Energy-Aware Ergodic Search: Continuous Long-Term Exploration for Multiagent Systems

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Abstract**——**

Index Terms**—Motion and Path Planning; Energy and Environment-Aware Autonomation.**

I. Introduction

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II. Problem Formulation

The problem addressed in this work is that of exploring a bounded space with multiple agents, continuosly, and proportionally to a spatial distribution. To achieve the latter, canonical ergodic search [1] derives the agent's control so that its trajectory maximizes an ergodic metric defined in the spectral domain [2].

Problem (Ergodic search)**.** Consider a bounded space $\mathcal{Q} \subset \mathbb{R}^D$ of dimension $D \in \mathbb{N}_{>0}$ and a spatial distribution ϕ. *Ergodic search problem* is the problem of deriving a control action $\mathbf{u}(t) \in \mathcal{U}$ so that the trajectory $\mathbf{q}(t) \in \mathcal{Q}$ is proportional to the distribution ϕ.

Here the notation $\mathbb{R}$ and $\mathbb{N}$ indicates reals and naturals, $\mathbb{N}_{>0}$ strictly naturals. Bold notation is used for vectors.

We extend the canonical erogidc search problem, to multi-agent continuous ergodic search, i.e., exploration with multiple robots under spatial distribution and battery constraints.

Problem (Multi-agent continuous ergodic search)**.** Consider a set of n agents $\boldsymbol{\alpha} := \{\alpha_1, \alpha_2, \ldots, \alpha_n\}$, a bounded space $\mathcal{Q}$, and a spatial distribution ϕ similarly to Problem II. We are interested in deriving each agent ${}^j\alpha$ control actions ${}^j\mathbf{u}(t)$ so that its trajectory ${}^j\mathbf{q}(t)$ is proportional to the distribution ϕ.

III. Methods

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A. Ergodic search

For the purposes of defining the spatial distribution, let us consider a Gaussian mixture model (GMM)

$$\phi(\boldsymbol{\delta}, \mathbf{q}(t)) := \sum_{k=1}^{n} \delta_k \, \mathcal{N}(\mathbf{q}(t) \,|\, \mu_k, \Sigma_k), \tag{1}$$

composed of n Gaussians. Each has a coveriance matrix $\Sigma_k \in \mathbb{R}^{D\times D}$, a center $\mu_k \in \mathcal{Q}$, and a positive mixing coefficient $\delta_k \in \boldsymbol{\delta}$ such that the sum of the δs is one.

The goal of ergodic search is to minimize an ergodic metric

$$\mathcal{E}(\mathbf{q}(t), \phi) := \frac{1}{2} \sum_{k\in\mathcal{K}} \Lambda_k (c_k - \phi_k)^2, \tag{2}$$

where ϕ_k are coefficients derived utilizing the Fourier series on the spatial distribution ϕ and c_k on the trajectory $\mathbf{q}$. $\mathcal{K}$ is a set of vectors in $\mathbb{N}^D$ built so that it contains the indices of all the frequencies, i.e., if there are k frequencies in 2D including the fundamental frequency, $\mathcal{K}$ is $\{\,[0 \;\cdots\; k]^T, [0 \;\cdots\; 0]^T\}, \ldots, \{\,[0 \;\cdots\; k]^T, [k \;\cdots\; k]^T\}$. Finally, Λ_k is a weight factor, i.e., if Λ_k is $(1 + \|k\|^2)^{(-D-1)/2}$ lower frequencies have more weight.

The coefficients c_k are derived using the Fourier series basis function. If we consider the trigonometric form

$$c_k(\mathbf{q}, t) := \int_{\mathcal{T}} \frac{1}{L^D} \prod_{d\in[D]} \big(\cos(k_d \, \mathbf{q}_d(\tau)\, \psi) - i \sin(k_d \, \mathbf{q}_d(\tau)\, \psi)\big) \, d\tau / t, \tag{3}$$

c_k is then evaluated per each k in $\mathcal{K}$ in Eq. (2). ψ is $2\pi/L$ for a given period $L \in \mathbb{R}_{>0}$, i is the imaginary unit, k_d is the dth item of k, and $\mathbf{q}_d$ the dth item of $\mathbf{q}$. $\mathcal{T}$ is built so that the integration is between $\tau = t_0$ and t, and the notation $[D]$ indicates positive naturals up to D.

For the purposes of deriving the coefficients ϕ_k, let us consider the GMM model in Eq. (1) on a search space $\mathcal{Q}$. The space is further bounded to a symmetric set $[-L/2, L/2]^D$ since the Gaussians are symmetric about the zero axes. The resulting new model is then

$$\Phi(\boldsymbol{\delta}, \mathbf{q}(t)) := \sum_{d\in[2^D]} \sum_{k=1}^{n} \delta_k \, \mathcal{N}(\mathbf{q}(t) \,|\, A_d\mu_k, A_d\Sigma_k A_d^T)/2^D, \tag{4}$$

where $A_d \in \mathbb{R}^{D\times D}$ are linear transformation matrices [2].

