \mathbb{R}^D$ of dimension D with $D \in \mathbb{N}_{>0}$ and a spatial distribution ϕ. *Ergodic search problem* is the problem of deriving a control action $\mathbf{u}(t) \in \mathcal{U} \subset \mathbb{R}^V$ with $V \in \mathbb{N}_{>0}$ so that the trajectory $\mathbf{q}(t) \in \mathcal{Q}$ is proportional to the spatial distribution ϕ.

Here the notation $\mathbb{R}$ and $\mathbb{N}$ indicates reals and naturals, $\mathbb{N}_{>0}$ strictly naturals. Bold notation is used for vectors.

We extend the canonical ergodic search problem to multi-agent continuous ergodic search, i.e., exploration with multiple agents under spatial distribution and battery constraints.

Problem II.2 (Multi-agent continuous ergodic search)**.** Consider a set of n agents $\boldsymbol{\alpha} := \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, a bounded space $\mathcal{Q}$, and a spatial distribution ϕ similar to Problem II.1. *Multi-agent continuous ergodic search problem* is the problem of deriving each agent ${}^j\alpha$ control action ${}^j\mathbf{u}(t)$ so that its trajectory ${}^j\mathbf{q}(t)$ is proportional to the spatial distribution ϕ on a continuous time horizon.

We will provide a solution to Problem II.2 (see Sec. III), assuming that there are one or more areas in $\mathcal{Q}$ – i.e., charging stations – where the agents $\boldsymbol{\alpha}$ can land and recharge the battery, e.g., using wireless charging (see Sec. IV).

[1]A. S., M. C., C. J. L., H. C., A. M. D., and I. A. are with the Department of Mechanical Engineering and Materials Science, Yale University, CT, USA. Email: adam.seewald@yale.edu;

III. Methods

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A. Ergodic search

For the purposes of defining the spatial distribution in both Problem II.1 and II.2, let us consider a Gaussian mixture model (GMM)

$$\phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, \mu_k, \Sigma_k), \tag{1}$$

composed of m Gaussians. Each has a covariance matrix $\Sigma_k \in \mathbb{R}^{D\times D}$, a center $\mu_k \in \mathcal{Q}$, and a positive mixing coefficient $\delta_k \in \boldsymbol{\delta}$ such that the sum of the δs is less or equal to one. They indicate how well is each Gaussian in the GMM considered.

The goal of the ergodic search is to minimize an ergodic metric [1]

$$\mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t)) := \frac{1}{2} \sum_{k\in\mathcal{K}} \Lambda_k \big(c_k(\mathbf{q}(t)) - \phi_k(\boldsymbol{\delta})\big)^2, \tag{2}$$

where ϕ_k are coefficients derived utilizing the Fourier series on the spatial distribution ϕ and c_k on the trajectory $\mathbf{q}(t)$. They are detailed in Equation (6) and (4) respectively. Λ_k is a weight factor, i.e., if

$$\Lambda_k = (1 + \|k\|^2)^{(-D-1)/2}, \tag{3}$$

lower frequencies have more weight [3]. $\mathcal{K} \in \mathbb{N}^D$ is a set of index vectors that covers $[K] \times \cdots \times [K] \in \mathbb{N}^{K^D}$ where K is a given number of frequencies including the fundamental frequency [2]. The notation $[K]$ indicates positive naturals up to K.

The coefficients c_k are derived using the Fourier series basis function. If we consider the trigonometric form, they can be expressed

$$c_k(\mathbf{q}(t)) := \int_{\mathcal{T}} \frac{1}{L^D} \prod_{d\in[D]_{>0}} \big(\cos(k_d\, \mathbf{q}_d(\tau)\, \psi) - i \sin(k_d\, \mathbf{q}_d(\tau)\, \psi)\big)\, d\tau / t, \tag{4}$$

where ψ is $2\pi/L$ for a given period $L \in \mathbb{R}_{>0}$, i is the imaginary unit, k_d is the dth item of k, and $\mathbf{q}_d$ the dth item of $\mathbf{q}$.

$\mathcal{T}$ is built so that the integration is between $\tau = t_0$ and t, and the notation $[D]_{>0}$ indicates strictly positive naturals up to D.

c_k is evaluated per each k in $\mathcal{K}$ in Eq. (2).

For the purposes of deriving the coefficients ϕ_k, let us consider the GMM model in Eq. (1) on a search space $\mathcal{Q}$.

The space is further bounded to a symmetric set $[-L/2, L/2]^D$ since the Gaussians are symmetric about the zero axes. The resulting new model is then

$$\Phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{d\in[2^D]_{>0}} \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, A_d\mu_k, A_d\Sigma_k A_d^T)/2^D, \quad (5)$$

where $A_d \in \mathbb{R}^{D\times D}$ are linear transformation matrices [2]. Let us call the integrand in Eq. (4) $c : \mathcal{Q} \longrightarrow \mathbb{R}^K$. It maps the space to the spectral domain. The equivalent of Eq. (4) for the spatial distribution can be then expressed

$$\phi_k(\boldsymbol{\delta}) := \int_{\mathcal{Q}} \Phi(\boldsymbol{\delta}, \mathbf{q})\, c(\mathbf{q}) \; d\mathbf{q}. \quad (6)$$

$\mathcal{Q}$ is built so that the integration is within the points of the bounded symmetric set $\mathbf{q} \in [-L/2, L/2]^D$.

