

# Energy-Aware Dynamic Planning Algorithm for Autonomous UAVs

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**Abstract**—abstract

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## I. INTRODUCTION

Many scenarios involving unmanned aerial vehicles (UAVs), such as precision agriculture, search and rescue, and surveillance, require high autonomy but have limited energy budgets. A typical example of these scenarios is a UAV flying a path and performing some on-board computational tasks. For instance, the UAV might detect ground patterns and notify other ground-based actors with little human interaction. We refer to such computational tasks that can be dynamically replanned and adapted as *computations*. We are interested in the energy optimization of the path and computations under uncertainty (atmospheric interferences) and refer to it as energy-aware dynamic planning. Such planning would find optimal tradeoffs between the path, computations, and energy requirements. Current generic planning solutions for outdoor UAVs do not plan the path and computations dynamically, nor are they energy-aware. They are often semi-autonomous: the path and computations are static and usually defined using planning software [1] (for instance [2] and [3]). Such a state of practice has prompted us to propose an *energy-aware dynamic planning algorithm* for UAVs. The algorithm combines and generalizes some of the past body of knowledge on mobile robot planning problems and addresses the increasing *computational demands* and their relation to

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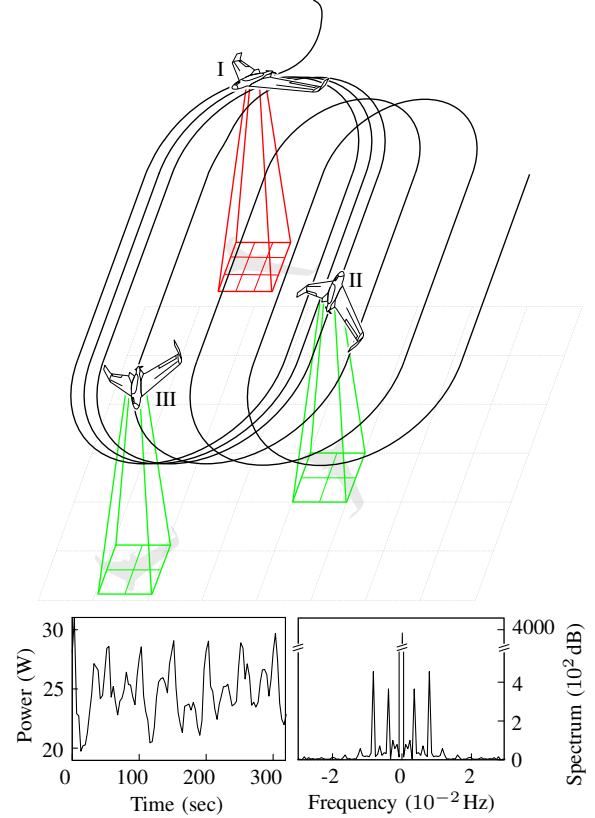


Fig. 1. An initial plan of an agricultural scenario replanned dynamically in terms of both the path (II travels a more accurate survey than III) and computations (I elaborates more images per second than II). In bottom left, the collected energy data of the UAV flying the scenario, along the power spectrum (bottom right) where the first frequency has been filtered.

energy consumption, path, and autonomy for the UAV planning problem.

Planning algorithms literature for mobile robots includes topics such as trajectory generation and path planning. Generally, the algorithms select an energy-optimized trajectory [4], e.g., by maximizing the operational time [5]. However, they apply to a small number of robots [6] and focus exclusively on planning the trajectory [7], despite compelling evidence for the energy consumption also being significantly influenced by computations [8]. Given the availability of powerful GPU-equipped mobile hardware [9], the use of computations is expected to increase in the near future [10]–[12]. More complex planning, which includes a broader concept of

the plan being a set of tasks and a path, all focus on the trajectory [8], [13] and apply to a small number of robots [14], [15]. For UAVs specifically, rotorcrafts have also gained interest in terms of algorithms for energy-optimized trajectory generation [16], [17].

Unlike most of the past planning algorithms literature, our algorithm plans both the path and computations. To model the path we use multiple trajectories and denote each with a mathematical function. In Figure 1, the path contains multiple circles and lines. To model the computations we use a profiling tool presented in previous work [18]. To guide the UAV we use a vector field [19] that converges smoothly to the planned trajectory. The use of vector fields for planning is widely discussed in the literature [19]–[24].

To achieve the energy-aware dynamic planning, we further introduce and formally proof a periodic energy model that accounts for the uncertainty. We use Fourier analysis to derive the model, and state estimation to address the uncertainty. Periodicity is often present due to repetitive patterns in the plan [25]. Indeed, UAV scenarios often iterate over a set of tasks and trajectories (e.g., monitoring or search and rescue). Given that the plan is periodic, we expect the energy consumption to approximately evolve periodically. In Figure 1, some collected data from a UAV flying a survey scenario along its power spectrum plot motivates our choice. The observation applies broadly; we briefly investigate periodic energy consumption evolutions in some ground-based mobile robots.

In the spirit of reducing costs and resources, we showcase the algorithm using the problem of dynamic planning for a precision agriculture fixed-wing UAV. Precision agriculture is often put into practice [26] with ground mobile robots used for harvesting [27]–[32], and UAVs for preventing damage and ensuring better crop quality [1], [33]. The plan is structured as follows. Trajectory-wise, the UAV flies in circles and lines covering a polygon. Computationally-wise, it detects obstacles using a convolutional neural network (CNN) and notifies grounded mobile robots employed for harvesting. The algorithm alters the plan; it controls the processing rate and the radius of the circles (affecting the distance between the lines). Figure 1 shows a slice of such plan. We observe that not only the path but also the computations significantly impact the energy, with a potential extension of up to 13 minutes over an hour by switching from the highest to the lowest level of computations (see Section V), in presence of a standard battery.

The remainder of the paper is organized as follows. The overview of dynamic planning, some preliminaries, and problem definition are provided in Section II. We show a suitable model for the energy in Section III, and

propose the algorithm in Section IV. In Section V, we present the results and showcases the performances. We propose some conclusions in Section VI. Finally, we provide some additional information and examples in Appendixes I–II.

## II. PLANNING OVERVIEW

The algorithm inputs a user-specified initial plan that consist of different stages. At each stage the UAV flies a trajectory, and does some computations. Per each stage the plan contains some parameters that allow to alter the path and computations along an energy budget. The alterations are bounded. There is one trajectory constraints set which bounds the path and multiple computations constraints sets, one per each computation parameter, that bound computations. In Figure 1, there are two parameters. The radius of the trajectory circle is relative to the trajectory (II and III). The quality of image detection is relative to the computations (I and II).

The algorithm outputs the control (the parameters) using output model predictive control (MPC) [34]. The UAV utilizes the control to influence its energy consumption to meet the energy budget. The control is data-driven. Energy sensor data estimates some coefficients of an energy model used to predict future energy consumption in presence of uncertainty. The energy budget is the battery capacity and other battery parameters. These are fixed values that are not replanned by the algorithm. Our goal is to complete the plan with the highest possible parameters as the UAV flies and its batteries drain.

