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Garki Hospital Abuja

Resource Person

Research Methodology Boot Camp

with Epi Info Training

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MS Epidemiology & Biostatistics

PhD Public Health (Epidemiology)

Target Audience Clinical Researchers, Post-Part 1 Residents, and Others

Important Information

- Limited slots are available on a first come, first served basis
- Laptop running Windows 10 required
- Organized as morning lecture sessions and afternoon hands on coaching sessions

Highlights

- Research Methodology
- Research Design
- Data Management
- Sample Size Calculations
- Test Statistics
- Interpretation of Results
- Report Writing
- Hands-on training sessions
- Statistical consulting sessions

For further details contact

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epimetrix health systems

Analysis of variance

Multiple group comparisons

- Consider 4 groups (A, B, C, D) to be compared on a particular variable:
  - Example: captopril, digoxin, both, or none. Effect on ejection fraction in HF.
- Would require six t-test comparisons A vs B, A vs C, A vs D, B vs C, B vs D, C vs D
- This would also increase the probability of a type I error.

Problem of multiple comparisons

- Each test is based upon the probability that the null hypothesis is true
- Each time we conduct a test, we run the risk of a Type I error
- For multiple *t* tests, the rate of error increases exponentially by the number of tests performed

Magnitude of the error

- Rate of type I errors = $1 - (1 - \alpha)^n$
  - where α = significance level and n = number of test comparisons
- For 6 comparisons the chance of a type I error = $1 - (1 - 0.05)^6 = 0.265$
- 73.5% confidence that difference is true

Analysis of variance

- Answers the question: “Do the group means differ from one another?”
- Does not tell us which pairs of groups are different.
- To determine which pairs are different requires *post-hoc* (after the fact) statistical tests.

Data required

- Independent variables: nominal (e.g. treatment groups)
- One-way ANOVA means that there is only one independent variable
- Dependent variables: continuous (mean values can be calculated)

Assumptions for ANOVA

- Robust test: even if assumptions not rigidly met, results may still be close to the truth (especially if groups are of equal size)
- Groups should be mutually exclusive (independent of each other)
- Dependent variable should be normally distributed
- Groups should have equal variances (homogeneity of variance)

Sources of variance

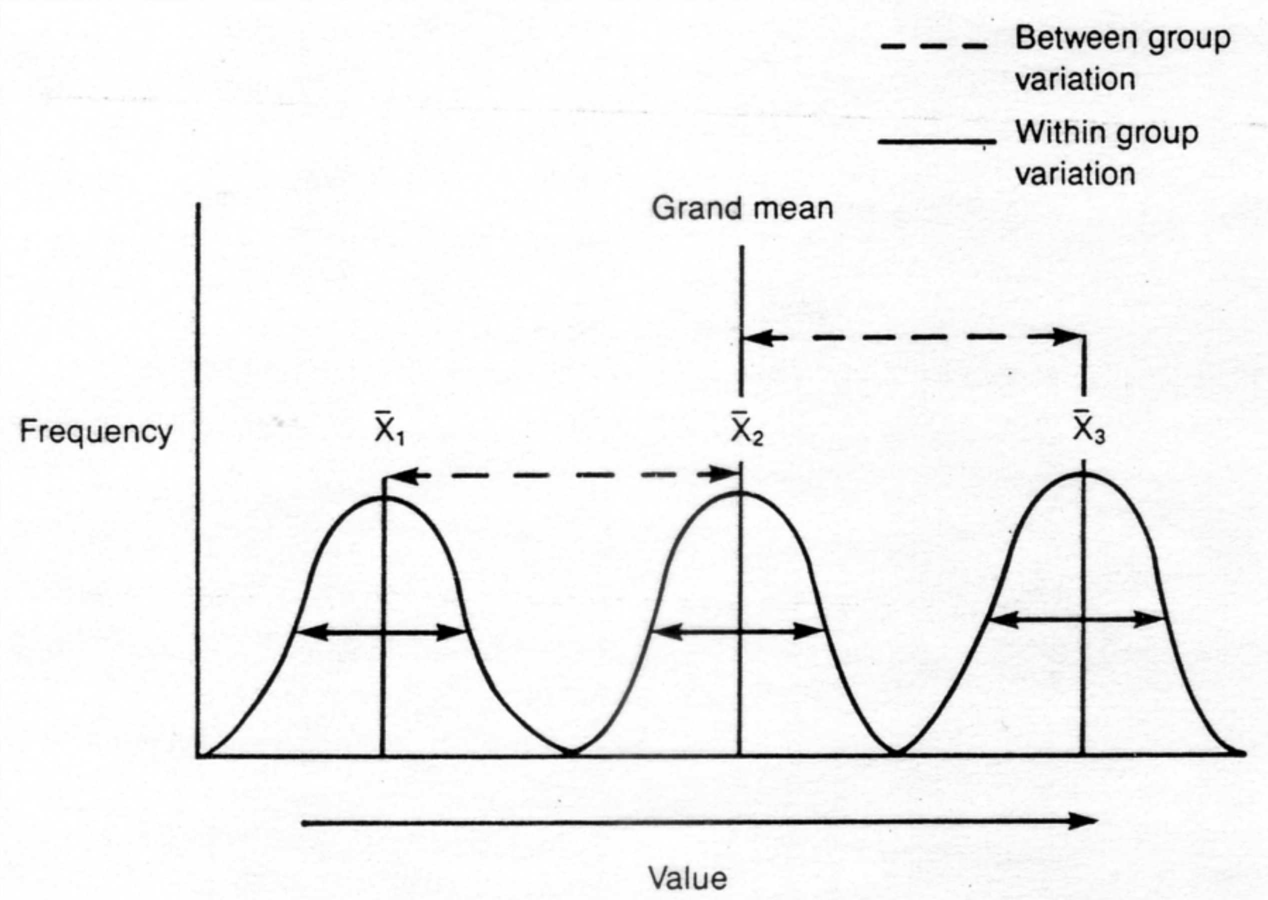
- According to the null hypothesis, all groups are equal and drawn from the same population (group means are identical):

$$\mu_1 = \mu_2 = \mu_3 = \mu$$

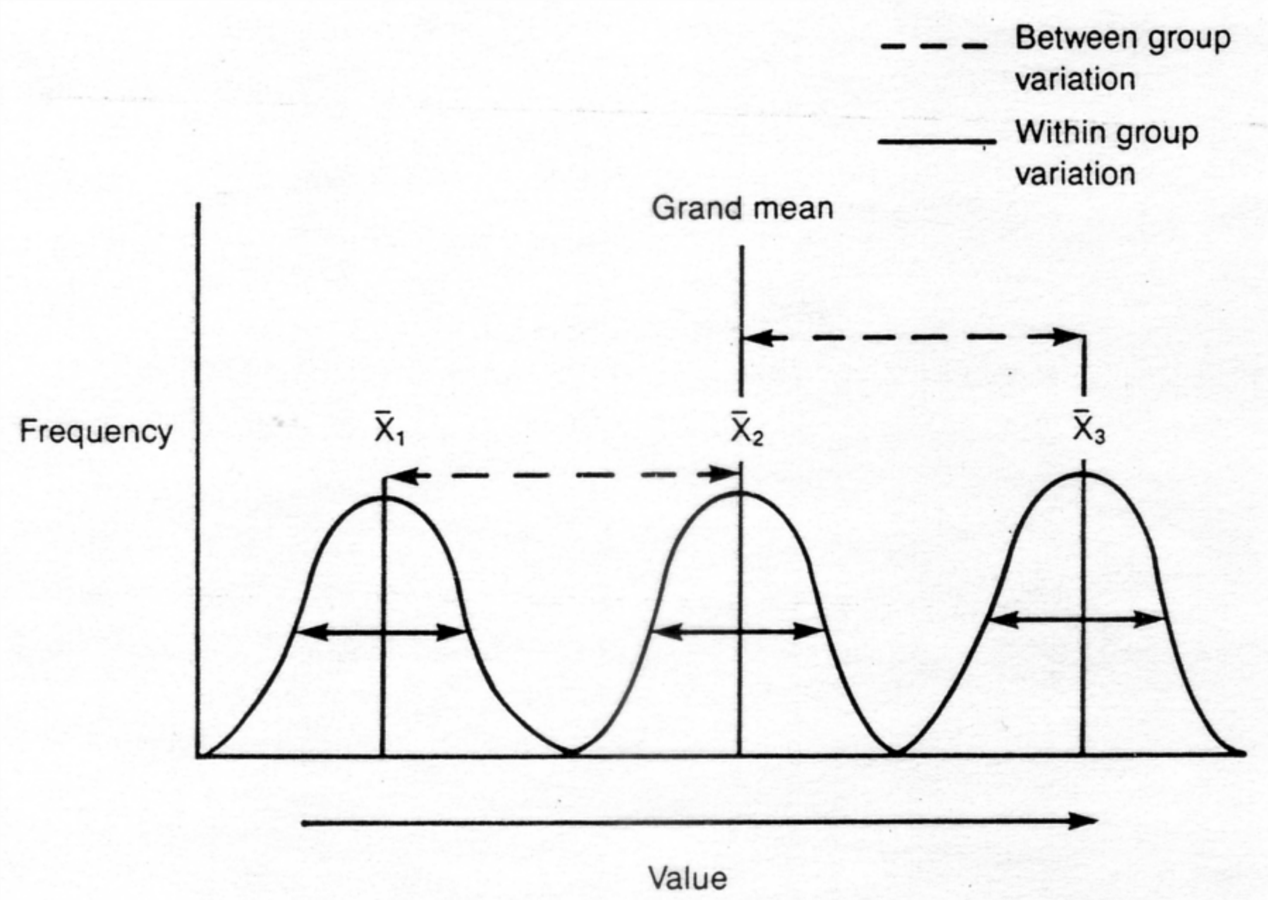
- Scores may vary from each other in their own group (within-group variation)
- The groups may vary from each other (between-group variation)
- Together the two types of variation add up to the total variation

Determining if group means differ

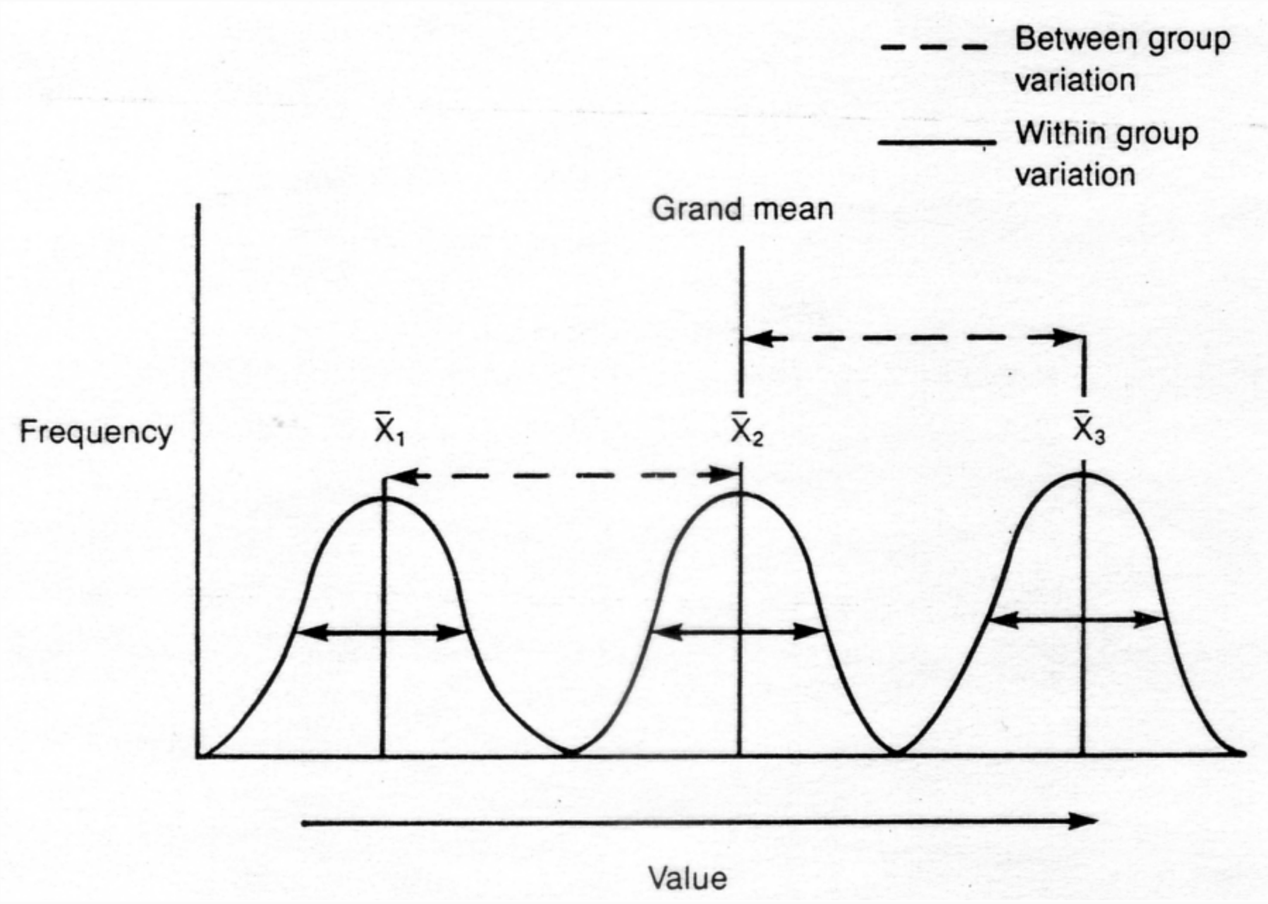
- Variance (σ^2) is used in statistics to measure variation, rather than standard deviation.
- Variance of each group is measured separately.
- All the subjects are then lumped together, and the variance of the total group is calculated.



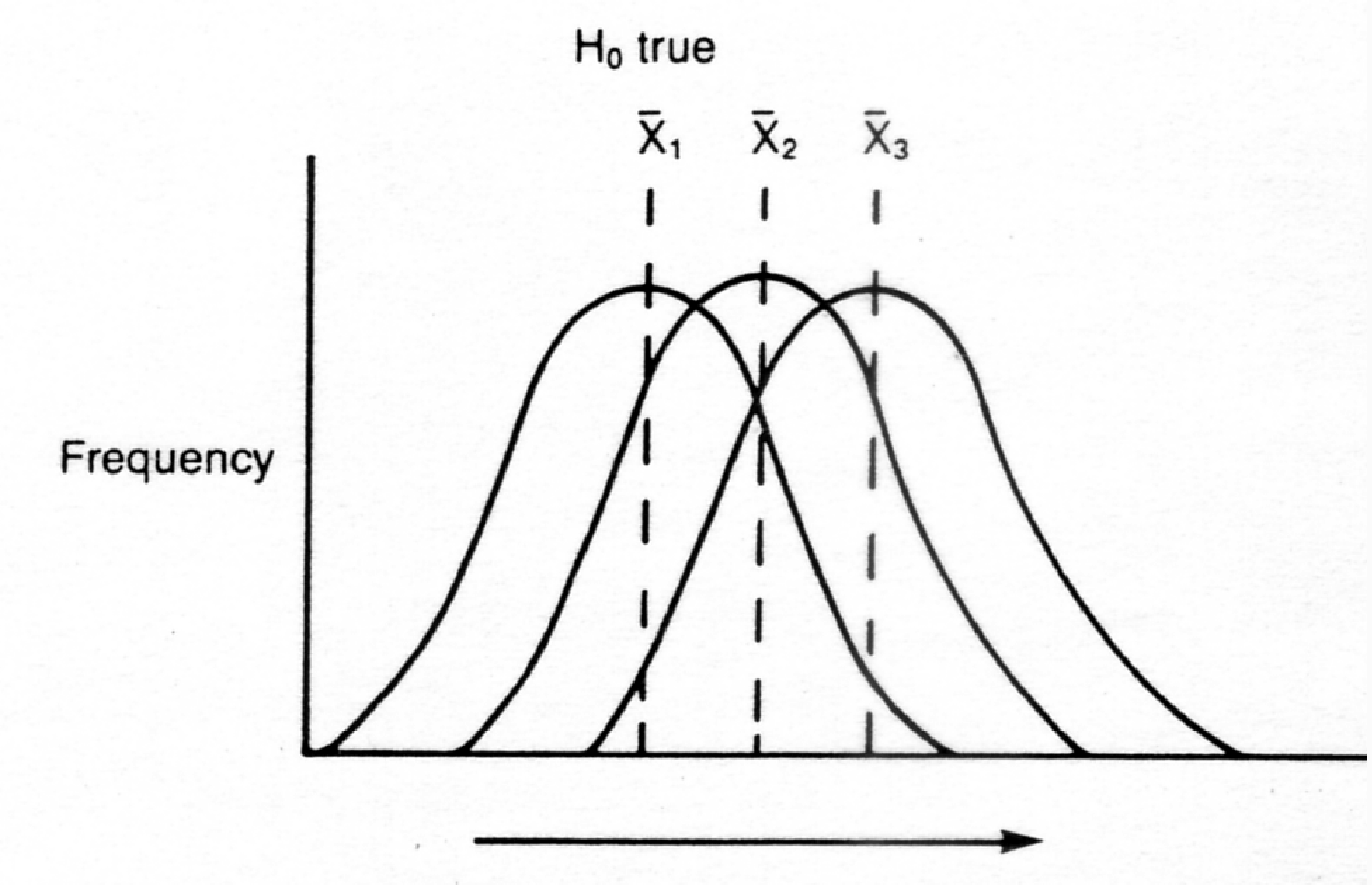
If the overall variance is about the same as the mean variance of the separate groups (within-group variation): the means of the separate groups are not different



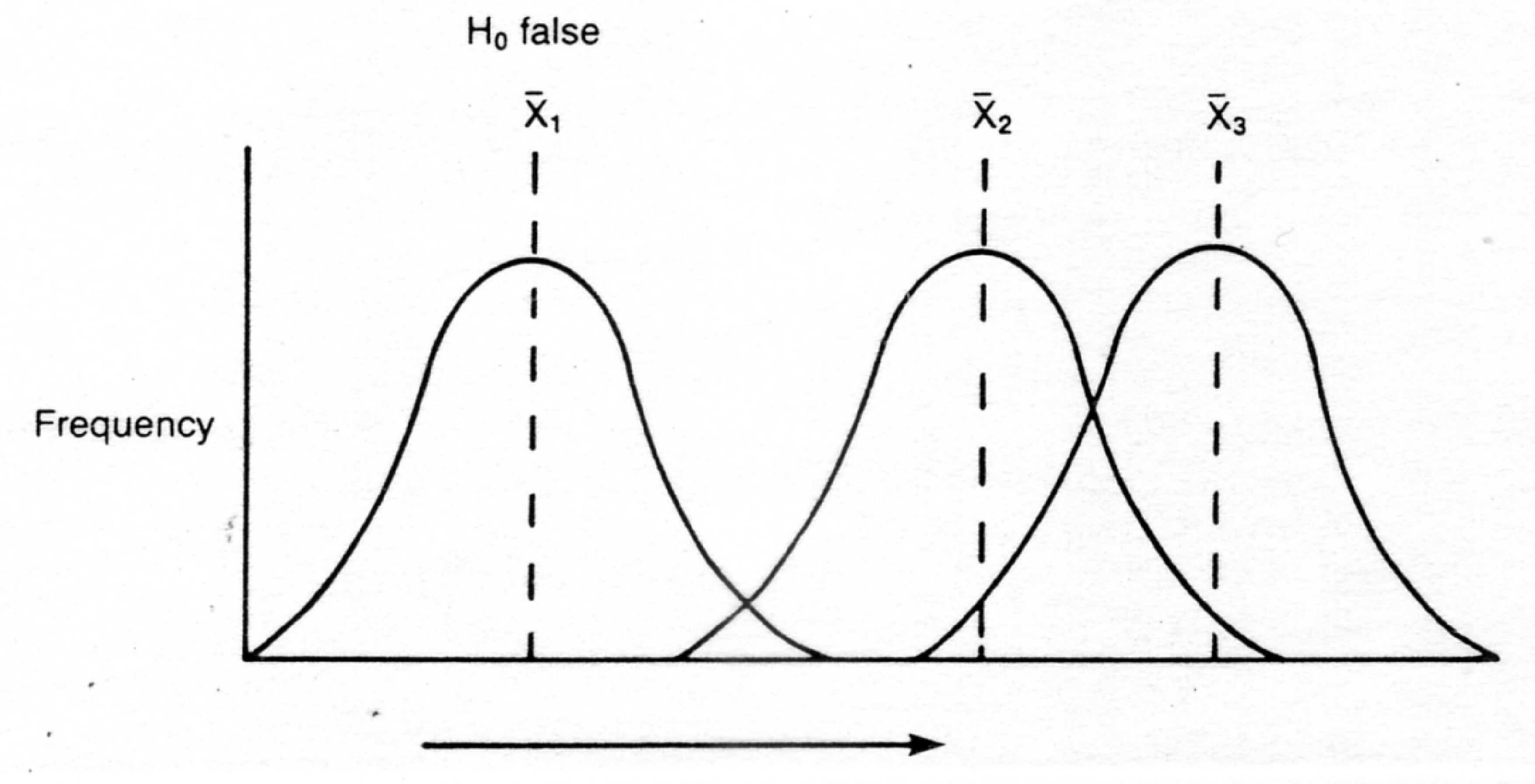
If within-group variation and total variation are equal, there is no between-group variation, because the combination of within-group variation and between group variation is the total variation



When the between group variance is greater (statistically greater) than the within-group variance, the means of the groups must be different



When the null hypothesis is true, the groups overlap to a large extent, and the within-group variation is greater than the between-group variation.



When the null hypothesis is false, the groups show little overlapping, and the distance between groups is greater (between > within)

Measure of variance: Sum of squares

- The sum of squares is the sum of the squared deviations of each of the scores around a respective mean.

$$\sigma^2 = \frac{\sum(x-\bar{x})^2}{N-1} = \frac{SS}{df}$$

Calculating sum of squares

	Group 1	Group 2	Group 3
	1	4	7
	2	5	8
	3	6	9
$\bar{x}_i$	2	5	8

Raw Scores	Deviations from $\bar{x}$	Squared Deviation
1	$1 - 5 = -4$	16
2	$2 - 5 = -3$	9
3	$3 - 5 = -2$	4
4	$4 - 5 = -1$	1
5	$5 - 5 = 0$	0
6	$6 - 5 = 1$	1
7	$7 - 5 = 2$	4
8	$8 - 5 = 3$	9
9	$9 - 5 = 4$	16
$\bar{x} = 5$	$\Sigma = 0$	$SS_{tot} = 60$

Total sum of squares

- The total sum of squares (SS_{tot}) represents the basis of the null hypothesis that all the subjects belong to one population, which is described by the grand mean.

Within sum of squares

Group 1

Raw Scores	Deviations from $\bar{x}$	Squared Deviation
1	$1 - 2 = -1$	1
2	$2 - 2 = 0$	0
3	$3 - 2 = 1$	1
$\bar{x} = 2$	$\Sigma = 0$	$SS_1 = 2$

Within sum of squares

Group 2

Raw Scores	Deviations from $\bar{x}$	Squared Deviation
4	$4 - 5 = -1$	1
5	$5 - 5 = 0$	0
6	$6 - 5 = 1$	1
$\bar{x} = 5$	$\Sigma = 0$	$SS_2 = 2$

Within sum of squares

Group 3

Raw Scores	Deviations from $\bar{x}$	Squared Deviation
7	$7 - 8 = -1$	1
8	$8 - 8 = 0$	0
9	$9 - 8 = 1$	1
$\bar{x} = 8$	$\Sigma = 0$	$SS_3 = 2$

Within sum of squares

$$\begin{aligned} \mathrm{SS_w} &= \mathrm{SS_1} + \mathrm{SS_2} + \mathrm{SS_3} \\ &= 2 + 2 + 2 \\ &= 6 \end{aligned}$$

- Represents portion of total variation within groups

Between-group sum of squares

Group	$x_i - \bar{x}$	$(x_i - \bar{x})^2$	n_i
1	2 - 5 = −3	9	3
2	5 − 5 = 0	0	3
3	8 − 5 = 3	9	3

$$
\begin{aligned}
\mathsf{SS_b} &= \mathsf{SS_1} + \mathsf{SS_2} + \mathsf{SS_3} \\
&= 3(9) + 3(0) + 3(9) \\
&= 54
\end{aligned}
$$

Between-group sum of squares

- Deviation of every score within group from overall mean is considered as average deviation.
- To represent each individual score, the average deviation is weighted by the number of the scores in the group.
- Thus, we weight the squared deviations by the number in the group.
- The weighted squared deviations are summed to provide the between-group sum of squares (SS_b).

One-way analysis of variance

Summary table

Source of variance	*SS*	*df*	*MS*	*F*	*p*
Between group	54	2	27	27	0.01
Within group	6	6	1		
Total	60	8			

Source of variation	Sum of Square	*df*	Mean Square	F statistic	p-value
Between	$\sum_{i=1}^{k} n_i \overline{Y}_i^2 - \frac{Y^2..}{n} = A$	$k-1$	$\frac{A}{k-1}$	$\frac{A/(k-1)}{B/(n-k)} = F$	$\Pr(F_{k-1,n-k} > F)$
Within	$\sum_{i=1}^{k} (n_i - 1) S_i^2 = B$	$n-k$	$\frac{B}{n-k}$		
Total	Between SS + Within SS				

Degrees of freedom

- The *df* for the between-group variance is equal to the number of groups (k) minus one: ($k - 1$)
- The *df* for the within-group variance is equal to the total number of subjects (n) minus the number of groups: ($n - k$)
- The mean square (the mean of the sum of squares or mean variance) is the sum of squares divided by the degrees of freedom: $MS = SS/df$

Testing group differences

- To determine whether the between-group difference is great enough to reject the null hypothesis, we compare it statistically to the within-group variance.

$$F = \frac{\text{Between } MS}{\text{Within } MS}$$

F ratio

- The *F* represents the ratio of between to within variance and is calculated as the between mean square divided by the within mean square.
- Interpretation of *F* based on the *F* distribution, which indicates the critical values of *F* knowing the *df*s for between and within mean squares (*F* ratio > 5 is usually significant).

One-way analysis of variance

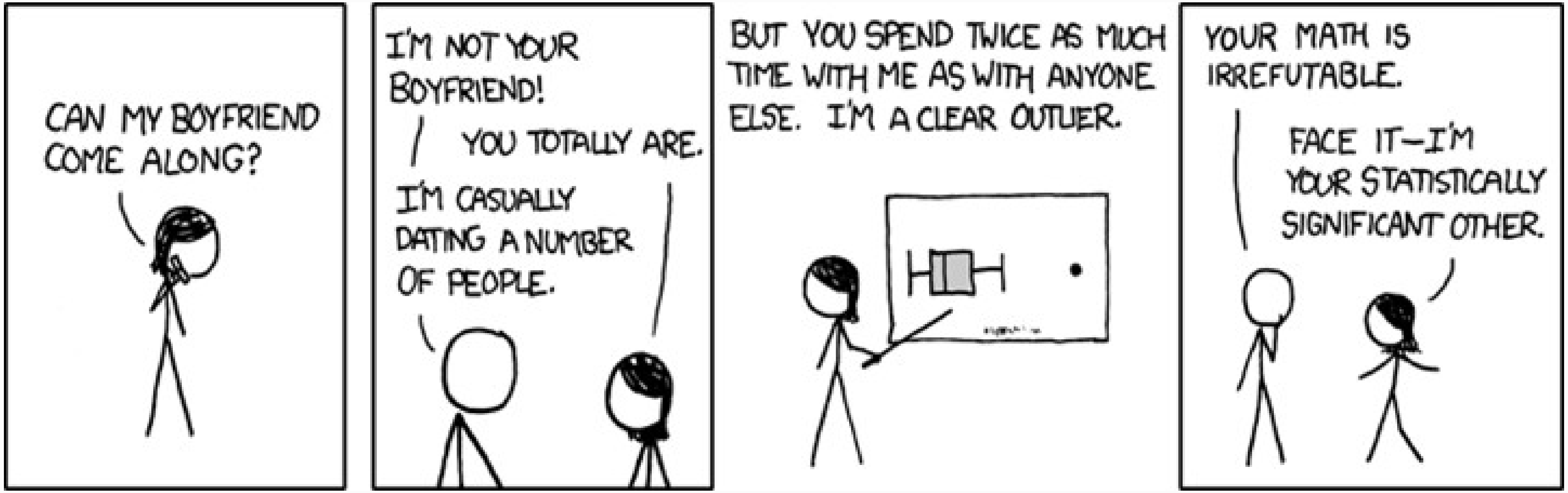
- Interpretation
  - When the ANOVA value has a p-value less than or equal to 0.05, it is said that the categories are significantly different; otherwise it is not.
- Post-hoc tests
  - Post hoc test becomes important when p-value indicates that the categories are significantly different.
  - This test enables us to identify which of the group (groups) are significantly different.

Multiple comparisons in one-way ANOVA

- When ANOVA leads to rejection of the hypothesis of no differences among group means: the question arises as to which pairs of means are significantly different
- Often, comparisons of interest are specified before looking at the actual data
- In this case, comparisons based on two-sample t-test could be considered appropriate
- Often the case that comparisons of interest are specified only after looking at the data, or there is a desire to carry out significance tests on all pairs of group means

Post-hoc tests

- Bonferroni *t* method (Dunn's multiple-comparison)
- Tukey's HSD
- Scheffe's procedure
- Newman-Keuls procedure
- Dunnett's procedure


CAN MY BOYFRIEND COME ALONG?
I'M NOT YOUR BOYFRIEND!
YOU TOTALLY ARE.
I'M CASUALLY DATING A NUMBER OF PEOPLE.
BUT YOU SPEND TWICE AS MUCH TIME WITH ME AS WITH ANYONE ELSE. I'M A CLEAR OUTLIER.
YOUR MATH IS IRREFUTABLE.
FACE IT—I'M YOUR STATISTICALLY SIGNIFICANT OTHER.