GRA6227 Business Optimization

BI Norwegian Business School, Fall 2021

Problem Set

Description & recommendations

- This document contains all the problem set that the student has to solve after the lectures.
- The 10 problems are partitioned according to their link with the techniques seen in the lectures
- It is important that the student tries to solve the different problems in the few days following the lectures. The solutions will be uploaded in itslearning and the learning assistants can help go through the proposed modelling.

Lecture 2

Problem 1 (Fourer. Chapter 01))

A steel company must decide how to allocate next weeks' time on a rolling mill. The mill takes unfinished slabs of steel as input, and can produce either of two semi-finished products, which we will call bands and coils.

The mill's two products come off the rolling line at different rates: 200 tons per hour for bands and 140 tons per hour for coils.

The two products have also different profitabilities: $25 per ton of bands and $30 per ton of coils.

To further complicate matters, the following weekly production amounts are the most that can be justified considering the currently booked orders: 6,000 maximum tons of bands and 4,000 tons of coils. Also, it should be noted that only 40 hours of production time are available this week.

a) Formulate a mathematical model for this problem with the goal of maximizing the profit of the company.
b) Convert the mathematical model into AMPL model (or similar modeling language).
c) How many tons of bands and how many tons of coils should be produced?

Problem 2

A farmer is producing two types of vegetables: cucumbers and onions. His goal is to produce the maximum weight of vegetables, knowing that the yield is $4kg/m^2$ for cucumbers and $5kg/m^2$ for onions. To crop his vegetables, the farmer must use two types of fertilizer, A and B.

- $8l$ of fertilizer A is available. $2l/m^2$ is needed for cucumbers and $1l/m^2$ for onions.
- $7l$ of fertilizer B is available. $1l/m^2$ is needed for cucumbers and $2l/m^2$ for onions.

Unfortunately, the farmer must also use an anti-parasite to prevent his crops from being degraded.

- $3l$ of anti-parasites is available. $1l/m^2$ is needed for onions.

a) Formulate a mathematical model for this problem.
b) Convert the mathematical model into AMPL model (or similar modelling language).
c) What is the optimal weight?

Problem 3

A bank has NOK6,500,000 in assets to allocate among investments in bonds, home mortgages, car loans and personal loans. Bonds are expected to produce return of 10%, mortgages 8.5%, car loans 9.5%, and personal loans 12.5%. To make sure the portfolio is not too risky, the bank wants to restrict personal loans to no more than the 25% of the total portfolio. The bank also wants to ensure that more money is invested in mortgages than personal loans. The bank also wants to invest more in bonds than personal loans.

a) Formulate a mathematical model for this problem with the objective of maximizing the expected return on the portfolio.
b) Convert the mathematical model into AMPL model (or similar modelling language).
c) What is the optimal solution?

Problem 4

A factory produces juice or cider from apples. The price of a ton of apples is 1500€. Each ton of apples can yield 500 liters of juice or 250 liters of cider. The maximum possible sales (in liters) and selling prices are given in table below.

	Maximum sales (liters)	Selling prices(€/liter)
Juice	5000	4
Cider	2000	8

By fermentation and distillation, the factory can also produce cider from apple juice and Calvados from cider. Fermenting a liter of apple juice produces 0.6 liters of cider and distilling a liter of cider gives 0.4 liters of Calvados. The factory can sell a maximum of 500 liters of Calvados, at a selling price of 10€/liter.

d) Formulate a mathematical model for this problem with the goal of maximizing the profit of the factory.
e) Convert the mathematical model into AMPL model (or similar modeling language).
f) How much should be produced for each product type?

Lecture 3

Problem 5 (Williams. Problem 12.5 (Manpower planning))

A company is undergoing a number of changes that will affect its manpower requirements in future years. Owing to the installation of new machinery, fewer unskilled but more skilled and semi-skilled workers will be required. In addition to this, a downturn in trade is expected in the next year, which will reduce the need for workers in all categories. The estimated manpower requirements for the next three years are as follows:

	Unskilled	Semi-skilled	Skilled
Current strength	2000	1500	1000
Year 1	1000	1400	1000
Year 2	500	2000	1500
Year 3	0	2500	2000

The company wishes to decide its policy with regard to the following over the next three years:

1. Recruitment
2. Retraining
3. Redundancy
4. Short-time working.

There is a natural wastage of labor. A fairly large number of workers leave during their first year. After this, the rate is much smaller. Taking this into account, the wastage rates can be taken as follows:

	Unskilled (%)	Semi-skilled (%)	Skilled (%)
Less than one year's service	25	20	10
More than one year's service	10	5	5

There has been no recent recruitment and all workers in the current labor force have been employed for more than one year.

