Problem 1

(This problem is partly discussed in Nahmias, chapter 6.)

A product is produced at three factories (A, B, and C) and shipped to four markets (1, 2, 3, and 4).

Capacities at the three factories:

A	*B*	*C*
100	140	110

Demand at the four markets:

1	*2*	*3*	*4*
55	60	35	50

Variable cost per unit for production and shipping:

	To *1*	To *2*	To *3*	To *4*
From *A*	7	4	9	6
From *B*	3	8	5	4
From *C*	6	5	4	11

a) Use the given information and develop a linear programming model. Find the solution that minimizes total costs. All demand must be satisfied.

b) The company wants to produce this product at fewer factories. For each factory, there is a fixed cost associated with producing the product at the factory, as follows.

Fixed cost per factory:

A	*B*	*C*
80	90	100

The fixed cost is avoided if the product is not produced at the factory.
Modify your model from a), so that the sum of variable and fixed costs is minimized.

c) The company wants to test out an alternative approach, in which fixed costs are not included in the model. Instead, the company wants a solution in which each factory will produce either zero of the product, or a quantity that is at or above a minimum level.

The minimum level for each factory is as follows:

A	*B*	*C*
70	80	75

(That is, for factory A, the total quantity must be at least 70 units, or otherwise it must be zero.)

Modify your model from a) to create an alternative model that fits the above description.

d) After having studied the solution from c), the company wants to impose a minimum quantity of 20 units on the shipments between each factory and market. That is, the quantity must be at or above 20, or it must be zero.
For example, the quantity shipped from factory B to market 3 must be at least 20, or it must be 0.

Modify your model from c) so that it incorporates the new restrictions.

Problem 2

A company produces only one product and wants to find a production plan that minimises the sum of production costs and inventory holding costs.

A mathematical formulation of the production planning problem is as follows:

$$\text{Min } z = \sum_t h_t \cdot I_t + \sum_t v_t \cdot X_t$$

Subject to

$$I_t = I_{t-1} + X_t - d_t \qquad \text{for all } t$$

$$X_t \le K_t \qquad \text{for all } t$$

$$X_t \ge 0 \qquad \text{for all } t$$

$$I_t \ge 0 \qquad \text{for all } t$$

h_t is inventory holding cost per unit on inventory per period.

v_t is variable production cost per unit produced.

d_t is demand per period.

K_t is production capacity per period.

The following data are given for an 8-period problem:

t	h	v	d	K
1	4	11	292	600
2	4	16	118	600
3	4	13	321	600
4	4	13	386	200
5	4	18	343	200
6	5	17	452	700
7	5	20	250	700
8	5	19	258	700

Initial inventory is zero.

a) Find the optimal solution.

b) The model in a) does not take into account the fact that a fixed cost is incurred every period the product is produced. The company thus believes that the number of periods in which the product is produced, should be reduced.
Formulate and solve a model that finds the minimum total costs when the following fixed costs ("setup costs") are included:

t	s
1	2500
2	2500
3	2500
4	3300
5	3300
6	4400
7	3300
8	2500

c) Assume that a second product is introduced.
Data for the second product are given in the following table:

t	h	v	s	d
1	3	12	5000	100
2	3	12	5000	100
3	3	12	5000	100
4	3	12	5000	100
5	3	12	5000	200
6	3	12	5000	200
7	3	12	5000	200
8	3	12	5000	200

Initial inventory is zero.

The production capacity given in a) must now be shared between the two products.
Assume for both products that that one unit produced consumes one unit of capacity.
Find a production plan that satisfies demand for both products and minimises total costs.

Problem 3

A manufacturing company – Company X - purchases a given component from a supplier.

Purchasing price is 10 per unit.

To keep the component on inventory at the Company X's production site costs 3 per unit per month.

Company X's estimated consumption of the component for the six first months is given in the following table:

Month	1	2	3	4	5	6
Demand	270	480	520	540	660	770

The supplier has a limited capacity, as given by the following table:

Month	1	2	3	4	5	6
Supplier's capacity	650	650	650	500	650	650

The company wants to set up a plan for the purchases throughput the 6-month period, i.e., how much to purchase in each month. All estimated consumption must be covered in the plan.

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Before month 1, there is an inventory of 150 unit.

Company X requires that the inventory level by the end of month 6 must be at least 100 units.

a) Formulate an optimization model that minimizes the sum of purchasing and inventory holding costs for the entire period.

b) In month 3, a second supplier will have 200 units of the same type of component available for company X, at a price of 12,50 per unit. Should company X purchase anything from the second supplier? If so, how much? Modify the optimization model so that it fits this new situation.