### A. Plan definition

Let us adopt the following mathematical notation. Given an integer  $a$ ,  $[a]$  is the set  $\{0, 1, \dots, a\}$ ,  $[a]^+$  the set  $[a]/\{0\}$ .  $c_{i,j}$  specifies the  $j$ -th parameter of the  $i$ -th parameters set  $c_i$ .  $c_{i,j}^0$  is the optimal value of the parameter  $c_{i,j}$  w.r.t. a given cost function  $l$ , and  $\underline{c}_{i,j}, \bar{c}_{i,j}$  its lower and upper bounds. The set  $\langle c_{i,1}, c_{i,2}, \dots, c_{i,n} \rangle$  is an ordered list of  $n$  parameters.

Let us assume that the trajectory at stage  $i$  can be altered with  $\rho$  trajectory parameters and the computations with  $\sigma$  computations parameters. We then express the trajectory as a continuous twice differentiable function  $\varphi_i : \mathbb{R}^2 \times \mathbb{R}^\rho \rightarrow \mathbb{R}$  of a point and the trajectory parameters. The function returns a metric of the distance between the point and the nominal trajectory. We express the computations as the value of the computations parameters. We discuss the concrete meaning of the value of trajectory parameters in Subsection III-A, and computations parameters in Subsection III-B.

**Definition II.1** (Stage, plan, triggering, and final point). The  $i$ -th stage  $\Gamma_i$  at time instant  $k$  of a plan  $\Gamma$  is defined

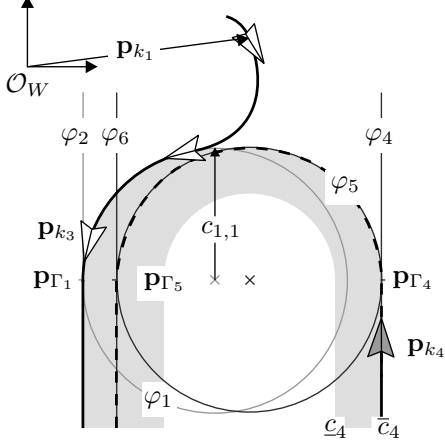


Fig. 2. Illustrative figure shows notation on a small initial slice of the plan in Figure 1.

as the ordered list

$$\begin{aligned} \Gamma_i := & \{ \langle \varphi_i(\mathbf{p}_k, c_{i,1}, \dots, c_{i,\rho}), c_{i,\rho+1}, \dots, c_{i,\rho+\sigma} \rangle \\ & | \exists \mathbf{p}_k, \varphi_i(\mathbf{p}_k, c_{i,1}, \dots, c_{i,\rho}) \in \mathcal{C}_i, \\ & \forall j \in [\sigma]^+, c_{i,\rho+j} \in \mathcal{S}_{i,j} \}, \end{aligned} \quad (1)$$

where  $\mathcal{C}_i := [\underline{c}_i, \bar{c}_i] \subseteq \mathbb{R}$  is the trajectory constraints set, and  $\mathcal{S}_{i,j} := [c_{i,\rho+j}, \bar{c}_{i,\rho+j}] \subseteq \mathbb{Z}_{\geq 0}$  the  $j$ -th computation constraints set.  $\mathbf{p}_k$  is a point of a UAV flying at an assigned altitude  $h \in \mathbb{R}_{>0}$  w.r.t. some inertial navigation frame  $\mathcal{O}_W$ . In Figure 2,  $\varphi_1, \dots, \varphi_6$  are trajectories.  $\varphi_1$  and  $\varphi_5$  are circles, while  $\varphi_2, \varphi_4$ , and  $\varphi_6$  are lines. They are all relative to different stages  $\Gamma_1, \Gamma_2, \dots$ . The constraints set  $\mathcal{C}_1, \mathcal{C}_2, \dots$  forms the area where the trajectories  $\varphi_1, \varphi_2, \dots$  can be altered with the parameters  $c_{i,1}, \dots, c_{i,\rho}$  (gray area in the figure). This area is bounded by  $\underline{c}_i, \bar{c}_i$ , and can be different per each stage (in Figure 2, the area relative to  $\Gamma_4$  is bounded by  $\underline{c}_4, \bar{c}_4$ ).

The *plan* is a finite state machine (FSM)  $\Gamma$  where the state-transition function  $\delta: \bigcup_i \Gamma_i \times \mathbb{R}^2 \rightarrow \bigcup_i \Gamma_i$  maps a stage and a point to the next stage

$$\delta(\Gamma_i, \mathbf{p}_k) := \begin{cases} \Gamma_{i+1} & \text{if } \mathbf{p}_k = \mathbf{p}_{\Gamma_i} \\ \Gamma_i & \text{otherwise} \end{cases}.$$

The point  $\mathbf{p}_{\Gamma_i}$  that allows the transition between  $\Gamma_i$  and  $\Gamma_{i+1}$  is called *triggering point*. In Figure 2,  $\mathbf{p}_{\Gamma_1}$  allows the transition between  $\Gamma_1$  and  $\Gamma_2$ ,  $\mathbf{p}_{\Gamma_4}$  between  $\Gamma_4$  and  $\Gamma_5$ , and  $\mathbf{p}_{\Gamma_5}$  between  $\Gamma_5$  and  $\Gamma_6$ . The last triggering point  $\mathbf{p}_{\Gamma_l}$  relative to last stage  $\Gamma_l$  is called *final point*.

A slice of the plan in Figure 3 shows the transition between the stages with the FSM. The triggering point  $\mathbf{p}_{\Gamma_{i-1}}$  allows the transition to stage  $\Gamma_i$ . The UAV remains in the stage with any generic point  $\mathbf{p}_{k_1}$ . It eventually enters the stage  $\Gamma_{i+1}$  with the triggering point  $\mathbf{p}_{\Gamma_i}$  and so on, until it reaches the final point.

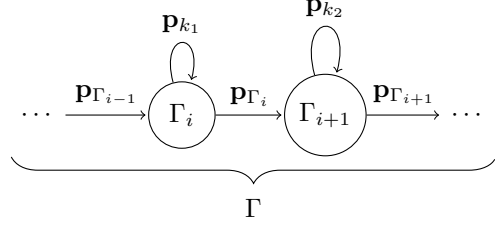


Fig. 3. The plan defined as a FSM

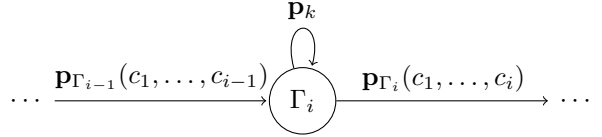


Fig. 4. Detail of the stage  $\Gamma_i$  in the FSM

Generally, one can express the triggering points in function of the  $i$ -th trajectory parameters  $c_{i,1}, \dots, c_{i,\rho}$ , or any previous trajectory parameters, propagating the information therein if necessary. We show this in Figure 4, where, for simplicity, we neglected the  $\sigma = 0$  computations parameters, so  $c_i = c_{i,1}, \dots, c_{i,\rho}$ .

We refer the reader to an example in Appendix II-B for a detailed implementation. In the example, the first trajectory parameter is propagated to all the following trajectories and triggering points.

We store the initial plan in the plan specification, the format is described in Appendix II.

### B. Problem formulation

In order to formulate the problem, we considerate the next assumption.

