Problem 1 (25%)

A city has two hospitals, each with several ambulances. Ambulance service is deemed adequate if there is only a 10% chance that no ambulance will be available when an ambulance call is received by a hospital. The average length of an ambulance service call is 20 minutes. Given this information, queueing theory indicates that hospital 1 can be assigned up to 4.9 calls per hour and that hospital 2 can be assigned up to 5.5 calls per hour.

The city has been divided to 12 districts. The average number of calls per hour emanating from each district is given in Table 1. The table also shows the travel time (in minutes) needed to get from each district to each hospital.

Table 1: Travel distance from each district to each hospital and average number of calls per districts

District	Hospital 1	Hospital 2	Demand (Calls/hour)
1	5	8	0.5
2	6	9	0.6
3	7	10	0.4
4	5	7	0.3
5	6	8	0.4
6	7	9	0.6
7	9	5	0.7
8	10	6	0.9
9	11	7	1
10	7	3	0.2
11	8	4	0.6
12	9	5	0.1

1. Create an AMPL model that:
   - Determines the proper assignment of districts' calls to hospitals.
   - Minimizes the average travel time needed to respond to a call

2. What is the optimal average travel time and how the districts are assigned to the hospitals?

Problem 2 (35%)

During the next four months a company must meet on time the demands shown in the table below for pairs of shoes.

Table 2: Forecasted demand

	Month 1	Month 2	Month 3	Month 4
Forecasted demand	3000	5000	2000	1000

At the beginning of month 1, there are 500 pairs of shoes on hand, and the company has 100 workers. A worker is paid €1500 per month. Each worker can work up to 160 hours per month before he or she receives overtime. A worker can work up to 20 hours of overtime per month and is paid €13 per hour for overtime labor. It takes four hours of labor and €15 of raw material to produce a pair of shoes. At the beginning of each month, workers can be hired or fired. Each hired worker costs €1600, and each

fired worker costs €2000. At the end of each month, a holding cost of €3 per pair of shoes left in inventory is incurred.

The company wants to use linear programming to determine its optimal production schedule and labor policy. For your convenience, all the data provided above are summarized in Table 3.

Table 3: Scalar data of the problem

Initial inventory of shoes	500
Initial number of workers	100
Regular hours/worker/month	160
Maximum overtime hours/worker/month	20
Hiring cost/worker	€1,600
Firing cost/worker	€2,000
Regular wages/worker/month	€1,500
Overtime wage rate/hour	€13
Labor hours/pair of shoes	4
Raw material cost/pair of shoes	€15
Holding cost/pair of shoes in inventory/month	€3

1. Create an AMPL model that:
   - Determines the workforce planning (hiring, firing and overtime)
   - Determines the production planning (Production quantities given the available capacity)
   - Minimizes the total cost

2. How does the optimal solution look like? Are there any assumptions in your model or in the input data that should cause you to cautiously consider the suggested decisions?

Problem 3 (40%)

Land and water resources are central to agriculture and rural development and are intrinsically linked to global challenges of food insecurity and poverty, climate change adaptation and mitigation, as well as degradation and depletion of natural resources that affect the livelihoods of millions of rural people across the world.

In this context, a farmer wishes to design a cyclic production plan of six crops (beans, cotton, maize, onions, tomatoes, and wheat) on his 10 hectares. The requirements of crops activities vary throughout the year. In this problem, it will be assumed that such requirements are given in a monthly basis and known with certainty.

Crop rotation is the practice of cultivating various crops in succession on the same land. With the assistance of an agronomist, the farmer has a calendar, shown in Table 4, that lists the year periods in which crops can be grown. An entry in this table indicates the fraction of a month a particular crop will occupy the land. A value of 1 means land is to be occupied for the corresponding crop for the entire month. A value less than 1 indicates that land is used either during the last portion of a month when the crop is sown or during the first portion of a month when the crop is harvested, and the land is cleared for another crop. For example, the season of beans starts in November and finishes in April. In November, the value of 0.25 indicates that the land allocated to beans will be used in the last quarter of the month. The beans allocated land cannot be used for other crops until the end of April.

