Fast and Efficient Bayesian Analysis of Structural Vector Autoregressions Using the R package bsvars version 3.1

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Abstract

The R package **bsvars** provides a wide range of tools for empirical macroeconomic and financial analyses using Bayesian Structural Vector Autoregressions. It uses frontier econometric techniques and compiled code written using `cpp` to ensure fast and efficient estimation of these multivariate dynamic structural models, possibly with many variables, complex identification strategies, and non-linear characteristics. The models can be identified using adjustable exclusion restrictions, heteroskedasticity, or non-normal shocks and feature a flexible three-level equation-specific local-global hierarchical prior distribution for the estimated level of shrinkage for autoregressive and structural parameters. Additionally, the package facilitates predictive and structural analyses such as impulse responses, forecast error variance and historical decompositions, forecasting, verification of heteroskedasticity and hypotheses on autoregressive parameters, and analyses of structural shocks, volatilities, and fitted values. These features differentiate **bsvars** from existing R packages that either focus on a specific structural model, do not consider heteroskedastic shocks, or lack the implementation using compiled code.

Keywords: Bayesian inference, Structural VARs, Gibbs sampler, exclusion restrictions, heteroskedasticity, non-normal shocks, forecasting, structural analysis, R.

Install the package:	`install.packages("bsvars")`
Load the package:	`library(bsvars)`
CRAN repository:	cran.r-project.org/package=bsvars
Website:	bsvars.github.io/bsvars
Development repository:	github.com/bsvars/bsvars
Suggestions and bug reporting:	github.com/bsvars/bsvars/issues
Discussions:	github.com/bsvars/bsvars/discussions
Social media at Mastodon:	@bsvars@fosstodon.org
Social media at Bluesky:	@bsvars.bsky.social
email:	bsvars@pm.me

1. Introduction

Since the publication of the seminal paper by Sims (1980) Structural Vector Autoregressions (SVARs) have become benchmark models for empirical macroeconomic analyses. Subsequently, they have found numerous applications in other fields and are now indispensable in everyday work at central banks, treasury departments and other economic governance institutions, as well as in finance, insurance, banking, and economic consulting.

The great popularity of these multivariate dynamic structural models was gained because they incorporate the reduced and structural forms into a unified framework. On the one hand, they capture the essential properties of macroeconomic and financial time series such as persistence, dynamic effects, system modelling, and potentially time-varying conditional variances. On the other hand, they control for the structure of an economy, system, or market through the contemporaneous effects and, thus, they identify contemporaneously and temporarily uncorrelated shocks that can be interpreted structurally. All these features make it possible to estimate the dynamic causal effects of the shocks on the measurements of interest. These effects are interpreted as the propagation of the well-isolated and unanticipated cause – a contemporaneously and temporarily independent shock – within the considered system of variables throughout the predictable future.

This flexibility comes at a cost of dealing with local identification of the model, sharply growing dimension of the parameter space with the increasing number of variables, and the estimation of latent variables. Bayesian inference provides original solutions to each of these challenges often deciding on the feasibility of the analyses with a demanded model including many variables, conditional heteroskedasticity, and sophisticated identification of the structural shocks. In this context, Bayesian estimation using Markov Chain Monte Carlo methods grants the certainty of reliable estimation of the parameters of interest through the straightforward process of diagnosing the algorithm's convergence but it might incur substantial computational cost.

The paper at hand and the corresponding package **bsvars** by Woźniak (2024) for R (R Core Team 2021) provide tools for empirical macroeconomic and financial analyses using Bayesian SVARs. It addresses the considered challenges by choosing a convenient model formulation, applying frontier econometric techniques, and relying on compiled code written using cpp to ensure fast and efficient estimation. Additionally, it offers a great flexibility in choosing the model specification and identification pattern, modifying the prior assumptions, and accessing interpretable tabulated or plotted outputs.