**Assumption II.1** (Plan periodicity). Given an initial plan  $\Gamma$  consisting of  $l$  stages, a generic starting point  $\mathbf{p}$ , the current levels of the parameters  $c_{1,1}, \dots, c_{1,\rho}$ , and a shift  $\mathbf{d} := (x_d, y_d)$ , there exists a constant  $n \in \mathbb{Z}_{>0}$  ( $n < l, l/n \in \mathbb{Z}$ ) such that

$$\begin{aligned} \varphi_{(i-1)n+j}(\mathbf{p} + (i-1)\mathbf{d}, c_1) &\approx \\ \varphi_{in+j}(\mathbf{p} + i\mathbf{d}, c_1), \quad \forall i \in [l/n - 1]^+, j \in [n]^+. \end{aligned}$$

Physically, this means that the plan is approximately periodic. We will see the assumption being eased for non-periodic plans in Section V.

**Definition II.2** (Period). The period  $T \in \mathbb{R}_{>0}$  is defined as the time between  $\varphi_{(i-1)n+j}$  and  $\varphi_{in+j}$  in Assumption II.1.

The algorithm measures the time between the stages and assumes the initial period is one. We note that periods might be different for different  $j$ s due to atmospheric interferences.

From Assumption II.1 follows the next proposition.

**Proposition II.2** (Necessary and sufficient condition for periodicity). Suppose Assumption II.1 holds, then the plan  $\Gamma$  is periodic if

$$\begin{aligned} & \varphi_{(i-1)n+j}(\mathbf{p} + (i-1)\mathbf{d}, c_1) - \\ & \varphi_{in+j}(\mathbf{p} + i\mathbf{d}, c_1) = d_j, \quad \forall i \in [l/n - 1]^+, j \in [n]^+. \end{aligned}$$

Moreover, if the plan  $\Gamma$  is periodic then

$$\text{rank} \begin{bmatrix} \varphi_1 & \varphi_{n+1} & \cdots & \varphi_{(l-n)+1} \\ \vdots & \vdots & \ddots & \vdots \\ \varphi_n & \varphi_{2n} & \cdots & \varphi_l \end{bmatrix} = 2,$$

where the trajectories in the first column are evaluated on  $\mathbf{p}, c_1$ , in the second on  $\mathbf{p} + \mathbf{d}, c_1$ , and in the last on  $\mathbf{p} + (l/n - 1)\mathbf{d}, c_1$ .

*Proof.* ( $\implies$ ) The expression implies that  $\varphi_{(i-1)n+j}$  is the same but shifted function in space  $\varphi_{(i-1)n+j} = \varphi_{in+j} + d_j$  and  $d_j$  is the  $j$ -th constant difference. This is equivalent to Assumption II.1.

( $\impliedby$ ) The column rank of the matrix in the proposition is the dimension of its column space. It denotes the size of the set of all the possible linear combination of the column vectors. We note that there is a limited number (two) of linear combinations of the column vectors being these linearly dependent (from  $\implies$ ). ■

The proposition is a necessary ( $\impliedby$ ) and sufficient ( $\implies$ ) condition for the periodicity. The algorithm finds  $n$  initializing it to one and finding the value which satisfies the proposition.

**Problem II.1** (UAV planning problem). Consider an initial plan  $\Gamma$  defined in Definition II.1 that satisfies Assumption II.1. We are interested in the planning of the path parameters  $c_{i,1}, \dots, c_{i,\rho}$  and computations parameters  $c_{i,\rho+1}, \dots, c_{i,\rho+\sigma}$  and in the guidance to the path resulting from such plan for the UAV planning problem.

### III. PERIODIC ENERGY MODEL

We model the energy using energy coefficients  $\mathbf{q} \in \mathbb{R}^m$  that characterize the energy signal. We refer to the instantaneous energy consumption evolution simply as the energy signal. The coefficients are derived from Fourier analysis (the size of the energy coefficients vector  $m$  is related to the order of a Fourier series) and estimated using a state estimator (Kalman filter).

We proof a relation between the energy signal and the energy coefficients in Lemma III.1. We show after the main results how this approach allows us variability in terms of the systems behaving periodically, piece-wise periodically, or merely linearly with sporadic periodicity.

Once we illustrate the energy model, we enhance it with the energy contribution of the trajectory in Subsection III-A, and of the computations in Subsection III-B.

Let us consider a periodic signal of period  $T$ , and a Fourier series of an arbitrary order  $r \in \mathbb{Z}_{\geq 0}$  for the purpose of modeling of the energy signal

$$h(t) = a_0/T + (2/T) \sum_{j=1}^r (a_j \cos \omega j t + b_j \sin \omega j t), \quad (2)$$

where  $h : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$  maps time to the instantaneous energy consumption,  $\omega := 2\pi/T$  is the angular frequency, and  $a, b \in \mathbb{R}$  the Fourier series coefficients.

The energy signal can be modeled by Equation (2) and by the output of a linear model

$$\begin{aligned} \dot{\mathbf{q}} &= A\mathbf{q} + B\mathbf{u}, \\ y &= C\mathbf{q}, \end{aligned} \quad (3)$$

where  $y \in \mathbb{R}$  is the instantaneous energy consumption.

The state  $\mathbf{q}$  are the energy coefficients

$$\begin{aligned} \mathbf{q} &= [\alpha_0 \quad \alpha_1 \quad \beta_1 \quad \cdots \quad \alpha_r \quad \beta_r]^T, \\ A &= \begin{bmatrix} 0 & & & & & \\ & A_1 & & & & \\ & & \ddots & & & \\ & & & A_r & & \end{bmatrix}, \quad A_j \begin{bmatrix} 0 & \omega j \\ -\omega j & 0 \end{bmatrix}, \\ C &= (1/T) [1 \quad 1 \quad 0 \quad \cdots \quad 1 \quad 0], \end{aligned} \quad (4)$$

where  $\mathbf{q} \in \mathbb{R}^m$  with  $m = 2r+1$ ,  $A \in \mathbb{R}^{m \times m}$  is the state transmission matrix, and  $C \in \mathbb{R}^m$  is the output matrix. In matrix  $A$ , the top left entry is zero, the diagonal entries are  $A_1, \dots, A_r$ , the remaining entries are zeros.

The linear model in Equation (3) allows us to include the control in the model of Equation (2). In fact, the algorithm uses the model to find the control sequence that results in the optimal parameters configuration (using MPC).

**Lemma III.1** (Signals equality). Suppose control is zero, matrices  $A, C$  are described by Equation (4), and the initial guess  $\mathbf{q}_0$  is

$$\mathbf{q}_0 = [a_0 \quad a_1/2 \quad b_1/2 \quad \cdots \quad a_r/2 \quad b_r/2]^T.$$

Then, the signal  $h$  in Equation (2) is equal to the output  $y$  in Equation (3).

