Problem 1 (35%)

A supplier of construction material is starting up business in Eastern Europe, where a number of production plants will be built.

The supply chain director of the company needs to come up with a suggested supply network design, including the number of plants to build, as well as the locations of these plants.

Seven possible locations for the plants are identified. These are Bratislava, Lvov, Krakow, Sibiu, Kharkiv, Pleven, and Nizhniy. Not all sites need to be used.

The market is divided into 12 different regions. We name these regions from *A* to *L*. The estimated yearly demand (in 1000 tons per year) for these market regions are given in the following table:

Region

A	B	C	D	E	F	G	H	I	J	K	L
200	150	225	120	155	180	170	230	230	190	265	300

The potential production capacities for the different production sites are restricted by the local availability of raw materials and are as follows (in 1000 tons per year):

Bratislava	370
Lvov	200
Krakow	300
Sibiu	670
Kharkiv	750
Pleven	450
Nizhniy	300

The variable cost (cost per ton) is the sum of raw material, variable production cost, and transportation cost from a given production plant to a given market region.

The variable costs (in EUR per ton):

Region

From \ to	A	B	C	D	E	F	G	H	I	J	K	L
Bratislava	52	49	49	39	28	30	22	20	27	14	16	18
Lvov	48	37	38	27	24	28	24	14	29	13	29	25
Krakow	34	21	20	29	21	20	28	11	15	14	20	13
Sibiu	30	15	20	14	12	22	13	20	10	27	11	20
Kharkiv	27	15	15	29	23	11	21	13	29	47	20	47
Pleven	16	19	21	20	21	19	25	13	37	47	55	59
Nizhniy	19	30	19	10	11	25	28	30	47	50	66	63

a)

Create an AMPL model that can be used as a tool to find the cost optimal supply network design for the company. How many factories should be built, at which locations, and how much should be supplied from each factory to each market?

b)

Now assume that at each factory, there is also a "fixed" cost that does not depend on the factory's production volume.

The fixed costs are given in the following table (mill. EUR per year):

Bratislava	16
Lvov	13
Krakow	20
Sibiu	12
Kharkiv	18
Pleven	15
Nizhniy	17

The above costs are incurred only if the corresponding factory is built.

Modify your AMPL model to take into account the fixed costs and find the cost optimal network design. Try to avoid non-linearities in your model.

How many factories should be built, at which locations, and how much should be supplied from each factory to each market?

c)

Modify your AMPL model to take into account the following "single-sourcing" restriction: Each market should be supplied by only one production plant. Try to avoid non-linearities in your model.

How many factories should be built, at which locations, and how much should be supplied from each factory to each market? What is the additional cost of imposing such a "single-sourcing" policy?

d)

Modify your AMPL model from c) so that it allows a solution in which one of the markets is supplied by multiple production plants. Try to avoid non-linearities in your model.

How many factories should now be built, at which locations, and how much should be supplied from each factory to each market? How much are the total costs reduced compared to the solution in c)?

Problem 2 (35%)

A company is preparing the introduction of a new product and wants to develop an optimisation model that can help determine the best choice of online advertising channels.

The total market has been divided into 14 market segments and there are 20 different advertising channels available to reach the various segments. Advertising cost per channel is shown in the following table:

	Cost
Channel 1	12
Channel 2	45
Channel 3	86
Channel 4	56
Channel 5	72
Channel 6	73
Channel 7	82
Channel 8	40
Channel 9	25
Channel 10	53
Channel 11	28
Channel 12	27
Channel 13	53
Channel 14	77
Channel 15	64
Channel 16	33
Channel 17	75
Channel 18	84
Channel 19	75
Channel 20	83

The following table shows which channels can reach which segments of the market (1 means that the channel reaches the market):

	S. 1	S. 2	S. 3	S. 4	S. 5	S. 6	S. 7	S. 8	S. 9	S. 10	S. 11	S. 12	S. 13	S. 14
Ch. 1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Ch. 2	1	0	0	0	0	1	0	1	0	0	0	1	0	0
Ch. 3	0	0	0	0	1	0	0	1	0	0	0	0	0	0
Ch. 4	0	0	0	0	0	1	1	1	0	0	0	0	0	0
Ch. 5	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Ch. 6	1	0	0	0	1	0	0	0	0	0	0	0	0	0
Ch. 7	1	1	0	0	0	0	0	0	0	0	1	0	0	0
Ch. 8	0	0	0	1	0	1	0	0	0	0	0	1	0	1
Ch. 9	0	0	0	0	1	1	0	0	0	0	1	1	0	0
Ch. 10	0	0	0	0	0	0	0	1	1	0	0	1	0	0
Ch. 11	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Ch. 12	0	0	0	0	0	1	1	1	0	1	0	0	1	0
Ch. 13	0	0	1	0	0	1	1	0	0	0	0	0	0	0
Ch. 14	0	0	1	0	0	0	1	0	0	0	1	0	0	0
Ch. 15	0	1	0	0	0	0	0	1	0	0	0	0	0	0
Ch. 16	0	0	0	0	0	0	0	0	0	0	1	1	0	0
Ch. 17	0	1	0	0	1	0	0	0	0	0	0	0	0	0
Ch. 18	0	0	0	1	0	0	0	0	0	0	0	0	1	0
Ch. 19	0	0	0	0	0	0	1	1	0	0	0	0	0	1
Ch. 20	0	1	0	0	0	0	0	0	0	0	0	0	0	0

In the following, try to avoid non-linearities in your models.

a) Create an AMPL model that chooses the optimal mix of advertising channels in such a way that total costs are minimized and all market segments are covered by at least one advertising channel.