Recruitment

It is possible to recruit a limited number of workers from outside. In any one year, the numbers that can be recruited in each category are as follows:

Unskilled (%)	Semi-skilled (%)	Skilled (%)
500	800	500

Retraining

It is possible to retrain up to 200 unskilled workers per year to make them semi- skilled. This costs £400 per worker. The retraining of semi-skilled workers to make them skilled is limited to no more than one quarter of the skilled labor force at the time as some training is done on the job. Retraining a semi-skilled worker in this way costs £500.

Downgrading of workers to a lower skill is possible but 50% of such workers leave, although it costs the company nothing. (This wastage is additional to the 'natural wastage' described above).

Redundancy
The redundancy payment to an unskilled worker is £200 and to a semi-skilled or skilled worker is £500.

Overmanning
It is possible to employ up to 150 more workers over the whole company than are needed, but the extra costs per employee per year are as follows:

Unskilled (%)	Semi-skilled (%)	Skilled (%)
£1500	£2000	£3000

Short-time working
Up to 50 workers in each category of skill can be put on short-time working. The cost of this (per employee per year) is as follows:

Unskilled (%)	Semi-skilled (%)	Skilled (%)
£500	£400	£400

An employee on short-time working meets the production requirements of half a full-time employee. The company's declared objective is to minimize redundancy. How should they operate in order to do this?

If their policy were to minimize costs, how much extra would this save? Deduce the cost of saving each type of job each year.

Lecture 4

Problem 6 (Williams. Problem 12.4 (Factory planning 1 & 2))

An engineering factory makes seven products (PROD 1 to PROD 7) on the following machines: four grinders, two vertical drills, three horizontal drills, one borer and one planer. Each product yields a certain contribution to profit (defined as £/unit selling price minus cost of raw materials). These quantities (in £/unit) together with the unit production times (hours) required on each process are given below. A dash indicates that a product does not require a process.

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5	PROD 6	PROD 7
Contribution to profit	10	6	8	4	11	9	3
Grinding	0.5	0.7	-	-	0.3	0.2	0.5
Vertical drilling	0.1	0.2	-	0.3	-	0.6	-
Horizontal drilling	0.2	-	0.8	-	-	-	0.6
Boring	0.05	0.03	-	0.07	0.1	-	0.08
Planning	-	-	0.01	-	0.05	-	0.05

In the present month (January) and the five subsequent months, certain machines will be down for maintenance. These machines will be as follows:

January	1 Grinder
February	2 Horizontal drills
March	1 Borer
April	1 Vertical drill
May	1 Grinder and 1 Vertical drill
June	1 Planer and 1 Horizontal drill

There are marketing limitations on each product in each month. These are given in the following table:

	PROD 1	PROD 2	PROD 3	PROD 4	PROD 5	PROD 6	PROD 7
January	500	1000	300	300	800	200	100
February	600	500	200	0	400	300	150
March	300	600	0	0	500	400	100
April	200	300	400	500	200	0	100
May	0	100	500	100	1000	300	0
June	500	500	100	300	1100	500	60

It is possible to store up to 100 of each product at a time at a cost of £0.5 per unit per month. There are no stocks at present, but it is desired to have a stock of 50 of each type of product at the end of June.

The factory works six days a week with two shifts of 8 h each day. It may be assumed that each month consists of only 24 working days.

No sequencing problems need to be considered.

a) Formulate an AMPL model that can be used to maximize the total revenues. Include the given data. When and what should the factory make in order to maximize the total profit?

Instead of stipulating when each machine is down for maintenance in the factory planning problem, it is desired to find the best month for each machine to be down.

Each machine must be down for maintenance in one month of the six apart from the grinding machines, only two of which need be down in any six months.

b) Extend the model to allow it to make these extra decisions. How much is the extra flexibility of allowing down times to be chosen worth?

Lecture 6

Problem 7 (2019 Fall - Group Exam - Problem 3)

In maritime logistics, the ability to increase the average size of ships (vessels) is advantageous both of economic and environmental reasons. Larger ships have both a lower cost and a lower CO^2 emission per ton transported, as long as the cargo capacity is fully utilized. The ship/cargo size is however often restricted by limited storage capacities in ports, as in the following case.