*Proof.* The equivalence of the models is trivial and the equality of the signal and output is achieved by a proper choice of items of matrices  $A, C$  and the initial guess  $\mathbf{q}_0$ . We refer the reader to Appendix I for a formal proof of the equality where we justify the choices of the items of the matrices and of the initial guess. ■

Let us consider the discretized version  $A_d := A\Delta t + I$  of the matrix  $A$ . We denote the continuous and discrete time with the standard notation  $t, k$ . We use the forward Euler approximation with a small enough value of the time step  $\Delta t$ . We note that one can guarantee to have the

same outputs rather than an approximation with  $A_d := e^{At}$ .

Let us suppose that at time instant  $k$  the plan reached the  $i$ -th stage  $\Gamma_i$ .

The control along with the input matrix

$$\mathbf{u}_k = [c_{k,1} \ \cdots \ c_{k,\rho} \ c_{k,\rho+1} \ \cdots \ c_{k,\rho+\sigma}]^T, \\ B\mathbf{u} := B_i(\mathbf{u}_k - \mathbf{u}_{k-1}), \quad B_i = \begin{bmatrix} \nu_i \\ \mathbf{0} \end{bmatrix}, \quad (5)$$

where  $\mathbf{u} \in \mathbb{R}^n$  is the control with  $n = \rho + \sigma$ ,  $B \in \mathbb{R}^{m \times n}$ , and  $\mathbf{u}_{-1} = \mathbf{0}$ . Moreover, the top row of  $B$  contains gain factors  $\nu_i = [\nu_{i,1} \ \cdots \ \nu_{i,\rho} \ \nu_{i,\rho+1} \ \cdots \ \nu_{i,\rho+\sigma}]$ , quantifying the contribution of a given parameter to the instantaneous energy consumption. The other entries of  $B$  are zeros.

The first  $\rho$  gain factors quantifies the contribution of the  $i$ -th trajectory parameters. A first guess of these parameters is derived empirically. They are later refined by the algorithm. The following  $\sigma$  gain factors quantifies the contribution of the computations parameters. They are derived using the modeling tool.

#### A. Trajectory parameters energy contribution

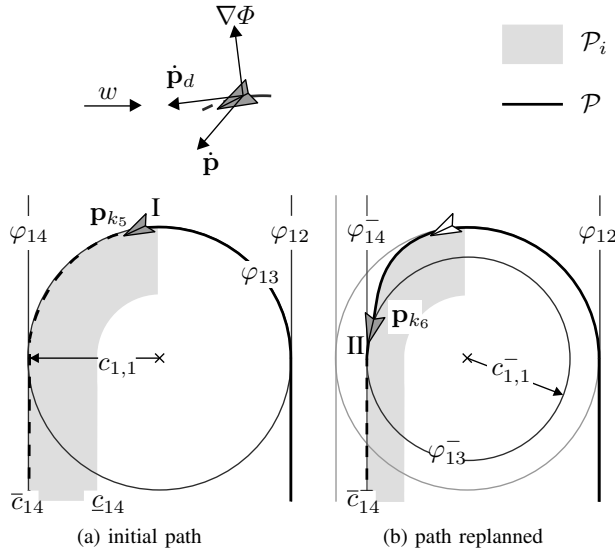


Fig. 5. Illustrative figure shows the alteration of the trajectory parameter  $c_{1,1}$ , the radius of the circle (the alteration of the path from Figure 1).

Equation (5) accounts for the energy due to the change of parameters  $\mathbf{u}_k - \mathbf{u}_{k-1}$ . For instance, when the trajectory  $\varphi_1$  is a circle (see Figure 5), a decrement in the trajectory parameter  $c_{1,1}$ —the radius of the circle—adds a negative contribution. It thus simulates the lowering of instantaneous energy consumption  $\nu_{1,1}(c_{1,1} - c_{1,1}^-) < 0$ , for a given  $\nu_{1,1} \in \mathbb{R}_{<0}$ . This is then summed to the first coefficient  $\alpha_0$ , shifting the modeled energy.

Let us assume that the set

$$\mathcal{P}_i := \{\mathbf{p}_k \mid \varphi_i(\mathbf{p}_k, c_{i,1}, \dots, c_{i,\rho}) \in \mathcal{C}_i\}, \quad (6)$$

delimits the area where the  $i$ -th trajectory  $\varphi_i$  is free to evolve using the trajectory parameters  $c_{i,1}, \dots, c_{i,\rho}$  (the gray area in Figure 5).  $\varphi_i$  is a function of the two coordinates and is equal to zero when a point  $\mathbf{p}_k$  is on the trajectory. Physically, this means the UAV is flying exactly over the nominal trajectory. The trajectory parameters allows to change the trajectory. They are a way to alter the nominal trajectory in the initial plan and thus alter the energy signal. In fact, the algorithm uses the set from Equation (6) to find the optimal trajectory parameters  $c_i^0$  (here we again neglect for simplicity the  $\sigma = 0$  computations parameters), such that  $\varphi_i(\mathbf{p}_k, c_i^0)$  has the highest instantaneous energy consumption (while still respecting the energy budget). The sequence of these parameters (control sequence) forms the path  $\mathcal{P}$ .

We derive the new position  $\mathbf{p}_{k+1}$  using the function  $\Phi := \varphi_i(\mathbf{p}_k, c_i^0)$ , computing its vector field  $\nabla\Phi := [\partial\Phi/\partial x \ \partial\Phi/\partial y]^T$ , and the direction to follow in the form of velocity vector [19]

$$\dot{\mathbf{p}}_d(\mathbf{p}_k) := E\nabla\Phi - k_e\Phi\nabla\Phi, \quad E = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix},$$

where  $E$  specifies the rotation (which influence the tracking direction), and  $k_e \in \mathbb{R}_{\geq 0}$  the gain to adjust the speed of convergence. The direction the velocity vector  $\dot{\mathbf{p}}_d$  is pointing at is generally different from the course heading  $\dot{\mathbf{p}}$  due to the atmospheric interferences, such as wind ( $w \in \mathbb{R}$  in Figure 5).

#### B. Computations parameters energy contribution

Let us recall from Definition II.1 that the  $i$ -th stage  $\Gamma_i$  of the plan  $\Gamma$  contains the computations parameters which characterize the computations. We assess the energy cost of these computations using `powprofiler`, the open-source modeling tool adapted from earlier work on computational energy analysis [18], [35], and energy estimation of a fixed-wing UAV [25].

We assume the UAV carries an embedded board that runs the computations. The tool measures the instantaneous energy consumption of a subset of possible computations parameters within the computations constraints sets and builds an energy model: a linear interpolation, one per each computation.

The computations are implemented by software components, such as ROS nodes in a ROS based system. The user implements these nodes in such a way that they change the computational load with node-specific ROS parameters, the computations parameters. In a generic software component system, the user maps the computational load to the arguments. In both cases, with ROS [36] or with generic software components

system [35], the tool performs automatic modeling. For instance, if the computation is a CNN object detector, the computation parameter  $c_{i,\rho+1}$  corresponds to frames-per-second (fps) rate. The tool then changes the detection frequency.

We note that while the trajectory can differ for each stage, the tasks remain the same. However, the user can inhibit or enable a computation varying its computation set.

Let us further define  $g : \mathbb{Z}_{\geq 0} \rightarrow \mathbb{R}_{\geq 0}$  as the instantaneous computational energy consumption value obtained using the tool.

