

Energy-Aware Planning-Scheduling for Autonomous Aerial Robots

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Abstract—The letter presents a planning-scheduling approach for battery-powered autonomous aerial robots, which plans in-flight a coverage path and simultaneously schedules onboard computational tasks. It derives a novel variable coverage motion robust to airborne constraints and an empirically motivated differential periodic energy model. The model is enhanced with the time and energy contribution of the path and tasks, utilizing a previously proposed automatic modeling tool. Accounting for uncertainty, an initial “plan-schedule” is replanned and rescheduled in the function of the battery, exploiting all the available resources yet avoiding possible in-flight failure in case of unexpected battery drops, e.g., due to adverse atmospheric conditions.

Index Terms—Aerial Systems: Perception and Autonomy, Optimization and Optimal Control, Planning, Scheduling and Coordination, Planning under Uncertainty

I. INTRODUCTION

USE CASES involving aerial robots span broadly. They compromise diverse planning and scheduling strategies and often require high autonomy under strict energy budgets. A typical instance is an aerial robot visiting each point in a given space [1], running some onboard computational tasks, a problem in the literature under coverage path planning (CPP) [2], [3]. Here, the aerial robot might detect ground patterns and notify other ground-based actors with little human interaction (see Figure 1). Such use cases arise in, e.g., precision agriculture [4], where harvesting involves ground [5]–[10], damage prevention aerial robots [11], [12]. In these and many others, the robot frequently mounts microcontroller and heterogeneous computing hardware [13] (i.e., with CPUs and GPUs) running power-demanding computational tasks [14]–[17]. We refer to computational tasks that can be scheduled with an energy impact as computations. We are interested in the simultaneous energy optimization of motion plans and computations schedules in-flight and refer to it as energy-aware planning-scheduling. Generic mobile robotics studies that deal with planning-scheduling energy awareness are scarce [18]–[21] and focused on ground-based robots [13], [20]–[23]. Yet, aerial robots are particularly affected by various energy considerations. Indeed it would be generally required to land to recharge the battery [24]. Such a state of practice

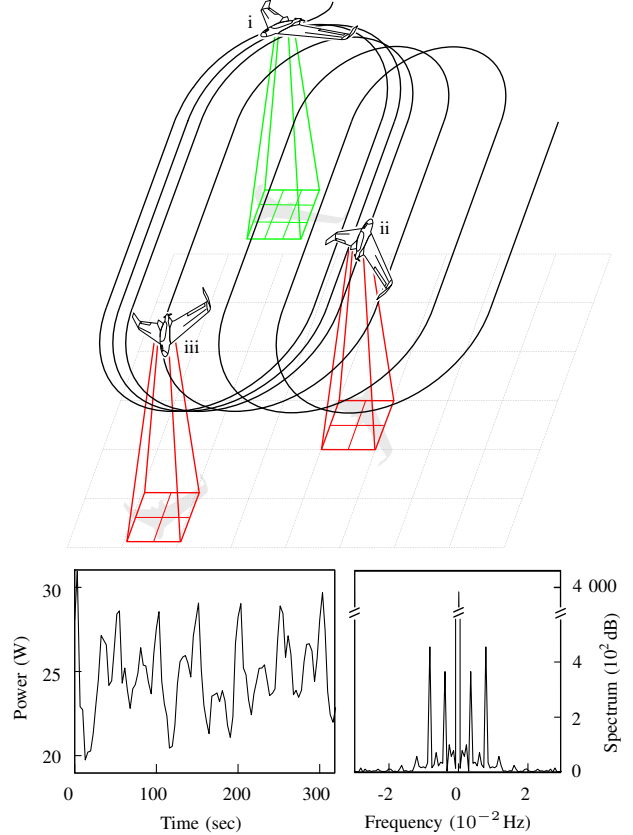


Fig. 1: Illustrative example of planning-scheduling. An initial plan (in i) is re-planned in-flight computations-wise (in ii) and motion-wise (in iii). Follow the collected energy data of a physical aerial robot flying the use case.

has prompted us to propose a planning-scheduling approach for autonomous aerial robots. It combines the past body of knowledge, addressing aerial robots’ peculiarities such as the atmospheric, battery, and turning radius constraints. Numerical simulations and experimental data of both static and dynamic plans-schedules show improved power savings and fault tolerance with the aerial robots remedying in-flight failures. Fig. 1 illustrates the intuition: an aerial robot flies full plan-schedule (i), that is optimized w.r.t. the battery (ii), and altered due to, e.g., unexpected battery defects (iii).

There are numerous planning algorithms applied to a variety of robots. An instance is an algorithm selecting an energy-optimized trajectory [25] by, e.g., maximizing the operational time [26]. Many apply to a small number of robots [27] and focus exclusively on planning the trajectory [28], despite evidence of the energy influence of consumptions [13], [18], [19], [21]. In view of the availability of powerful heterogeneous computing hardware [29], the use of computations is further expected to increase in the foreseeable future [30]–[32]. In this context, planning-scheduling energy awareness is a recent research direction. Early studies (2000–2010) varied

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hardware-dependent aspects, e.g., frequency, voltage, along with motion aspects, e.g., motor and travel velocities [13], [18], [22], [33] whereas the literature from the past decade derives energy-aware plans-schedules in broader terms. These include simultaneous considerations for planning-scheduling in perception [21], localization [20], navigation [23], [34], [35], and anytime planning [19]. We extend the relevant literature to the case of CPP with aerial robots.

Our focus is on fixed wings, i.e., airborne robots where wings provide lift, propellers forward thrust, and control surfaces maneuvering. Here, motion and computations energies are within an order of magnitude from each other [36]. Indeed there are other classes where planning-scheduling energy awareness leads to irrelevant savings, i.e., when the motion energy contribution far outreaches the computations and vice-versa. The occurrence frequently happens with rotary-wing aerial robots (e.g., quadrotors or quadcopters, hexacopters, etc.) and lighter-than-air aerial robots (e.g., blimps). It is a common theme with wider planning-scheduling literature, focusing on energy-efficient ground-based robots such as Pioneer 3DX [13], [23], [34], [35], ARC Q14 [20], [21], and Pack-Bot UGV [22].

