Energy-Aware Ergodic Search: Continuous Long-Term Exploration for Multiagent Systems

Adam Seewald[1], Marvin Chancán[1], Cameron J. Lerch[1], Hector Castillo[1], Aaron M. Dollar[1], and Ian Abraham[1]

***Abstract*—**

***Index Terms*—Motion and Path Planning; Energy and Environment-Aware Autonomation.**

I. Introduction

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II. Problem Formulation

The problem addressed in this work is that of exploring a bounded space with multiple agents, continuosly, and proportionally to a spatial distribution. To achieve the latter, canonical ergodic search [1] derives the agent's control so that its trajectory maximizes an ergodic metric defined in the spectral domain [2].

Problem II.1 (Ergodic search)**.** Consider a bounded space $\mathcal{Q} \subset \mathbb{R}^D$ of dimension D with $D \in \mathbb{N}_{>0}$ and a spatial distribution ϕ. *Ergodic search problem* is the problem of deriving a control action $\mathbf{u}(t) \in \mathcal{U} \subset \mathbb{R}^V$ with $V \in \mathbb{N}_{>0}$ so that the trajectory $\mathbf{q}(t) \in \mathcal{Q}$ is proportional to the distribution ϕ.

Here the notation $\mathbb{R}$ and $\mathbb{N}$ indicates reals and naturals, $\mathbb{N}_{>0}$ strictly naturals. Bold notation is used for vectors.

We extend the canonical erogidc search problem, to multi-agent continuous ergodic search, i.e., exploration with multiple robots under spatial distribution and battery constraints.

Problem II.2 (Multi-agent continuous ergodic search)**.** Consider a set of n agents $\boldsymbol{\alpha} := \{\alpha_1, \alpha_2, \dots, \alpha_n\}$, a bounded space $\mathcal{Q}$, and a spatial distribution ϕ similarly to Problem II.1. We are interested in deriving each agent ${}^j\alpha$ control actions ${}^j\mathbf{u}(t)$ so that its trajectory ${}^j\mathbf{q}(t)$ is proportional to the distribution ϕ.

III. Methods

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A. Ergodic search

For the purposes of defining the spatial distribution, let us consider a Gaussian mixture model (GMM)

$$\phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, \mu_k, \Sigma_k), \tag{1}$$

composed of m Gaussians. Each has a coveriance matrix $\Sigma_k \in \mathbb{R}^{D \times D}$, a center $\mu_k \in \mathcal{Q}$, and a positive mixing coefficient $\delta_k \in \boldsymbol{\delta}$ such that the sum of the δs is less or equal to one. They indicate how well is each Guassian in the GMM considered.

The goal of ergodic search is to minimize an ergodic metric

$$\mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t), t) := \frac{1}{2} \sum_{k \in \mathcal{K}} \Lambda_k \big(c_k(\mathbf{q}(t), t) - \phi_k(\boldsymbol{\delta}, t) \big)^2, \tag{2}$$

where ϕ_k are coefficients derived utilizing the Fourier series on the spatial distribution ϕ and c_k on the trajectory $\mathbf{q}(t)$. They are detailed in Equation (6) and (4) respectively. Λ_k is a weight factor, i.e., if

$$\Lambda_k = (1 + \|k\|^2)^{(-D-1)/2}, \tag{3}$$

lower frequencies have more weight [3]. $\mathcal{K} \in \mathbb{N}^D$ is a set index vectors that covers $[K] \times \dots \times [K] \in \mathbb{N}^{K^D}$ where K is a given number of frequencies including the fundamental frequency. The notation $[K]$ indicates positive naturals up to K.

The coefficients c_k are derived using the Fourier series basis function. If we consider the trigonometric form

$$c_k(\mathbf{q}(t), t) := \int_{\mathcal{T}} \frac{1}{L^D} \prod_{d \in [D]_{>0}} \begin{aligned}&\big(\cos(k_d \, \mathbf{q}_d(\tau) \, \psi) \\ &- i \sin(k_d \, \mathbf{q}_d(\tau) \, \psi) \big) \, d\tau / t,\end{aligned} \tag{4}$$

c_k is then evaluated per each k in $\mathcal{K}$ in Eq. (2). ψ is $2\pi/L$ for a given period $L \in \mathbb{R}_{>0}$, i is the imaginary unit, k_d is the dth item of k, and $\mathbf{q}_d$ the dth item of $\mathbf{q}$.