A port terminal contains 32 storage tanks of varying sizes. 20 different products are stored in the tanks. The products have different densities (in tons per m^3) and demands, as shown in the table on the next page.

For each product, the maximum time between replenishments (deliveries by ship) is determined by the products' storage capacity and its daily demand.

	Density	**Demand**
Product 01	1,46	2633
Product 02	1,49	2355
Product 03	1,27	1035
Product 04	1,45	2538
Product 05	1,66	2981
Product 06	1,79	437
Product 07	1,28	459
Product 08	1,72	1479
Product 09	1,90	1810
Product 10	1,62	774
Product 11	1,82	1760
Product 12	1,48	148
Product 13	1,30	1864
Product 14	1,40	1344
Product 15	1,90	792
Product 16	1,77	728
Product 17	1,87	552
Product 18	1,54	144
Product 19	1,22	1063
Product 20	1,50	672

The current allocation is shown in the next page.

Tank	Volume (m3)	Product
Tank 01	2500	Product 13
Tank 02	3800	Product 15
Tank 03	3400	Product 8
Tank 04	1600	Product 18
Tank 05	1800	Product 3
Tank 06	6600	Product 2
Tank 07	5200	Product 1
Tank 08	2200	Product 3
Tank 09	4000	Product 7
Tank 10	4300	Product 1
Tank 11	5000	Product 4
Tank 12	4000	Product 11
Tank 13	2500	Product 9
Tank 14	2100	Product 17
Tank 15	4200	Product 19
Tank 16	5600	Product 16
Tank 17	5500	Product 14
Tank 18	5800	Product 12
Tank 19	7000	Product 10
Tank 20	7700	Product 14
Tank 21	3400	Product 8
Tank 22	5200	Product 5
Tank 23	4600	Product 13
Tank 24	6600	Product 4
Tank 25	7000	Product 5
Tank 26	5600	Product 20
Tank 27	3000	Product 19
Tank 28	2600	Product 16
Tank 29	2600	Product 13
Tank 30	4800	Product 9
Tank 31	6700	Product 6
Tank 32	1200	Product 3

In the current setup, product 4 has been allocated Tank 11 (5000 m^3) and Tank 24 (6600 m^3), which gives a total capacity of 11600 m^3.

Since Product 4 has a density of 1,45 tons per m^3 its capacity in tons is 16820 tons.

Product 4 has a daily demand of 2538 tons, so that 16820 tons covers 16820 / 2538 = 6,63 days. Hence, there needs to be a transport of Product 4 at least every 6,63 days on average.

The cycle time for Product 4 is said to be 6,63 days.

The bottleneck product in the current setup is Product 11, which has been allocated Tank 12.

Capacity in tons = 4000 × 1,82 = 7280 tons.

Demand for Product 11 is 1760 tons per day. Hence, Product 11 must be replenished at least every 7280 / 1760 = 4,14 days on average.

The cycle time for Product 11 is 4,14 days.

Since this is the lowest cycle time for all products It determines the maximum time between deliveries for the whole terminal.

In this case, the terminal needs a replenishment on average every 4,14 days. This limits the total quantity that can be delivered and hence the maximum vessel size that can be used efficiently.

To minimize costs and emissions we want to maximize the time between replenishments, that is, maximize the minimum cycle time.

In the following, try to avoid non-linearities in your models.

a) Assume first that that the company pays a fixed fee per storage tank used (independent of tank size) and hence wants to simply minimize the number of storage tanks used, not explicitly taking into account demand or trying to calculate cycle times. Assume just that each product should have a minimum of 6500 tons storage capacity. Formulate an AMPL model and find the solution that minimizes the number of tanks used. How many tanks are needed?

b) Now consider the situation explained in the introduction. Assume that all tanks are used. The only goal is to maximize the time between deliveries to the terminal, that is, maximize the minimum cycle time (the cycle time for the bottleneck product). The optimal solution to this problem will help utilizing large ships with low costs and low emissions per ton transported. Formulate and solve an AMPL model to find the optimal solution. Which product is the bottleneck product in your solution and what is the minimum cycle time?

Lecture 7

Problem 8 (2019 Fall - Group Exam - Problem 1)

A supplier of construction material is starting up business in Eastern Europe, where a number of production plants will be built.

The supply chain director of the company needs to come up with a suggested supply network design, including the number of plants to build, as well as the locations of these plants.

