Math 8410 Spectral Methods for PDEs (Fall 2023)

Instructors: Shaun Lui (shaun.lui@umanitoba.ca) and Mikael Slevinsky (richard.slevinsky@umanitoba.ca).
Tentative Time: Tues, Thurs 3 - 4:15 (CDT)
Location: MH416 and Zoom link (Meeting ID: 675 4106 0780 Passcode: 626774)
Textbook: Course notes will be provided.
References:

1. J. Shen T. Tao and L.-L. Wang, Spectral methods. Algorithms, analysis and applications, Springer, 2011.
2. L.N. Trefethen, Spectral Methods in Matlab, SIAM, 2000.
3. L.N. Trefethen, Approimation Theory and Approximation Practice (Extended Ed.), SIAM, 2020.
4. S. Olver, R. M. Slevinsky, and A. Townsend, Fast algorithms using orthogonal polynomials, *Acta Numerica*, 29: 573–699, 2020.

Grading Scheme: There are 4 Homeworks (each contributing 17% toward the grade) and a project (32%).
Prerequisite: Undergraduate analysis and PDEs. Some exposure to numerical analysis is desirable, but not necessary. Some of the homework questions will require computer programming (MATLAB or Julia, etc.)

Course Content:

1. Part I: Introduction to Spectral Methods (Shaun Lui)
   (a) Trigonometric and orthogonal polynomials (truncation, interpolation and quadrature error estimates, aliasing, Lebesgue constants)
   (b) Fourier spectral (FFT), spectral Galerkin and spectral tau methods
   (c) Spectral collocation for Poisson equation with Dirichlet BCs (convergence and condition number estimates)
   (d) Neumann problems and fourth-order PDEs
   (e) Other topics (Ultraspherical spectral methods, time-dependent PDEs)
2. Part II: Fast Algorithms for Orthogonal Polynomials (Mikael Slevinsky)

(a) Synthesis and analysis

(b) Chebyshev polynomials and the fast discrete sine and cosine transforms

(c) Modification algorithms for orthogonal polynomials

(d) Fast approximation of the connection coefficients

(e) Multivariate orthogonal polynomials via Koornwinder's construction

(f) Time evolution with exponential integrators

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