

Chapter 11 Practice Questions

Question 11.1:

Let's begin with the biomass-leaf litter system discussed in the chapter:

$$N_1(t+1) = rN_1(t)(1 - N_2(t))$$

$$N_2(t+1) = N_1(t) + pN_2(t)$$

Just like in a continuous-time system, we can use these equations to calculate equilibrium values for N_1 and N_2 . But, unlike in continuous-time where we obtained this equilibrium by setting $\frac{dN}{dt} = 0$, here, we need to do something else. Describe, in words, how we could go about finding this equilibrium.

Question 11.2:

Now, use this process to calculate the equilibrium for these two equations. Hint – it might be a good idea to double-check the answer to question 11.1 before proceeding.

Question 11.3:

You can use the following code to simulate and plot dynamics for this system over 1000 time steps, with parameter values $r = 2$ and $p = 0.5$:

```
# functions for n1(t+1) and n2(t+1)
f1 = function(r, N1, N2) {r*N1*(1-N2)}
f2 = function(p, N1, N2) {N1 + p*N2}

# set r and p
r = 2
p = 0.5
```

```

# make empty vectors for storing results
n1 = rep(NA, 1000)
n2 = rep(NA, 1000)

# set starting abundances
n1[1] = ???
n2[1] = ???

# simulate for 1000 time steps
for(i in 1:999) {
  n1[i+1] = f1(r, n1[i], n2[i])
  n2[i+1] = f2(p, n1[i], n2[i])
}

# plot output
matplot(1:1000, cbind(n1,n2), type = "l",
        xlab="time", ylab="abundance")
legend("topright", c("n1", "n2"), col=1:2,
       lty=1:2, bty = "n")

```

Replace the two "???" signs above with your solutions for the equilibrial values for N_1 and N_2 . What behavior do you expect to see when you simulate these dynames? What behavior do you find? Again, you may want to double-check your answers from the previous question before running this code.

Question 11.4:

Now, try running the same code again, but with different starting values – $N_1(t = 1) = 0.1$, and $N_2(t = 1) = 0$. What do we expect to find now? Should the system eventually converge to the equilibrial values, or not? Now try running the simulation – what do you find? Why does this happen?

Question 11.5:

Finally, what can we do to make the system flatten out over time to the equilibrial values? Try making some changes to the script above, and see if your idea works.

Answers:

Question 11.1:

In this case, we need to set $N_1(t+1) = N_1(t)$, and $N_2(t+1) = N_2(t)$. This procedure works because, at equilibrium, we expect the values to stop changing over time – meaning that the value this time step should equal the value next time step.

Question 11.2:

Because N_1 and N_2 appear in both equations, finding the equilibrium here will be a bit trickier than it was in single-species systems. A good place to start is to use one equation to solve for N_2 as a function of N_1 , and then substitute this value into the other equation. It doesn't really matter which equation we choose, but in this case, we'll try using the equation for N_2 to solve for N_1 . To do so, we first set $N_2(t+1) = N_2(t) = N_2$, and $N_1(t+1) = N_1(t) = N_1$, which yields

$$N_2 = N_1 + pN_2$$

$$N_2 - pN_2 = N_1$$

$$N_2(1 - p) = N_1$$

$$N_2 = N_1/(1 - p)$$

Now, we can substitute this into the equation for N_1 , which yields

$$N_1 = rN_1(1 - N_2)$$

$$N_1 = rN_1(1 - N_1/(1 - p))$$

$$N_1 = rN_1 - N_1^2/(1 - p)$$

Diving both sides by N_1 gives

$$1 = r - rN_1/(1 - p)$$

$$1 + rN_1/(1 - p) = r$$

$$rN_1/(1 - p) = r - 1$$

$$N_1 = (r - 1)(1 - p)/r$$

$$N_1 = (1 - 1/r)(1 - p)$$

Finally, we can plug this value for N_1 back into the formula for N_2 , which yields

