

Equations for the sim

CIA Wanted

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We split horizontal and vertical dynamics.

1 Vertical dynamics

$$a_v = \frac{1}{m}(L - W), \quad (1)$$

where a_v is the vertical acceleration in ms^{-2} , m is the mass of the aircraft in Kg, and L and W are the lift and weight forces respectively in Newtons.

$$W = mg, \quad (2)$$

where $g = 9.8 \text{ms}^{-2}$ is the gravity acceleration.

$$L = c_l v_s^2, \quad (3)$$

where c_l is a coefficient to be determined experimentally, and v_s is the airspeed of the aircraft. We can assume that

$$v_s = c_{th}(t_h + \Delta t_h) + w_{bx}, \quad (4)$$

where $(t_h + \Delta t_h) \in [0, 100]$ is the throttle signal and c_{th} is a coefficient to be determined experimentally, and w_{bx} is the component of the wind vector along the horizontal axis of the vehicle. We set t_h to a nominal value, let us say 50 and we use Δt_h to control the altitude of the aircraft with a P controller, i.e.,

$$\Delta t_h = k_p(h_d - h_c) - k_{vv}v_v, \quad (5)$$

where k_p and k_{vv} are positive gain constants, h_d and h_c are the desired and current altitudes in meters respectively, and v_v is the vertical velocity in m/s.

1.1 Euler integration

$$\begin{cases} h(k+1) &= h(k) + v_s(k)\Delta T \\ v_v(k+1) &= v_v(k) + a_v(k)\Delta T, \end{cases} \quad (6)$$

where v_v is the vertical speed (do not confuse it with the airspeed v_s).

2 Horizontal kinematics

$$\begin{cases} \dot{p} &= c_{th}(t_h + \Delta t_h) \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + w, \\ \dot{\theta} &= u \end{cases}, \quad (7)$$

where p is the position of the aircraft in the plane, θ is the angle of the velocity vector in the plane, w is the wind vector, and u is the control action for steering the aircraft (path tracking controller).

2.1 Euler integration

$$\begin{cases} p(k+1) &= p(k) + \dot{p}(k)\Delta T \\ \theta(k+1) &= \theta(k) + \dot{\theta}(k)\Delta T, \end{cases} \quad (8)$$