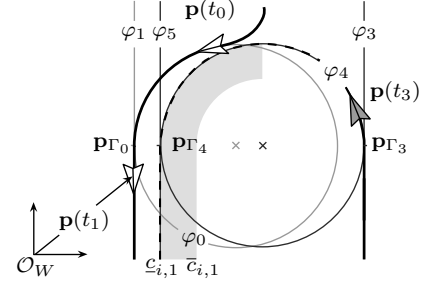
To guarantee energy awareness, our approach uses optimal control where both the paths and schedules variations are trajectories, varying between given bounds (e.g., physical constraints of the robot and computing hardware, the desired quality of the coverage, etc.). Numerous past planning-scheduling studies also employ optimization techniques [18], [20], [21], [33], whereas some others greedy [13], [19], [22] and reinforcement learning-based approaches [23], [34], [35]. Both the variations trajectories are derived for future time instants employing computations and motion energies and battery models. The energy model for the computations uses regression analysis from our earlier study on heterogeneous computing hardware energy modeling [37], [38], whereas the battery an equivalent circuit model (ECM) from the literature [39]–[41]. The motion wraps the past two in a cohesive model that uses differential and periodic modeling to predict future energy behavior of various plans-schedules. In Fig. 1, collected energy data (bottom left) and spectrum analysis (right) of a fixed-wing aerial robot flying CPP motivates the motion energy model: the evolution is periodic, an observation we exploit in Section III for the computations and motion energies and battery models.

The remaining sections of the letter are then organized as follows. Sec. II provides basic constructs, such as the concepts of the plan, stages, triggering and final points, and path functions, as well as the problem formulation. Sec. IV describes in detail the methodology of planning-scheduling. Sec. V presents the results and showcases the performances, and Sec. VI concludes and provides future perspectives. Appendices provide supplementary information.

II. PROBLEM FORMULATION

Before defining the problem of energy-aware planning-scheduling in Sec. II-B, Sec. II-A provides necessary preliminaries. For CPP and, e.g., pattern detections in the agricultural

Fig. 2: Defs. II.1–II.4 on a slice of the plan Γ . $\mathbf{p}_{\Gamma_1}, \dots$ are triggering points in which proximity happens change of stages Γ_1, \dots . Each contains a path function φ_1, \dots and parameters to alter the path and schedule $c_{1,1}, \dots$.



use case, we assume that aerial robot travels a *plan* with information about the path and the schedule.

A. Preliminaries

Definition II.1 (Plan). Given a generic point $\mathbf{p}(t) \in \mathbb{R}^2$ w.r.t. a reference frame \mathcal{O}_W of the aerial robot flying at a given altitude $h \in \mathbb{R}_{>0}$, the *plan* is a finite state machine (FSM) Γ , where the state-transition function $s : \bigcup_i \Gamma_i \times \mathbb{R}^2 \rightarrow \bigcup_i \Gamma_i$ maps a stage and a point to the next stage

$$s(\Gamma_i, \mathbf{p}(t)) := \begin{cases} \Gamma_{i+j} & \exists j \in \mathbb{Z}, \text{ if } \|\mathbf{p}(t) - \mathbf{p}_{\Gamma_i}\| < \varepsilon_i \\ \Gamma_i & \text{otherwise} \end{cases}.$$

Here, Γ_i is a *stage*, and \mathbb{Z} indicates the set of integers. At each stage, the aerial robot travels a path and runs a schedule on the computing hardware. Both are to be altered in Sec. IV within given boundaries with *path*- and *computations*-specific *parameters*. ε_i is a stage-dependent value detailed after Definition II.4.

Definition II.2 (Stage). The i th stage Γ_i at time instant t of a plan Γ is

$$\Gamma_i := \{\varphi_i(\mathbf{p}(t), c_i^\rho), c_i^\sigma \mid \forall j \in [\rho]_{>0}, c_{i,j} \in \mathcal{C}_{i,j}, \forall k \in [\sigma]_{>0}, c_{i,\rho+k} \in \mathcal{S}_{i,k}\},$$

where c_i^ρ and c_i^σ are ρ *path* and σ *computations parameters*. $\mathcal{C}_{i,j} := [\underline{c}_{i,j}, \bar{c}_{i,j}] \subseteq \mathbb{R}$ is the j th path parameter $c_{i,j}$ constraint set, and $\mathcal{S}_{i,k} := [\underline{c}_{i,\rho+k}, \bar{c}_{i,\rho+k}] \subseteq \mathbb{Z}_{\geq 0}$ is the k th computation parameter constraint set.

The notation $[\cdot]$ indicates positive naturals up to \cdot , i.e., $\{0, 1, \dots, \cdot\}$. For a set \mathbb{X} , $\mathbb{X}_{\geq 0}$ then indicates it is positive, $\mathbb{X}_{>0}$ strictly positive.

The function φ_i is a *path function* specifying the path. These are stage-dependent mathematical functions the aerial robot tracks as it travels the coverage. The notation $[\underline{\cdot}, \bar{\cdot}]$ indicates the upper/lower bounds of a parameter \cdot , i.e.,

$$\underline{\cdot} \leq \cdot \leq \bar{\cdot}. \quad (1)$$

Definition II.3 (Path functions). $\varphi_i : \mathbb{R}^2 \times \mathbb{R}^\rho \rightarrow \mathbb{R}$, $\forall i \in \{1, 2, \dots\}$ are *path functions*, forming the path. They are a function of $\mathbf{p}(t)$ and path parameters $c_i^\rho(t)$ and are continuous and twice differentiable.

The change of stages happens in the proximity of given points termed *triggering points*, whereas the plan is complete at the occurrence of the *final point*.

Definition II.4 (Triggering and final points). The *triggering point* \mathbf{p}_{Γ_i} allows the transition between stages. The *final point* is the last triggering point \mathbf{p}_{Γ_i} relative to the last stage Γ_i .

The stage-dependent value $\varepsilon_i \in \mathbb{R}_{\geq 0}$ in Def. II.1 expresses the radius of an imaginary circle over \mathbf{p}_{Γ_i} .

Fig. 2 illustrates the concepts in Defs. II.1–II.4. $\varphi_0, \dots, \varphi_5$ are path functions. φ_0 and φ_4 are circles, while φ_1, φ_3 , and φ_5 are lines. They are relative to different stages $\Gamma_1, \Gamma_2, \dots$ but Γ_0 , the starting stage. The constraint sets $\mathcal{C}_{1,1}, \mathcal{C}_{2,1}, \dots$ forms the area where it is possible to alter the paths $\varphi_1, \varphi_2, \dots$ with the parameters $c_{1,1}, c_{2,1}, \dots$; the gray area in the figure.

A convenient way of defining Γ is specifying a set of stages, a shift, and a final point. The set is termed *primitive stages* and iterated with the shift up to reaching the final point.

Definition II.5 (Primitive stages). Given the number of *primitive stages* $n \in \mathbb{Z}_{>0}$, a *shift* $\mathbf{d} \in \mathbb{R}^2$, and a final point \mathbf{p}_{Γ_f} , the stages $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ are primitive if they form the remainder of the plan with \mathbf{d} up to \mathbf{p}_{Γ_f} .

