Integer Non-Linear Programming (INLP) formulation for MRTA-Flood.

Nomenclature

Δ_{max}	Maximum allowed range in a single tour for each robot
τ_i	Time deadline for task $i \in T$
$\text{time}(i, j, h, s, r)$	time taken to execute scenario (i, j, h, s, r)
$\text{time}_i^{\text{complete}}$	Time at which task i is completed
C_{max}	Maximum capacity of each robot
$d(i, j)$	Distance between nodes i and j $(i, j \in V)$
D	Depot. Index of depot is considered as 0
H	Maximum number of decisions per tour for each robot
h	Index for each decision in a tour
i, j, k	Indices for nodes
M	Number of robots
N	Number of tasks
R	Set of robots
r	Index for robots
S	Maximum number of tours for each robot
s	Index for each tour
$t(i, j)$	Time to reach from node i to j $(i, j \in V)$
T	Set of tasks
$T_{\text{track}}(j, h, s, r)$	Dependent variable which tracks the time at which task $j \in T$ done by robot $r \in R$ during decision $h \in [1, H]$ of route $s \in [1, S]$
V	Set of tasks including the depot, [D,T]
$x(i, j, h, s, r)$	Binary decision variable which takes a value of 1 if robot $r \in R$ travels from node $i \in V$ to $j \in V$ during decision number $h \in [1, h]$ of route $s \in [1, S]$

The MRTA-Flood problem, due to its static nature, can be modeled as an offline INLP as explained below. It should be noted that this formulation is only applicable for scenarios with a fixed number of tasks and not for scenarios with dynamically generated tasks.

$$\min\ f_{\text{cost}} = \psi - u(r)e^{-d_\psi} \tag{1}$$

$$N_{\text{success}} = \sum_{i \in [1,N]} \eta_i \begin{cases} \eta_i = 1, \ if\ \text{time}_i^{\text{completed}} \leq \tau_i \\ \eta_i = 0, \ if\ \text{time}_i^{\text{completed}} > \tau_i \end{cases}$$

$$\psi = (N - N_{\text{success}})\,/N$$

$$u(\psi) = \begin{cases} 1 & \text{if } \psi = 0 \\ 0 & \text{otherwise} \end{cases}$$

$$d_\psi = \sum_{i=1}^{N_r} d_i^{\text{total}} / (\sqrt{2}\, N)$$

$$\sum_{j \in V} x(1, j, 1, s, r) = 1, \ \forall r \in R, \ \forall s \in [1, S] \tag{2}$$

$$\sum_{j \in V} x(j, 1, H, s, r) = 1, \ \forall r \in R, \ \forall s \in [1, S] \tag{3}$$

$$\sum_{i,j \in V} x(i, j, h, s, r) = 1, \ \forall r \in R, \tag{4}$$

$$\forall s \in [1, S], \ \forall h \in [1, H]$$

$$\sum_{i \in V} \sum_{h \in [1,H]} \sum_{s \in [1,S]} x(i, j, h, s, r) = 1, \ \forall j \in T \tag{5}$$

$$\sum_{j \in V} x(i, j, h, s, r) = \sum_{k \in V} x(k, i, h-1, s, r), \forall i \in V, \tag{6}$$

$$\forall h \in [2, H], \forall s \in [1, S], \ \forall r \in R$$

$$\sum_{h \in [1,H]} \sum_{i,j \in V} d(i, j) \times x(i, j, h, s, r) \leq \Delta_{max}, \tag{7}$$

$$\forall r \in R \ \forall s \in [1, S]$$

$$\sum_{h \in [1,H]} \sum_{i,j \in V} x(i, j, h, s, r) \leq C_{max}, \tag{8}$$

$$\forall r \in R \ \forall s \in [1, S]$$

$$T_{\text{track}}(j, 0, 0, r) = \sum_{k \in V} t(k, j) \times x(k, j, 0, 0, r), \tag{9}$$

$$\forall r \in R, \ j \in V$$

$$T_{\text{track}}(j, h, s, r) = \sum_{i \in V} (t(i, j) + \sum_{k \in V} T_{\text{track}}(k, h', s', r)) \times \tag{10}$$

$$x(i, j, h, s, r), \forall j \in V, \ s \in [1, S], \ h \in [1, H], \ r \in R$$

$$h' = \begin{cases} H - 1, \ if\ h = 0 \\ h - 1, \ otherwise \end{cases} \qquad s' = \begin{cases} s - 1, \ if\ h = 0 \\ s, \ otherwise \end{cases}$$

$$\text{time}_i^{\text{complete}} = \sum_{h \in [1,H]} \sum_{s \in [1,S]} \sum_{r \in R} T_{\text{track}}(j, h, s, r), \ \forall j \in T \tag{11}$$