Table 4: Calendar of land occupation

	beans	cotton	maize	onions	tomatoes	wheat
Jan	1	0	0	1	0	1
Feb	1	0	0	1	0	1
Mar	1	0.5	0	1	0	1
Apr	1	1	0	1	0	1
May	0	1	0.25	0.25	0	1
Jun	0	1	1	0	0	0
Jul	0	1	1	0	0.75	0
Aug	0	1	1	0	1	0
Sep	0	1	1	0	1	0
Oct	0	1	0.5	0	1	0
Nov	0.25	0.75	0	0.5	0.75	0.5
Dec	1	0	0	1	0	1

Water is a major requirement for agricultural production and can be supplied from surface water distribution or from groundwater. It is assumed that the total amount of water available to the farmer each month is restricted to 5000 m^3. Furthermore, there is an overall annual limit on the use of water equal to 50,000 m^3. The price of water is fixed for the entire year and amounts to \$0.01 per 1 m^3. The crop water requirements for the farm are given in Table 5.

Table 5: Crop water requirements [m^3/hectare])

	beans	cotton	maize	onions	tomatoes	wheat
Jan	438	0	0	452	0	535
Feb	479	0	0	507	0	802
Mar	505	197	0	640	0	556
Apr	142	494	0	453	0	59
May	0	1047	303	0	0	0
Jun	0	1064	896	0	0	0
Jul	0	1236	1318	0	120	0
Aug	0	722	953	0	241	0
Sep	0	89	205	0	525	0
Oct	0	0	0	0	881	0
Nov	272	0	0	0	865	373
Dec	335	0	0	305	0	456

Labor is also a critical factor in agricultural operations. Since labor is used for various activities (land preparation, planting, tending, and harvesting), each requiring a particular level of intensity, the use of labor is distributed over the year. Each crop pattern will have a set of labor requirements, which will change each month. The pattern of labor requirements in Table 6 corresponds to that of land use in Table 4. The numbers in the table refer to the number of labor hours needed that month to cultivate one hectare of each crop. The table also provides the number of available person-hours each month in the last column. For example, one hectare of onions requires 155 hours in April, corresponding to approximately 88% of the monthly working hours of a worker (155/176).

Table 6: Crop labor requirements [hour/hectare] and monthly hours [hour/person]

	beans	cotton	maize	onions	tomatoes	wheat	Hours
Jan	6	0	0	41	0	14	160
Feb	6	0	0	40	0	4	160
Mar	6	40	0	40	0	8	184
Apr	128	40	0	155	0	8	176
May	0	72	34	19	0	137	168
Jun	0	16	40	0	0	0	176
Jul	0	12	57	0	136	0	176
Aug	0	16	64	0	120	0	176
Sep	0	8	35	0	96	0	176
Oct	0	46	9	0	56	0	168
Nov	60	34	0	89	48	19	176
Dec	6	0	0	37	0	11	176

The farmer's family and outside laborers provide the labor. The cost of one person-year of permanent family labor is $4,144. Permanent family labor is 1.5, and this value can be interpreted as a sign of part-time work activity. Outside workers may be hired temporarily or permanently (either full- or part-time). While temporary work costs $4 per hour, permanent labor costs $5,180 per person-year. The number of working hours differs per month and is listed in the last column of Table 6.

Based on prior years, an estimate of the anticipated yield for each crop can be made beforehand. For instance, 1.75 tons of maize can be produced on a hectare of land. In addition, crop prices impact farm revenue, and pricing information is also assumed to be available. The prices and yields for the six crops are shown in Table 7.

Table 7: Crop yields and prices

	beans	cotton	maize	onions	tomatoes	wheat
Yield (ton/hectare)	1.00	1.50	1.75	6.00	6.00	1.50
Price ($/ton)	2000	3500	700	750	800	1000

For your convenience, the scalar data of the problem are presented in Table 8.

Table 8: Scalar data of the problem

Land surface (hectare)	10
Available family labor (person-year)	1.5
Family yearly wage ($/person-year)	4144
Outside permanent yearly wage ($/person-year)	5180
Temporary hourly wage ($/hour)	4
Water monthly limit (m^3)	5000
Water yearly limit (m^3)	50000
Water price ($/$m^3$)	0.01

1. Create an AMPL model that:
   - Allocates the available land to the six crops
   - Determines the amount of outside labor to hire temporarily or permanently.
   - Such that the total revenue of the farmer is maximized