

# Homework assignment 2 - EL2450

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## 1 Rate Monotonic

### 1.1

Rate Monotonic scheduling is a scheduling method that will predetermine the priority of each task proportional to the tasks activation frequency. The priority is determined at the task creation and will remain unchanged during the whole application.

### 1.2

A set of tasks  $J = \{J_1, J_2, \dots, J_n\}$  is schedulable with RM if

$$U = \sum_{i=1}^n \frac{C_i}{T_i} \leq n(2^{1/n} - 1) \quad (1)$$

where  $C_i$  is the computation time,  $T_i$  is the period,  $U$  is the utilization factor and  $n$  is the number of computations. When we have a sampling time of  $T = \{20, 29, 30\}ms$  and a computation time of 6 ms each we can see that,

$$\frac{6}{20} + \frac{6}{29} + \frac{6}{35} = 0.678 \quad (2)$$

and with a utilization factor of  $U = 0.780$  we can see that the set of tasks  $J$  should be schedulable with RM.

### 1.3

The pendulums are indeed stabilized. There is however a slight difference in the control performance. From what we can see is the settling time longer the shorter the pendulum gets. Which is correct according to the lab introduction.

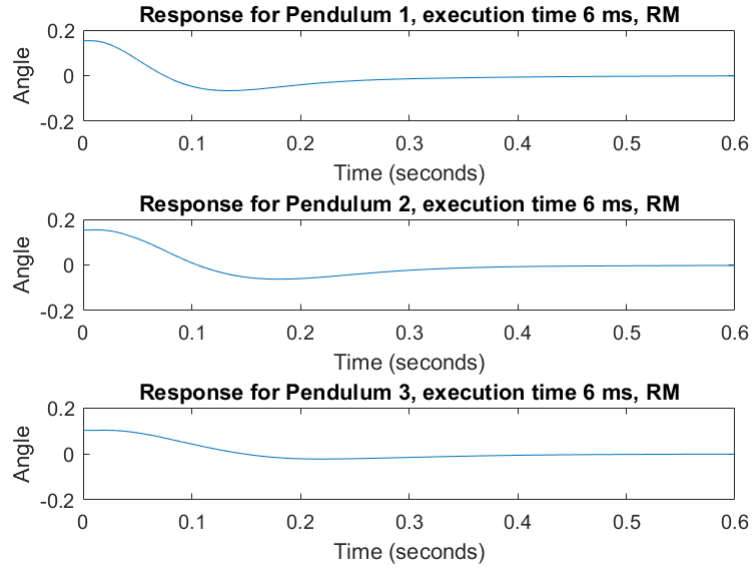


Figure 1: Pendulum angles for the three different pendulums

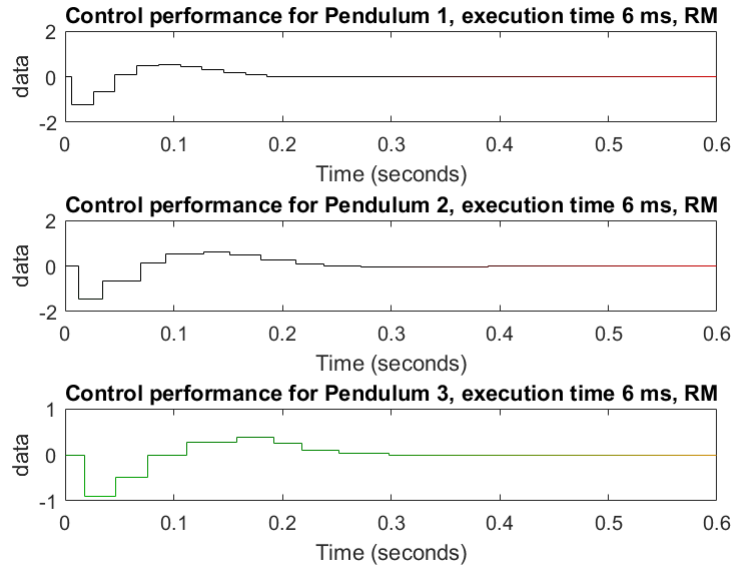


Figure 2: Control signal for pendulum

## 1.4

The schedule plot from Simulink corresponds to our own schedule. This can be seen in figure 3 and 4. Be aware that our schedule is ordered from shortest-longest pendulum and the one from Simulink is longest-shortest pendulum.