Here, c_i indicates a row vector that contains both the path and computations parameters in sequence, i.e., $c_i := [c_i^p \ c_i^c]'$, where \cdot' is the transpose of \cdot . The path parameters in the definition have a constant distance per each value in $[n]_{>0}$, i.e.,

$$\varphi_{(i-1)n+j}(\mathbf{p} + (i-1)\mathbf{d}, c_1^p) - \varphi_{in+j}(\mathbf{p} + i\mathbf{d}, c_1^p) = e_j, \quad (2)$$

holds $\forall i \in [l/n - 1]_{>0}, j \in [n]_{>0}$ assuming the total number of stages is known and is $l \in \mathbb{Z}_{>0}$. $e_j \in \mathbb{R}$ given a shift \mathbf{d} , initial point \mathbf{p} , and initial value of path parameters c_1^p .

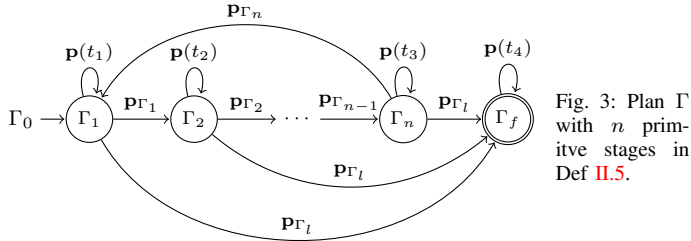


Fig. 3: Plan Γ with n primitive stages in Def II.5.

Fig. 3 illustrates the concepts in Def. II.5. A plan composed of n stages $\Gamma_1, \Gamma_2, \dots, \Gamma_n$ (containing primitive paths $\varphi_1, \varphi_2, \dots, \varphi_n$) is reiterated with the shift \mathbf{d} . $t_1 < \dots < t_4$ are time instant $\in \mathbb{R}_{\geq 0}$. Γ_f is the accepting stage, indicating the plan is completed, Γ_0 the initial stage where the aerial robot, e.g., awaits the starting command.

B. Problem formulation

The problem of planning-scheduling is composed of two sub-problems. One is to form a static plan that visits each point in space, the other to re-plan and re-schedule the plan in-flight in an energy-aware way.

Problem II.1 (Coverage problem). Consider a finite set of vertices of a polygon $v := \{v_1, v_2, \dots\}$ where each vertex $v_i := (x_{v_i}, y_{v_i}), \forall i \in [|v|]_{>0}$ is a point w.r.t. \mathcal{O}_W . Let $\underline{r} \in \mathbb{R}_{\geq 0}$, the minimum turning radius, and $\mathbf{p}(t_0)$, the starting point at the time instant t_0 , be given. The *coverage problem* is the problem of finding a plan Γ that covers the polygon.

For a set \mathbb{X} , the notation $|\mathbb{X}|$ indicates the cardinality of \mathbb{X} .

Problem II.2 (Planning problem). Consider an initial plan Γ in Def. II.1. The *planning problem* is the problem of finding the optimal configuration of path and computations parameters $c_i(t), \forall i \in \{1, 2, \dots\}$ under energy constraints and uncertainty at each time step t .

III. ENERGY MODELS

The solution to Pb. II.2 requires accurate energy models, predicting the impact of given path and computations parameters on the battery at future time instants. To this end, Secs. III-A–III-C provide models for the motion and computations energies and battery.

A. Energy model for the motion

Let us return to the collected energy data and spectrum analysis in Fig. 1, illustrating the energy of a coverage plan Γ with four primitive path functions iterated with a shift. Energy data exhibit periodic behavior, an observation further backed by the power spectrum analysis. It indicates that to model the energy, three frequencies are adequate.

An intuitive way of modeling the energy data is thus a Fourier series of a given order $r \in \mathbb{Z}_{\geq 0}$ and period $T \in \mathbb{R}_{>0}$

$$h(t) = a_0/T + (2/T) \sum_{j=1}^r (a_j \cos \omega j t + b_j \sin \omega j t), \quad (3)$$

where $h : \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ maps time to the instantaneous energy consumption, $\omega := 2\pi/T$ is the angular frequency, and $a, b \in \mathbb{R}$ the series coefficients.

The model in Equation (3) does not quantify the contribution of path and computations parameters c_i , where, e.g., different schedules results in different instantaneous energy. For this latter purpose, we use another model

$$\dot{\mathbf{q}}(t) = \mathbf{A}\mathbf{q}(t) + \mathbf{B}\mathbf{u}(t), \quad (4a)$$

$$y(t) = \mathbf{C}\mathbf{q}(t), \quad (4b)$$

where $y(t) \in \mathbb{R}$ is the instantaneous energy consumption. The state $\mathbf{q} \in \mathbb{R}^m$ with $m := 2r + 1$ contains energy coefficients

$$\mathbf{q}(t) = [\alpha_0(t) \ \alpha_1(t) \ \beta_1(t) \ \dots \ \alpha_r(t) \ \beta_r(t)]'. \quad (5)$$

The state transition matrix

$$\mathbf{A} = \begin{bmatrix} 0 & 0^{1 \times 2} & \dots & 0^{1 \times 2} \\ 0^{2 \times 1} & \mathbf{A}_1 & \dots & 0^{2 \times 2} \\ \vdots & \vdots & \ddots & \vdots \\ 0^{2 \times 1} & 0^{2 \times 2} & \dots & \mathbf{A}_r \end{bmatrix}, \quad \mathbf{A}_j := \begin{bmatrix} 0 & \omega j \\ -\omega j & 0 \end{bmatrix}, \quad (6)$$

where $\mathbf{A} \in \mathbb{R}^{m \times m}$ contains r sub-matrices \mathbf{A}_j and $0^{i \times j}$ is a zero matrix of i rows and j columns. In matrix \mathbf{A} , the top left entry is zero, the diagonal entries are $\mathbf{A}_1, \dots, \mathbf{A}_r$, the remaining entries are zeros.

The output matrix

$$\mathbf{C} = (1/T) \begin{bmatrix} 1 & \overbrace{1 \ 0 \ \dots \ 1 \ 0}^{2r} \end{bmatrix}, \quad (7)$$

where $\mathbf{C} \in \mathbb{R}^m$ (the first value in the first column is one, the pattern one–zero is then repeated $2r$ times).

Under favorable conditions, models in Eqs. (3–4) are equal.

Lemma III.1 (Signal, output equality). Given \mathbf{u} a zero vector, matrices \mathbf{A}, \mathbf{C} described by Eqs. (6–7), and the initial guess $\mathbf{q}(t_0) = \mathbf{q}_0$ at initial time instant t_0

$$\mathbf{q}_0 = [a_0 \ a_1/2 \ b_1/2 \ \dots \ a_r/2 \ b_r/2]'$$

h in Eq. (3) is equal to y in Eq. (4).