b) The company believes that the effects of channel 7 and 8 reinforce each other so that if one of these channels is chosen than the other channel must also be chosen. In other words, we want to forbid solutions where channel 7 is chosen and channel 8 is not chosen, and vice versa.
How much does this requirement increase the total costs compared with the solution in a)?

c) Disregard the information given in b), but assume the following:
If at least one of channel 11 and channel 12 is chosen, then at least one of channel 16 and channel 17 must be chosen.
How much does this requirement increase the total costs compared with the solution in a)?

d) Disregard the information given in b) and c), but assume the following:
If <u>both</u> channel 12 and channel 13 are chosen, then also channel 18 must be chosen.
How much does this requirement increase the total costs compared with the solution in a)?

e) In the solution in a), some of the market segment are covered more than once (that is, they are covered by more than one channel). Based on the model in a), add constraints that ensure that at least six market segments are covered more than once.

f) The following table shows expected revenue per segment.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
571	384	773	844	934	113	313	190	629	965	924	505	973	779

Assume that the revenue per segment is achieved if the segment is covered at least once.
Assume that there are not multiple revenues per segment.
Modify the model in a) so that total revenues are maximized, given a total advertising budget of 200.

Problem 3 (30%)

In maritime logistics, the ability to increase the average size of ships (vessels) is advantageous both of economic and environmental reasons. Larger ships have both a lower cost and a lower CO^2 emission per ton transported, as long as the cargo capacity is fully utilized. The ship/cargo size is however often restricted by limited storage capacities in ports, as in the following case.

A port terminal contains 32 storage tanks of varying sizes. 20 different products are stored in the tanks. The products have different densities (in tons per m^3) and demands, as shown in the following table:

	Density	Demand
Product 01	1,46	2633
Product 02	1,49	2355
Product 03	1,27	1035
Product 04	1,45	2538
Product 05	1,66	2981
Product 06	1,79	437
Product 07	1,28	459
Product 08	1,72	1479
Product 09	1,90	1810
Product 10	1,62	774
Product 11	1,82	1760
Product 12	1,48	148
Product 13	1,30	1864
Product 14	1,40	1344
Product 15	1,90	792
Product 16	1,77	728
Product 17	1,87	552
Product 18	1,54	144
Product 19	1,22	1063
Product 20	1,50	672

For each product, the maximum time between replenishments (deliveries by ship) is determined by the products's storage capacity and its daily demand.

The current allocation is as follows:

Tank	Volum (m3)	Product
Tank 01	2500	Product 13
Tank 02	3800	Product 15
Tank 03	3400	Product 8
Tank 04	1600	Product 18
Tank 05	1800	Product 3
Tank 06	6600	Product 2
Tank 07	5200	Product 1
Tank 08	2200	Product 3
Tank 09	4000	Product 7
Tank 10	4300	Product 1
Tank 11	5000	Product 4
Tank 12	4000	Product 11
Tank 13	2500	Product 9
Tank 14	2100	Product 17
Tank 15	4200	Product 19
Tank 16	5600	Product 16
Tank 17	5500	Product 14
Tank 18	5800	Product 12
Tank 19	7000	Product 10
Tank 20	7700	Product 14
Tank 21	3400	Product 8
Tank 22	5200	Product 5
Tank 23	4600	Product 13
Tank 24	6600	Product 4
Tank 25	7000	Product 5
Tank 26	5600	Product 20
Tank 27	3000	Product 19
Tank 28	2600	Product 16
Tank 29	2600	Product 13
Tank 30	4800	Product 9
Tank 31	6700	Product 6
Tank 32	1200	Product 3

In the current setup,
product 4 has been allocated
Tank 11 (5000 m^3) and Tank 24 (6600 m^3),
which gives a total capacity of 11600 m^3.

Since Product 4 has a density of 1,45 tons per m^3
its capacity in tons is 16820 tons.

Product 4 has a daily demand of 2538 tons, so that
16820 tons covers 16820 / 2538 = 6,63 days.
Hence, there needs to be a transport of
Product 4 at least every 6,63 days on average.
The cycle time for Product 4 is said to be 6,63 days.

The bottleneck product in the current setup is
Product 11, which has been allocated Tank 12.
Capacity in tons = 4000 × 1,82 = 7280 tons.
Demand for Product 11 is 1760 tons per day.
Hence, Product 11 must be replenished at least every
7280 / 1760 = 4,14 days on average.
The cycle time for Product 11 is 4,14 days.
Since this is the lowest cycle time for all products
It determines the maximum time between
deliveries for the whole terminal.
In this case, the terminal needs a replenishment
on average every 4,14 days. This limits the total
quantity that can be delivered and hence the
maximum vessel size that can be used efficiently.

To minimize costs and emissions we want to maximize the time between replenishments, that is, maximize the minimum cycle time.

In the following, try to avoid non-linearities in your models.

a) Assume first that that the company pays a fixed fee per storage tank used (independent of tank size) and hence wants to simply minimize the number of storage tanks used, not explicitly taking into account demand or trying to calculate cycle times. Assume just that each product should have a minimum of 6500 tons storage capacity. Formulate an AMPL model and find the solution that minimizes the number of tanks used. How many tanks are needed?

b) Now consider the situation explained in the introduction. Assume that all tanks are used. The only goal is to maximize the time between deliveries to the terminal, that is, maximize the minimum cycle time (the cycle time for the bottleneck product). The optimal solution to this problem will help utilizing large ships with low costs and low emissions per ton transported. Formulate and solve an AMPL model to find the optimal solution. Which product is the bottleneck product in your solution and what is the minimum cycle time?