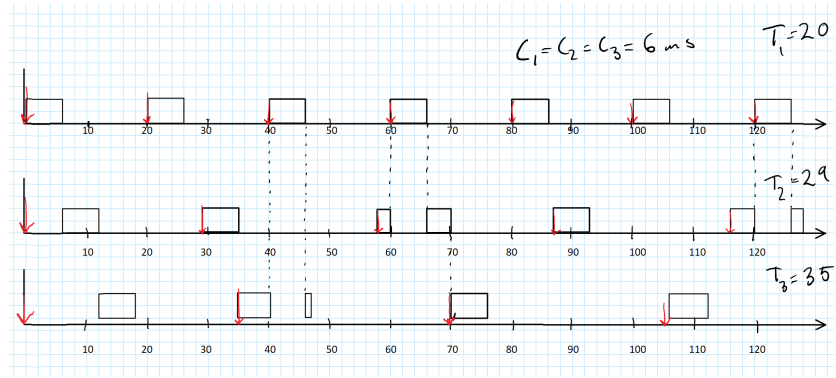


Figure 3: Our schedule for 6 ms computation time

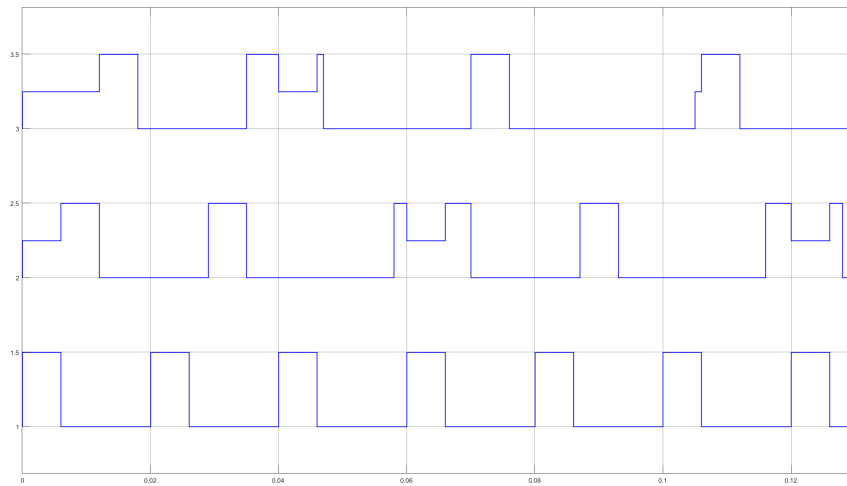


Figure 4: Schedule from Simulink for 6 ms

## 1.5

The longest pendulum becomes unstable after a few seconds due to missed deadlines.