To define the nominal control and the output matrix, we exploit the effect of variation of path and computations parameters on the energy.

Lemma III.2 (Parameters, energy relation). Given $c_i(t)$ parameters at two following time instants $t \in \{t_j, t_{j+1}\} \subset \mathbb{R}_{\geq 0}$ s.t. $t_j < t_{j+1}$ for an arbitrary stage Γ_i , a change in parameters $c_i(t_j) \neq c_i(t_{j+1})$ results in different overall and instantaneous energies for path and computations parameters respectively.

Appendices A–B contains proofs of Lems. III.1–III.2.

Using the same notation from Lemma III.2, the nominal control in Eq. (4)

$$\mathbf{u}(t_{j+1}) := \hat{\mathbf{u}}(t_{j+1}) - \hat{\mathbf{u}}(t_j), \quad (8)$$

for all time instants. $\hat{\mathbf{u}}$ is then a scale transformation

$$\hat{\mathbf{u}}(t) := \text{diag}(\nu_i)c_i(t) + \tau_i \quad (9)$$

where $\text{diag}(\cdot)$ is a diagonal matrix with items of a set \cdot on the diagonal and zeros elsewhere. $\nu_i := [\nu_{i,1} \ \dots \ \nu_{i,n}]'$ and $\tau_i := [\tau_{i,1} \ \dots \ \tau_{i,n}]'$ are scaling factors with $n := \rho + \sigma$ that transform parameters domain (see Def. II.2) to time and power domains.

Let us assume that the time evolves linearly. Path parameters c_i^p can be transformed into a time measure with scaling factors

$$\nu_{i,j} = (\bar{t} - \underline{t}) / (\bar{c}_{i,j} - \underline{c}_{i,j}) / \rho, \quad (10a)$$

$$\tau_{i,j} = (\underline{c}_{i,j}(\bar{t} - \underline{t}) / (\bar{c}_{i,j} - \underline{c}_{i,j}) + \underline{t}) / \rho, \quad (10b)$$

$\forall j \in [\rho]_{>0}$ where \bar{t}, \underline{t} are time measures needed to complete the coverage with configurations $\bar{c}_i^p, \underline{c}_i^p$.

Similarly to Eq. (10), computations parameters c_i^σ can be transformed into an instantaneous energy measure with scaling factors

$$\nu_{i,j} = (g(\bar{c}_{i,j}) - g(\underline{c}_{i,j})) / (\bar{c}_{i,j} - \underline{c}_{i,j}), \quad (11a)$$

$$\tau_{i,j} = \underline{c}_{i,j}(g(\bar{c}_{i,j}) - g(\underline{c}_{i,j})) / (\bar{c}_{i,j} - \underline{c}_{i,j}) + g(\underline{c}_{i,j}), \quad (11b)$$

$\forall j \in [\rho + 1, n]$. The function g is detailed in Sec. III-B and quantifies the power of the computing hardware.

The input matrix then includes the change in energy, i.e.,

$$B = \begin{bmatrix} 0^{1 \times \rho} & 1 & \dots & 1 \\ 0^{1 \times \rho} & 0 & \dots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0^{1 \times \rho} & 0 & \dots & 0 \end{bmatrix}, \quad (12)$$

where $B \in \mathbb{R}^{m \times n}$ contains zeros except the first row where the first ρ columns are still zeros and the remaining σ are ones.

B. Energy model for the computations

Models for heterogeneous computing hardware in the literature often rely on analytical expressions [42]–[45] or different techniques including regressional analysis [37], [46], [47], aiding the selection of hardware or software specific parameters. This section summarizes an energy model from our early studies [37], [38] that relies on regressional analysis aiding to quantify the computations energy of any combination of computations c_i^σ withing the bounds (see Def. II.2).

The model relies on an automatic modeling and profiling tool [37] named `powprofiler` distributed [48] under the open-source M.I.T. license. It is segmented into two layers. In the *measurement layer* the tool measures a discrete set of computations parameters and infers the energy of the remaining in the *predictive layer* via a piece-wise linear regression.

Let us assume there is at least one measuring device, i.e., shunt or internal power resistor, multimeter, or amperometer, quantifying the power drain of a specific component, e.g., CPU, GPU, memory, etc., or of the entire computing hardware.

Definition III.1 (Measurement layer). Given a measuring device, computations parameters, and initial and final time instants, the *measurement layer* is the function $\mathbf{g} : \mathbb{Z}_{>0} \times \mathbb{Z}^\sigma \times \mathcal{T} \rightarrow \mathbb{R}$ that returns an energy measure.

Here, the notation \mathcal{T} encloses all the time intervals from initial t_0 to final t_f , i.e., $\mathcal{T} := [t_0, t_f]$.

Definition III.2 (Predictive layer). Given a measuring device and computations parameters the *predictive layer* is the function $\mathbf{g} : \mathbb{Z}_{>0} \times \mathbb{Z}^\sigma \rightarrow \mathbb{R}$ that returns and energy measure.

The energy measures in Defs. III.1–III.2 can be either average or overall energy. Additionally, the tool supports the measures of battery state of charge (SoC) detailed in Sec. III-C. The function g in Def. III.2 is contained in the computations scaling factors in Eq. (11) assuming the computations energy behaves linearly between \underline{c}_i^σ and \bar{c}_i^σ , otherwise

$$g(c_i^\sigma) = (\mathbf{g}(\lceil c_i^\sigma \rceil, \mathcal{T}_1) - \mathbf{g}(\lfloor c_i^\sigma \rfloor, \mathcal{T}_2)) / (c_i^\sigma - \lfloor c_i^\sigma \rfloor) + \mathbf{g}(\lfloor c_i^\sigma \rfloor, \mathcal{T}_2), \quad (13)$$

where the notation $\lceil c_i^\sigma \rceil, \lfloor c_i^\sigma \rfloor$ indicates two adjacent measurement layers, and $\mathcal{T}_1, \mathcal{T}_2$ are the corresponding two time intervals. The measuring device in both \mathbf{g} and g is implicit.

C. Battery model

The battery model predicts the battery SoC in function of a given load for future time instants. There are multiple possible models in the literature [49], with varying accuracy, complexity, and easy of implementation ranging from accurate but costly physical models [50], [51], to abstract models [39]–[41], [52] that have compelling trade-offs in terms of ease of implementation and accuracy.

We model a Li-ion battery of an aerial robot flying with an abstract model referred to as “Rint” equivalent circuit model (ECM) in the literature [39]–[41].