Seven possible locations for the plants are identified. These are Bratislava, Lvov, Krakow, Sibiu, Kharkiv, Pleven, and Nizhniy. Not all sites need to be used.

The market is divided into 12 different regions. We name these regions from *A* to *L*. The estimated yearly demand (in 1000 tons per year) for these market regions are given in the following table:

Region											
A	B	C	D	E	F	G	H	I	J	K	L
200	150	225	120	155	180	170	230	230	190	265	300

The potential production capacities for the different production sites are restricted by the local availability of raw materials and are as follows (in 1000 tons per year):

Bratislava	Lvov	Krakow	Sibiu	Kharkiv	Pleven	Nizhniy
370	200	300	670	750	450	300

The variable cost (cost per ton) is the sum of raw material, variable production cost, and transportation cost from a given production plant to a given market region.

The variable costs (in EUR per ton):

	Region											
From\to	A	B	C	D	E	F	G	H	I	J	K	L
Bratislava	52	49	49	39	28	30	22	20	27	14	16	18
Lvov	48	37	38	27	24	28	24	14	29	13	29	25
Krakow	34	21	20	29	21	20	28	11	15	14	20	13
Sibiu	30	15	20	14	12	22	13	20	10	27	11	20
Kharkiv	27	15	15	29	23	11	21	13	29	47	20	47
Pleven	16	19	21	20	21	19	25	13	37	47	55	59
Nizhniy	19	30	19	10	11	25	28	30	47	50	66	63

a) Create an AMPL model that can be used as a tool to find the cost optimal supply network design for the company. How many factories should be built, at which locations, and how much should be supplied from each factory to each market?

Now assume that at each factory, there is also a "fixed" cost that does not depend on the factory's production volume. The fixed costs are given in the following table (mill. EUR per year):

Bratislava	Lvov	Krakow	Sibiu	Kharkiv	Pleven	Nizhniy
16	13	20	12	18	15	17

The above costs are incurred only if the corresponding factory is built.

Modify your AMPL model to consider the fixed costs and find the cost optimal network design. Try to avoid non-linearities in your model.

b) How many factories should be built, at which locations, and how much should be supplied from each factory to each market?

Modify your AMPL model to take into account the following "single-sourcing" restriction: Each market should be supplied by only one production plant. Try to avoid non-linearities in your model.

c) How many factories should be built, at which locations, and how much should be supplied from each factory to each market? What is the additional cost of imposing such a "single-sourcing" policy?

Modify your AMPL model from c) so that it allows a solution in which one of the markets is supplied by multiple production plants. Try to avoid non-linearities in your model.

d) How many factories should now be built, at which locations, and how much should be supplied from each factory to each market? How much are the total costs reduced compared to the solution in c)?

Lecture 8

Problem 9 (2019 Fall - Group Exam - Problem 1)

A company is preparing the introduction of a new product and wants to develop an optimization model that can help determine the best choice of online advertising channels.

The total market has been divided into 14 market segments and there are 20 different advertising channels available to reach the various segments. Advertising cost per channel is shown in the following table:

	Cost
Channel 1	12
Channel 2	45
Channel 3	86
Channel 4	56
Channel 5	72
Channel 6	73
Channel 7	82
Channel 8	40
Channel 9	25
Channel 10	53
Channel 11	28
Channel 12	27
Channel 13	53
Channel 14	77
Channel 15	64
Channel 16	33
Channel 17	75
Channel 18	84
Channel 19	75
Channel 20	83

The following table shows which channels can reach which segments of the market (1 means that the channel reaches the market):

	S. 1	S. 2	S. 3	S. 4	S. 5	S. 6	S. 7	S. 8	S. 9	S. 10	S. 11	S. 12	S. 13	S. 14
Ch. 1	1	0	0	0	0	0	0	0	0	0	0	0	0	0
Ch. 2	1	0	0	0	0	1	0	1	0	0	0	1	0	0
Ch. 3	0	0	0	0	1	0	0	1	0	0	0	0	0	0
Ch. 4	0	0	0	0	0	1	1	1	0	0	0	0	0	0
Ch. 5	0	0	1	1	1	0	1	0	0	0	0	0	0	0
Ch. 6	1	0	0	0	1	0	0	0	0	0	0	0	0	0
Ch. 7	1	1	0	0	0	0	0	0	0	0	1	0	0	0
Ch. 8	0	0	0	1	0	1	0	0	0	0	0	1	0	1
Ch. 9	0	0	0	0	1	1	0	0	0	0	1	1	0	0
Ch. 10	0	0	0	0	0	0	0	1	1	0	0	1	0	0
Ch. 11	0	0	0	0	0	0	0	0	0	0	0	0	1	0
Ch. 12	0	0	0	0	0	1	1	1	0	1	0	0	1	0
Ch. 13	0	0	1	0	0	1	1	0	0	0	0	0	0	0
Ch. 14	0	0	1	0	0	0	1	0	0	0	1	0	0	0
Ch. 15	0	1	0	0	0	0	0	1	0	0	0	0	0	0
Ch. 16	0	0	0	0	0	0	0	0	0	0	1	1	0	0
Ch. 17	0	1	0	0	1	0	0	0	0	0	0	0	0	0
Ch. 18	0	0	0	1	0	0	0	0	0	0	0	0	1	0
Ch. 19	0	0	0	0	0	0	1	1	0	0	0	0	0	1
Ch. 20	0	1	0	0	0	0	0	0	0	0	0	0	0	0

In the following, try to avoid non-linearities in your models.

a) Create an AMPL model that chooses the optimal mix of advertising channels in such a way that total costs are minimized, and all market segments are covered by at least one advertising channel.

b) The company believes that the effects of channel 7 and 8 reinforce each other so that if one of these channels is chosen than the other channel must also be chosen. In other words, we want to forbid solutions where channel 7 is chosen and channel 8 is not chosen, and vice versa.
How much does this requirement increase the total costs compared with the solution in a)?

c) Disregard the information given in b), but assume the following:
If at least one of channel 11 and channel 12 is chosen, then at least one of channel 16 and channel 17 must be chosen.

How much does this requirement increase the total costs compared with the solution in a)?

d) Disregard the information given in b) and c), but assume the following:
If both channel 12 and channel 13 are chosen, then also channel 18 must be chosen.

How much does this requirement increase the total costs compared with the solution in a)?

e) In the solution in a), some of the market segments are covered more than once (that is, they are covered by more than one channel). Based on the model in a), add constraints that ensure that at least six market segments are covered more than once.

f) The following table shows expected revenue per segment.

1	2	3	4	5	6	7	8	9	10	11	12	13	14
571	384	773	844	934	113	313	190	629	965	924	505	973	779

Assume that the revenue per segment is achieved if the segment is covered at least once. Assume that there are not multiple revenues per segment.

Modify the model in a) so that total revenues are maximized, given a total advertising budget of 200.

Lecture 11

Problem 10 (2019 Fall - Individual Exam - Problem 1)

In Istanbul metropolitan municipality, optimization modeling has been used to determine the best possible locations of fire stations. The following model is a simplified version of one of the models that were used in the analysis:

Minimize	$\sum_j c_j X_j$	
Subject to:	$\sum_j a_{i,j} X_j \geq 1$	for each district i
	$X_j \in \{0,1\}$	for each candidate location j

a) Give a brief interpretation of each of the elements of the above model.

b) Add a constraint to ensure the following: If all of the candidate locations 3, 8 and 9 are chosen then also candidate location 14 must be chosen.

c) Add a constraint to ensure the following: If candidate location 5 is chosen then at most two of the following candidates can be chosen: candidate location 6, candidate location 7, candidate location 12, and candidate location 18. Otherwise, there should be no restriction on these.

d) Istanbul is a city with many cultural heritage sites that necessitate special consideration in terms of fire-fighting capacity. Hence, some of the districts require coverage by multiple fire stations. Add a constraint which ensures that district 10 is covered by at least three stations.

e) Assume that the municipality requires that there are some districts that are covered by at least three stations. Modify the model to ensure that there are at least five such districts. It should be left to the optimization to decide *which* districts are covered by at least three stations.

In the following, disregard the information b) through e).

f) Extend the initial model based on the following features of the actual problem:
   - For each district i, based on historical records, the demand for fire protection capacity is estimated to be f_i.
   - All demand must be covered.
   - There can be at most one fire station per candidate location.
   - Fire stations are classified into different capacity classes. Each station in fire station class k has a capacity of r_k. It has also a fixed cost c_k. Hence, it is necessary to determine not only the location of fire stations, but also their capacities.
   - The goal of the model is to minimize total fixed costs.

g) The municipality operates on a budget restriction that makes the above assumption of 100% demand coverage unrealistic. It is therefore decided to try another approach, covering as much demand as possible, given a total cost budget of B. Demand, capacity and cost parameters are as in f). Create a model that fits this new problem description.