Energy-Aware Ergodic Search: Continuous Exploration for Multi-Agent Systems with Battery Constraints

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***Abstract*—Exploring space without interruption is important in scenarios such as search and rescue and precision agriculture. Ergodic search derives continuous and uninterrupted trajectories in these scenarios so that the robots spend more time in areas with high information density. However, existing literature on ergodic search does not consider the robot's energy constraints. If the robots are battery-powered, it is physically not possible to explore the space on a single battery charge. Our paper tackles this challenge, integrating ergodic search methods with energy-aware coverage. We trade off battery usage and coverage quality, aiming to maintain uninterrupted exploration of a given space by at least one agent. Our approach derives an abstract battery model for future state-of-charge estimation and extends canonical ergodic search to ergodic search under battery constraints. Empirical data from simulations and real-world experiments demonstrate the effectiveness of our energy-aware ergodic search, which ensures continuous and uninterrupted exploration and guarantees spatial coverage.**

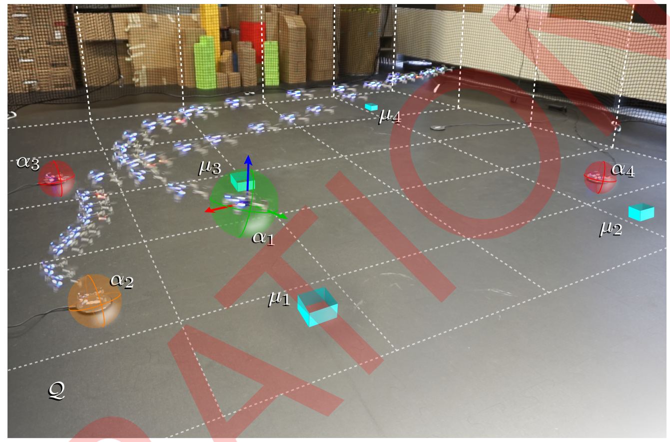

Fig. 1: A set of agents explore a space $\mathcal{Q}$, focusing on areas with high information density μ_1, μ_2, μ_3, and μ_4, employing ergodic search. The exploration is continuous and uninterrupted, guaranteeing that there is always one agent exploring – α_1, whereas α_2, α_3, and α_4 are recharging. The colors of the spheres indicate the state of charge.

I. Introduction

Robotic exploration is a recurring problem in different scenarios. It typically involves coverage path planning (CPP), which deals with deriving robots' trajectories that traverse every point in a given space [1–3]. Within CPP, ergodic search is a recent and promising direction [4–17], as it enhances the efficiency of traditional CPP by optimizing the time a robot spends in a given region w.r.t. an information measure. As a result, ergodic search derives trajectories so that the robots spend more time in areas with high information density while quickly passing areas with low information density [13, 18]. The user can specify areas of interest, e.g., where the robots should spend more time exploring in a search and rescue scenario [14], where the robots should collect more data in a precision agriculture scenario [16], etc. Figure 1 shows our setup, which resembles a search and rescue scenario. Four agents explore a space where the high information density is represented by cyan boxes. Some agents are actively exploring, while others are grounded and recharging.

Prior literature has been studying ergodic search in manipulation [8], tactile sensing [15], stochastic dynamics [7, 17], distributed information [5], time optimality [14], and active learning [4]. Ergodic search for multi-agent systems [9, 10] has been applied in conjunction with low-information sensors [10–12], swarms control [9], obstacles [11], and decentralized systems [12]. Ergodic search has been proven successful in use cases involving urban environments [13] and information gathering [6]. While prior literature includes ergodic search methods in a variety of settings, energy constraints have not been studied yet. Partly due to these constraints, the uninterrupted exploration that considers a spatial distribution is currently hindered.

Canonical ergodic search indeed derives continuous exploration [5, 15, 19], but it is physically not possible to continue exploring on a single battery charge. Scenarios involving CPP, however, often require that the space is covered continuously. This work enhances the current ergodic search literature by incorporating more traditional energy-aware CPP approaches [20–25], battery- and energy-aware planning [26–29], and planning of energy trade-offs [30, 31]. It answers the question: *Is it possible to tradeoff battery and coverage quality so that there is at least one agent exploring at all times*?

To answer the question, our approach derives an abstract battery model [32] for battery state of charge (SoC) estimation at future time instances. We first adapt canonical ergodic search to multi-agent ergodic search [9, 10]. We then utilize the formulation to propose energy-aware ergodic search, i.e., ergodic search under battery constraints. The exploration is continuous and uninterrupted, employing a finite horizon framework reminiscent of a model predictive controller formulation [29]. Experimental data from simulations and real-world experiments show the impact of our newly proposed energy-aware ergodic search. We show with empirical evidence that we can effectively explore a space, there is always one agent exploring, the spatial distribution is satisfied, and the exploration is continuous and uninterrupted (see Section IV). The detailed results from the experimental

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IN PREPARATION

evaluation and the code to replicate our approach are made available on the project repository webpage[1].