The gain factors that quantifies the contribution of the computations parameters

$$\begin{bmatrix} \nu_{i,\rho+1} \\ \vdots \\ \nu_{i,\rho+\sigma} \end{bmatrix}^T = \begin{bmatrix} g(c_{i,\rho+1}) \\ \vdots \\ g(c_{i,\rho+\sigma}) \end{bmatrix} / [c_{i,\rho+1} \quad \cdots \quad c_{i,\rho+\sigma}].$$

Moreover, let  $g(\{\emptyset\})$  be zero.

#### IV. ALGORITHM

The main purpose of the algorithm is to output a valid control sequence  $\mathbf{u} := \{\mathbf{u}_0, \mathbf{u}_1, \dots\}$  at each time step given an initial plan  $\Gamma$  and to direct the UAV on a valid path: to solve Problem II.1.

**Definition IV.1** (Valid control sequence and path). A control sequence  $\mathbf{u}$  is valid if for every stage  $\Gamma_{i-1}$ ,  $i \in [l]^+$  there exist a control  $\mathbf{u}_k$  that produce the next stage  $\Gamma_i$ .

The path  $\mathcal{P}$  resulting from such control sequence is a valid path

$$\begin{aligned} \mathcal{P} := \{ \mathbf{p}_k \mid \exists \mathbf{u}_k, n \in \mathbb{Z}_{>0}, \\ \langle \varphi_{i-1}(\mathbf{p}_{k-n}), \mathbf{u}_{k-n} \rangle \in \Gamma_{i-1} \\ \implies \langle \varphi_i(\mathbf{p}_k), \mathbf{u}_k \rangle \in \Gamma_i \}, \end{aligned}$$

where for simplicity  $\langle \varphi_i(\mathbf{p}_k), \mathbf{u}_k \rangle := \langle \varphi_i(\mathbf{p}_k, c_{k,1}, \dots, c_{k,\rho}), c_{k,\rho+1}, \dots, c_{k,\rho+\sigma} \rangle$ .

We assume that at least one period in the initial plan is valid.

**Assumption IV.1.** Given an initial plan  $\Gamma$ , we assume that  $\exists i \in [l/n - 1]^+$ ,  $j \in [n]^+$  s.t.  $\Gamma_{(i-1)n+j}, \dots, \Gamma_{in+j}$  is valid.

Let us proof that if the plan is periodic and one period is valid, then the plan is valid.

**Theorem IV.2** (Valid periodic plan). Suppose Assumption IV.1, Lemma III.1, and Proposition II.1 hold (a period of the plan is valid, its energy evolution is described by Equation (3), and the plan is periodic). Then the plan is valid and its energy evolution modeled by the periodic energy model in Section III-A.

*Proof.* The proof is based on mathematical induction. Base case:

Induction step:

■

##### A. Output constraints set

We stated earlier the output  $y_k$ —the instantaneous energy consumption—evolves in  $\mathbb{R}_{\geq 0}$ . This is generally untrue. Physical UAVs are bounded by strict energy budgets due to battery limitations.

Let us hence consider the state of charge (SoC) of such battery with a simplistic difference equation [25]

$$\text{SoC}_k = - \left( V - \sqrt{V^2 - 4R_r \tilde{V} y_k V^{-1}} \right) / 2R_r Q_c,$$

where  $V \in \mathbb{R}$  is the internal battery and  $\tilde{V} \in \mathbb{R}$  the stabilized voltage,  $R_r \in \mathbb{R}$  the resistance, and  $Q_c \in \mathbb{R}$  the constant nominal capacity. We define the output constraints set

$$\mathbb{Y}_k := \{ y_k \mid y_k \in [0, \text{SoC}_k Q_c V] \subseteq \mathbb{R}_{\geq 0} \},$$

and  $\max \mathbb{Y}_k$  is the maximum discharge capacity by the internal battery voltage—the maximum instantaneous energy consumption.

##### B. Deployment algorithm

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1: procedure STEP( $\mathbf{q}_{k-1}, \mathbf{u}_{k-1}^a, \mathbf{u}_{k-2}^a, P_{k-1}$ )
2:    $\mathbf{u}_{k-1} \leftarrow \mathbf{u}_{k-1}(\max \mathbf{u}_{k-1}^a, \mathbf{u}_{k-2}^a)$ 
3:    $\mathbf{u}_{k-1}^0 \leftarrow \arg \max_{\mathbf{u}} \sum_{i=k-1}^{k+N-2} l(\mathbf{q}_i, \mathbf{u}_i) + V_f(\mathbf{q}_{k+N-1})$ 
4:    $\hat{\mathbf{q}}_k^- \leftarrow A\hat{\mathbf{q}}_{k-1} + B\mathbf{u}_{k-1}^0$ 
5:   if  $C\hat{\mathbf{q}}_k^- \notin \mathbb{Y}_k$  then
6:      $\mathbf{u}_{k-1}^a \leftarrow \mathbf{u}_{k-1}^a / \{\max \mathbf{u}_{k-1}^a\}$ 
7:     return STEP( $\mathbf{q}_{k-1}, \mathbf{u}_{k-1}^a, \mathbf{u}_{k-2}^a, P_{k-1}$ )
8:   else
9:     if  $|y_k^a - C\hat{\mathbf{q}}_k^-| \leq \varepsilon$  then
10:       $\hat{\mathbf{q}}_k \leftarrow \hat{\mathbf{q}}_k^-$ 
11:       $P_k \leftarrow P_k^-$ 
12:    else
13:       $P_k^- \leftarrow AP_{k-1}A^T + Q$ 
14:       $K \leftarrow P_k^- C^T / (CP_k^- C^T + R)$ 
15:       $\hat{\mathbf{q}}_k \leftarrow \hat{\mathbf{q}}_k^- + K(y_k^a - C\hat{\mathbf{q}}_k^-)$ 
16:       $P_k \leftarrow (I + KC)P_k^-$ 
17:    end if
18:     $\mathbf{u}_k^a \leftarrow \mathbf{u}_{k-1}^a$ 
19:    return ( $\hat{\mathbf{q}}_k, P_k, \mathbf{u}_k^a$ )
20:  end if
21: end procedure