ϕ_k is evaluated per each k in $\mathcal{K}$ in Eq. (2).

Let us first formulate the solution for Problem II.1, borrowed by canonical ergodic search. If the agent's dynamics is described by a generic differential equation $\dot{\mathbf{q}}(t) = f(\mathbf{q}(t), \mathbf{u}(t))$, an optimal control problem (OCP) that selects an ergodic control action can be formulated as [4]

$$\begin{aligned}
&\min_{\mathbf{q}(t),\mathbf{u}(t)} \int_{\mathcal{T}} \mathbf{u}(\tau)^T R \mathbf{u}(\tau)\, d\tau + \mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t)), && (7a)\\
&\text{s.t. } \dot{\mathbf{q}} = f(\mathbf{q}(t), \mathbf{u}(t)), && (7b)\\
&\quad \mathbf{q}(t) \in \mathcal{Q},\ \mathbf{u}(t) \in \mathcal{U}, && (7c)\\
&\quad \mathbf{q}(t_0), \mathbf{q}(t_f) \text{ are given}, && (7d)
\end{aligned}$$

where the ergodic metric is derived in Eq. (2), $R \in \mathbb{R}^{V\times V}$ is a control penalizing diagonal positive-definite matrix, and t_0, t_f are respectively the first and last time instants. $\mathcal{T}$ is $[t_0, t_f)$.

To formulate the solution to Problem II.2, let us first extend the OCP in Eq. (7) to multiagent systems. Eq. (7a) becomes

$$\min_{\square} \sum_{k=1}^{n} \left(\int_{\mathcal{T}_k} {}^k\mathbf{u}(\tau)^T R_k \, {}^k\mathbf{u}(\tau)\, d\tau + \mathcal{E}(\boldsymbol{\delta}, {}^k\mathbf{q}(t)) \right), \quad (8)$$

where the ergodic metric and the control penalizing term R_k are now agent-specific. The term $\square$ is ${}^1\mathbf{q}(t), {}^2\mathbf{q}(t), \ldots, {}^n\mathbf{q}(t), {}^1\mathbf{u}(t), {}^2\mathbf{u}(t), \ldots {}^n\mathbf{u}(t)$. $\mathcal{T}_k$ is $[{}^kt_0, {}^kt_f)$, i.e., different agents might have different duration.

Let us consider a vector $\mathbf{b} \in \mathbb{R}^3$ – which is detailed later in Section III-B – whose trajectory $\mathbf{b}(t)$ describes the battery metrics' evolution in time. If $\mathbf{b}_{\text{SoC}}$ is the value of the vector that expresses the battery state of charge (SoC), the expression in Eq. (8) might select ergodic metrics corresponding to trajectories that are impossible to traverse in the $\mathbf{b}_{\text{SoC}} \in (0, 1]$ domain, i.e., one or more agents' state at $\mathbf{q}(t_f)$ will not satisfy Eq. (7d).

In order to satisfy the battery SoC domain and always keep at least one agent exploring, an OCP must satisfy an additional constrain

$$\exists k \in [n] \text{ s.t. } {}^k\mathbf{b}_{\text{SoC}}(t_f) \in (0, b_f], \quad (9)$$

where $b_f \in (0, 1) \subset \mathbb{R}_{>0}$ is a given desired battery SoC at the final time instant.

Finally, let us consider the realistic assumption that the optimization horizon $N \in \mathbb{R}_{>0}$ is known and is, e.g., an empirically collected value that corresponds to one of the agents' discharge times (see Sec. IV).

The OCP that provides a solution to Problem II.2 can be formulated as

$$\begin{aligned}
&\min_{\square,\boldsymbol{\delta}} \sum_{k=1}^{n} \int_{\mathcal{T}_k} {}^k\mathbf{u}(\tau)^T R_k \, {}^k\mathbf{u}(\tau)\, d\tau - \sum_{k=1}^{m} \delta_k, && (10a)\\
&\text{s.t. } {}^1\dot{\mathbf{q}}(t) = f_1({}^1\mathbf{q}(t), {}^1\mathbf{u}(t)), \ldots, {}^n\dot{\mathbf{q}}(t) = f_n({}^n\mathbf{q}(t), {}^n\mathbf{u}(t)), && (10b)\\
&\quad {}^1\mathbf{q}(t), \ldots, {}^n\mathbf{q}(t) \in \mathcal{Q},\ {}^1\mathbf{u}(t), \ldots, {}^n\mathbf{u}(t) \in \mathcal{U}, && (10c)\\
&\quad \exists k \in [n] \text{ s.t. } {}^k\mathbf{b}_{\text{SoC}}(t_f) \in (0, b_f], && (10d)\\
&\quad \forall k \ \mathcal{E}(\boldsymbol{\delta}, {}^k\mathbf{q}(t)) \leq \gamma, && (10e)\\
&\quad g_1({}^1\mathbf{q}(t), {}^1\mathbf{u}(t)) \leq 0, \ldots, g_n({}^n\mathbf{q}(t), {}^n\mathbf{u}(t)) \leq 0, && (10f)\\
&\quad g_\delta(\boldsymbol{\delta}) \leq 0, && (10g)\\
&\quad {}^1\mathbf{q}(t_0), {}^1\mathbf{q}(t_f), \ldots, {}^n\mathbf{q}(t_0), {}^n\mathbf{q}(t_f), b_f, \gamma \text{ are given}, && (10h)
\end{aligned}$$