The remainder of the paper is structured as follows. Sec. II formulates the problem of multi-agent continuous ergodic search. Sec. III discusses the methods for both the canonical ergodic search and a battery model enhanced ergodic search. Sec. V concludes and proposes future directions.

II. Problem Formulation

This work addresses the problem of exploring a bounded space with multiple agents, continuously, and proportionally to a spatial distribution. In the remainder of the text, we will use the term "continuously" to indicate that there is at least one agent that is exploring the space at all times.

Canonical ergodic search [4–8, 18, 19] does not deal with continuous exploration. It derives an agent's control – or analogously multiple agents control [9–13] – so that its trajectory maximizes an ergodic metric defined in the spectral domain [33].

Problem II.1 (Ergodic search)**.** Consider a bounded space $\mathcal{Q} \subset \mathbb{R}^D$ of dimension $D \in \mathbb{N}_{>0}$ and a spatial distribution ϕ s.t. $\int_{\mathcal{Q}} \phi \, d\mathbf{q}$, $\phi(\mathbf{q}) \geq 0 \;\; \forall \mathbf{q} \in \mathcal{Q}$. *Ergodic search problem* is the problem of deriving a control action $\mathbf{u}(t) \in \mathcal{U} \subset \mathbb{R}^V$ with $V \in \mathbb{N}_{>0}$ so that the trajectory $\mathbf{q}(t) \in \mathcal{Q}$ is proportional to the spatial distribution ϕ via an ergodic metric.

Here the notation $\mathbb{R}$ and $\mathbb{N}$ indicates reals and naturals, $\mathbb{N}_{>0}$ strictly naturals. Bold notation is used for vectors.

Definition II.1 (Ergodic metric)**.** Consider a time average distribution that describes where the robot spends more time $\int_{\mathcal{T}} \Delta\big(s - \mathbf{q}(t)\big)\, dt \,/t_f$, where $s \in \mathcal{Q}$ is a spatial statistics [12], Δ is a Dirac delta function, and $t_f \in \mathbb{R}_{>0}$ the final time instant. An *ergodic metric* from Problem II.1 relates the average distribution and ϕ.

We will derive an ergodic metric in Equation (2) in Sec. III-A.

We extend the canonical ergodic search problem to multi-agent continuous ergodic search, i.e., uninterrupted exploration with multiple agents under spatial distribution and battery constraints.

Problem II.2 (Multi-agent continuous ergodic search)**.** Consider a set of n agents $\boldsymbol{\alpha} := \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, a bounded space $\mathcal{Q}$, and a spatial distribution ϕ similar to Problem II.1. *Multi-agent continuous ergodic search problem* is the problem of deriving each agent ${}^j\alpha$ control action ${}^j\mathbf{u}(t)$ so that its trajectory ${}^j\mathbf{q}(t)$ is proportional to the spatial distribution ϕ and $\exists \alpha_k \in \boldsymbol{\alpha}$, ${}^k\mathbf{q}(t)$ s.t. the problem is satisfied, i.e., there is at least one agent exploring at all times.

We will provide a solution to Problem II.2 (see Sec. III), assuming that there are one or more areas in $\mathcal{Q}$ – namely, charging stations – where the agents $\boldsymbol{\alpha}$ can land and recharge the battery, using, e.g., wireless charging (see Sec. IV).

[1] github.com/adamseew/enerergo

III. Methods

In this section, we discuss the methods utilized in this work for continuous, uninterrupted exploration with multiple agents and proportionally to a spatial distribution

We discuss how to achieve the latter in Sec. III-A and the former in Sec. III-B.

A. Ergodic search

To derive an agent's trajectory proportionally to a spatial distribution, canonical ergodic search first requires defining the distribution ϕ.

For this purpos, in both Problem II.1 and II.2, let us consider a Gaussian mixture model (GMM)

$$\phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, \mu_k, \Sigma_k), \tag{1}$$

composed of m Gaussians. Each has a covariance matrix $\Sigma_k \in \mathbb{R}^{D \times D}$, a center $\mu_k \in \mathcal{Q}$, and a positive mixing coefficient $\delta_k \in \boldsymbol{\delta}$ such that the sum of the δs is less or equal to one. They indicate how well is each Gaussian in the GMM considered.

The goal of the ergodic search is to minimize an ergodic metric [18] (see Definition II.1)

$$\mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t)) := \frac{1}{2} \sum_{k \in \mathcal{K}} \Lambda_k \big(c_k(\mathbf{q}(t)) - \phi_k(\boldsymbol{\delta})\big)^2, \tag{2}$$

where ϕ_k are coefficients derived utilizing the Fourier series on the spatial distribution ϕ and c_k on the trajectory $\mathbf{q}(t)$. They are detailed in Eq. (6) and (4) respectively. Λ_k is a weight factor. That is, if

$$\Lambda_k = (1 + \|k\|^2)^{(-D-1)/2}, \tag{3}$$

lower frequencies have more weight [5]. $\mathcal{K} \in \mathbb{N}^D$ is a set of index vectors that covers $[K] \times \cdots \times [K] \in \mathbb{N}^{K^D}$ where K is a given number of frequencies including the fundamental frequency [33]. The notation $[K]$ indicates positive naturals up to K.