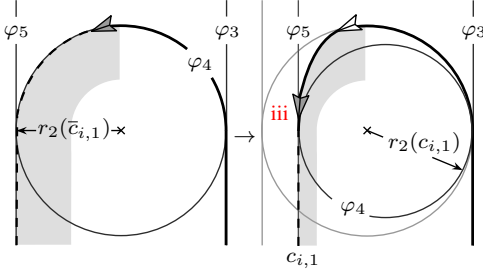
Lemma III.3 (Battery SoC). Given the internal battery voltage $V \in \mathbb{R}$ measured in volts, resistance R_r in ohms, constant nominal capacity Q_c in amperes per hour, and a battery coefficient k_b , the *battery SoC* evolves

$$\dot{b}(y(t)) = -k_b \left(V - \sqrt{V^2 - 4R_r y(t)} \right) / (2R_r Q_c).$$

Appendix C contains a proof of Lem. III.3.

Eq. (4) states that the output y evolves in \mathbb{R} . Let us re-evaluate the output constraint with Lem. III.3.

Fig. 4: The alteration of the path parameter $c_{i,1}$, the radius of the circle (it corresponds to the alteration of the plan in Fig. 1).



Definition III.3 (Output constraint).

$$\mathcal{Y}(t) := \{y \mid y \in [0, b(y(t))Q_c V] \subseteq \mathbb{R}_{\geq 0}\},$$

is the *output constraint*, where $b(y(t))Q_c V$ is the maximum instantaneous energy consumption.

IV. PLANNING-SCHEDULING

This section solves Problems II.1–II.2, i.e., derives a plan in Sec. IV-A, and re-plans the plan energy-wise in Sec. IV-B.

A. Coverage path planning

There are various approaches in the literature to solve CPP problems, such as Pb. II.1. Approaches that ensure the completeness of the cover are NP-hard [53] and use cellular decomposition, dividing the free-space into sub-regions to be easily covered [2], [3].

An intuitive way to solve the problem is with a back-and-forth motion, sweeping the space delimited by v we term \mathcal{Q}^v . Although abundant in both mobile ground-based [2], [54], [55] and aerial [56]–[59] robotics literature, the motion, called *boustrophedon motion* [2], is unsuitable for aerial robots broadly, especially for fixed-wing aerial robots. These robots have reduced maneuverability [60]–[63] and are generally unable to fly quick turns [64].

To address fixed wings and aerial robots generally, this section details a different motion with a wide turning radius. It is similar to another motion in the literature termed *Zamboni motion* [56], but additionally allows variable CPP at the very core of this work. The motion, termed *Zamboni-like motion*, is composed of four primitive paths (see Def. II.5): two lines φ_1, φ_2 and two circles φ_3, φ_4 (see Fig. 5). Let us assume the vertices v_1, v_2, \dots are ordered from the top-left-most vertex in clockwise order, the aerial robot can overfly the edges formed by the vertices, and $v_x|_{v_y}$ indicates an edge formed by vertices v_x, v_y .

Algorithm 1 illustrates the procedure to generate the plan Γ that covers \mathcal{Q}^v per each discretized time step, i.e., $\mathcal{T} := \{t_0, t_0 + h, \dots, t_f\}$ for a given step $h \in \mathbb{R}_{>0}$.

To implement the variable CPP, the radius r_2 of the second circle $\varphi_{|\Gamma|+4}$ on Line 13

$$r_2(c_{i,1}) := \sqrt{r^2 + c_{i,1}}, \quad (14)$$

is in function of a path parameter $c_{i,1} \in (r^2 - r^2, 0]$ and r is a given ideal turning radius along with the minimum radius (see Pb. II.1). The center than also changes

$$\varphi_{|\Gamma|+4} := (x - x_{\mathbf{p}_{|\Gamma|+3}} + r_2)^2 + (y - y_{\mathbf{p}_{|\Gamma|+3}})^2 - r_2^2, \quad (15)$$

where $(x_{\mathbf{p}}, y_{\mathbf{p}}) =: \mathbf{p}$ for any point \mathbf{p} . Fig. 4 illustrates the concept of $c_{i,1}$ altering the CPP. The radius of the first circle