22: procedure EADMPA( $\mathcal{M}, \mathbf{p}_0, \mathbf{q}_0$ )
23:    $k \leftarrow 0$ 
24:    $\mathbf{u}_{k-1}^a \leftarrow \{\emptyset\}$ 
25:    $\mathbf{p}_k \leftarrow \mathbf{p}_0$ 
26:    $\mathbf{q}_k \leftarrow \mathbf{q}_0$ 

```

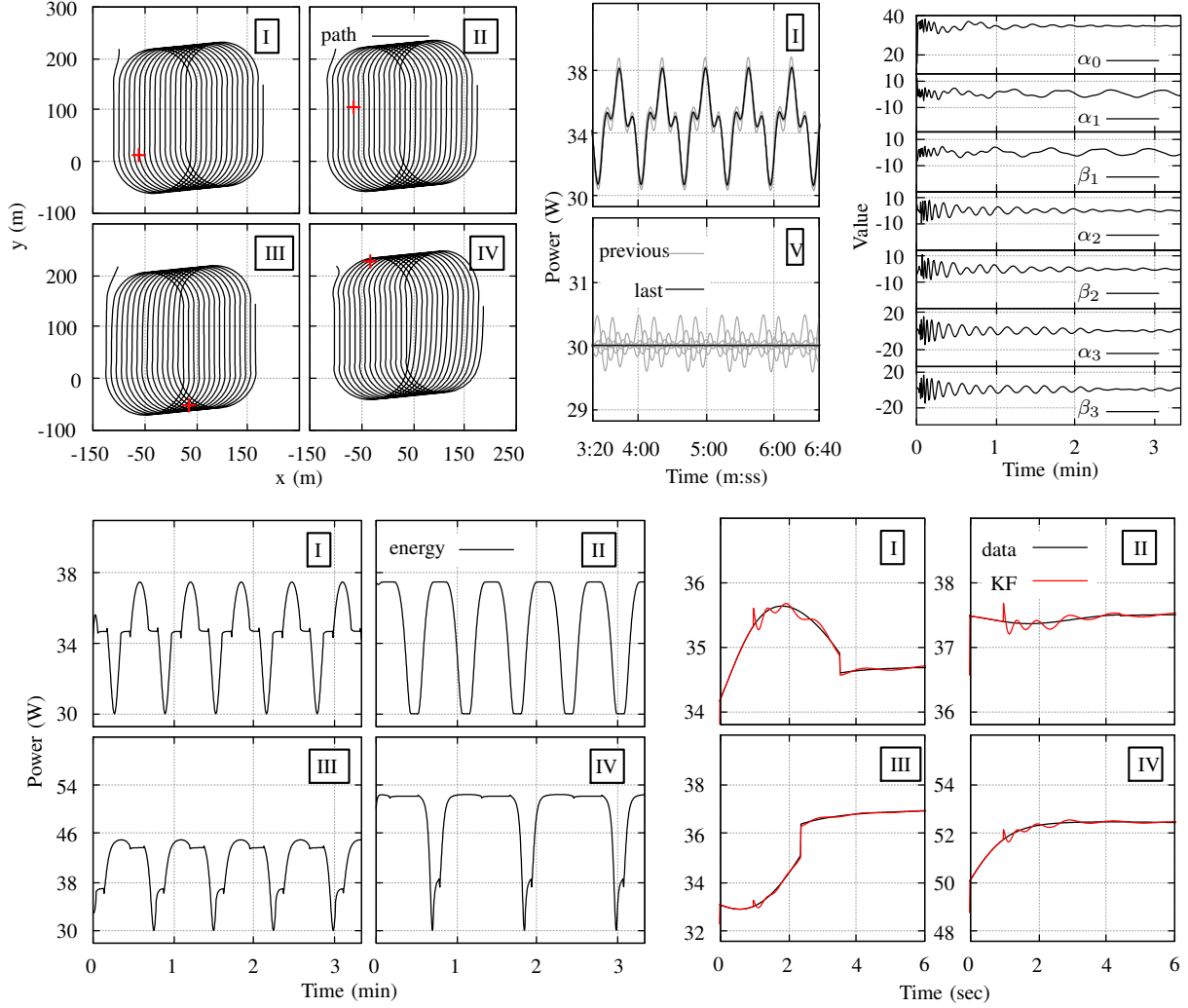


Fig. 6.