The coefficients c_k are derived using the Fourier series basis function. If we consider the trigonometric form, they can be expressed

$$c_k(\mathbf{q}(t)) := \int_{\mathcal{T}} \frac{1}{L^D} \prod_{d \in [D]_{>0}} \begin{aligned} &\big(\cos(k_d \, \mathbf{q}_d(\tau)\, \psi) \\ &- i \sin(k_d \, \mathbf{q}_d(\tau)\, \psi)\big) \, d\tau / t, \end{aligned} \tag{4}$$

where ψ is $2\pi/L$ for a given period $L \in \mathbb{R}_{>0}$, i is the imaginary unit, k_d is the dth item of k, and $\mathbf{q}_d$ the dth item of $\mathbf{q}$.

$\mathcal{T}$ is built so that the integration is between $\tau = t_0$ and t, and the notation $[D]_{>0}$ indicates strictly positive naturals up to D.

c_k is evaluated per each k in $\mathcal{K}$ in Eq. (2).

To derive the coefficients ϕ_k, let us consider the GMM model in Eq. (1) on a search space $\mathcal{Q}$. The space is further bounded to a symmetric set $[-L/2, L/2]^D$ since the Gaussians are symmetric about the zero axes. The resulting new model is then

$$\Phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{d \in [2^D]_{>0}} \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, A_d \mu_k, A_d \Sigma_k A_d^T)/2^D, \tag{5}$$

IN PREPARATION

where $A_d \in \mathbb{R}^{D\times D}$ are linear transformation matrices [33].

Let us call the integrand in Eq. (4) $c : \mathcal{Q} \longrightarrow \mathbb{R}^K$. It maps the space to the spectral domain. The equivalent of Eq. (4) for the spatial distribution can be then expressed

$$\phi_k(\boldsymbol{\delta}) := \int_{\mathcal{Q}} \Phi(\boldsymbol{\delta}, \mathbf{q})\, c(\mathbf{q})\; d\mathbf{q}. \tag{6}$$

$\mathcal{Q}$ is built so that the integration is within the points of the bounded symmetric set $\mathbf{q} \in [-L/2, L/2]^D$.

ϕ_k is evaluated per each k in $\mathcal{K}$ in Eq. (2).

Let us first formulate the solution to Problem II.1, utilizing a formulation borrowed from canonical ergodic search. If the agent's dynamics is described by a generic differential equation $\dot{\mathbf{q}}(t) = f\big(\mathbf{q}(t),\, \mathbf{u}(t)\big)$, an optimal control problem (OCP) that selects an ergodic control action can be formulated as [17]

$$\min_{\mathbf{q}(t),\mathbf{u}(t)} \int_{\mathcal{T}} \mathbf{u}(\tau)^T R\mathbf{u}(\tau)\, d\tau + \mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t)), \tag{7a}$$

$$\text{s.t. } \dot{\mathbf{q}} = f(\mathbf{q}(t), \mathbf{u}(t)), \tag{7b}$$

$$\mathbf{q}(t) \in \mathcal{Q},\ \mathbf{u}(t) \in \mathcal{U}, \tag{7c}$$

$$\mathbf{q}(t_0), \mathbf{q}(t_f) \text{ are given}, \tag{7d}$$

where the ergodic metric is derived in Eq. (2), $R \in \mathbb{R}^{V\times V}$ is a control penalizing diagonal positive-definite matrix, and t_0, t_f are respectively the first and last time instants. $\mathcal{T}$ is $[t_0, t_f)$.

To formulate the solution to Problem II.2, let us first extend the OCP in Eq. (7) to multi-agent systems [10]. Eq. (7a) becomes

$$\min_{\square}\ \frac{1}{n}\sum_{k=1}^{n}\left(\int_{\mathcal{T}_k} {}^k\mathbf{u}(\tau)^T R_k\, {}^k\mathbf{u}(\tau)\, d\tau + \mathcal{E}(\boldsymbol{\delta}, {}^k\mathbf{q}(t))\right), \tag{8}$$

where the ergodic metric and the control penalizing term R_k are now agent-specific. The term $\square$ is ${}^1\mathbf{q}(t), {}^2\mathbf{q}(t), \dots,$ ${}^n\mathbf{q}(t), {}^1\mathbf{u}(t), {}^2\mathbf{u}(t), \dots {}^n\mathbf{u}(t)$. $\mathcal{T}_k$ is $[{}^kt_0, {}^kt_f)$, i.e., different agents might have different duration.

Let us consider a vector $\mathbf{b} \in \mathbb{R}^3$ – which is detailed later in Sec. III-B – whose trajectory $\mathbf{b}(t)$ describes the battery metrics' evolution in time. If $\mathbf{b}_{\text{SoC}}$ is the value of the vector that expresses the battery SoC, the expression in Eq. (8) might select ergodic metrics corresponding to trajectories that are impossible to traverse in the $\mathbf{b}_{\text{SoC}} \in (0, 1]$ domain, i.e., one or more agents' state at $\mathbf{q}(t_f)$ will not satisfy Eq. (7d).

In order to satisfy the battery SoC domain and always keep at least one agent exploring, an OCP must satisfy an additional constrain

$$\exists k \in [n] \text{ s.t. } {}^k\mathbf{b}_{\text{SoC}}(t_f) \in (0, b_f], \tag{9}$$

where $b_f \in (0, 1] \subset \mathbb{R}_{>0}$ is a given desired battery SoC at the final time instant.

Finally, let us consider the realistic assumption that the optimization horizon $N \in \mathbb{R}_{>0}$ is known and is, e.g., an empirically collected value that corresponds to one of the agents' discharge times (see Sec. IV).

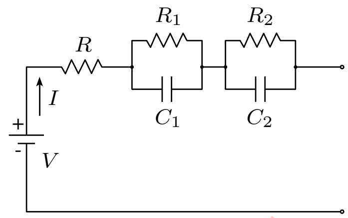

Fig. 2: Abstract equivalent circuit model for state-of-charge estimation [39]. The model consists of a second-order resistor-capacitor circuit with two resistors R_1 and R_2 and two capacitors C_1 and C_2 in two separate circuit elements. An additional resistor R is also employed.

The OCP that provides a solution to Problem II.2 can be formulated as

$$\min_{\square}\ \frac{1}{n}\sum_{k=1}^{n}\int_{\mathcal{T}_k} {}^k\mathbf{u}(\tau)^T R_k\, {}^k\mathbf{u}(\tau)\, d\tau, \tag{10a}$$

$$\text{s.t. } {}^1\dot{\mathbf{q}}(t) = f_1({}^1\mathbf{q}(t), {}^1\mathbf{u}(t)), \dots, {}^n\dot{\mathbf{q}}(t) = f_n({}^n\mathbf{q}(t), {}^n\mathbf{u}(t)), \tag{10b}$$

$${}^1\mathbf{q}(t), \dots, {}^n\mathbf{q}(t) \in \mathcal{Q},\ {}^1\mathbf{u}(t), \dots, {}^n\mathbf{u}(t) \in \mathcal{U}, \tag{10c}$$

$$\exists k \in [n] \text{ s.t. } {}^k\mathbf{b}_{\text{SoC}}(t_f) \in (0, b_f], \tag{10d}$$

$$\forall k\ \mathcal{E}(\boldsymbol{\delta}, {}^k\mathbf{q}(t)) \le \gamma, \tag{10e}$$

$$g_1(\boldsymbol{\delta}, {}^1\mathbf{q}(t), {}^1\mathbf{u}(t)) \le 0, \dots, g_n(\boldsymbol{\delta}, {}^n\mathbf{q}(t), {}^n\mathbf{u}(t)) \le 0, \tag{10f}$$

$${}^1\mathbf{q}(t_0), {}^1\mathbf{q}(t_f), \dots, {}^n\mathbf{q}(t_0), {}^n\mathbf{q}(t_f), b_f, \gamma \text{ are given}, \tag{10g}$$

where constraints in Eq. (10f) are optional and express additional requirements, e.g., that there is always at least one agent exploring $\mathcal{Q}$, the agents explore the space two-by-two, etc. (see Sec. IV).

In Eq. (10), the ergodic metric is integrated into the constraint as proposed in [14].

Furthermore, the evolutions of the agents' states in time are described by generic differential equations ${}^k\dot{\mathbf{q}}(t) = f_k({}^k\mathbf{q}(t), {}^k\mathbf{u}(t))\ \forall k \in [n]$ [12].

B. Battery modeling

To derive a battery model for continuous exploration – a model that allows us to predict when an agent is exploring and when it conversely should be recharging the battery – let us consider an abstract equivalent circuit model (ECM). These models are commonly employed in battery metrics estimation for robots and other applications, especially if equipped with rechargeable battery cells [29, 34–38].

The ECM model we employ is a second-order resistor-capacitor (RC) circuit model [32], as illustrated in Fig. 2 [39].

Formally, it can be formulated as [32]

$$\dot{\mathbf{b}}(t) = \begin{bmatrix} -1/(R_1C_1) & 0 & 0 \\ 0 & -1/(R_2C_2) & 0 \\ 0 & 0 & 0 \end{bmatrix} \mathbf{b}(t) + \begin{bmatrix} 1/C_1 \\ 1/C_2 \\ -\zeta/Q \end{bmatrix} I(t), \tag{11}$$

where $\zeta \in \mathbb{R}$ is a battery coefficient [29], $R_1, R_2 \in \mathbb{R}$ and $C_1, C_2 \in \mathbb{R}$ are the resistors and capacitors relative to the first and second RC elements in the ECM measured in ohms and farad respectively. $Q \in \mathbb{R}$ is the battery nominal capacity measured in amperes per hour.

$I \in \mathbb{R}$ is then the internal current which is load-dependent, e.g., the current required to run the motors, actuators, etc. It is measured in amperes.

The state $\mathbf{b} := \begin{bmatrix} V_1 & V_2 & \mathbf{b}_{\text{SoC}} \end{bmatrix} \in \mathbb{R}^3$ contains three battery metrics. $V_1, V_2 \in \mathbb{R}$ are the voltages measured in volts across the first and second RC elements, and $\mathbf{b}_{\text{SoC}} \in (0, 1]$ is

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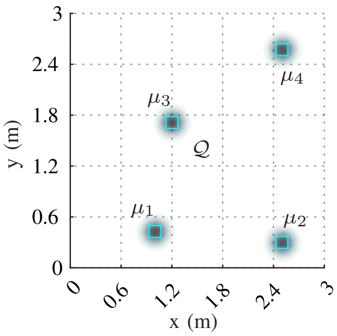

Fig. 3: Information spatial distribution and search space in our experimental evaluation. The distribution consists of four Gaussians in a Gaussian mixture model ϕ. The Gaussians are centered in μ_1, μ_2, μ_3, and μ_4, as depicted by the cyan empty squares. The search space $\mathcal{Q}$ is a three-by-three area. The resulting ergodic trajectories are expected to be s.t. the robot spends more time close to the Gaussians.

the normalized battery SoC that evolves from fully charged – or from a given initial value $\mathbf{b}_{\text{SoC}}(t_0)$ – to discharged.

The battery voltage at the extremes of the ECM $V_e \in \mathbb{R}$, measured in volts, can be then formulated as [32]

$$V_e(t) = V(\mathbf{b}_{\text{SoC}}(t)) - V_1(t) - V_2(t) - I(t)R, \qquad (12)$$

where $R \in \mathbb{R}$ is the single resistor measured in ohm in Fig. 2, and V is the open circuit voltage. It is a nonlinear function of the battery SoC and can be retrieved from the battery's data sheet [37].

The values of R_1, C_1, R_2, C_2, R are identified so that the model output and the physical behavior of the agents are matched as closely as possible [32] (see Sec. IV).

The battery model allows us to determine the battery SoC $\mathbf{b}_{\text{SoC}}$ in Eq. (10d), which is in turn utilized to find the control action $\mathbf{u}(t)$ so that there is at least one agent exploring $\mathcal{Q}$ at all times. This means that when the solution of the OCP in Eq. (10) is evaluated, the battery model in Eq. (11) is integrated for the duration of the horizon, whereas the recharging is approximated linear with the expression $\mathbf{b}_{\text{SoC}} = \eta\,\mathbf{b}_{\text{SoC}} + \theta$ for given $\eta, \theta \in \mathbb{R}$ determined empirically.

IV. Experimental Results

In this section, we discuss our experimental setup and results. Our experiments are implemented first in simulation using MATLAB (R), and physical experiments are implemented in Python and conducted using a set of Crazyflie 2.0 micro aerial vehicles (MAVs).

In both cases, the source code[1] is released under the popular non-commercial open-source license CC BY-NC-SA 4.0. The solution of the OCP in Eq. (10) relies on two external open-source components from the literature: the popular nonlinear programming solver IPOPT [40] and a software framework for nonlinear optimization called CasADi [41].

Each MAV is equipped with a positioning and wireless charging decks. Precise positioning of MAVs is achieved via two HTC SteamVR Base Station 2.0 units. Each MAV is then equipped with a one-cell 250 mAh 3.7 volts LiPo battery.

We evaluate our approach under two different scenarios. In both the scenarios, we use a three-by-three-meter space $\mathcal{Q}$. The spatial distribution ϕ contains four Gaussians in the GMM in Eq. (1), as illustrated in Fig. 3 (the four cyan empty squares).

Intermittent exploration

In the first scenario, an intermediate step towards uninterrupted and continuous exploration, one MAV is positioned on top of a wireless charging station at coordinates (1.5,1.5). The horizon is set to five minutes and is derived empirically along with battery and recharging coefficients. The battery values used in the scenario are those proposed in [32]. The number of frequencies K is set to nine, as in [33]. The ergodic trajectories at four different horizons t_0, t_4, t_8, and t_{12} are shown in Fig. 4. For horizons following the first, the past ergodic trajectories are illustrated in gray.

The MAV starts exploring the space (filled blue square in the figure) and derives the ergodic trajectory for the first horizon t_0. At the end of the horizon (filled blue dot), the MAV returns to the charging station. Once the battery is recharged – formally, once the constraint in Eq. (9) is satisfied – the exploration proceeds. The exploration derived in this way is, however, not uninterrupted, as it has periods of "no exploration" in which the agent recharges the battery. We show a continuous and uninterrupted exploration achieved by a multi-agent system in the following.

Continuous, uninterrupted exploration

In the second extensive scenario, four MAVs α_1, α_2, α_3, and α_4 are placed on top of four wireless charging stations. The optional constraints in Eq. (10f) are built so that each MAV covers two Gaussians at a time that are respectively

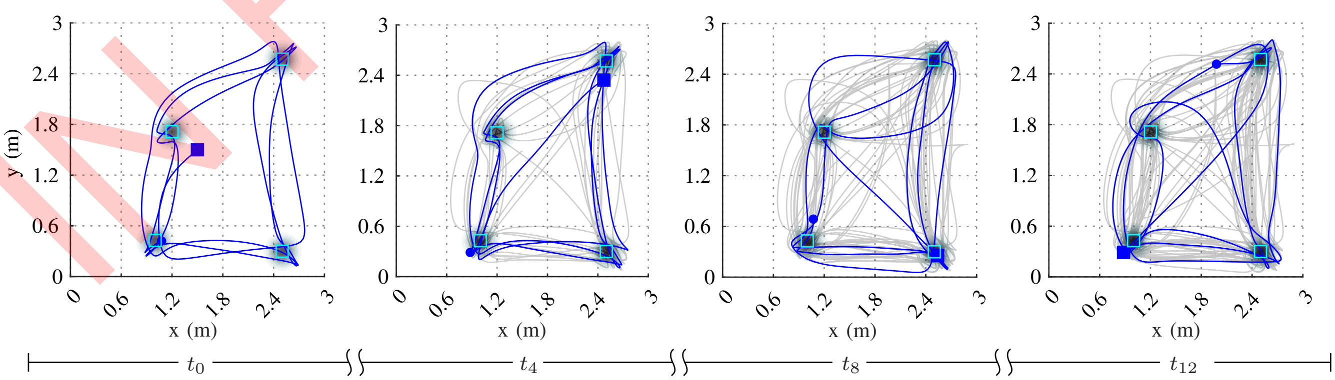

Fig. 4: Experimental evaluation of intermittent exploration. One agent α_1 is placed in the center of the search space $\mathcal{Q}$ at coordinates (1.5,1.5), on top of a wireless charging station (blue filled square). The agent explores the space in the first horizon t_0 (left of the figure), spending most of the time close to the Gaussians – the areas with high information density. The agent then returns to the charging station to recharge the battery. It proceeds for the following horizon, resuming the previous exploration. The figure depicts horizons t_0, t_4, t_8, and t_{12}. The exploration is not uninterrupted, conversely to Fig. 5.