Algorithm 1 Zamboni-like motion for CPP

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1: for all  $i \in \mathcal{T}$  do
2:   if  $\mathbf{p}(i) = \mathbf{p}_{\Gamma_i}$  in Def. II.4 then return  $\Gamma$ 
3:   if  $\mathbf{p}(i) = \mathbf{p}_{\Gamma_j}$  then
4:      $j \leftarrow j + 1$ 
5:     if  $j \notin [n]_{>0}$  then
6:        $j \leftarrow 1$ 
7:      $\varphi_{|\Gamma|+1} \leftarrow$  line in Def. II.3 parallel to  $v_1|_{v_{|v|}}$  that inter-
       sects  $\mathbf{p}_{|\Gamma|}$ 
8:      $\mathbf{p}_{|\Gamma|+1} \leftarrow$  other intersection of  $\varphi_{|\Gamma|+1}$  and  $v$ 
9:      $\varphi_{|\Gamma|+2} \leftarrow$  circle whose left most point lays on  $\mathbf{p}_{|\Gamma|+1}$ 
10:     $\mathbf{p}_{|\Gamma|+2} \leftarrow$  other inter. of  $\varphi_{|\Gamma|+2}$  and  $v$ 
11:     $\varphi_{|\Gamma|+3} \leftarrow$  line par. to  $\varphi_{|\Gamma|+1}$  that inter.  $\mathbf{p}_{|\Gamma|+2}$ 
12:     $\mathbf{p}_{|\Gamma|+3} \leftarrow$  other inter. of  $\varphi_{|\Gamma|+3}$  and  $v$ 
13:     $\varphi_{|\Gamma|+4} \leftarrow$  circle in Eq. (15) whose right most point lays
       on  $\mathbf{p}_{|\Gamma|+3}$ 
14:     $\mathbf{p}_{|\Gamma|+4} \leftarrow$  other inter. of  $\varphi_{|\Gamma|+4}$  and  $v$ 
15:     $\Gamma \leftarrow \Gamma \cup \{\Gamma_{|\Gamma|+1}, \dots, \Gamma_{|\Gamma|+4}\}$  in Defs. II.1–II.2

```

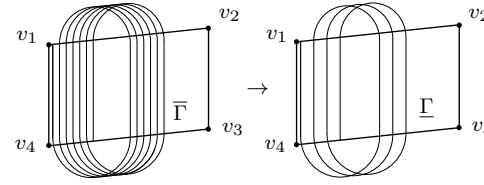


Fig. 5: Zamboni-like motion: $(\bar{\Gamma})$ with four primitive paths (Lines 9–14 in Alg. 1) can be re-planned (Γ) with r_2 in Eq. (14).

on Line 9 is then $r_1 := r + x_d/2$ (i.e., the radiuses of the two circles ensure that the primitive paths are shifted of d in Def. II.5).

Alg. 1 initialize j to minus one and builds the first four primitive functions $\varphi_1, \dots, \varphi_4$. The remaining Γ is built with the shift d up to the given final point \mathbf{p}_{Γ_i} . Initial point is \mathbf{p}_{Γ_1} , placed s.t. the line φ_1 is at the same distance from an eventual previous line, e.g., $x_{\mathbf{p}_{\Gamma_1}} = x_{v_1} + x_d/2$ in Fig. 5.

B. Coverage re-planning-scheduling

Past literature in planning-scheduling often relies on optimal control and optimization related approaches [18], [20], [21], [33]. We similarly derive an optimal control problem returning the trajectory of parameters $c_i(\mathcal{T})$ with $\mathcal{T} := [t_0, t_f]$ (see Def. III.1). Since the final time instant and the exact value of the state \mathbf{q} are not known, we use a technique in the literature named output model predictive control (MPC) that derives the configuration for a finite horizon on an estimated state $\hat{\mathbf{q}}$, i.e., $t_f := t_0 + N$ for a given $N \in \mathbb{R}_{>0}$.

An optimal control problem (OCP) that selects the highest configuration of c_i and respects the constraints, with $\mathbf{q}(t)$ and $c_i(t)$ the state and parameters trajectories, $l : \mathbb{R}^m \times \mathcal{C}_i \times \mathcal{S}_i \times \mathbb{R}_{\geq 0} \rightarrow \mathbb{R}$ and $l_f : \mathbb{R}^m \times \mathbb{R}_{>0} \rightarrow \mathbb{R}$ given initial and final cost functions

$$\max_{\mathbf{q}(t), c_i(t)} l_f(\mathbf{q}(t_f), t_f) + \int_{t_0}^{t_f} l(\mathbf{q}(t), c_i(t), t) dt, \quad (16a)$$

$$\text{s.t. } \dot{\mathbf{q}} = f(\mathbf{q}(t), c_i(t), t), \quad (16b)$$

$$c_{i,j}(t) \in \mathcal{C}_{i,j}, c_{i,\rho+k}(t) \in \mathcal{S}_{i,k} \forall j \in [\rho]_{>0}, k \in [\sigma]_{>0}, \quad (16c)$$

$$\mathbf{q}(t) \in \mathbb{R}^m, y(t) \in \mathcal{Y}(t), \quad (16d)$$

$$\mathbf{q}(t_0) = \hat{\mathbf{q}}_0 \text{ given (last estimated state), and} \quad (16e)$$

$$b(y(t_0)) = b_0 \text{ given.} \quad (16f)$$

Algorithm 2 Coverage re-planning-scheduling