$\mathcal{T}$ is built so that the integration is between $\tau = t_0$ and t, and the notation $[D]_{>0}$ indicates strictly positive naturals up to D.

For the purposes of deriving the coefficients ϕ_k, let us consider the GMM model in Eq. (1) on a search space $\mathcal{Q}$. The space is further bounded to a symmetric set $[-L/2, L/2]^D$ since the Gaussians are symmetric about the zero axes. The resulting new model is then

$$\Phi(\boldsymbol{\delta}, \mathbf{q}) := \sum_{d \in [2^D]_{>0}} \sum_{k=1}^{m} \delta_k \, \mathcal{N}(\mathbf{q} \,|\, A_d \mu_k, A_d \Sigma_k A_d^T) / 2^D, \tag{5}$$

where $A_d \in \mathbb{R}^{D \times D}$ are linear transformation matrices [2]. Let us call the integrand in Eq. (4) $c : \mathcal{Q} \longrightarrow \mathbb{R}^K$. It maps the

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[1]A. S., M. C., C. J. L., H. C., A. M. D., and I. A. are with the Department of Mechanical Engineering and Materials Science, Yale University, CT, USA. Email: adam.seewald@yale.edu;

space to spectral domain. The equivalent of Eq. (4) for the spatial distribution can be then expressed

$$\phi_k(\boldsymbol{\delta}, t) := \int_{\mathcal{Q}} \Phi(\boldsymbol{\delta}, \mathbf{q})\, c(\mathbf{q})\; d\mathbf{q}. \tag{6}$$

$\mathcal{Q}$ is build so that the integration is withing the points of the bounded symmetric set $\mathbf{q} \in [-L/2, L/2]^D$.

Let us first formulate the solution for Problem II.1, borrowed by canonical ergodic search [4]. If the robot's dynamics is described by a generic differential equation $\dot{\mathbf{q}}(t) = f\big(\mathbf{q}(t), \mathbf{u}(t), t\big)$, an optimal control problem (OCP) that selects an ergodic control action can be formulated

$$\min_{\mathbf{q}(t),\mathbf{u}(t)} \mathcal{E}(\boldsymbol{\delta}, \mathbf{q}(t), t_f) + \int_{\mathcal{T}} \mathbf{u}(\tau)^T R \mathbf{u}(\tau)\, d\tau, \tag{7a}$$

$$\text{s.t. } \dot{\mathbf{q}} = f(\mathbf{q}(t), \mathbf{u}(t), t), \tag{7b}$$

$$\mathbf{q}(t) \in \mathcal{Q},\; \mathbf{u}(t) \in \mathcal{U}, \tag{7c}$$

$$\mathbf{q}(t_0), \mathbf{q}(t_f) \text{ given}, \tag{7d}$$

where the ergodic metric is derived in Eq. (2), $R \in \mathbb{R}^{V \times V}$ is a control penalizing diagonal positive-definite matrix, and t_0, t_f are respectively the first and last time instants. $\mathcal{T}$ is $[t_0, t_f)$.

To formulate the solution to Pb. II.2, let us first extend the OCP in Eq. (7) to multiagent systems. Eq. (7a) becomes

$$\min_{\square} \sum_{k=1}^{n} \left(\mathcal{E}(\boldsymbol{\delta}, {}^k\mathbf{q}(t), t) + \int_{\mathcal{T}_k} {}^k\mathbf{u}(\tau)^T R_k\, {}^k\mathbf{u}(\tau)\, d\tau \right), \tag{8}$$

where the ergodic metrics and the control penalizing term R_k are now agent-specifc. The term $\square$ is ${}^1\mathbf{q}(t), {}^2\mathbf{q}(t), \ldots, {}^n\mathbf{q}(t), {}^1\mathbf{u}(t), {}^2\mathbf{u}(t), \ldots {}^n\mathbf{u}(t)$. $\mathcal{T}_k$ is $[{}^kt_0, {}^kt_f)$, i.e., different agents might have different duration.

