# MF2007 Workshop Part1

Adam Lang, Andreas Fröderberg, Gabriel Andersson Santiago

January 22, 2016

## 1 Ex. 1

The model can be described with the following differential equation,

$$f - d\dot{x} = m\ddot{x} \Leftrightarrow \ddot{x} = \frac{1}{m}(f - d\dot{x}) \tag{1}$$

In this system we have one energy storing element, the mass, $m$. The transfer function can be derived by first Laplace transforming the differential equation,

$$\mathscr{L}\{\ddot{x} = \frac{1}{m}(f - d\dot{x})\} \Rightarrow s^2Y = (\frac{1}{m}(U - dsY) \tag{2}$$

Then can the transfer function be found as,

$$G(s) = \frac{1}{ms^2 + ds} \tag{3}$$

We can also derive a state space model from the differential equations where,

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} = \begin{bmatrix} x \\ \dot{x} \end{bmatrix}, \tag{4}$$

so that,

$$\dot{\mathbf{x}} = \begin{bmatrix} x_2 \\ \frac{1}{m}(F - dx_2) \end{bmatrix}. \tag{5}$$

This will then give us the state space model as,

$$\begin{cases} \dot{\mathbf{x}} = \begin{bmatrix} 0 & 1 \\ 0 & -\frac{d}{m} \end{bmatrix} \mathbf{x} + \begin{bmatrix} 0 \\ \frac{1}{m} \end{bmatrix} \mathbf{u} \\ \\ \mathbf{y} = \begin{bmatrix} 0 & 1 \end{bmatrix} \mathbf{x} \end{cases} \tag{6}$$