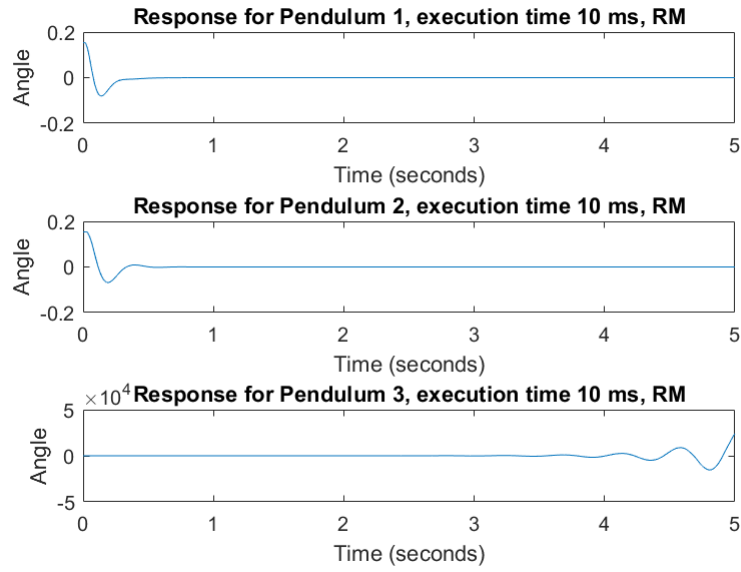


Figure 5: Pendulum angles for the three different pendulums

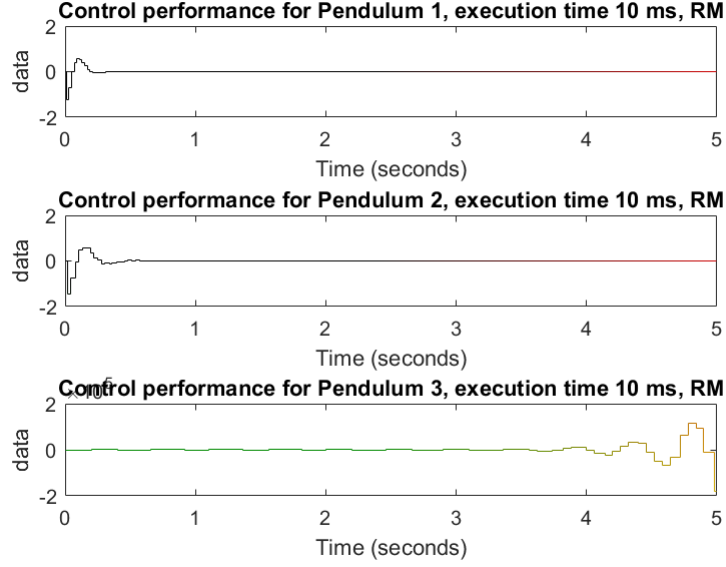


Figure 6: Control signal for pendulum

Deadlines are missed for the long pendulum which means that the tasks are not schedulable. This is verified by calculating the equation

$$U \leq n(2^{(1/n)} - 1) = 3 * (2^{(1/3)} - 1) = 0.779 \quad (3)$$

where

$$U = \sum_{i=1}^n \frac{C_i}{D_i} = \frac{10}{20} + \frac{10}{29} + \frac{10}{35} = 1.13 \quad (4)$$

This means that it is not schedulable due to

$$1.13 > 0.779 \quad (5)$$

Our schedule for a computation time of 10 ms can be seen in figure NUMBER as well can the schedule from Simulink be seen in figure NUMBER. Our schedule is equal to the one from Simulink.

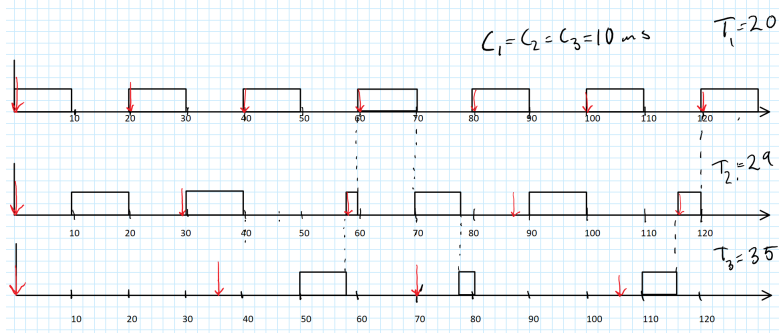


Figure 7: Our schedule for 10 ms computation time

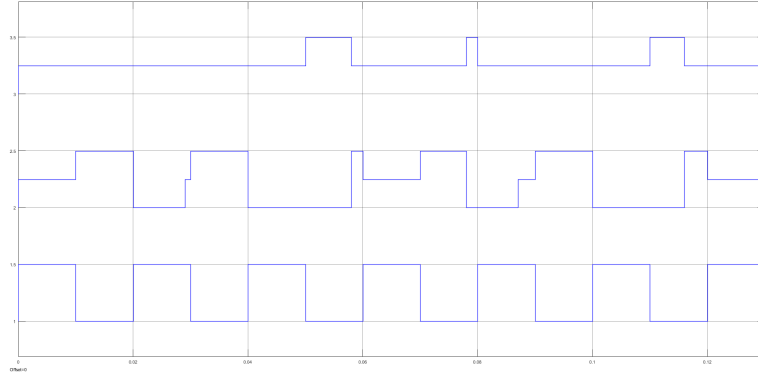


Figure 8: Simulink schedule for 10 ms computation time

## 2 Earliest Deadline First

### 2.1

Earliest deadline first scheduling executes the task with the shortest time left of its deadline  $d_k$ . This means that the schedule dynamically changes depending on the different task's deadlines. The advantages of EDF compared to RM is that EDF fully uses the processor computational power while RM is limited to  $U = n(2^{1/n} - 1)$ . Disadvantage is

## 2.2

Tasks are schedulable with *EarliestDeadlineFirst* if

$$U \leq 1 \quad (6)$$

which in this case is

$$U = \frac{6}{20} + \frac{6}{29} + \frac{6}{35} = 0.678 < 1 \quad (7)$$

This means that the tasks are schedulable which we verified with our schedule that is shown in 2.4.

## 2.3

The pendulums have stabilized. The shortest one is the hardest to control just as with Rate monotonic scheduling. There is not a huge difference in the control signal but you can see the sampling time differences. Since pendulum 3 is sampled the slowest it also takes longest time for it to be completely stable.

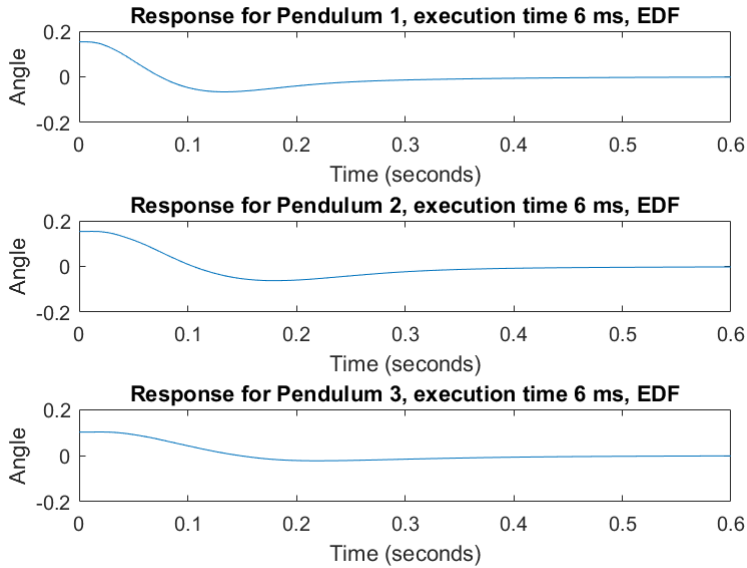


Figure 9: Pendulum angles for the three different pendulums

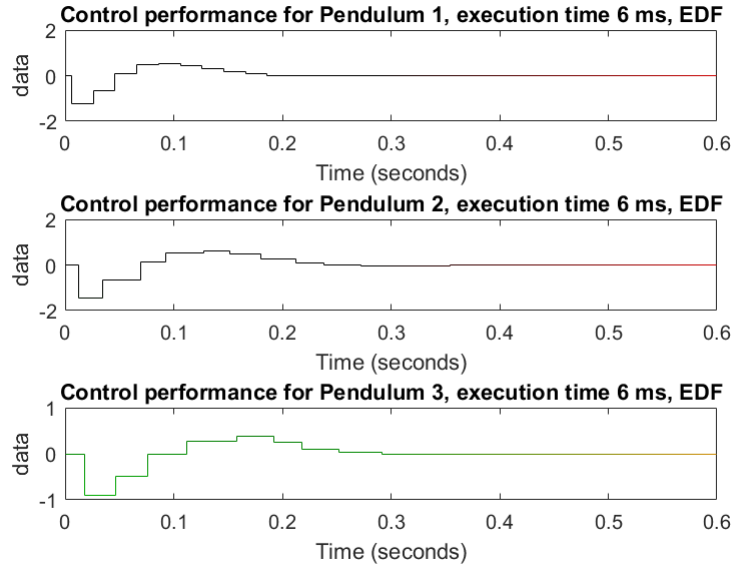


Figure 10: Control signal for the three different pendulums

## 2.4

Our schedule for EDF with 6 ms can be seen in figure NUMBER. The schedule from Simulink can be seen in figure NUMBER. The result from Simulink agrees with our schedule; all tasks are carried out before the next sampling time.

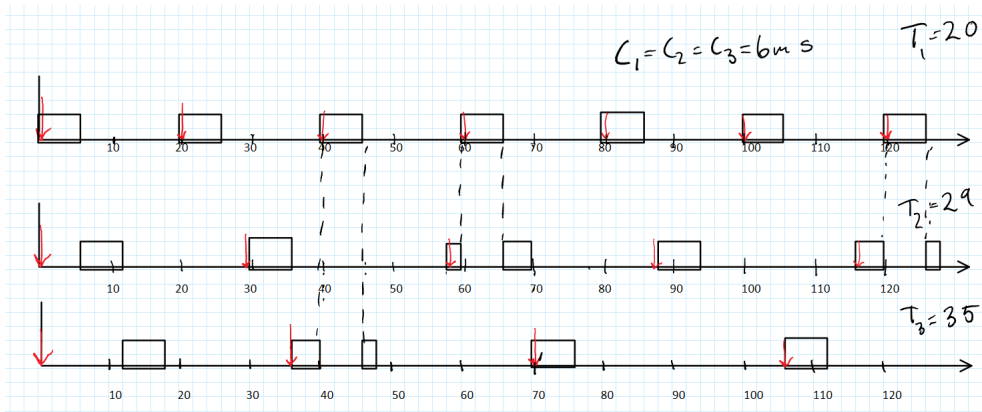


Figure 11: Our schedule for 6 ms computation time, EDF



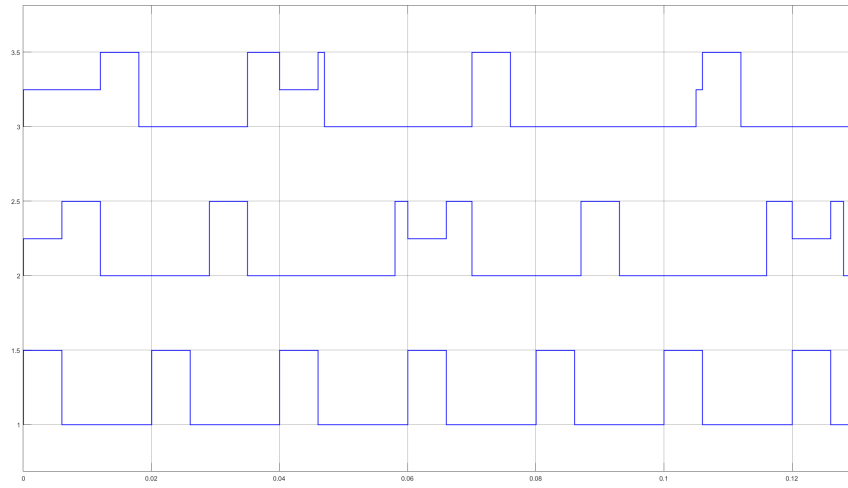


Figure 12: Simulink schedule for 6 ms computation time, EDF

## 2.5

Even though deadlines are missed with EDF the pendulums still stabilizes.

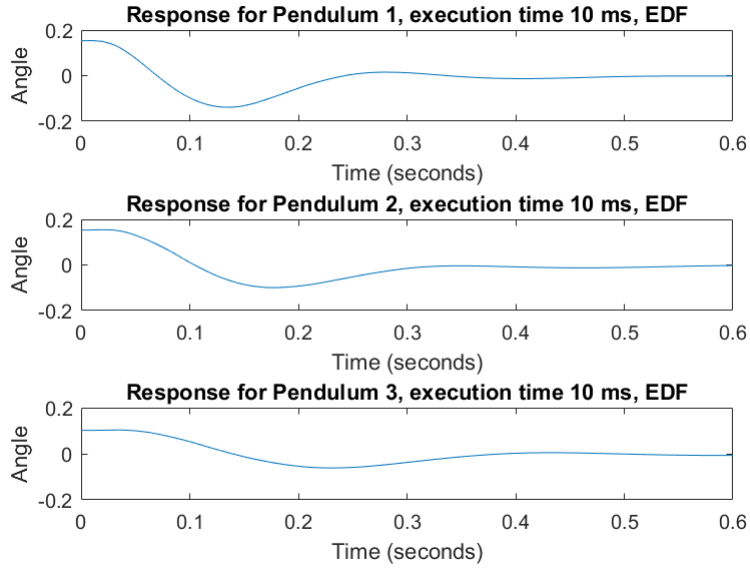


Figure 13: Pendulum angles for the three different pendulums, EDF

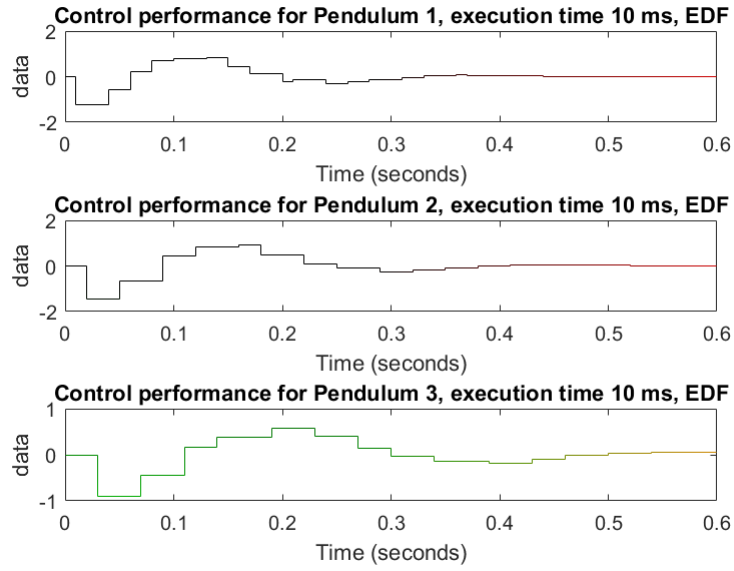


Figure 14: Control signal for the three different pendulums, EDF

The tasks are not schedulable with a computation time of 10 ms due to

$$U = \sum_{i=1}^n \frac{C_i}{D_i} = \frac{10}{20} + \frac{10}{29} + \frac{10}{35} = 1.13 > 1 \quad (8)$$

which we also verified when we created our schedule. Our schedule for a computation time of 10 ms with EDF is shown in figure NUMBER. Simulink schedule for a computation time of 10 ms is shown in figure NUMBER. Our schedule matches the one from Simulink.

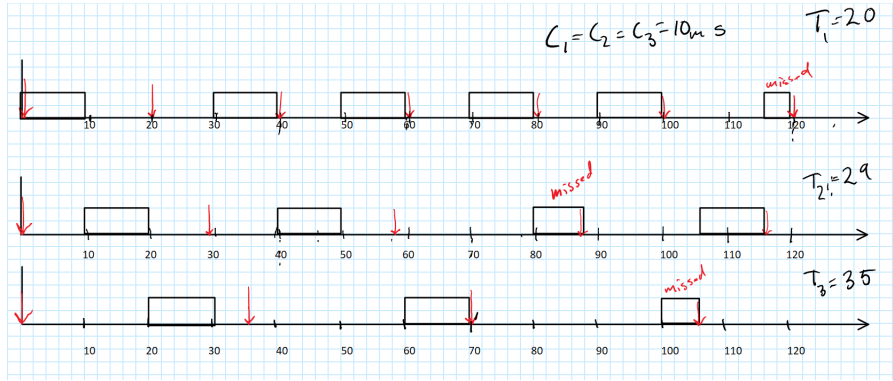


Figure 15: Our schedule for 10 ms computation time, EDF

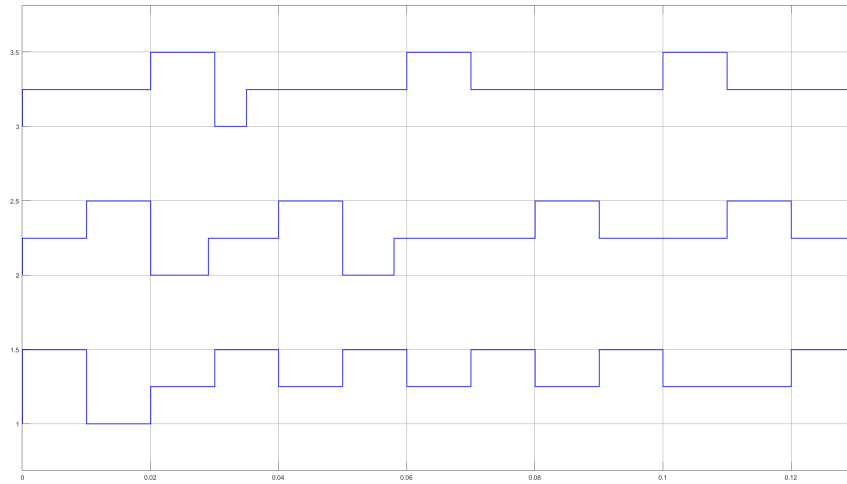


Figure 16: Simulink schedule for 10 ms computation time, EDF

## 2.6

The controller performs better with Earliest Deadline First due to every pendulum stabilizes which pendulum 3 does not with Rate Monotonic.

## 3 Network Control System

### 3.1

We have the following system

$$\dot{x}(t) = Ax(t) + Bu(t)y(t) = Cx(t) \quad (9)$$

where

$$\begin{aligned} A &= 0 \\ B &= I \end{aligned}$$

and the discrete controller

$$u(kh) = -Kx(kh), \quad k = 0, 1, 2, \dots$$

discretizing the system in equation 9 will give that the plant can be written

$$x((k+1)h) = \Phi x(kh) + \Gamma_0(\tau_k)u(kh) + \Gamma_1(\tau_k)u((k-1)h) \quad (10)$$

Given that  $A = 0, B = I$  and that  $\tau = \tau_{sc} + \tau_{ca} \leq h$  we can calculate the matrices,

$$\begin{aligned} \Phi &= e^{Ah} = 1 \\ \Gamma_0(\tau_k) &= \int_0^{h-\tau_k} e^{As} B ds = I(h - \tau) \\ \Gamma_1(\tau_k) &= \int_{h-\tau_k}^h e^{As} B ds = I\tau \end{aligned}$$

Placing that back into the system in equation 10

$$x(kh + \tau) = x(kh) + \begin{bmatrix} 1 & 0 \\ 0 & I\tau \end{bmatrix} \begin{bmatrix} x(kh) \\ u(kh + \tau) \end{bmatrix} + \begin{bmatrix} I\tau \\ 1 \end{bmatrix} u(k) \quad (11)$$

### 3.2

The system will become unstable when a delay of 0.4 seconds is used. The settling time for 0.39 is quite long but the system does stabilize. Figure 17 shows different time delay settings in Simulink. It clearly shows that a delay of 0.3 seconds still makes the system stabilize quickly while a delay of 0.4 makes it unstable.

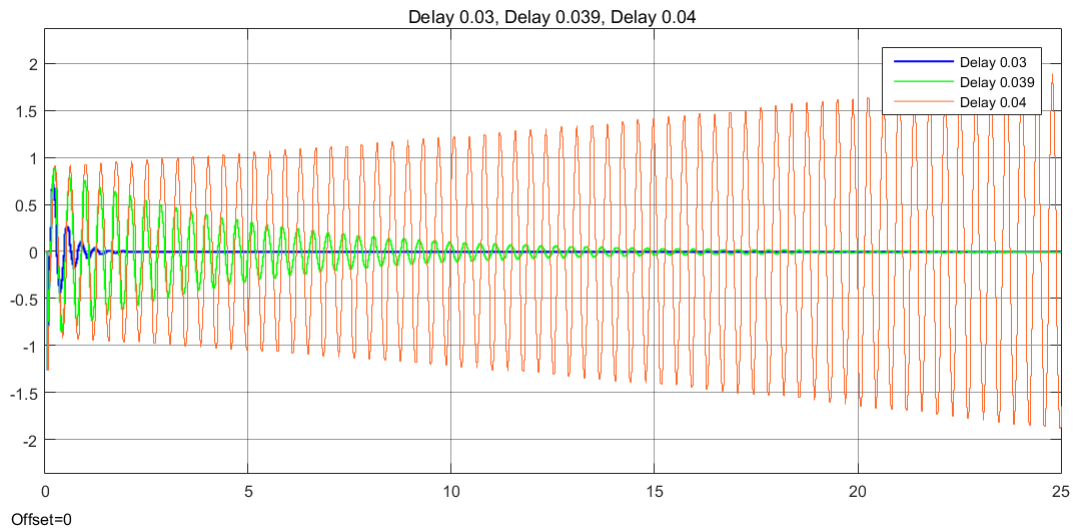


Figure 17: Control signal for different delay settings

## 4 Discrete Event System

### 4.1

Since the state machines M1 and M2 have the same states they are represented as the same state machine. This can be seen in Figure 17.

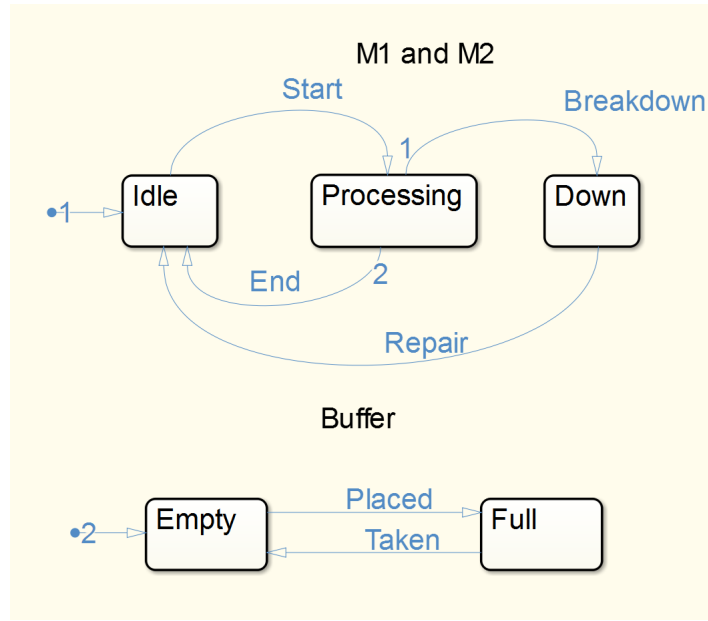


Figure 18: State machine for M1, M2 and the buffer

### 4.2

Each state in Figure 18 represents the current state in M1, M2 and the buffer. There are 18 different states available

$$3^2 * 2 = 18 \quad (12)$$

since both machines has 3 states and the buffer has 2. The form  $x, y, z$  is used where  $x = M1, y = Buffer, z = M2$ . For example

$$IEI = Idle, Empty, Idle$$

The events of each transition are not shown in Figure 18 due to too many transitions and the events are obvious.



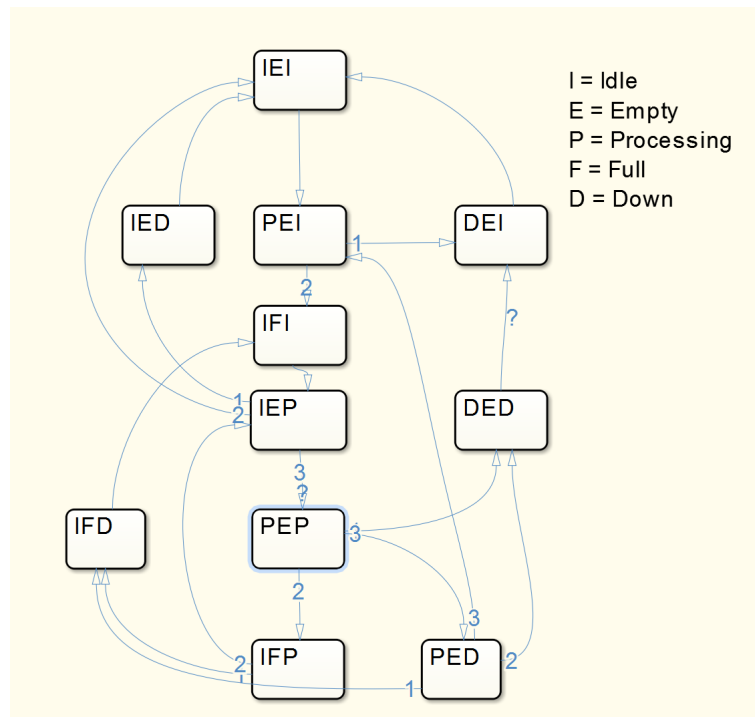


Figure 20: State machine for M1, M2 and the buffer

## 4.4

A controller to make sure the requirements are fulfilled when using the complete Discrete Event System from 4.2 needs to have information of which state each machine is in. Then by using that information *Guards* can be used on each transition along with the event to ensure it goes to the right state. A *Guard* is a form of If-statement used in state machines to set requirements for transitions.