```

27: while  $k \leq t_f$  do
28:    $\mathbf{u}_k^a \leftarrow \{\mathbf{u}_k^a \mid (\varphi_i(\mathbf{p}_k, \mathbf{c}_i), \Psi(\mathbf{s}_i)) \in \lambda(\mathbf{p}_k)\}$ 
29:    $(\mathbf{q}_k, P_k, \mathbf{u}_k^a) \leftarrow \text{STEP}(\mathbf{q}_k, \mathbf{u}_k^a, \mathbf{u}_{k-1}^a, P_k)$ 
30:    $\mathbf{p}_k \leftarrow \mathbf{p}_k \dot{\mathbf{p}}_d(\mathbf{p}_k)/v$ 
31:    $\mathbf{u}_{k-1}^a \leftarrow \mathbf{u}_k^a$ 
32:    $k \leftarrow k + 1$ 
33: end while
34: end procedure

```

Per each time step  $k$  (the final time  $t_f$  is unknown), the algorithm updates the state—the position at line 30 and the energy coefficients at line 29—and the control input. Note that the position can be computed directly from Equation (III-A). If the velocity is  $v \in \mathbb{R}_{\geq 0}$ , and the starting point  $\mathbf{p}_0$ ,  $\mathbf{p}_{k+1} = \mathbf{p}_k \dot{\mathbf{p}}_d(\mathbf{p}_k)/v$ .

In detail, initial guess for  $P_0 \in \mathbb{R}^{j \times j}$  is positive definite and derived empirically, for  $\mathbf{q}_0$  the initial measurement is distributed to the coefficients (see Theorem IV.2). Line 2 selects the maximum possible control

from the current control input. Line 3 uses robust output feedback model predictive control (MPC) [34] to select the optimal control  $\mathbf{u}^0$  for a given horizon  $N \in \mathbb{Z}_{>0}$  from the cost function

$$\begin{aligned}
 l(\mathbf{q}_k, \mathbf{u}_k) &:= (1/2)(\mathbf{q}_k^T Q \mathbf{q}_k + \mathbf{u}_k^T R \mathbf{u}_k), \\
 V_f(\mathbf{q}_k) &:= (1/2)(\mathbf{q}_k^T P_f \mathbf{q}_k),
 \end{aligned} \tag{7}$$

where matrices  $Q \in \mathbb{R}^{j \times j}$ ,  $R \in \mathbb{R}^{l \times l}$  are positive definite.

Follows a check if the mission can finish without the eventuality of battery discharge (output constraints satisfaction) at line 5, with the control input being eventually updated and the process reiterated at line 7.

Before the next step, state estimator—the discrete-time Kalman filter [37] at lines 13–16—predicts the state  $\mathbf{q}$  if the modeled instantaneous energy consumption diverges from the sensor's value  $y_k^a$  more than a given  $\varepsilon \in \mathbb{R}_{\geq 0}$ , or sensor measurements are unavailable ( $y_k^a = 0$ ).

## V. RESULTS

\*

## VI. CONCLUSION AND FUTURE WORK

\*

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APPENDIX I  
PROOF OF LEMMA III.1

We propose a formal proof of Lemma III.1. The proof justifies the choice of the items of the matrices  $A, C$  and of the initial guess  $\mathbf{q}_0$  in Equation (4). We write these elements such that the coefficients of the series  $a_0, \dots, b_r$  are the same as the coefficients of the state  $\alpha_0, \dots, \beta_r$ .

Let us re-write the Fourier series expression in Equation (2) in its complex form with the well-known Euler's formula  $e^{it} = \cos t + i \sin t$ . With  $t = \omega j t$ , we find the expression for  $\cos \omega j t = (e^{i\omega j t} + e^{-i\omega j t})/2$  and  $\sin \omega j t = (e^{i\omega j t} - e^{-i\omega j t})/(2i)$  by substitution of  $\sin \omega j t$  and  $\cos \omega j t$  respectively. This leads [38]

$$h(t) = a_0/T + (1/T) \sum_{j=1}^r e^{i\omega j t} (a_j - ib_j) + (1/T) \sum_{j=1}^r e^{-i\omega j t} (a_j + ib_j), \quad (8)$$

where  $i$  is the imaginary unit.

The solution at time  $t$  can be expressed  $\mathbf{q} = e^{At} \mathbf{q}_0$ . Both the solution and the system in Equation (3) are well established expressions derived using standard textbooks [38], [39]. To solve the matrix exponential  $e^{At}$ , we use the eigenvectors matrix decomposition method [40].

The method works on the similarity transformation  $A = VDV^{-1}$ . The power series definition of  $e^{At}$  implies  $e^{At} = Ve^{Dt}V^{-1}$  [40]. We consider the non-singular matrix  $V$ , whose columns are eigenvectors of  $A$ ;  $V := [v_0 \ v_1^0 \ v_1^1 \ \dots \ v_r^0 \ v_r^1]$ . We then consider the diagonal matrix of eigenvalues  $D = \text{diag}(\lambda_0, \lambda_1^0, \lambda_1^1, \dots, \lambda_r^0, \lambda_r^1)$ .  $\lambda_0$  is the eigenvalue associated to the first item of  $A$ .  $\lambda_j^0, \lambda_j^1$  are the two eigenvalues associated with the block  $A_j$ . We can write  $Av_j = \lambda_j v_j \ \forall j = \{1, \dots, m\}$ , and  $AV = VD$ .

We apply the approach in terms of Equation (3), under the assumptions;  $\dot{\mathbf{q}} = A\mathbf{q}$ . The linear combination of the initial guess and the generic solution

$$F\mathbf{q}(0) = \gamma_0 v_0 + \sum_{k=0}^1 \sum_{j=1}^r \gamma_j v_j^k$$

$$F\mathbf{q}(t) = \gamma_0 e^{\lambda_0 t} v_0 + \sum_{k=0}^1 \sum_{j=1}^r \gamma_j e^{\lambda_j^k t} v_j^k$$

where  $F = [1 \ \dots \ 1]$  is a properly sized vector of ones.

Let us consider the second expression. It represents the linear combination of all the coefficients of the state

at time  $t$ . It can also be expressed in the following form

$$F\mathbf{q}(t)/T = \gamma_0 e^{\lambda_0 t} v_0/T + (1/T) \sum_{j=1}^r \gamma_j e^{\lambda_j^0 t} v_j^0 + (1/T) \sum_{j=1}^r \gamma_j e^{\lambda_j^1 t} v_j^1. \quad (9)$$

We proof that the eigenvalues  $\lambda$  and eigenvectors  $V$  are such that Equation (9) is equivalent to Equation (8).

The matrix  $A$  is a block diagonal matrix, so we can express its determinant as the multiplication of the determinants of its blocks  $\det(A) = \det(0) \times \det(A_1) \times \dots \times \det(A_r)$ . We proof the first determinant and the others separately.

First, we proof that the first term of both the Equation (8) and (9) matches. We find the eigenvalue from  $\det(0) = 0$ , which is  $\lambda_0 = 0$ . The corresponding eigenvector can be chosen arbitrarily ( $0 - \lambda_0)v_0 = [0 \ \dots \ 0] \ \forall v_0$ , thus we choose  $v_0 = [1 \ 0 \ \dots \ 0]^T$ . We find the value  $\gamma_0$  of the vector  $\gamma$  so that the terms are equal  $\gamma_0 = [a_0 \ 0 \ \dots \ 0]$ .

Then, we proof that all the terms in the sum of both the Equations (8) and (9) match.

For the first block  $A_1$ , we find the eigenvalues from  $\det(A_1 - \lambda I) = 0$ . The polynomial  $\lambda^2 + \omega^2$ , gives two complex roots—the two eigenvalues  $\lambda_1^0 = i\omega$  and  $\lambda_1^1 = -i\omega$ . The eigenvector associated with the eigenvalue  $\lambda_1^0$  is  $v_1^0 = [0 \ -i \ 1 \ 0 \ \dots \ 0]^T$ . The eigenvector associated with the eigenvalue  $\lambda_1^1$  is  $v_1^1 = [0 \ i \ 1 \ 0 \ \dots \ 0]^T$ . Again, we find the values  $\gamma_1$  of the vector  $\gamma$  such that the equivalences

$$\begin{cases} e^{i\omega t} (a_1 - ib_1) &= \gamma_1 e^{i\omega t} v_1^0 \\ e^{-i\omega t} (a_1 + ib_1) &= \gamma_1 e^{i\omega t} v_1^1 \end{cases}$$

hold. They hold for  $\gamma_1 = [b_1 \ a_1]$ . The proof for the remaining  $r - 1$  blocks is equivalent.

The initial guess is build such that the sum of the coefficients is the same in both the signals. In the output matrix, the frequency  $1/T$  accounts for the period in Equations (8–9) and (2). At time instant zero, the coefficients  $b_j$  are not present and the coefficients  $a_j$  are doubled for each  $j = 1, 2, \dots, r$  (thus we multiply by a half the corresponding coefficients in  $\mathbf{q}_0$ ). To match the outputs  $h(t) = y(t)$ —or equivalently  $F\mathbf{q}(t)/T = C\mathbf{q}(t)$ —we define  $C = 1/T [1 \ 1 \ 0 \ \dots \ 1 \ 0]$ . We thus conclude that the lemma holds.

We note that the signal would still be periodic with another linear combination of coefficients (for instance,  $C = d [1 \ 0 \ 1 \ \dots \ 0 \ 1]$ , or  $d [1 \ \dots \ 1]$  for a constant value  $d \in \mathbb{R}$ ). Even if both the signals  $h$  and  $y$  would be still periodic, they would not be equal. ■

## APPENDIX II PLAN SPECIFICATION

### A. Format

The algorithm reads the plan  $\Gamma$  from a file—the plan specification—with a specific file format.

The first stage  $\Gamma_1$  is expressed in the first line of the file

1:  $k_e; r; \varphi_1(\mathbf{p}_k, c_{1,1}, \dots, c_{1,\rho})$

where  $k_e$  is a convergence value,  $r$  a rotation, and  $\varphi_1$  the trajectory. The rotation is expressed in binary format with zero being the counter-clockwise rotation, one the clockwise rotation.

The next line specifies the triggering point  $\mathbf{p}_{\Gamma_1}$

2:  $x, y$

where  $x$  and  $y$  are the two coordinates of the point. The UAV reaches the point within a given radius (a parameter of the algorithm) and enters the next stage  $\Gamma_2$ . The coordinates can be expressed in function of the trajectory parameters  $c_{1,1}, \dots, c_{1,\rho}$ .

The line

3:  $[\underline{c}_1, \bar{c}_1]$

specifies the lower  $\underline{c}_1$  and upper  $\bar{c}_1$  bounds of the trajectory  $\varphi_1$ .

The following  $\sigma$  lines

4:  $[\underline{c}_{1,j}, \bar{c}_{1,j}]$

specifies the lower  $\underline{c}_{1,j}$  and upper  $\bar{c}_{1,j}$  bounds of the  $j$ -th computation parameter set. They are in ascending order.

This means that for the first stage, the algorithm can alter the trajectory satisfying  $\varphi_1 \in \mathcal{C}_1$ . It can alter the computations individually. In fact the  $j$ -th computation parameter has to be in  $\mathcal{S}_{1,j}$ .

Follows the entries for stage  $\Gamma_2$  (equivalent to lines 1–4),  $\Gamma_3$  and so on, until the last stage. We assume that the last triggering point is the final UAV position.

### B. Example

We discuss how to specify an initial plan with an example of a survey scenario, equivalent to the one in Figure 1.

Recall that the plan is composed of a set of stages (Section II). It has a variable number of entries per each stage. The first line

1:  $0.0003; 0; (\mathbf{x}+45)^2 + (\mathbf{y}-146)^2 - 4900 + \mathbf{c1}$

corresponds to the trajectory  $\varphi_1(\mathbf{p}_k, c_{1,1}) := (x+45)^2 + (y-146)^2 - 4900 + c_{1,1}$  in the initial stage  $\Gamma_1$ . The trajectory is a circle. The gain is  $k_e = 3 \cdot 10^{-4}$ , and the rotation is counter-clockwise—we use the rotation matrix  $E$  from Equation (III-A). We note that line 1 further specifies the parameter  $\mathbf{c1}$  of the radius of the circle. It corresponds to  $c_{1,1}$  in Definition II.1.

The following line

2:  $-\text{sqrt}(4900 + \mathbf{c1}) - 45, 146$

specifies the triggering point  $\mathbf{p}_{\Gamma_1}$ . **sqrt** is the square root. The UAV reaches the triggering point within a given  $\varepsilon$  (the tolerance), and the algorithm switches to the next stage  $\Gamma_2$ . We note that the triggering point is in function of the parameter  $\mathbf{c1}$ . This is necessary; an change in a parameter in one stage affects the overall flight.

Let us assume the scenario has two computations. One computation is the CNN object detection algorithm, and it can be planned by one parameter  $c_{1,2}$ , the fps rate. Another computation is the variable key-size encryption algorithm. It can be planned by one parameter  $c_{1,2}$ , the key-size. Then in the plan

3:  $[-3 \cdot 10^3, 0]$

4:  $[2, 10]$

5:  $[32, 448]$

line 3 corresponds to the trajectory  $\varphi_1$ 's constraints set  $\mathcal{C}_1 = [-3 \cdot 10^3, 0]$ . The algorithm selects  $c_{1,1} \in \mathcal{C}_1$ . Line 4 corresponds to the computation constraints set  $\mathcal{S}_{1,1}$  for the computation parameter  $c_{1,2}$ . Line 4 corresponds to the computation constraints set  $\mathcal{S}_{1,2}$  for the computation parameter  $c_{1,2}$ . Lines 3–5 must follow the ascending order of the parameters. Lines 1–5 all describe the stage  $\Gamma_1$ .

Once the UAV reaches the triggering point (line 2) and there are no further lines, the algorithm terminates (the last triggering point is the final point). If there are further lines, the stage switches to  $\Gamma_2$

6:  $0.05; 1; \mathbf{x} + \text{sqrt}(4900 + \mathbf{c1}) + 45$

7:  $-\text{sqrt}(4900 + \mathbf{c1}) - 45, 11$

8:  $[-3 \cdot 10^3, 0]$

9:  $[2, 10]$

10:  $[32, 448]$

line 6 corresponds to the trajectory  $\varphi_2(\mathbf{p}_k, c_{1,1}) := x + \sqrt{4900 + c_{1,1}} + 45$ , a linear equation that intersects the triggering point (line 2). The gain is  $k_e = 5 \cdot 10^{-2}$ , and the rotation is opposite to line 1. The algorithm computes  $-E$  from the rotation matrix in Equation III-A.

The gain is different from the one used in  $\varphi_1$  as different equations require different convergence rate. When we follow a circle it is useful to turn the rotation direction in advance. When we follow a straight line, it is preferable to turn the rotation direction closer to the line. The rotation is likewise different. In fact, the rotation matrix  $E$  points upwards in the space. It guides the UAV in the counter-clockwise direction (see Figure 5; the UAV is traveling counter-clockwise). The rotation matrix  $-E$  points downwards and guides the UAV in the clockwise direction.

The triggering point of the stage  $\Gamma_2$  at line 7 also depends on the parameter  $\mathbf{c1}$ . Lines 8–10 specifies the constraints sets. They are expressed the same way as lines 3–5. They have to be specified explicitly for

each stage, although they don't change. The current implementation of the algorithm has no other means to distinguish different constraints sets.

Again, if there are further lines and UAV reaches the triggering point (line 7), the algorithm switches the stage to  $\Gamma_3$