The objective function 1 is the same as 1. Constraint 2 ensures that every journey starts from the depot, and similarly, constraint 3 ensures that every journey ends at the depot. Constraint 4 ensures that a robot makes a single transition during decision-making. Constraint 5 ensures that all the task locations are visited once. Constraint 6 ensures that the starting location of a transition is the same as the ending location of the previous transition for a robot. Constraint 7 enforces the maximum range constraint for a journey, while constraint 8 ensures that the maximum number of packages delivered by a robot during a journey does not exceed its maximum capacity. Constraint 9, 10, and 11 are used to compute the time of completion for all the tasks.

Multivariate Analysis of Variance (MANOVA) Tables

In all three case studies (generalizability & scalability, ablation study, and study on dynamic tasks), we had two metrics which are the % task completion and the cost. They are performed for different numbers of tasks, number of robots, and method. Here we refer to these entities as independent variables, while the metrics, % task completion, and cost as the dependent variables. We perform a Multivariate Analysis of Variance (MANOVA) in all three studies to find if one more of the independent variables has an effect on the dependent variables. We perform four popular MANOVA tests, which are 1) Wilk's test, 2) Pillai's trace, 3) Hotelling- Lawley trace, and 4) Roy's greatest root. The null hypothesis (H_0) of these tests is that there are no significant differences in the means of the dependent variables across all levels of each of the independent variables, and there are no interactions among the independent variables. The alternate hypothesis (H_a) is that there are differences in the means of the dependent variables based on the levels of at least one of the factors, or there are interactions among the factors that lead to significant differences in the means. Considering a 5% significance level, if the p-value of the tests is less than 0.05, then we can reject the null hypothesis H_0. It can be seen that from all three tables below, the p-value is 0, which means we can reject the null hypothesis. We further perform the Analysis of Variance (ANOVA) with the dependent variable being the cost. The tables for this can be found in appendix 6.

Table 5 MANOVA table for Generalizability & Scalability

Intercept	Value	Num DF	Den DF	F Value	p-value (Pr > F)
Wilks' lambda	0.0305	2.0000	4883.0000	77628.3248	0.0000
Pillai's trace	0.9695	2.0000	4883.0000	77628.3248	0.0000
Hotelling-Lawley trace	31.7953	2.0000	4883.0000	77628.3248	0.0000
Roy's greatest root	31.7953	2.0000	4883.0000	77628.3248	0.0000
Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.1278	6.0000	9766.0000	2924.4875	0.0000
Pillai's trace	1.0798	6.0000	9768.0000	1910.3917	0.0000
Hotelling-Lawley trace	5.1975	6.0000	6508.8892	4229.4631	0.0000
Roy's greatest root	4.8635	3.0000	4884.0000	7917.8343	0.0000
Tasks	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9370	2.0000	4883.0000	164.2539	0.0000
Pillai's trace	0.0630	2.0000	4883.0000	164.2539	0.0000
Hotelling-Lawley trace	0.0673	2.0000	4883.0000	164.2539	0.0000
Roy's greatest root	0.0673	2.0000	4883.0000	164.2539	0.0000
Tasks:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.7424	6.0000	9766.0000	261.3356	0.0000
Pillai's trace	0.2682	6.0000	9768.0000	252.0978	0.0000
Hotelling-Lawley trace	0.3326	6.0000	6508.8892	270.6432	0.0000
Roy's greatest root	0.2818	3.0000	4884.0000	458.7968	0.0000
Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9238	2.0000	4883.0000	201.4846	0.0000
Pillai's trace	0.0762	2.0000	4883.0000	201.4846	0.0000
Hotelling-Lawley trace	0.0825	2.0000	4883.0000	201.4846	0.0000
Roy's greatest root	0.0825	2.0000	4883.0000	201.4846	0.0000
Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9173	6.0000	9766.0000	71.8018	0.0000
Pillai's trace	0.0832	6.0000	9768.0000	70.6552	0.0000
Hotelling-Lawley trace	0.0897	6.0000	6508.8892	72.9561	0.0000
Roy's greatest root	0.0834	3.0000	4884.0000	135.8417	0.0000
Tasks:Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9870	2.0000	4883.0000	32.2216	0.0000
Pillai's trace	0.0130	2.0000	4883.0000	32.2216	0.0000
Hotelling-Lawley trace	0.0132	2.0000	4883.0000	32.2216	0.0000
Roy's greatest root	0.0132	2.0000	4883.0000	32.2216	0.0000
Tasks:Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9918	6.0000	9766.0000	6.7244	0.0000
Pillai's trace	0.0082	6.0000	9768.0000	6.7169	0.0000
Hotelling-Lawley trace	0.0083	6.0000	6508.8892	6.7325	0.0000
Roy's greatest root	0.0074	3.0000	4884.0000	12.1192	0.0000

Table 6 MANOVA table for Ablation study

Intercept	Value	Num DF	Den DF	F Value	p-value (Pr > F)
Wilks' lambda	0.0522	2.0000	2987.0000	27093.2064	0.0000
Pillai's trace	0.9478	2.0000	2987.0000	27093.2064	0.0000
Hotelling-Lawley trace	18.1407	2.0000	2987.0000	27093.2064	0.0000
Roy's greatest root	18.1407	2.0000	2987.0000	27093.2064	0.0000
Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.6411	4.0000	5974.0000	371.7172	0.0000
Pillai's trace	0.3591	4.0000	5976.0000	326.9958	0.0000
Hotelling-Lawley trace	0.5593	4.0000	3583.3603	417.6056	0.0000
Roy's greatest root	0.5585	2.0000	2988.0000	834.4254	0.0000
Tasks	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.7039	2.0000	2987.0000	628.3977	0.0000
Pillai's trace	0.2961	2.0000	2987.0000	628.3977	0.0000
Hotelling-Lawley trace	0.4208	2.0000	2987.0000	628.3977	0.0000
Roy's greatest root	0.4208	2.0000	2987.0000	628.3977	0.0000
Tasks:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.6755	4.0000	5974.0000	323.6896	0.0000
Pillai's trace	0.3351	4.0000	5976.0000	300.7448	0.0000
Hotelling-Lawley trace	0.4647	4.0000	3583.3603	346.9923	0.0000
Roy's greatest root	0.4280	2.0000	2988.0000	639.4372	0.0000
Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9417	2.0000	2987.0000	92.4278	0.0000
Pillai's trace	0.0583	2.0000	2987.0000	92.4278	0.0000
Hotelling-Lawley trace	0.0619	2.0000	2987.0000	92.4278	0.0000
Roy's greatest root	0.0619	2.0000	2987.0000	92.4278	0.0000
Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.8936	4.0000	5974.0000	86.3891	0.0000
Pillai's trace	0.1069	4.0000	5976.0000	84.3752	0.0000
Hotelling-Lawley trace	0.1184	4.0000	3583.3603	88.4241	0.0000
Roy's greatest root	0.1131	2.0000	2988.0000	168.8979	0.0000
Tasks:Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9975	2.0000	2987.0000	3.6784	0.0254
Pillai's trace	0.0025	2.0000	2987.0000	3.6784	0.0254
Hotelling-Lawley trace	0.0025	2.0000	2987.0000	3.6784	0.0254
Roy's greatest root	0.0025	2.0000	2987.0000	3.6784	0.0254
Tasks:Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9383	4.0000	5974.0000	48.2865	0.0000
Pillai's trace	0.0617	4.0000	5976.0000	47.5689	0.0000
Hotelling-Lawley trace	0.0656	4.0000	3583.3603	49.0149	0.0000
Roy's greatest root	0.0647	2.0000	2988.0000	96.6289	0.0000

Table 7 MANOVA table for Dynamic tasks

Intercept	Value	Num DF	Den DF	F Value	p-value (Pr > F)
Wilks' lambda	0.0670	2.0000	2087.0000	14529.8736	0.0000
Pillai's trace	0.9330	2.0000	2087.0000	14529.8736	0.0000
Hotelling-Lawley trace	13.9242	2.0000	2087.0000	14529.8736	0.0000
Roy's greatest root	13.9242	2.0000	2087.0000	14529.8736	0.0000
Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.6594	4.0000	4174.0000	241.5775	0.0000
Pillai's trace	0.3456	4.0000	4176.0000	218.0578	0.0000
Hotelling-Lawley trace	0.5091	4.0000	2503.3604	265.6016	0.0000
Roy's greatest root	0.4940	2.0000	2088.0000	515.7584	0.0000
Tasks	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9865	2.0000	2087.0000	14.3146	0.0000
Pillai's trace	0.0135	2.0000	2087.0000	14.3146	0.0000
Hotelling-Lawley trace	0.0137	2.0000	2087.0000	14.3146	0.0000
Roy's greatest root	0.0137	2.0000	2087.0000	14.3146	0.0000
Tasks:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.9385	4.0000	4174.0000	33.6204	0.0000
Pillai's trace	0.0618	4.0000	4176.0000	33.3073	0.0000
Hotelling-Lawley trace	0.0651	4.0000	2503.3604	33.9442	0.0000
Roy's greatest root	0.0581	2.0000	2088.0000	60.6108	0.0000
Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.6981	2.0000	2087.0000	451.1827	0.0000
Pillai's trace	0.3019	2.0000	2087.0000	451.1827	0.0000
Hotelling-Lawley trace	0.4324	2.0000	2087.0000	451.1827	0.0000
Roy's greatest root	0.4324	2.0000	2087.0000	451.1827	0.0000
Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.7175	4.0000	4174.0000	188.4107	0.0000
Pillai's trace	0.2953	4.0000	4176.0000	180.8693	0.0000
Hotelling-Lawley trace	0.3758	4.0000	2503.3604	196.0550	0.0000
Roy's greatest root	0.3199	2.0000	2088.0000	333.9785	0.0000
Tasks:Robots	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.8491	2.0000	2087.0000	185.4025	0.0000
Pillai's trace	0.1509	2.0000	2087.0000	185.4025	0.0000
Hotelling-Lawley trace	0.1777	2.0000	2087.0000	185.4025	0.0000
Roy's greatest root	0.1777	2.0000	2087.0000	185.4025	0.0000
Tasks:Robots:Method	**Value**	**Num DF**	**Den DF**	**F Value**	**p-value (Pr > F)**
Wilks' lambda	0.8729	4.0000	4174.0000	73.3600	0.0000
Pillai's trace	0.1289	4.0000	4176.0000	71.8956	0.0000
Hotelling-Lawley trace	0.1435	4.0000	2503.3604	74.8489	0.0000
Roy's greatest root	0.1272	2.0000	2088.0000	132.8442	0.0000

Analysis of Variance (ANOVA) Tables

The results from MANOVA (in appendix 6), suggest that we can reject the null hypothesis. Therefore for further analysis, we perform the Analysis of Variance (ANOVA) with the dependent variable being the cost, and the independent variables being the number of tasks, number of robots and method. The null hypothesis (H_0) here is that there are no significant differences in the means of the dependent variable across all levels of each of the independent variables, and there are no interactions among the independent variables. The alternate hypothesis (H_a) is that there are differences in the mean of the dependent variable based on the levels of at least one of the factors, or there are interactions among the factors that lead to significant differences in the means. From Tables 8, 9, and 10, we can see that the independent variables individually have an effect on the dependent variable (p-values being less than 0.05), and also the combination of all the independent variables for the three cases (p-value less than 0.05). This shows that one or more of the means of the dependent variable (cost) for the combination of the dependent variables is different from the other combination. Since we are mostly interested in comparing the performance of the methods, especially for comparing CAM against the other methods, we perform a pairwise T-test to compare the mean cost of CAM for all the combinations of the dependent variable excluding the method. The tables for the pairwise T-test are provided in appendix 6.

Table 8 ANOVA table for Generalizability & Scalability

Variable	sum_sq	df	F	p-value (Pr>F)
Method	453.275623	4.0	4016.859454	$0.000000e+00$
Tasks	20.345619	1.0	721.199105	$1.457873e-148$
Tasks:Method	29.639409	4.0	262.659919	$2.965318e-205$
Robots	16.049315	1.0	568.906330	$3.296777e-119$
Robots:Method	15.108710	4.0	133.891087	$7.583359e-109$
Tasks:Robots	0.127646	1.0	4.524718	$3.345715e-02$
Tasks:Robots:Method	6.666839	4.0	59.080509	$8.128978e-49$

Table 9 ANOVA table for Ablation study

Variable	sum_sq	df	F	p-value (Pr>F)
Method	236.718872	2.0	2928.393895	$0.000000e+00$
Tasks	42.943459	1.0	1062.487001	$1.140731e-199$
Tasks:Method	13.052630	2.0	161.471032	$2.575986e-67$
Robots	12.647211	1.0	312.911398	$1.132536e-66$
Robots:Method	15.693698	2.0	194.143072	$5.374917e-80$
Tasks:Robots	0.617930	1.0	15.288543	$9.432582e-05$
Tasks:Robots:Method	1.298421	2.0	16.062463	$1.151845e-07$

Table 10 ANOVA table for study on scenarios with Dynamic tasks

Variable	sum_sq	df	F	p-value (Pr>F)
Method	81.540165	2.0	1395.571158	$0.000000e+00$
Tasks	7.838654	1.0	268.319290	$7.914200e-57$
Tasks:Method	8.546661	2.0	146.277283	$3.520929e-60$
Robots	7.463293	1.0	255.470583	$2.437315e-54$
Robots:Method	6.201535	2.0	106.140123	$1.257572e-44$
Tasks:Robots	1.072099	1.0	36.698232	$1.631320e-09$
Tasks:Robots:Method	1.003857	2.0	17.181146	$3.972430e-08$

T-test Tables

Table 11 p-values from T-test for comparison of CAM against baseline methods

# Tasks	# Robots	AM	BiG-MRTA	Feas-RND	INLP
50	5	6.0443×10^{-39}	2.2911×10^{-30}	5.04476×10^{-54}	1.65×10^{-24}
	10	3.87716×10^{-40}	3.1511×10^{-57}	6.83462×10^{-82}	0.001538131
100	10	3.71798×10^{-28}	8.85257×10^{-16}	1.01763×10^{-43}	
	20	1.37528×10^{-63}	9.83231×10^{-49}	1.29662×10^{-91}	
200	20	5.55875×10^{-21}	0.023012349	2.68507×10^{-37}	
	40	3.49909×10^{-64}	4.8054×10^{-42}	9.54508×10^{-92}	
500	50	2.51409×10^{-19}	1.17685×10^{-06}	4.98936×10^{-59}	
	100	3.45292×10^{-34}	1.26852×10^{-19}	1.94255×10^{-52}	
1000	100	1.80324×10^{-27}	1.98111×10^{-05}	1.72093×10^{-79}	
	200	6.17581×10^{-31}	4.07881×10^{-10}	3.08636×10^{-50}	

Table 12 p-values from T-test for comparison of CAM against the ablation models

# Tasks	# Robots	$CAM_{E_{FF}}$	$CAM_{d_{FF}}$
50	5	3.1298×10^{-42}	2.13817×10^{-40}
	10	1.51745×10^{-28}	7.58281×10^{-60}
100	10	2.0319×10^{-32}	4.18355×10^{-30}
	20	1.30132×10^{-36}	2.51495×10^{-69}
200	20	1.71362×10^{-27}	8.13367×10^{-23}
	40	5.34323×10^{-47}	1.89727×10^{-75}
500	50	1.81505×10^{-31}	1.19863×10^{-28}
	100	6.84588×10^{-48}	7.05606×10^{-41}
1000	100	1.93374×10^{-60}	3.16895×10^{-68}
	200	3.85799×10^{-54}	7.19298×10^{-46}

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Table 13 p-values from T-test for comparison of CAM against the baseline methods for scenarios with dynamically generated tasks

# Tasks	# Robots	AM	BiG-MRTA
100	10	1.99268×10^{-26}	1.66921×10^{-06}
	20	1.98077×10^{-48}	2.00251×10^{-39}
200	20	8.64098×10^{-25}	2.33846×10^{-06}
	40	3.96435×10^{-49}	7.78208×10^{-40}
500	50	3.58201×10^{-15}	0.084545291
	100	2.8818×10^{-30}	3.51699×10^{-18}