Math 535: Real Groups (Lie Theory II)
Spring Term, 2023

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Course Website	`http://www.math.ubc.ca/~lior/teaching/2223/535_W23/`
Contact me at	MAT 229B — 604-827-3031 – `lior@math.ubc.ca`
My Website	`http://www.math.ubc.ca/~lior/`
Class	TBA + Zoom
Office Hours	TBA + Zoom
Textbook	None required; see below for recommendations
(Informal) Prerequisites	Group theory, Lie algebras, point-set topology, functional analysis

About the course

This will be a graduate course on the structure and representation theory of Lie groups. The course will have four parts:

1. Topological groups, representation theory, and the Peter–Weyl Theorem.
2. Lie groups: basic definitions and differential geometry
3. Compact Lie groups: maximal tori, roots and weights, representation theory, Weyl character and integration formulae.
4. Real Lie groups: Lie algebras, Poincare–Birkhoff–Witt, Cartan subalgebras, roots and weights, parabolic subgroups, introduction to infinite-dimensional representations.

As a graduate course there are no strict pre-requisites, but course is easier to follow for students with a general graduate background including real analysis and point-set topology, functional analysis, some algebra, and undergraduate group theory and linear algebra.

- Specific background: while we will develop the notion of a Lie algebra from scratch as well as the theory of roots, weights, etc, the course will be easier going for students who have seen previously seen Lie algebras and the classification of complex (semi)simple Lie algebras, such as in MATH 534.

Textbooks

The books [2, 4] give general introductions to Lie groups and are the main references for parts (1),(2),(4). The book [1] is a complete reference to the central part of the course – the compact case. The early sections of [3] review the structure theory of real semisimple groups and the representation theory of compact groups, but the main focus is on infinite-dimensional representations.

Evaluation and grading

The grade will be based on up to six regular problem sets. There will be no final exam.

References

[1] Bröcker–tom Dieck, *Representations of Compact Lie Groups*

[2] Knapp, *Lie Groups Beyond an Introduction*

[3] Knapp, *Representation Theory of Semisimple Groups: An Overview Based on Examples*

[4] Warner, *Foundations of Differentiable Manifolds and Lie Groups*