```

1: for all  $i \in \mathcal{T}$  do
16:  $\mathbf{q}(\mathcal{K} \setminus \{i + N\}), c_j(\mathcal{K}) \leftarrow \text{solve NLP } \arg \max_{\mathbf{q}(k), c_j(k)} l_f(\mathbf{q}(i + N), i + N) + \sum_{k \in \mathcal{K}} l_d(\mathbf{q}(k), c_j(k), k)$  in Eq. (16)
    on  $\mathcal{K} = \{i, i + h, \dots, i + N\}$ 
17:  $k \leftarrow i$ 
18: while  $b_d(y(k)) > 0$  do
19:   if  $k \in \mathcal{K}$  then
20:      $\mathbf{q}(k + h) \leftarrow \text{solve model in Eq. (4a)}$ 
21:      $b_d(y(k + h)) \leftarrow \text{solve model in Lem. III.3}$ 
22:      $k \leftarrow k + h$ 
23:    $t_b \leftarrow k - i$ 
24:    $t_s \leftarrow (\text{diag}(\nu_j^p) c_j^p(i) + \tau_j^p) [1 \quad 1 \quad \dots \quad 1]$ 
25:    $t_r \leftarrow (t_s / \bar{t})(\bar{t} - i)$ 
26:   if  $t_r < t_b$  then
27:      $c_j^p(i) \leftarrow \text{find } c_j^p \text{ with } t_c \in [0, t_b], \text{ otherwise take } c_j^p$ 
28:    $\hat{\mathbf{q}}(i + h) \leftarrow \text{estimate } \mathbf{q} \text{ in Eq. (4a) with energy sensor } \Upsilon(i)$ 
29:    $\hat{y}(i + h) \leftarrow \text{derive } y \text{ from Eq. (4b) with est. state } \hat{\mathbf{q}}(i + h)$ 

```

Here, Eq. (16b) is the differential periodic energy model in Eq. (4). Eq. (16c) are the parameters constraints sets in Def. II.2. Eq. (16d) is the state and output constraints in Lem. III.3 that requires to evolve the battery model. Eq. (16e) is the state guess estimated via state estimation with the very first estimate given. Eq. (16f) is the battery SoC, estimated via a flight controller.

Alg. 2 implements Eq. (16) for the purpose of energy-aware re-planning-scheduling of Γ from Alg. 1, i.e., Lines 16–29 continue after Line 15 in Alg. 1.

Line 16 in Alg. 2 contains a transformed version of the OCP in Eq. (16) into a non-linear program (NLP) that can be easily solved with available NLP solvers [65]. Its solution leads to both trajectories of parameters and states for future N instants. Here, the sets \mathcal{K}, \mathcal{T} have possibly a different step h , tuning the precision. The functions l_d, b_d are merely the discretized versions of Eq. (16a) and Lem. III.3, with, e.g., Runge-Kutta methods, Euler method, etc [66].

Lines 17–23 estimate the time needed to completely drain the battery, exploiting the SoC already predicted previously on Line 16. The coverage is then replanned accordingly on Lines 24–27 using Lem. III.2 and scaling factors from Eq. (10). Lines 28–29 estimate the energy model's state with current energy sensor reading Υ (with, e.g., linear Kalman filter [67]).

V. RESULTS

Fig. 1 details the data of a physical flight in standard atmospheric conditions. Fig. 6 extends the flight with NVIDIA (R) Jetson Nano heterogeneous computing hardware aided by a flight simulation implemented in MATLAB (R). Upper-case roman numerals I, II indicate the plans are static (i.e., solely Alg. 1), lower-case i, ii exploit planning-scheduling in the letter. The computing hardware carries a camera as a peripheral and is evaluated independently of the aerial robot with `powprofiler` (see Sec. III-B). The scheduler

varies a computation parameter $c_{i,2}$ relative to ground hazards detection rate from two to ten frames-per-second (FPS). The detection uses PedNet, a Convolutional Neural Network (CNN) [68], implemented through Robot Operating System (ROS) middleware [69] as well as the scheduler itself. The planner varies the path parameter $c_{i,1}$ in Eq. (14) between zero and -1000 (i.e., the planner-scheduler is the concrete implementation of Algs. 1–2).

Fig. 6a illustrate same plan Γ under different conditions. Flights I–i have a constant wind speed of five meters per second, wind direction of zero degrees, and an initial parameters $c_{i,1}, c_{i,2}$ values of zero and ten (i.e., full r_2 and detection). Flights II–ii (see added gray background for clarity) the same but a wind direction of 90 degrees and the initial parameters values of -1000 and two (i.e., minimum r_2 and detection).

Fig. 6b illustrate the power ($y(i)$ on Line 28 in Alg. 2), and the energy model ($C\mathbf{q}(i)$ on Line 20). Flight i simulates a battery (green line) drop at approximately one minute and a half and four minutes and a half. Planner-scheduler optimizes the path in the proximity of the drops to ensure that the flight is completed, whereas it maximizes the parameter $c_{i,2}$ when the battery is discharging, respecting the output constraint (Def. III.3). Flight ii simulates the opposite scenario. The lowest configuration of parameters and no battery defects. The path parameter increases as soon as the algorithm estimated enough data (two periods T) and the computation parameter decreases mapping the battery discharge rate. For both cases scaling factors are derived empirically, the horizon N is set to six seconds similarly to relevant literature, order r is three (see Fig. 1) and the costs l_d, l_f are merely squared controls. The figure further details the energy model (see detail view for I–II) on an initial slice of the model ($\hat{y}(i)$), power ($\Upsilon(i)$), and period (T). The bottom detail of I illustrates the evolutions of the state \mathbf{q} in time, concluding that approximately two periods are sufficient to obtain a consistent state estimate.

VI. CONCLUSIONS AND FUTURE DIRECTIONS

The letter provides a planning-scheduling approach for autonomous aerial robots powered by a limited power source, extending relevant past literature. It proposes a novel coverage motion for variable CPP robust to aerial robots constraints such as the turning radius of fixed wings. Energy modeling in the letter exploits collected empirical data of the fixed-wing aerial robot flying static CPP and further incorporates energy of the computing hardware via the `powprofiler` tool. The approach compromises two algorithms: one derives a static coverage plan, whereas the other re-plans-schedules the plan on a finite horizon via MPC. It evolves the state of the energy model while optimizing the battery usage. The plan compromise multiple stages, where at each the aerial robot flies a path and does some computations, allowing further extensibility in terms of constructs and approaches.