B. Battery modeling

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IV. Experimental Results

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V. Conclusion and Future Directions

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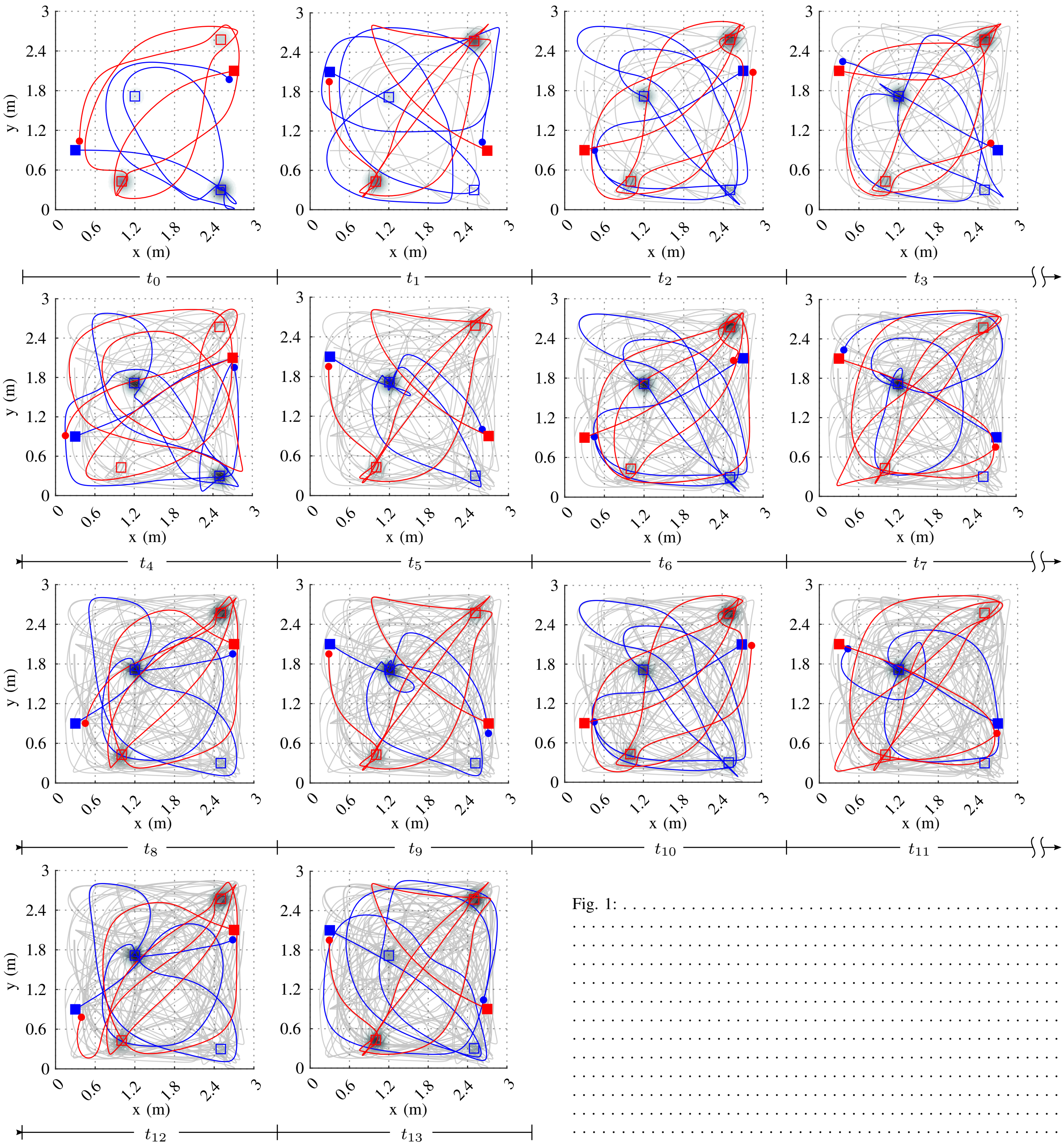

Fig. 1: