Problem X

The following shows a formulation for a production planning problem known as the Capacitated Lot Sizing Problem for one product:

$$\text{Min } z = \sum_t h_t \cdot I_t + \sum_t v_t \cdot X_t + \sum_t s_t \cdot Y_t \qquad (1)$$

Subject to

$$I_t = I_{t-1} + X_t - d_t \qquad \text{for all } t \qquad (2)$$

$$X_t \leq K_t \cdot Y_t \qquad \text{for all } t \qquad (3)$$

$$Y_t \in \{0,1\} \qquad \text{for all } t \qquad (4)$$

$$X_t \geq 0 \qquad \text{for all } t \qquad (5)$$

$$I_t \geq 0 \qquad \text{for all } t \qquad (6)$$

h_t is inventory holding cost per unit on inventory per period.

v_t is variable production cost per unit produced.

d_t is demand per period.

s_t is setup cost per setup.

K_t is production capacity per period.

Problem X (continued)

a) Build an optimization model (in AMPL or similar tool) that represents the above problem. Find the optimal solution for the following data set. Initial inventory is 0.

	h(t)	v(t)	s(t)	d(t)	K(t)
t=1	2	12	50 000	18 000	40 000
t=2	2	12	50 000	16 000	40 000
t=3	2	12	50 000	18 000	40 000
t=4	2	12	50 000	15 000	40 000
t=5	2	14	50 000	13 000	40 000
t=6	2	14	50 000	11 000	40 000

h_t is inventory holding cost per unit on inventory per period.

v_t is variable production cost per unit produced.

s_t is setup cost per setup.

d_t is demand per period.

K_t is production capacity per period.

Problem X (continued)

b) Extend the model to include multiple products, which use a joint production capacity.
Find the optimal solution for the following data set (for 2 products).
Initial inventory for both products is 0.

	h(p,t)	h(p,t)	v(p,t)	v(p,t)	s(p,t)	s(p,t)	d(p,t)	d(p,t)	K(t)
	p=1	p=2	p=1	p=2	p=1	p=2	p=1	p=2	
t=1	2	3	12	15	50 000	30 000	18 000	12 000	40 000
t=2	2	3	12	15	50 000	30 000	16 000	9 000	40 000
t=3	2	3	12	15	50 000	30 000	18 000	12 000	40 000
t=4	2	3	12	15	50 000	30 000	15 000	8 000	40 000
t=5	2	3	14	18	50 000	30 000	13 000	14 000	40 000
t=6	2	3	14	18	50 000	30 000	11 000	16 000	40 000

Problem X (continued)

c) Extend the multi-product model to include a setup time that consumes production capacity each time a product is produced.

Find the optimal solution for the following setup times:

setup times	
p=1	p=2
3000	2000

Problem X (continued)

d) The model in c) is a starting point for a company that wants to optimise its *campaign planning*. A campaign means that the company reduces the price of a product in a given period. The price reduction will then lead to increased demand for the given product in the given period.
Imagine that a campaign implies a price reduction of 10%, and that the corresponding demand then increases by 20%. For simplicity, assume that demand for other periods and other products is not affected.
Assume that the company has decided to run at most 3 campaigns during the planning horizon (but that it has not been decided for which products and which periods the campaigns will be run).
Extend the above model so that it will simultaneously suggest a production plan and the products and periods for which the campaigns should be run, so that total profit is maximized.

Prices without reductions:

Standard prices	
p=1	p=2
35,00	32,00