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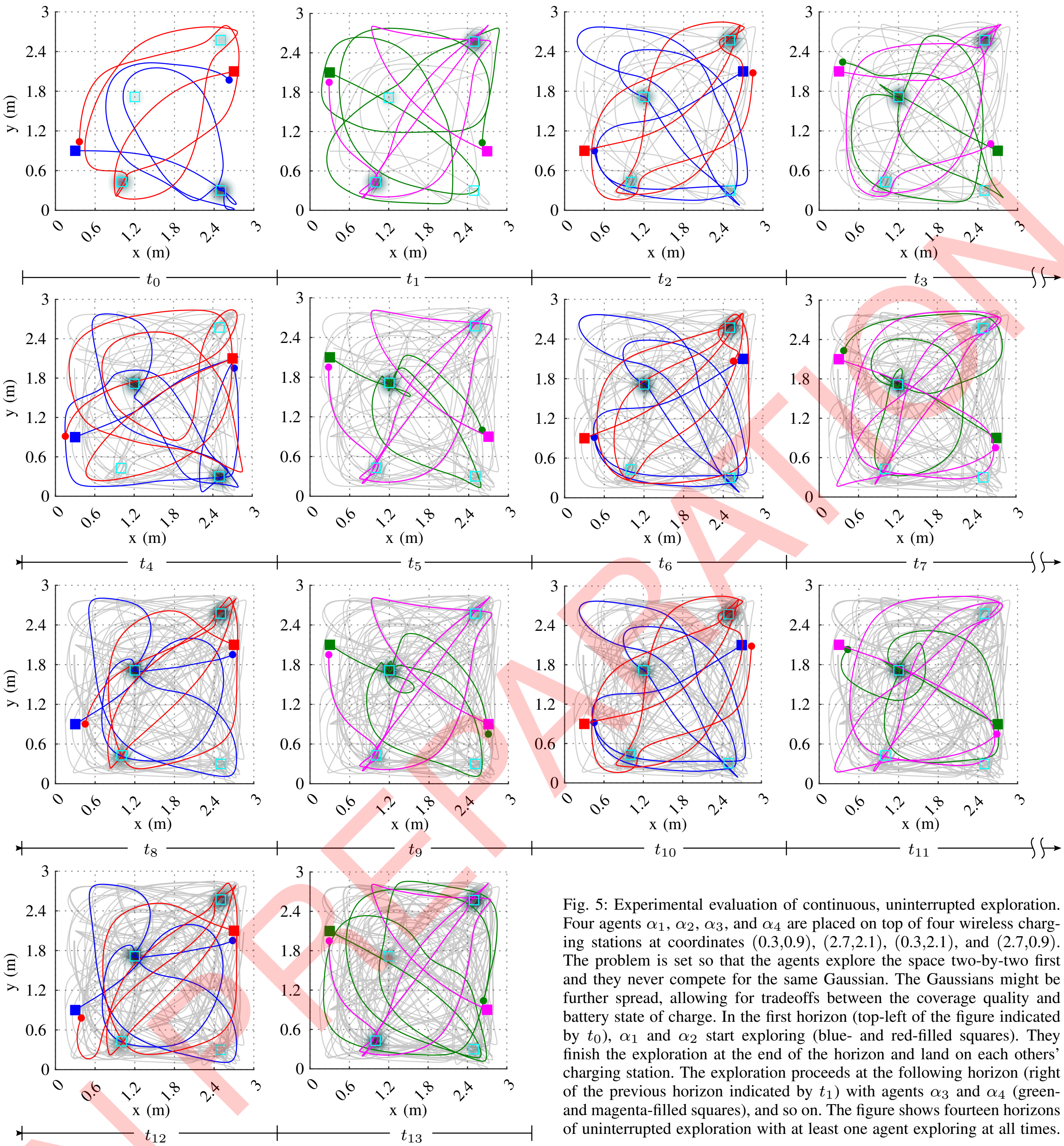

Fig. 5: Experimental evaluation of continuous, uninterrupted exploration. Four agents α_1, α_2, α_3, and α_4 are placed on top of four wireless charging stations at coordinates (0.3,0.9), (2.7,2.1), (0.3,2.1), and (2.7,0.9). The problem is set so that the agents explore the space two-by-two first and they never compete for the same Gaussian. The Gaussians might be further spread, allowing for tradeoffs between the coverage quality and battery state of charge. In the first horizon (top-left of the figure indicated by t_0), α_1 and α_2 start exploring (blue- and red-filled squares). They finish the exploration at the end of the horizon and land on each others' charging station. The exploration proceeds at the following horizon (right of the previous horizon indicated by t_1) with agents α_3 and α_4 (green- and magenta-filled squares), and so on. The figure shows fourteen horizons of uninterrupted exploration with at least one agent exploring at all times.

IN PREPARATION

farthest (μ_3 and μ_2 are the centers of Gaussians covered by the "red" and "dark-green" agents, μ_1 and μ_4 are the centers of the Guassians covered by the "blue" and "magenta" agents in Fig. 5). This means that the MAVs will never compete for the same Gaussian. The battery constraint is edited so that there are two MAVs first, i.e.,

$$\exists_{=1}\, k_1, k_2 \in [n] \text{ s.t. } {}^{k_1}\mathbf{b}_{\text{SoC}}, {}^{k_2}\mathbf{b}_{\text{SoC}} \in (0, b_f], \tag{13}$$

where the notation $\exists_{=1}$ indicates the unique existential quantification.

There is an additional constraint on the final point in Eq (10g), set so that the agents have to be in the proximity of a charging station.

The cost function in Eq. (10a) is further enhanced with the mixing coefficient $\boldsymbol{\delta}$ in Eq. (1), allowing us to find the tradeoffs between the single Gaussians, the different agents, and the battery SoC. Namely, the cost is

$$\min_{\square,\boldsymbol{\delta}} \; \frac{1}{n}\sum_{k=1}^{n}\int_{\mathcal{T}_k} {}^{k}\mathbf{u}(\tau)^T R_k\, {}^{k}\mathbf{u}(\tau)\, d\tau - \frac{1}{n}\sum_{k=1}^{m}\delta_k. \tag{14}$$

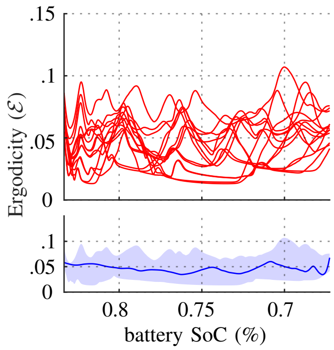

Fig. 6: Ergodicity $\mathcal{E}$ in Eq. (2) as a function of the battery state of charge. The top plot shows the evolution of the ergodicity for all the horizons in Fig. 5. The bottom shows the average ergodicity. Initially, the MAVs are at their charging stations. As they start exploring, they move to the areas with high information density – ergodicity decreases. As they approach the end of the horizon, they start moving to the charging stations – ergodicity increases.

