## Recap of Multiple Linear Regression

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1. In your own words, explain what each line of code below does. I have marked each relevant line of code with a comment, which correspond to the description you will need to make below the code chunk. However, for the first action, where there is a question, comment directly next to it. This is to get you used to commenting your code in class.

Part a: Upon completing the below actions, it stores the updated data set with new columns to a new object called star\_df\_homework. In this specific line, it assigns what we read in, star\_df, to star\_df\_homework. The subsequent pipe is signaling that we will be doing something else after. Remember that these could all be on the same line if desired, but its easier to read and comment the current way.

Part b: This line uses the filter function to select only data entries that have complete information, i.e., ones with no missing information/ NA's

Part c: This line also uses the filter function to select the data for only entries where the student is in second grade

Part d: This line creates a dummy variable called small using the mutate function and filling the new dummy variable with either TRUE or FALSE for that observation, corresponding to whether the class size is small or not (TRUE meaning class is small, FALSE meaning class is not small (i.e., regular or regular+aide))

Part e: another dummy variable but this one is called regular and it gets filled with either TRUE or FALSE, corresponding to whether or not class size is regular

Part f: another dummy variable called regular\_plus and this one is also filled with either TRUE or FALSE, corresponding to whether or not class size is regular+aide.

Part g: creating another dummy variable called sum that holds 0's or 1's. It adds up the values of three previous dummy variables, but it can only ever be one of the classes at a time, so the this variable will always have the value 1.

2. When you run the code below to generate simple tables, why are there so fewer observations in the second line of code? What does each value in the variable star mean?

The first line is for the full raw dataset, while the second uses our filtered data set (of complete cases and grade 2). Therefore, it mechanically must have fewer observations, as we removed data. The variable star is the treatment variable and captures what class the student was

assigned to: small, regular, or regular+aide, and the corresponding number of students in each treatment arm.

```
table(star_df$star)
##
##
        regular regular+aide
                                       small
##
            9192
                          9589
                                        8015
table(star_df_homework$star)
##
##
        regular regular+aide
                                       small
                                        1694
##
            1945
                          2033
```

Make sure to choose the correct dataset for this and future sections.

3. Tabulate the variable school, as we did in the previous question but just for the *analysis dataset*, to make sure you know the values it takes on. Then regress math on school. Interpret the coefficients. What's the omitted category? Do you find the results surprising? If so, why? What might be an omitted variable?

```
table(star_df_homework$school)
##
##
  inner-city
                    rural
                            suburban
                                           urban
##
         1199
                     2665
                                 1476
                                             332
lm(math ~ school, star_df_homework)
##
## Call:
## lm(formula = math ~ school, data = star_df_homework)
##
## Coefficients:
##
      (Intercept)
                       schoolrural
                                     schoolsuburban
                                                         schoolurban
##
           561.56
                              30.18
                                              16.93
                                                                20.33
```

The school variable is a categorical variable indicating the school location type. There are four categories: inner-city, suburban, rural and urban.

The ommitted category in the regression is inner-city, meaning that the intercept represents the average math score of students in inner-city schools and that the other coefficients should be interpreted relative to the expected math score of students in inner-city schools. In particular, on average, students in rural schools score 30 points higher than those in inner-city schools, those in suburban schools score almost 17 points higher, and those in urban schools score about 20 points higher. Inner-cities tend to be poorer areas which might explain that they score so much lower than students in other locations. An omitted variable might be some student- or school-evel variable of disadvantage.

4. Compute the share of students qualifying for free lunch (i.e. lunch equals "free") by school location category. Hint: one option is to use the group\_by() function then %>% to summarise(mean(variable == "value")). What do you observe? Add lunch to the previous question's regression. How do the coefficients change?

```
star_df_homework %>%
  group_by(school) %>%
  summarise(mean(lunch == "free"))
```

```
## # A tibble: 4 x 2
```

As expected, the share of students qualifying for free lunches, a common indicator of disadvantage, is significantly higher in inner-city schools (almost 90% of such students) while it is between 34% and 44% of students in other locations.

```
lm(math ~ school, star_df_homework)
##
## Call:
## lm(formula = math ~ school, data = star_df_homework)
##
## Coefficients:
##
      (Intercept)
                       schoolrural
                                    schoolsuburban
                                                         schoolurban
##
           561.56
                             30.18
                                              16.93
                                                               20.33
lm(math ~ school + lunch, star_df_homework)
##
## Call:
## lm(formula = math ~ school + lunch, data = star df homework)
##
##
  Coefficients:
##
      (Intercept)
                       schoolrural
                                    schoolsuburban
                                                         schoolurban
                                                                       lunchnon-free
##
          559.228
                            18.750
                                              4.289
                                                              10.114
                                                                               22.517
```

We observe that the coefficients on the rural, suburban and urban decrease dramatically once the free lunch status of students are taken into account. Note that students not qualifying for free lunch score significantly higher, on average, than those who do, controlling for the school's location category.

5. Regress math on star and interpret the coefficients. Then, regress math on star, gender, ethnicity, lunch, degree, experience and school. Recalling that this is a randomized experiment, does it look like the randomization was well done?

```
lm(math ~ star, star_df_homework)

##
## Call:
## lm(formula = math ~ star, data = star_df_homework)
##
## Coefficients:
## (Intercept) starregular+aide starsmall
## 577.696 2.696 8.942
```

The interpretation of the coefficients is as usual a comparison of conditional means. That is, students in small classes score, on average 8.94 points higher than those in regular classes (the omitted category) while students in regular + aide classes score only 2.70 points higher.