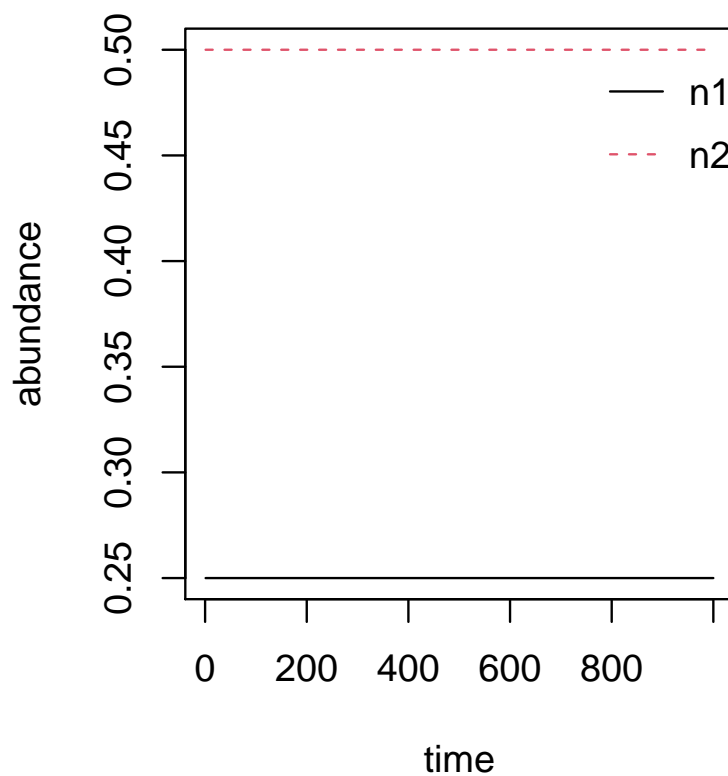
$$N_2 = N_1 / (1 - p)$$

$$N_2 = (1 - 1/r)(1 - p) / (1 - p)$$

$$N_2 = (1 - 1/r)$$

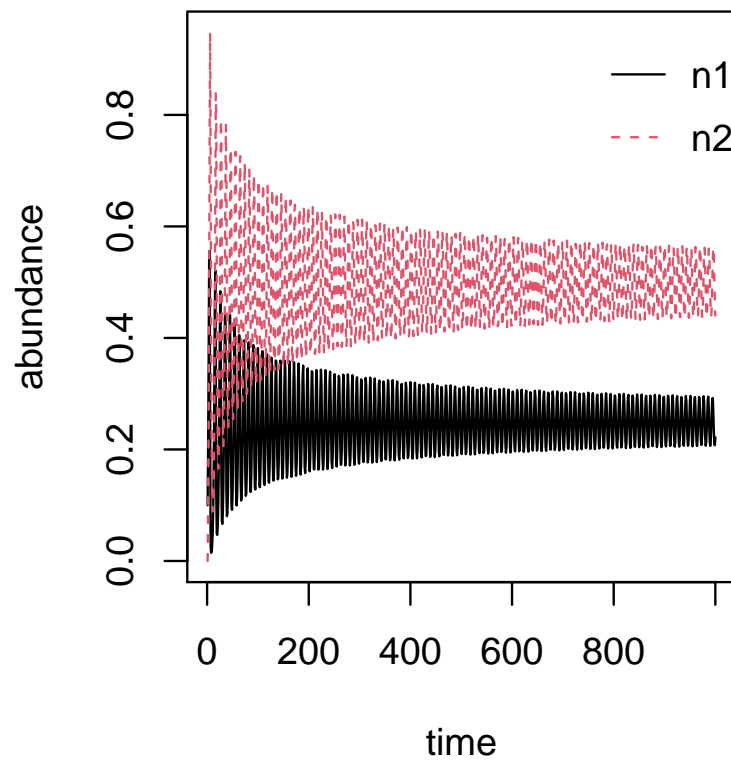
Question 11.3:

Since we expect the system to be stationary at equilibrium, we would expect to see two flat lines – i.e. no changes in N_1 or N_2 over time. Indeed, if we input the right equilibrium values, we get the following plot:



Question 11.4:

In this case, the system never seems to go to equilibrium – it just keeps oscillating forever. The reason for this is that the growth rate is so fast, that the system ends up overshooting the equilibrium altogether. And, for the growth rate that we chose here, these over-shootings keep on happening over and over again, so that we never flatten out to a fixed value. This is very similar to the limit cycles that we saw for single species systems.



Question 11.5:

There are probably several different answers that would work here. But, the simplest is to just decrease r to a smaller growth rate – this will minimize the extent to which the system overshoots equilibrium. For example, if we set $r = 1.5$, we find:

