1 Exercise 1

1.1

This is the tap that connects the upper tank to the water reservoir. If the gain is set to 0, this means that there is no contribution from this when the upper tank is being emptied, i.e. the tap is closed.

1.2

The transfer functions was implemented in the presented code.

1.3

The step block, the initial value for the step block is set to 0 meaning that the step will start with this value. The step time is set to 100 meaning that the step will come at $t = 100$ and the final value is set to 10 meaning that the step will step from 0 to 10.
The constant block, this will give a constant output of 40.
The summation block, this will add together both the step- and constant block and the output will be the sum of these to. The reference block will therefore at time 100 have a step from 40 to 50

1.4

The function and the transfer function was added to the script.

1.5

ζ	χ	ω_0	T_r	M	$T_{settling}$
0.5	0.7	0.1	8.4	14.0	45,4
0.5	0.7	0.2	4.9	26.9	37.0
0.5	0.8	0.2	5.0	31.7	26.5

The last control parameters will give values that is within the requirements. Therefore will this be the best controller for us.

1.6

The crossover frequency for the open loop system, $F(s)G(s)$ can be found by solving for $0dB$ when $s = j\omega$

$$|F(j\omega)G(j\omega)| = 1 \tag{1}$$

This will give the crossover frequency $\omega_c = 0.362\,rad/s$.

1.7

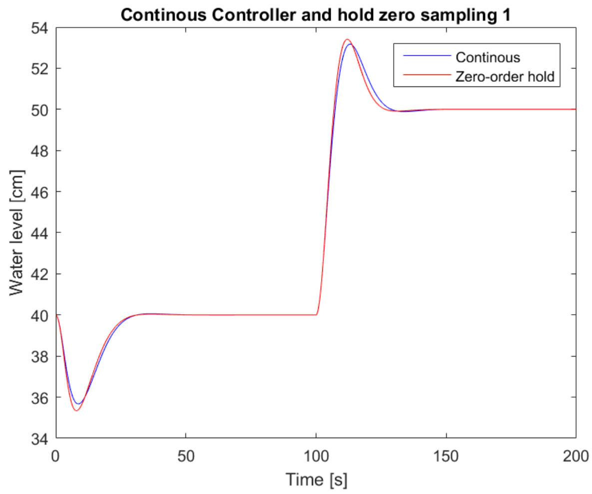

Figure 1: Step response from the continuous controller and the ZOH

As seen in figure **??**, the system with the zero order hold is a bit quicker although there is more overshoot.

1.8

When comparing the ZOH to the discrete controller with different sampling times the control performance decreases around a sampling time of five seconds. The system does not fulfil the requirements at five seconds but the system is still stable. This can be seen in Figure **??**.

1.9

If we allow a phase deterioration of 20° we can calculate the sampling frequency as

$$\omega_s \approx \frac{2\pi\omega_{cutoff}}{0.35}, \tag{2}$$

which gives us the sampling time as,

$$T = \frac{2\pi}{\omega_s} = 0.1539 \tag{3}$$

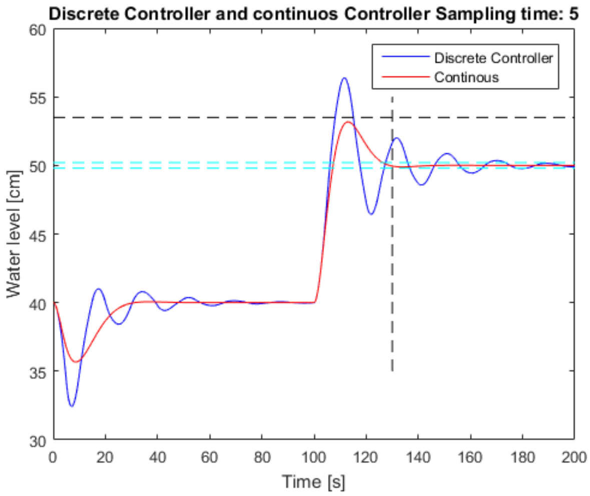

Figure 2: Step response of ZOH and continuous system

1.10

Around a sampling time of three oscillatory behaviour starts to show as seen in Figure ??.

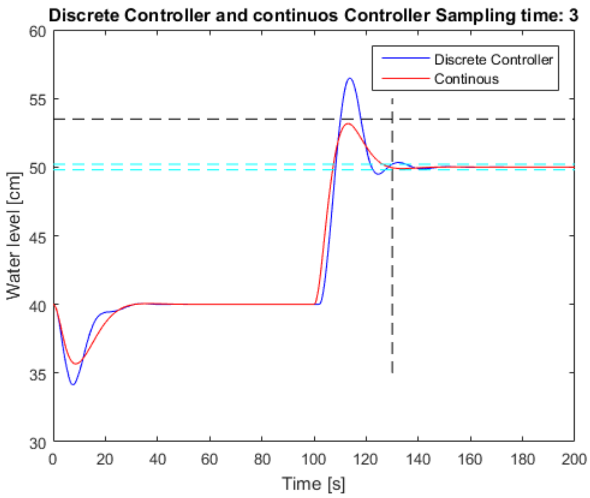

Figure 3: Step response of ZOH and continuous system with sampling time 3

1.11

The control performance is bad with a sampling time of 4 second which is shown in Figure ??.

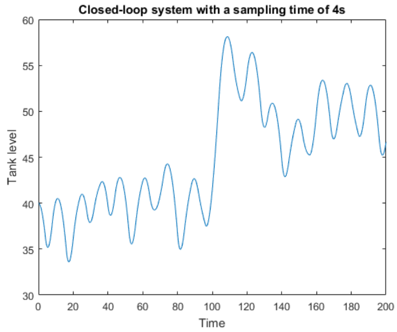

Figure 4: Step response of closed loop system with $T_s = 4$

1.12

The coefficients for the discrete systems were derived by using the c2d command in MATLAB which transforms a continuous time system to a discrete time system. The coefficients are the following:

a_1	a_2	b_1	b_2
0.0916	0.0739	-1.4497	0.5254

1.13

The poles needs to be inside the unit circle for the system to be stable.

1.14

The pole polynomial is

$$z^4 + d_0 z^3 + d_1 z^2 + d_2 z + d_3 \tag{4}$$

with the following values

d_0	d_1	d_2	d_3
-1.2061	0.5495	-0-0924	0-0051

1.15

We have the following closed loop system

$$Gc(s) = \frac{F_d G_d}{1 + F_d G_d} \tag{5}$$

which can be rewritten to

$$Gc(s) = \frac{F_{num} * G_{num}}{F_{den} * G_{den} + F_{num} * G_{num}} \tag{6}$$

The equation for the poles in the closed loop system is therefore

$$F_{den} * G_{den} + F_{num} * G_{num} = 0 \tag{7}$$

Replacing the terms in the equation with its corresponding polynomial and setting this equal to the pole polynomial of the discrete-time closed loop system we get

$$(z-1)(z+r)(z^2+b_1z+b_2)+(c_0z^2+c_1z+c_2)(a_1z+a_2) = z^4+d_0z^3+d_1z^2+d_2z+d_3 \tag{8}$$

By expanding the left side of the equation following polynomial is derived

$$z^4+z^3(a_1c_0+b_1+r-1)+z^2(a_1c_1+a_2c_0+b_1r+b_2-b_1-r)+z(a_1+c_2+a_2c_1+b_2r-b_1r-b_2)+a_2c_2-b_2r \tag{9}$$

This gives the corresponding expression for the right side's d_i values

d_0	d_1	d_2	d_3
$a_1c_0 + b_1 + r - 1$	$a_1c_1 + a_2c_0 + b_1r + b_2 - b_1 - r$	$a_1 + c_2 + a_2c_1 + b_2r - b_1r - b_2$	$a_2c_2 - b_2r$

Which can be represented in matrix form.

$$\begin{bmatrix} 1 & a_1 & 0 & 0 \\ b_1 - 1 & a2 & a1 & 0 \\ b_2 - b_1 & 0 & a_2 & a_1 \\ -b_2 & 0 & 0 & a_2 \end{bmatrix} \begin{bmatrix} r \\ c_0 \\ c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} d_0 - b_1 + 1 \\ d_1 - b_2 + b_1 \\ d_2 + b_2 \\ d_3 \end{bmatrix} \tag{10}$$

1.16

By using the solver command in Matlab the values for r, c_0, c_1 and c_2 could be calculated.

r	c_0	c_1	c_2
0.4543	8.6154	-10.3654	3.2973

The poles of the closed loop system are

p_1	p_2	p_3	p_4
0.1353	0.1353	0.4677+0.2453i	0.46677-0.2435i

which is the same as in Question 11.

1.17

Using the discrete designed controller instead of the transformed continuous controller improves the control of the system. This is probably due to the controller is designed for a discrete system and the poles have been placed to match the poles in the stable continuous system. Comparison can be seen in Figure **??**.

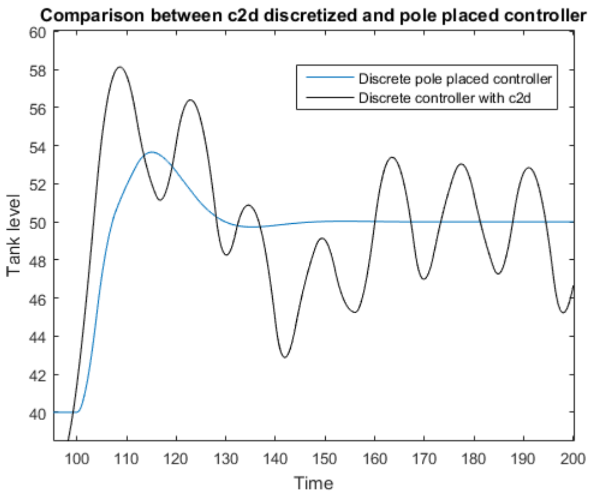

Figure 5: Comparison between discrete designed controller and the transformed continuous controller

1.18

An input difference of 0-100 with a 10-bit A/D converter gives a quantization level of

$$\frac{100}{2^{10}} = 0.0977 \tag{11}$$

1.19

The model in Simulink with quantizer blocks is shown in Figure **??**

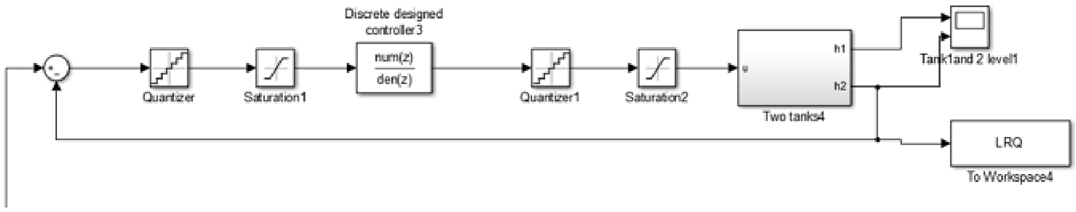

Figure 6: Simulink model of the system with quantization blocks

1.20

When a quantization level of 0.7813 was used the control performance started to be degraded. This corresponds to a 7 bit A/D converter.

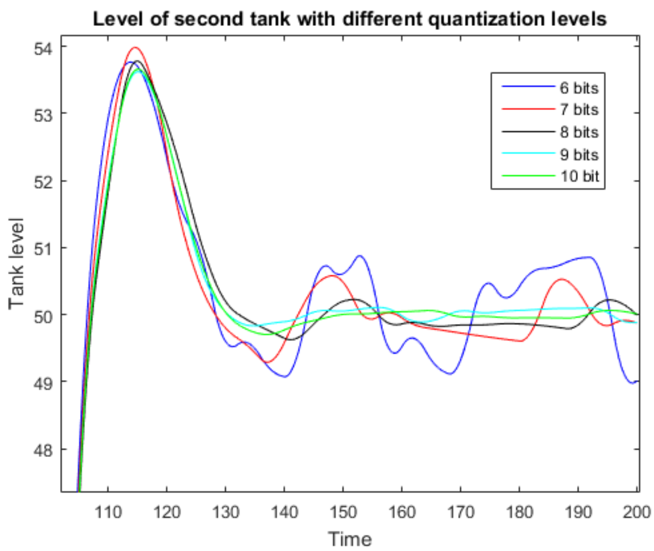

Figure 7: Comparison between different quantization levels