##

```
## Call:
## lm(formula = math ~ star + gender + ethnicity + lunch + degree +
##
       experience + school, data = star df homework)
##
##
   Coefficients:
##
         (Intercept)
                        starregular+aide
                                                    starsmall
                                                                        gendermale
           557.64898
                                  2.01899
                                                                          -1.07263
##
                                                      7.89579
##
   ethnicityamindian
                          ethnicityasian
                                                ethnicitycauc
                                                                ethnicityhispanic
##
            -21.91483
                                 42.16429
                                                     18.61906
                                                                          15.04510
##
      ethnicityother
                            lunchnon-free
                                                 degreemaster
                                                                         degreephd
##
            54.67051
                                 18.40555
                                                     -1.29047
                                                                           0.97823
##
    degreespecialist
                               experience
                                                  schoolrural
                                                                   schoolsuburban
##
            12.11634
                                 -0.05146
                                                      3.99491
                                                                          -4.65503
##
         schoolurban
##
            -3.97039
```

The coefficients on the treatment variable star decrease very slightly compared to the previous simple regression. This is expected considering this was a randomized experiment and therefore we can suppose that the randomization seems to have worked. If the randomization had been very poor, then accounting for all these factors should alter the coefficients on star more substantially.

6. (Optional) What's the adjusted  $R^2$  from the previous multiple regression? How do you interpret it? What might you deduce about the importance of observable individual, teacher and school characteristics in explaining educational outcomes?

```
summary(reg_all)
```

```
##
## Call:
  lm(formula = math ~ star + gender + ethnicity + lunch + degree +
##
       experience + school, data = star_df_homework)
##
  Residuals:
##
                        Median
                                     3Q
        Min
                  1Q
                                              Max
##
   -137.882
            -27.033
                        -1.892
                                 25.174
                                         144.834
##
## Coefficients:
##
                      Estimate Std. Error t value Pr(>|t|)
                                   1.68995 329.979 < 2e-16 ***
## (Intercept)
                      557.64898
## starregular+aide
                        2.01899
                                   1.30760
                                              1.544 0.122635
## starsmall
                        7.89579
                                   1.36425
                                              5.788 7.52e-09 ***
## gendermale
                       -1.07263
                                   1.08674
                                            -0.987 0.323675
## ethnicityamindian -21.91483
                                  28.97355
                                            -0.756 0.449457
## ethnicityasian
                       42.16429
                                  11.90591
                                             3.541 0.000401 ***
## ethnicitycauc
                       18.61906
                                   1.76609
                                             10.543
                                                    < 2e-16 ***
                      15.04510
## ethnicityhispanic
                                  14.53924
                                              1.035 0.300810
## ethnicityother
                       54.67051
                                  13.71246
                                              3.987 6.78e-05 ***
                                                    < 2e-16 ***
## lunchnon-free
                       18.40555
                                   1.25646
                                            14.649
## degreemaster
                       -1.29047
                                   1.17370
                                            -1.099 0.271603
## degreephd
                        0.97823
                                   7.08286
                                             0.138 0.890157
## degreespecialist
                                   5.50130
                                              2.202 0.027674 *
                       12.11634
## experience
                       -0.05146
                                   0.06398
                                            -0.804 0.421259
## schoolrural
                        3.99491
                                   2.09764
                                              1.904 0.056898
## schoolsuburban
                       -4.65503
                                   1.90889
                                            -2.439 0.014775 *
## schoolurban
                       -3.97039
                                   2.91817
                                            -1.361 0.173703
```

```
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 40.86 on 5655 degrees of freedom
## Multiple R-squared: 0.1489, Adjusted R-squared: 0.1465
## F-statistic: 61.85 on 16 and 5655 DF, p-value: < 2.2e-16</pre>
```

The adjusted  $R^2$  of the previous regression is about 0.15, which means that that model explains about 15% of the variance in students' math scores. This implies that 85% of the variance in math scores remains unaccounted for. In simple terms, class size, gender, ethnicity, free lunch status, teacher experience and degree, and school location, explain very little of the differences in math scores between the students in this dataset. Extrapolating a bit, one may argue that most of the differences in educational outcomes do not appear to be explained by these factors. This DOES NOT mean that they may not have important causal effects on educational outcomes.

7. Regress math on gender and experience (the teacher's experience). Interpret the coefficients. How would these regression results look like visually?

```
lm(math ~ gender + experience, star_df_homework)

##
## Call:
## lm(formula = math ~ gender + experience, data = star_df_homework)
##
## Coefficients:
## (Intercept) gendermale experience
## 581.35255 -0.91383 0.03453
```

The intercept coefficient corresponds to the average math score of female students, keeping teacher experience constant. Recall that because we have both a numeric variable and a dummy variable as regressors, the intercept can be interpreted as the intercept for the line of the omitted category, in this case female. The coefficient on gender represents the expected difference in math scores between male and female students, holding teacher experience constant. The coefficient is negative but small, implying that on average male students score slightly lower than their female peers, holding teacher experience constant. Graphically, this implies that the line for male students lies below that for females. The coefficient on experience corresponds to the expected change in math score associated, on average, to an increase in a teacher's experience by 1 year, accounting for student gender. As such, an additional year of teacher experience is associated, on average, with a 0.03 increase in math scores, keeping gender constant. This is a very small effect, since you would need 100 years of increased experience to expect math scores to increase by 3 points.