where constraints in Eq. (10f) and (10g) are optional and express additional requirements, e.g., that there is always at least one agent exploring $\mathcal{Q}$, the agents explore the space two-by-two, etc. (see Sec. IV).

In Eq. (10), the ergodic metric is integrated into the constraint as proposed in [5]. The cost contains further mixing coefficient $\boldsymbol{\delta}$ in Eq. (1) – so that one can find tradeoffs between the single Guassians, the different agents, and the battery SoC (see Sec IV).

B. Battery modeling

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IV. Experimental Results

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V. Conclusion and Future Directions

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References

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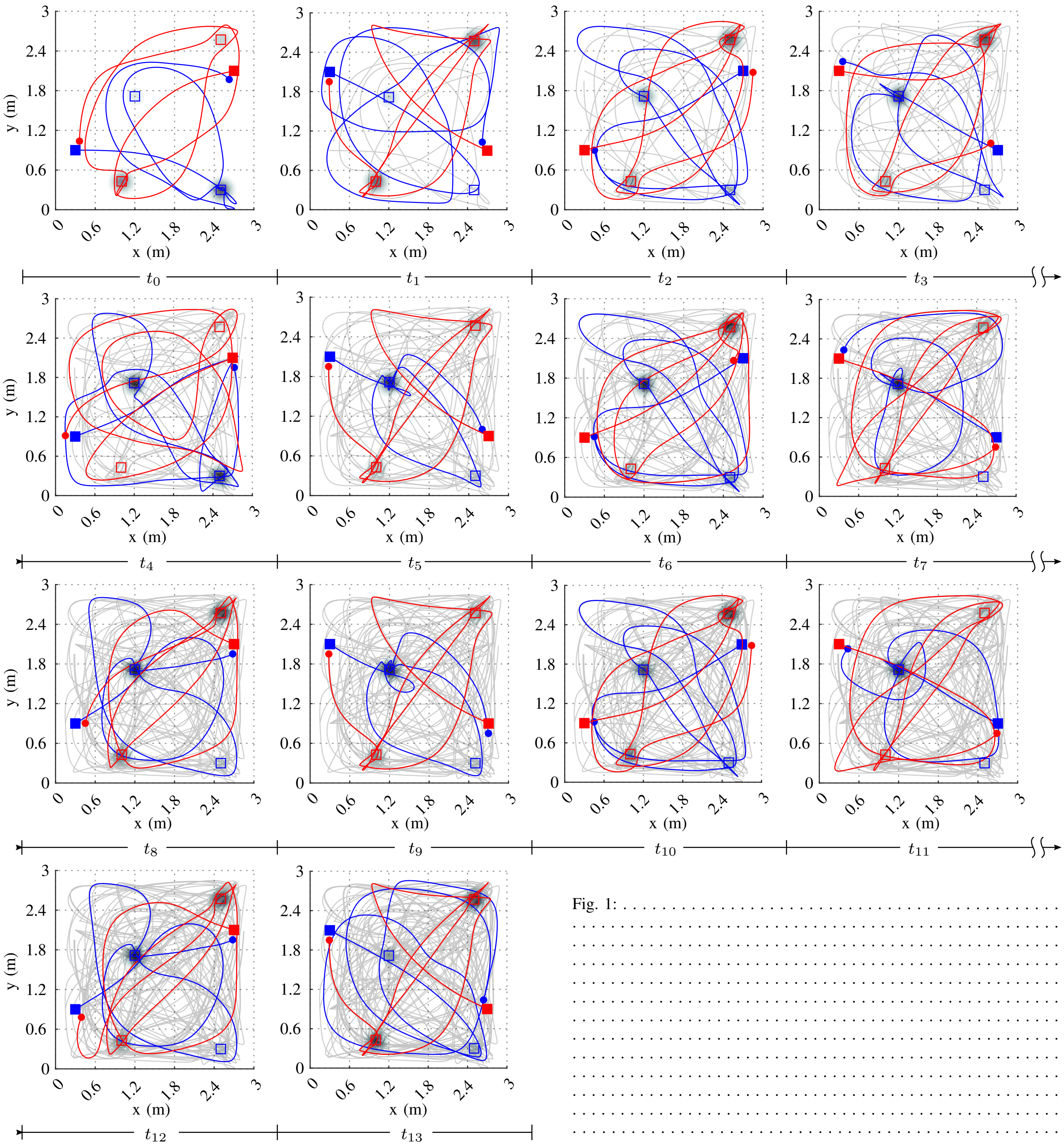

Fig. 1: . . . .