Indeed, we are currently extending the results to a standard flight controller. The guidance on the coverage needs alike further investigation, as well as the study of the implications of planning-scheduling on other energy-critical mobile robots. Here, our initial investigation led to possible savings [70], in line with relevant literature [20], [21].

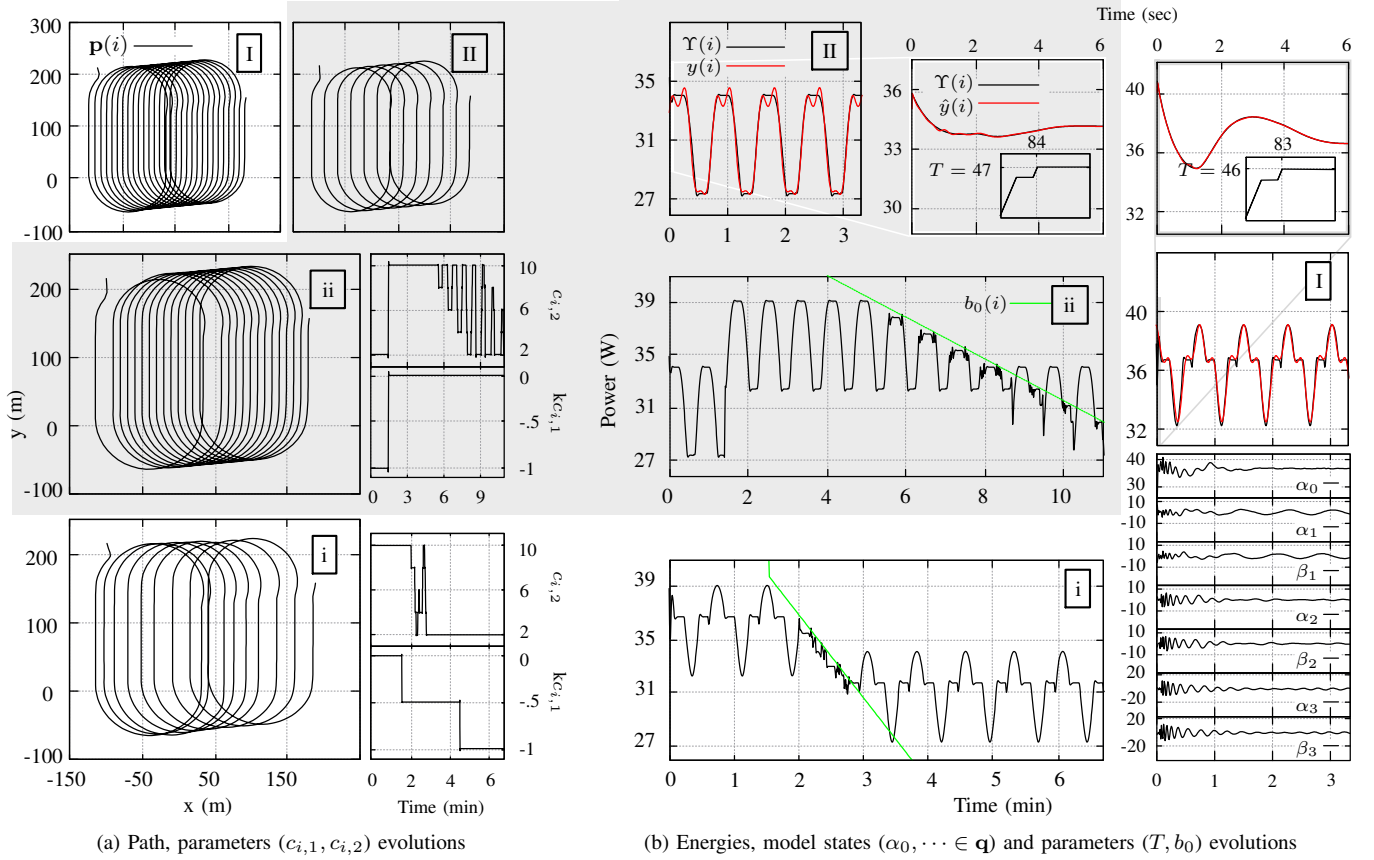


Fig. 6: CPP with novel Zamboni-like motion (I,II) and Planning-scheduling of CPP and ground hazards detections with PedNet CNN (i,ii) in terms of the path, energies, and plans-schedules under different conditions (I-i,II-ii), i.e., wind speed and direction, battery behavior, and parameters initial values.

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APPENDIX A

PROOF OF LEM. III.1

We propose a formal proof of Lem. III.1. The proof justifies the choice of the items of the matrices A, C and of the initial guess \mathbf{q}_0 in Eq. (??). We write these elements such that the coefficients of the series a_0, \dots, b_r are the same as the coefficients of the state α_0, \dots, β_r .

Let us re-write the Fourier series expression in Eq. (3) in its complex form with the well-known Euler's formula $e^{it} = \cos t + i \sin t$. With $t = \omega j t$, we find the expression for $\cos \omega j t = (e^{i\omega j t} + e^{-i\omega j t})/2$ and $\sin \omega j t = (e^{i\omega j t} - e^{-i\omega j t})/(2i)$ by substitution of $\sin \omega j t$ and $\cos \omega j t$ respectively. This leads [71]

$$h(t) = a_0/T + (1/T) \sum_{j=1}^r e^{i\omega j t} (a_j - ib_j) + (1/T) \sum_{j=1}^r e^{-i\omega j t} (a_j + ib_j), \quad (17)$$

where i is the imaginary unit.

The solution at time t can be expressed $\mathbf{q} = e^{At} \mathbf{q}_0$. Both the solution and the system in Eq. (4) are well established expressions derived using standard textbooks [71], [72]. To solve the matrix exponential e^{At} , we use the eigenvectors matrix decomposition method [73].

The method works on the similarity transformation $A = VDV^{-1}$. The power series definition of e^{At} implies $e^{At} = Ve^{Dt}V^{-1}$ [73]. We consider the non-singular matrix V , whose columns are eigenvectors of A ; $V := [v_0 \ v_0^1 \ v_1^1 \ \dots \ v_r^0 \ v_r^1]$. We then consider the diagonal matrix of eigenvalues $D = \text{diag}(\lambda_0, \lambda_0^1, \lambda_1^1, \dots, \lambda_r^0, \lambda_r^1)$. λ_0 is the eigenvalue associated to the first item of A . λ_j^0, λ_j^1 are the two eigenvalues associated with the block A_j . We can write $Av_j = \lambda_j v_j \ \forall j = \{1, \dots, m\}$, and $AV = VD$.

We apply the approach in terms of Eq. (4), under the assumptions made in the lemma (the control is a zero vector); $\dot{\mathbf{q}} = A\mathbf{q}$. The linear combination of the initial guess and the generic solution

$$F\mathbf{q}(0) = \gamma_0 v_0 + \sum_{k=0}^1 \sum_{j=1}^r \gamma_j v_j^k \quad (18)$$

$$F\mathbf{q}(t) = \gamma_0 e^{\lambda_0 t} v_0 + \sum_{k=0}^1 \sum_{j=1}^r \gamma_j e^{\lambda_j^k t} v_j^k$$

where $F = [1 \ \dots \ 1]$ is a properly sized vector of ones.

Let us consider the second expression in Eq. (18). It represents the linear combination of all the coefficients of the state at time t . It can also be expressed in the following form

$$F\mathbf{q}(t)/T = \gamma_0 e^{\lambda_0 t} v_0/T + (1/T) \sum_{j=1}^r \gamma_j e^{\lambda_j^0 t} v_j^0 + (1/T) \sum_{j=1}^r \gamma_j e^{\lambda_j^1 t} v_j^1. \quad (19)$$

We proof that the eigenvalues λ and eigenvectors V are such that Eq. (19) is equivalent to Eq. (17).

The matrix A is a block diagonal matrix, so we can express its determinant as the multiplication of the determinants of its blocks $\det(A) = \det(0) \times \det(A_1) \times \dots \times \det(A_r)$. We proof the first determinant and the others separately.

Thereby we start by proofing that the first terms of the Eq. (17) and (19) match. We find the eigenvalue from $\det(0) = 0$, which is $\lambda_0 = 0$. The corresponding eigenvector can be chosen arbitrarily $(0 - \lambda_0)v_0 = [0 \ \dots \ 0] \ \forall v_0$, thus we choose $v_0 = [1 \ 0 \ \dots \ 0]$. We find the value γ_0 of the vector γ so that the terms are equal, $\gamma_0 = [a_0 \ 0 \ \dots \ 0]$.

Then, we proof that all the terms in the sum of both the Eqs. (17) and (19) match.

For the first block A_1 , we find the eigenvalues from $\det(A_1 - \lambda I) = 0$. The polynomial $\lambda^2 + \omega^2$, gives two complex roots—the two eigenvalues $\lambda_1^0 = i\omega$ and $\lambda_1^1 = -i\omega$. The eigenvector associated with the eigenvalue λ_1^0 is $v_1^0 = [0 \ -i \ 1 \ 0 \ \dots \ 0]^T$. The eigenvector associated with

the eigenvalue λ_1^1 is $v_1^1 = [0 \quad i \quad 1 \quad 0 \quad \cdots \quad 0]^T$. Again, we find the values γ_1 of the vector γ such that the equivalences

$$\begin{cases} e^{i\omega t}(a_1 - ib_1) &= \gamma_1 e^{i\omega t} v_1^0 \\ e^{-i\omega t}(a_1 + ib_1) &= \gamma_1 e^{i\omega t} v_1^1 \end{cases}$$

hold. They hold for $\gamma_1 = [b_1 \quad a_1]$.

The proof for the remaining $r - 1$ blocks is equivalent.

The initial guess is build such that the sum of the coefficients is the same in both the signals. In the output matrix, the frequency $1/T$ accounts for the period in Eq. (17) and (19) and (3). At time instant zero, the coefficients b_j are not present and the coefficients a_j are doubled for each $j = 1, 2, \dots, r$ (thus we multiply by a half the corresponding coefficients in \mathbf{q}_0). To match the outputs $h(t) =$

$y(t)$ —or equivalently $F\mathbf{q}(t)/T = C\mathbf{q}(t)$ —we define $C = (1/T) [1 \quad 1 \quad 0 \quad \cdots \quad 1 \quad 0]$. We thus conclude that the signal and the output are equal, hence the lemma holds. ■

We note for practical reasons that the signal would still be periodic with another linear combination of coefficients (for instance, $C = d [1 \quad 0 \quad 1 \quad \cdots \quad 0 \quad 1]$, or $d [1 \quad \cdots \quad 1]$ for a constant value $d \in \mathbb{R}$).

APPENDIX B

PROOF OF LEM. III.2

APPENDIX C

PROOF OF LEM. III.3