# Equations for the sim

CIA Wanted

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We split horizontal and vertical dynamics.

# 1 Vertical dynamics

$$a_v = \frac{1}{m}(L - W), \tag{1}$$

where $a_v$ is the vertical acceleration in $\mathrm{ms}^{-2}$, $m$ is the mass of the aircraft in Kg, and $L$ and $W$ are the lift and weight forces respectively in Newtons.

$$W = mg, \tag{2}$$

where $g = 9.8\,\mathrm{ms}^{-2}$ is the gravity acceleration.

$$L = c_l v_s^2, \tag{3}$$

where $c_l$ is a coefficient to be determined experimentally, and $v_s$ is the airspeed of the aircraft. We can assume that

$$v_s = c_{th}\,(t_h + \Delta t_h) + w_{bx}, \tag{4}$$

where $(t_h + \Delta_h) \in [0, 100]$ is the throttle signal and $c_{th}$ is a coefficient to be determined experimentally, and $w_{bx}$ is the component of the wind vector along the horizontal axis of the vehicle. We set $t_h$ to a nominal value, let us say 50 and we use $\Delta t_h$ to control the altitude of the aircrat with a P controller, i.e.,

$$\Delta t_h = k_p(h_d - h_c), \tag{5}$$

where $k_p$ is a positive gain constant, and $h_d$ is the desired altitude and $h_c$ is the current altitude of the aircraft in meters.

## 1.1 Euler integration

$$\begin{cases} h(k+1) & = h(k) + v_s(k)\Delta T \\ v_v(k+1) & = v_s(k) + a_v(k)\Delta T, \end{cases} \tag{6}$$

where $v_v$ is the vertical speed (do not confuse it with the airspeed $v_s$).

# 2 Horizontal kinematics

$$\begin{cases} \dot{p} & = c_{th}(t_h + \Delta t_h) \begin{bmatrix} \cos(\theta) \\ \sin(\theta) \end{bmatrix} + w \\ \dot{\theta} & = u \end{cases}, \tag{7}$$

where $p$ is the position of the aircraft in the plane, $\theta$ is the angle of the velocity vector in the plane, $w$ is the wind vector, and $u$ is the control action for steering the aircraft (path tracking controller).

### 2.1 Euler integration

$$\begin{cases} p(k+1) & = p(k) + \dot{p}(k)\Delta T \\ \theta(k+1) & = \theta(k) + \dot{\theta}(k)\Delta T, \end{cases} \tag{8}$$