A similar approach is undertaken in prior literature [16], where the ergodic objective is dynamic as more information is gathered, rather than the battery status is changed.

The number of frequencies and battery and recharging coefficients are those used in the previous scenario, whereas the horizon is now set to two and a half minutes. The ergodicity metric in Eq. (2) is set to be lower or equal to 0.05 via the constraint in Eq. (10e), in line with similar literature [14].

The results are shown in Fig. 5. The figure is to be read from left to right and from top to bottom, with the horizon being indicated under each subfigure (meaning that t_0 is the first horizon, t_1 is the second horizon, etc.). Initially, two MAVs are selected via the solution to the OCP in Eq. (10), α_1 "blue" and α_2 "red". They are located at coordinates (0.3,0.9) and (2.7,2.1) respectively, denoted by the blue and red filled squares. The energy-aware ergodic trajectories ${}^1\mathbf{q}(t)$ and ${}^2\mathbf{q}(t)$ are selected so that the MAVs land at each other's charging stations, i.e., ${}^1\mathbf{q}(t_f) = {}^2\mathbf{q}(t_0)$ and vice-versa. The mixing coefficients for α_1 are such that $\delta_2 > \delta_3$, meaning that the agent α_1 explores in more detail the area delimited by the Gaussian centered in μ_2. This is indicated by the darker coloring of the different Gaussians, which is proportional to the optimal value of $\boldsymbol{\delta}$.

An analogous situation is to be observed with agent α_2. At the end of the optimization horizon, both agents land in the proximity of each other's charging stations (the red and blue filled dots at the end of the trajectories for respectively α_2 and α_1), meaning that the constraint in Eq. (10g) is evaluated within

$$\|{}^{k_2}\mathbf{q}(t_f) - {}^{k_1}\mathbf{q}(t_0)\| \leq \varepsilon, \tag{15}$$

where $\varepsilon \in \mathbb{R}_{>0}$ and $k_1, k_2 \in [n]$ are given.

Once the two agents α_1 and α_2 land, they start recharging. The formulation of the OCP in Eq. (10) is such that the other two agents α_3 "dark-green" and α_4 "magenta" are selected. They are located at coordinates (0.3,2.1) and (2.7,0.9) respectively. They proceed on the respective energy-aware ergodic trajectories and land at each other's charging stations, with the past trajectory being indicated in the background in gray. The figure shows fourteen horizons. The actual exploration in the scenario, however, is continuous.

Ergodicity against battery staate of charge

We report the evolution of the value of the ergodicity metric in Eq. (2) in time as a function of the battery SoC in Fig. 6. The experimental data are from continuous, uninterrupted exploration (see Fig. 5).

The top of the figure shows the evolution per each horizon in Fig. 5 in red, whereas the bottom shows the averaged value in blue. We can observe that the exploration starts at an initial value of ergodicity, which mostly depends on the distance from the charging stations to the components of the GMM (i.e., high information density). The ergodicity decreases as the agents move towards the Gaussians in the spatial distribution GMM. It oscillates as the agents move from one Gaussian to another. In the first half of the horizon, the average ergodicity continues to descend as more information is gathered. In the second, the ergodicity increases, peaking at the end, as the discharged agents return to the charging stations for recharging (i.e., low information density).

V. Conclusion and Future Directions

This work enhances prior literature on ergodic search – a method to derive robots' trajectories that visit areas with high information density – with energy-aware coverage. It answers the question of whether is it possible to explore a space uninterruptedly, with at least one agent at all times. Although prior literature has been investigating ergodic search, battery constraints have not been studied. Yet, it is not possible to continue exploring uninterruptedly on a single battery charge. Our paper tackles this challenge. Our methods are to derive an abstract battery model and extend the canonical ergodic search to energy-aware ergodic search. Continuous exploration is achieved using an optimization framework, which resembles a model predictive controller formulation. Experimental data indicate the effectiveness of the battery-constrained exploration with a single agent first. Uninterrupted coverage is achieved with a multi-agent system so that there is always one agent exploring and the spatial distribution is satisfied – a statement that we prove with empirical evidence.

A limitation of the current methods is that the charging stations are in fixed positions. To enable real-world use cases, we are currently extending the methods to mobile charging stations, which arise in scenarios such as environmental surveying. Furthermore, energy optimality is not addressed by our methods due to the ergodic metric's non-convexity and high non-linearity. Future work will explore the possibilities of guaranteeing energy optimality under tight battery constraints. The physical implementation of such an energy-optimal method will also need to be investigated for feasibility.

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