B. Battery modeling

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IV. Experimental Results

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V. Conclusion and Future Directions

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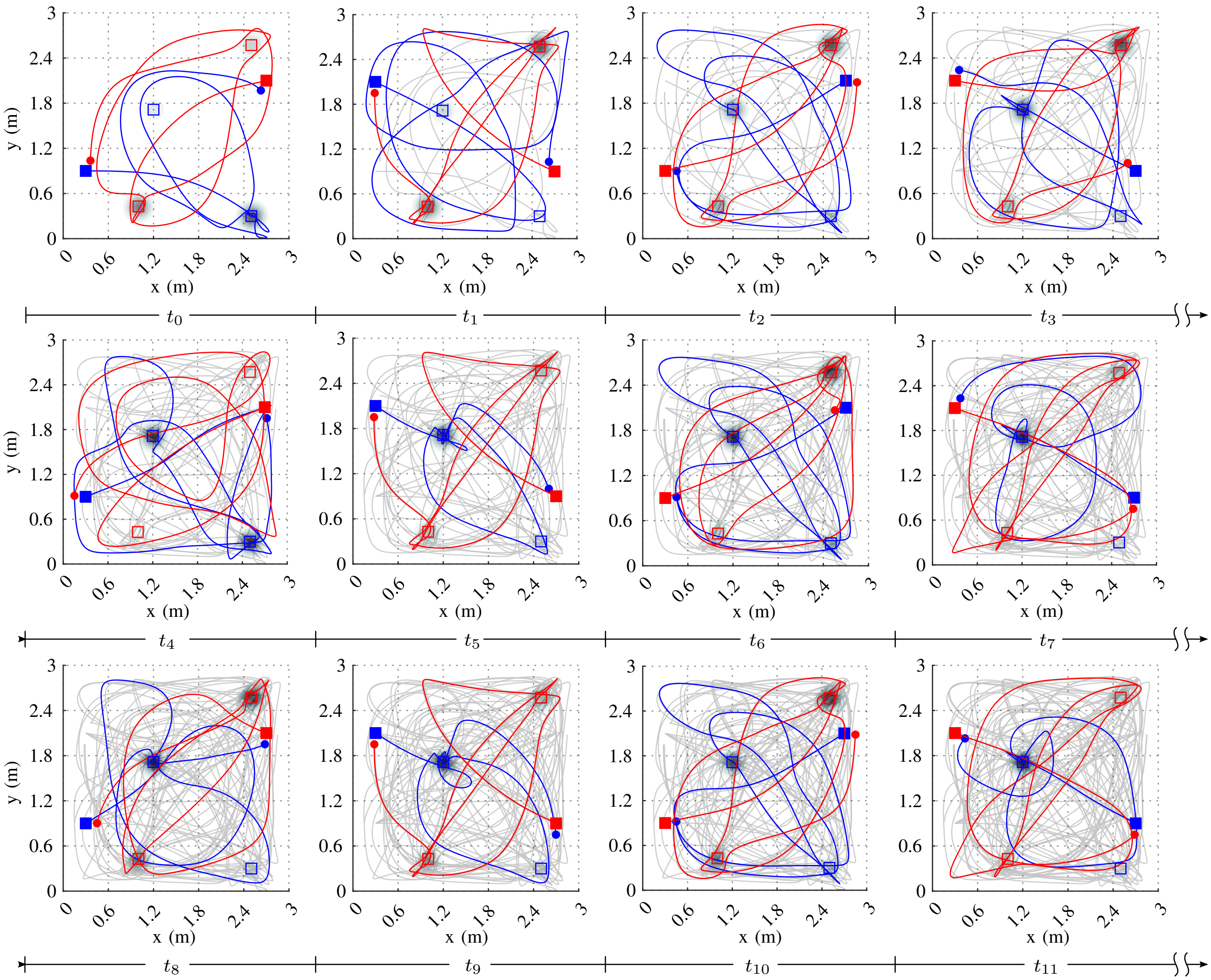
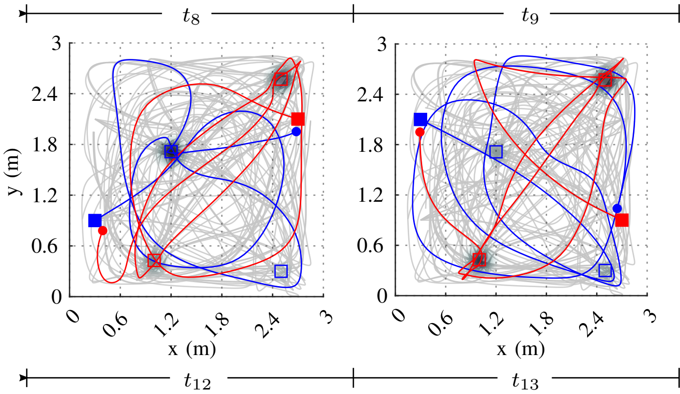


y (m)
x (m)
0
0.6
1.2
1.8
2.4
3
t_0
t_1
t_2
t_3
t_4
t_5
t_6
t_7
t_8
t_9
t_{10}
t_{11}



y (m)
x (m)
0
0.6
1.2
1.8
2.4
3
t_{12}
t_{13}

Fig. 1: . . . .