```
11: 0.0003; 90; (x+sqrt(4900+c1)-30)^2
    + (y-11)^2-5625
12: -sqrt(4900+c1)+105, 11
13: [-3*10^3, 0]
14: [2, 10]
15: [32, 448]
```

line 11 corresponds to the trajectory  $\varphi_3(\mathbf{p}_k, c_{1,1}) := (x + \sqrt{4900 + c_{1,1}} - 30)^2 + (y - 11)^2 - 5625$ , a circle of fixed size that changes the coordinates in function of the parameter  $\mathbf{c1}$ . The rotation is counter-clockwise, alike  $\varphi_1$ .

The lines

```
16: 0.05; 90; x+sqrt(4900+c1)-105
17: -sqrt(4900+c1)+105, 147
18: [-3*10^3, 0]
19: [2, 10]
20: [32, 448]
21: 0.0003; 90; (x-sqrt(4900+c1)+105)^2
    + (y-147)^2-4900+c1
```

```
22: -3*sqrt(4900+c1)+105, 147
23: [-3*10^3, 0]
```

defines the stages  $\Gamma_4$  and  $\Gamma_5$ . Line 16 corresponds to the trajectory  $\varphi_4(\mathbf{p}_k, c_{1,1}) := x + \sqrt{4900 + c_{1,1}} - 105$ . Line 21 corresponds to the trajectory  $\varphi_4(\mathbf{p}_k, c_{0,1}) := (x - \sqrt{4900 + c_{0,1}} + 105)^2 + (y - 147)^2 - 4900 + c_{0,1}$ .

We note that the survey scenario example changes the radius of the circular trajectory  $\varphi_1$  through the parameter  $c_{1,1}$ . It also changes the radius of  $\varphi_5$  in the same way (recall Assumption II.1). The other trajectories ( $\varphi_2, \varphi_3, \varphi_4$ ) displacement alters the distance between the lines in the survey. For simplicity, we suppose line 22 describes the final point. We further suppose the user does not desire to process any computation in the last stage  $\Gamma_5$ . To inhibit the computations

```
24: [0, 0]
25: [0, 0]
```

If there are any following stages in the survey, they are constructed the same way. The stages  $\Gamma_i$  for  $i := \{0, 4, 8, \dots\}$  contain trajectory circle with variable radius. For  $i := \{1, 3, 5, \dots\}$  trajectory line with variable displacement. For  $i := \{2, 6, 10, \dots\}$  trajectory circle with fixed radius and variable displacement.