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***Abstract*— abstract abstract abstract abstract abstract abstract abstract abstract abstract abstract abstract abstract abstract abstract**

I. Introduction

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II. State of the Art

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III. Model

The model presented in this section deals with the motion and the energy using two distinct systems, subject to the same process noise w (i.e., the atmospheric interferences such as wind).

The motion defines a trajectory and a position $\mathbf{p}$; the trajectory is expressed through explicit trajectory equation φ, while the position determines a vehicle frame $\mathcal{O}_V$ in the 2D Euler space relative to an inertial frame $\mathcal{O}_W$. For the sake of simplicity, it is assumed that the z-axis is constrained to a specific altitude h, with the system being free to evolve in the x, y-axes (i.e., the drone is flying, performing a mission, at a given height). A motion guidance action is derived using vector field design, enabling the convergence to the desired trajectory anywhere in $\mathcal{O}_W$.

The energy $\mathbf{q}$ spent to perform such guidance action is later derived using Fourier analysis and its state-space representation. This component is decomposed in the energy needed for the actuation (i.e., the mechanical energy), and the computations (i.e., the computational energy). A control action, which varies quality of service (QoS) levels of the software being executed, is derived for this purpose. The main goal is to tune the explicit energy equation allowing to maximize the computational and mechanical outcome of the mission while trying to minimize the eventuality of battery discharge.

A. Motion and Guidance Model

The position $\mathbf{p}$, derived from [1], is described by the following non-holonomic model:

$$\begin{cases} \dot{\mathbf{p}}(t) &= s\Psi(\psi(t)) + w(t), \\ \dot{\psi}(t) &= u(t), \end{cases} \tag{1}$$

where $\mathbf{p}(t) \in \mathbb{R}^2$ is a point in $\mathcal{O}_W$, $s \in \mathbb{R}$ describes the airspeed assumed constant, $\psi \in (-\pi, \pi]$ the attitude yaw angle and $\Psi(\psi(t)) = \begin{bmatrix}\cos\psi(t) & \sin\psi(t)\end{bmatrix}^T$, and $u \in \mathbb{R}$ the guidance action which express the angular velocity $\dot{\psi}$ of $\mathcal{O}_V$.

Let us define a generic continuously differentiable function $\varphi : \mathbb{R}^2 \to \mathbb{R}$. The desired trajectory $\mathcal{P}$ can be hence defined:

$$\mathcal{P} := \{\mathbf{p} : \|\varphi(\mathbf{p})\| < c + \varepsilon\}, \tag{2}$$

as the set of points that follows the trajectory $\varphi(\mathbf{p}) = c$ within a given $\varepsilon \in \mathbb{R}^{\geq 0}$.

Given a set of discrete values c_i, the space relative to $\mathcal{O}_W$ can be covered by all the trajectories $\varphi(\mathbf{p}) = c_i$ within specific boundaries $c_m \leq c_i \leq c_M$.

The concept of a trajectory is used later to design a controller that selects c_i value with the highest energy value under the energy budget constraints defined in the next subsection. Here we design a guidance action u that allows following such trajectory. The guidance is derived using the vector field approach presented in [1]. The algorithm heads to $\mathcal{P}$ using a vector field based on the minimization of the norm in Equation (2).

Let us define $\varPhi := \varphi(\mathbf{p})$. The vector field is hence defined as a desired velocity vector

$$\dot{\mathbf{p}}_d(\mathbf{p}) := E\nabla\varPhi - k_e\varPhi\nabla\varPhi, \;\; E = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \tag{3}$$

where $\nabla\varPhi \in \mathbb{R}^2$ is defined as the gradient of φ at the point $\mathbf{p}$ (i.e., it's vector field), E specifies the tracking direction, and $k_e \in \mathbb{R}^{\geq 0}$ the gain which adjusts the speed of convergence of the vector field to the desired trajectory.

The direction the velocity vector $\dot{\mathbf{p}}$ is pointing at is generally different from the course heading $\chi \in (-\pi, \pi]$ due to the atmospheric interferences.

Let us further define $\hat{\mathbf{p}} := \mathbf{p}/\|\mathbf{p}\|$, the desired course heading rate $\dot{\chi}_d$ is computed by sensing the position $\mathbf{p}$, the ground velocity $\dot{\mathbf{p}}$, and is expressed

$$\begin{aligned} \dot{\chi}(\mathbf{p}, \dot{\mathbf{p}}) = -E\frac{\dot{\mathbf{p}}_d}{\|\dot{\mathbf{p}}_d\|^2} \cdot \\ \left(E\hat{\dot{\mathbf{p}}}_d\hat{\dot{\mathbf{p}}}_d^T \left((E - k_e\varPhi)H(\varPhi)\dot{\mathbf{p}} - k_e\nabla\varPhi^T\dot{\mathbf{p}}\nabla\varPhi\right)\right)^T, \end{aligned} \tag{4}$$

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where $H(\cdot)$ is defined as the Hessian operator, with the physical meaning being that the curvature of the desired trajectory has to be known in order to be tracked.

Under the assumption of the airspeed $s = \|w(t)\|$ for $t > 0$ (i.e., the constant airspeed being greater than the norm of the wind), the guidance action defines how fast the drone converges to the desired trajectory and can be expressed

$$u(\dot{\mathbf{p}}, \mathbf{p}, \psi) = \frac{\|\mathbf{p}\|}{s \cos \beta} \left(\dot{\chi}_d(\dot{\mathbf{p}}, \mathbf{p}) + k_d \hat{\dot{\mathbf{p}}}^T E \hat{\dot{\mathbf{p}}}_d \right), \quad (5)$$

where $k_d \in \mathbb{R}^{\geq 0}$ is the gain which adjusts the speed of convergence of the drone to the vector field $\dot{\mathbf{p}}_d$ to the desired trajectory, $\beta = \cos^{-1} \left(\hat{\dot{\mathbf{p}}}^T \Psi(\psi) \right)$ is the slideslip angle, and $\dot{\chi}_d$ is given in Equation (4).

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B. Energy Model

We consider a Fourier series of an arbitrary order r to evaluate the energy spent performing a given motion and software configuration

$$f(t) = \sum_{n=0}^{r} a_n \cos \frac{nt}{\xi} + b_n \sin \frac{nt}{\xi}, \quad (6)$$

where $\xi \in \mathbb{R}$ is the characteristic time, and $a_n, b_n \in \mathbb{R}$ for $n \in \{0, \ldots, r\}$ the Fourier series coefficients. The Fourier analysis allows to account for the periodicity of the mission, with a trajectory $\mathcal{P}$ being reitared over time.

The non-linear model in Equation (6) can be expressed using an equivalent linear time-varying state-space model, expressed in the following form:

$$\begin{cases} \dot{\mathbf{q}}(t) &= A\mathbf{q}(t) + B\mathbf{u}(t) + w(t), \\ y(t) &= C\mathbf{q}(t) + v(t), \end{cases} \quad (7)$$

where $y(t) \in \mathbb{R}^{\geq 0}$ is the energy evolution of the controlled system, $w(t)$ the same process noise from the system in Equation (1), $v(t) \in \mathbb{R}^{\geq 0}$ the measurement noise, and the control $\mathbf{u}$ along with the input matrix B are defined subsequently. The state $\mathbf{q}$ respresents the evolution of the Fourier series coefficients in time, which can be expressed along the state transition matrix, and the output matrix as follows

$$\begin{aligned} \mathbf{q}(t) &= \left[\begin{array}{c|ccccc} a_0 & a_1 & b_1 & \cdots & a_r & b_r \end{array} \right], \\ A &= \left[\begin{array}{c|ccc} 1 & \mathbf{0} & \cdots & \mathbf{0} \\ \hline \mathbf{0} & A_1 & \cdots & \mathbf{0} \\ \vdots & \vdots & \vdots & \vdots \\ \mathbf{0} & \mathbf{0} & \cdots & A_r \end{array} \right], \; A_n = \begin{bmatrix} 0 & n \\ -n^2 & 0 \end{bmatrix}, \quad (8) \\ C &= \left[\begin{array}{c|ccccc} 1 & 1 & 0 & \cdots & 1 & 0 \end{array} \right], \end{aligned}$$

where $\mathbf{q} \in \mathbb{R}^l$ given $l := 2r + 1$, $A \in \mathbb{R}^{l \times l}$ is the state transmission matrix, $C \in \mathbb{R}^l$ is the output matrix respectively. Furthermore, the first row and column of the matrix A contain a zeros row vectors and columns vectors respectively.

Motivated by the fact that the system of interest is sampled at discrete time, and for the sake of simplicity, we consider the discretized version of Equations (1 and 7) by employing the following expression:

$$\begin{cases} \mathbf{p}_{k+1} &= s\Psi(\psi_k) + w_k, \\ \psi_{k+1} &= u_k, \end{cases} \quad (9b)$$

$$\begin{cases} \mathbf{q}_{k+1} &= A\mathbf{q}_k + B\mathbf{u}_k + w_k, \\ y_k &= C\mathbf{q}_k + v_k. \end{cases} \quad (9a)$$

As the system was observed to behave stochastically, with the process and measurement noise evolving in a normal distribution, a Kalman filter [2], [3] is employed to predict the state $\hat{\mathbf{q}}$. Such a state, along with the predicted output $\hat{y}$, differs from the model state $\mathbf{q}$ and measured output y in Equation (9b) due to the presence of uncertainty.

The prediction is done using the following expression:

$$\hat{\mathbf{q}}_{k+1}^{-} = A\hat{\mathbf{q}}_k + B\mathbf{u}_k, \quad (10a)$$

$$P_{k+1}^{-} = AP_kA^T + Q, \quad (10b)$$

where $\hat{\mathbf{q}}_k^{-}, \hat{\mathbf{q}}_k \in \mathbb{R}^l$ depicts the estimate of the state before and after measurement (or simply estimate), and $P_k, P_k^{-} \in \mathbb{R}^{l \times l}$ the error covariance matrix (i.e., the variance of the estimate before measurement).

The estimation of the state and the update of the predicted output is done using the following expression:

$$K_k = (CP_{k+1}^{-}C^T + R)^{-1}(P_{k+1}^{-}C^T), \quad (11a)$$

$$\hat{\mathbf{q}}_{k+1} = \hat{\mathbf{q}}_{k+1}^{-} + K_k(y_k - C\hat{\mathbf{q}}_{k+1}^{-}), \quad (11b)$$

$$P_{k+1} = (I - G_{k+1}C)P_{k+1}^{-}, \quad (11c)$$

$$\hat{y}_k = C\hat{\mathbf{q}}_{k+1}, \quad (11d)$$

where $K_k \in \mathbb{R}^l$ is the gain of the Kalman filter, and I the identity matrix. The noise covariance matrixes $Q \in \mathbb{R}^{l \times l}, R \in \mathbb{R}$ indicates the process noise and sensor noise covariance respectively, and $\hat{y}_k \in \mathbb{R}$ is the estimated energy.

Equation (10) converges to the predicted energy evolution as follows. An initial guess of the values $\hat{\mathbf{q}}_0, P_0$ is derived empirically from collected data. It is worth considering that an appropriate guess of these parameters allows the system to converge to the desired energy evolution in a shorter amount of time. The tuning parameters Q, R are also derived from the collected data and may differ due to i.e., different sensors used to measure the instantaneous energy consumption, or different atmospheric conditions accounting for the process noise.

At time $k = 0$, the initial estimate before measurement of the state and of the error covariance matrix is updated in Equation (10a) and (10b) respectively. The value of $\hat{\mathbf{q}}_1$ is then used in Equation (11b) to estimate the current state along with the measurment from the sensor y_0, where the sensor noise covariance matrix R accounts for the amount of uncertainty in the measurement. The estimated ouput $\hat{y}_0$ is then obtained from Equation (11d). The algorithm is iterative. At time $k = 1$ the values $\hat{\mathbf{q}} = 1, P_1$ computed at previous step are used to estimate the values $\hat{\mathbf{q}}_2, P_2$, and y_1.

Two different components of the overall energy consumption are being modeled in our analysis, an approach that has been extensively reviewed in our previous work [4]–[7]. A mechanical energy model accounts for the energy by reason of the physical system being moved in space, whereas a computational energy model accounts for the computations. The main goal is to derive a control action $\mathbf{u}$ for the current state $\hat{\mathbf{q}}$ from the optimal control law $\kappa(\hat{\mathbf{q}})$. This is achieved by solving online a finite horizon optimal control problem by the hand of a model predictive control (MPC) derived later in this section.

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C. Computational Model

The energy due to the computational units of the system is obtained with `powprofiler`, an open-source modeling tool that measures empirically a number of software configuration and build an energy model accordingly [4]. A multivariate linear interpolation is derived automatically by `powprofiler`, while being accessed online at the hand of a lookup table in the optimal control algorithm presented later. The robotics system in such an analysis composes a number of computationally expensive ROS nodes, allowing to vary the amount of computations changing node-specific quality of service (QoS) values. For the sake of simplicity and to allow an easy integration in an existing ROS system, the QoS values are utilizing ROS middleware, meaning that defined ROS parameters are intended specifically as QoS ranges.

Let us define $\mathrm{QoS}_i \in \mathbb{Z}^{\geq 0}, \forall i \in \{0, \ldots, r\}$ the i-th QoS range, $g_c(\mathrm{QoS}_i) \in \mathbb{R}^{\geq 0}$ the instantaneous energy value obtained interogating `powprofiler` online, and $\mathbf{u}_c$ the set of r QoS values the system is composed of. The computational energy component can be hence described using the following expression:

$$E_c(\mathbf{u}_c) = \sum_{i=0}^{r} g_c(\mathrm{QoS}_i), \tag{12}$$

TODO

IV. Evaluation

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V. Conclusion and Future Work

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