More specifically, the package uses the SVAR models featuring a standard reduced form VAR equation following Bańbura, Giannone, and Reichlin (2010) (see also Woźniak 2016) and a structural equation linking the reduced form error term to the structural shocks via the structural matrix as in Lütkepohl, Shang, Uzeda, and Woźniak (2024) and Chan, Koop, and Yu (2024). The normal prior distribution for the autoregressive parameters implements the interpretability of the Minnesota prior by Doan, Litterman, and Sims (1984) by centring it around the mean that reflects unit-root nonstationarity or stationarity of the variables and a adjustable level of shrinkage depending on the equation and exhibiting exponential decay with the increasing autoregressive lag order. The prior distribution for the structural matrix is generalised-normal by Waggoner and Zha (2003a) which preserves the shape of the likelihood function (see Woźniak and Droumaguet 2015). Both of these priors are combined with the flexible three-level equation-specific local-global hierarchical prior distribution for

Figure 1: The hexagonal package logo features an impulse response that can be fully reproduced using the **bsvars** package following a script available at: github.com/donotdespair/naklejki/blob/master/bsvars/bsvars.R

the estimated level of shrinkage as in Lütkepohl *et al.* (2024) improving the model fit. The estimation of the level of shrinkage was shown to substantially improve forecasting performance of VAR models by Giannone, Lenza, and Primiceri (2015). Additionally, these specification choices lead to an efficient equation-by-equation Gibbs sampler for the posterior distribution of the autoregressive and structural parameters proposed by Chan *et al.* (2024) and Waggoner and Zha (2003a) respectively.

All the structural models in the **bsvars** package can be identified using highly adaptable exclusion restrictions proposed by Waggoner and Zha (2003a) and use the structural matrix sign normalisation by Waggoner and Zha (2003b). However, one might also choose not to impose any restrictions on the structural matrix and identify it through heteroskedasticity following the ideas by Rigobon (2003) or by non-normal shocks as proposed by Lanne and Lütkepohl (2010). Therefore, the package offers a range of models for conditional variance including a homoskedastic model with time-invariant variances. The list of heteroskedastic specifications is opened by Stochastic Volatility (SV) model in two versions: non-centred as in Lütkepohl *et al.* (2024) and centred used by Chan *et al.* (2024), and is followed by the Markov-switching heteroskedasticity (MSH) model proposed by Brunnermeier, Palia, Sastry, and Sims (2021) with the Bayesian implementations following Woźniak and Droumaguet (2015) in two versions: with stationary Markov process and its sparse version that facilitates the estimation of the number of states and non-parametric interpretations. Similarly, a model with non-normal shocks is implemented in two versions of the normal mixture (MIX) model: with a finite number of components as in Frühwirth-Schnatter (2006) and in its sparse representation serving as an approximation of a non-parametric infinite mixture as proposed by Malsiner-Walli, Frühwirth-Schnatter, and Grün (2016).

The R package **bsvars** implements a wide range of tools for structural and predictive analyses. The former encompass the methods comprehensively revised by (Kilian and Lütkepohl 2017, Chapter 4: SVAR Tools) and include the impulse response functions, forecast error variance decompositions, historical decompositions, as well as the basic analysis of fitted values, structural shocks, conditional standard deviations and regime

probabilities for MSH and MIX models. The predictive analysis includes Bayesian forecasting implemented through an algorithm sampling from the predictive density of the unknown future values of the dependent variables. Methods `summary()` and `plot()` support the user in the interpretations and visualisation of these types of analyses.

A distinguishing feature of the package are the Bayesian model diagnostic tools for the verification of heteroskedasticity and hypotheses on autoregressive parameters using Savage-Dickey Density Ratios (SDDRs) by Verdinelli and Wasserman (1995). The SDDRs represent the Bayes factors for sharp hypotheses and are reliable, precisely estimated, and straightforward to compute once the posterior sample is available. As shown by Lütkepohl and Woźniak (2020) and Lütkepohl *et al.* (2024), SDDRs are a pragmatic tool for the Bayesian assessment of a null hypothesis of homoskedasticity in heteroskedastic models allowing to verify partial or global identification through heteroskedasticity. A general implementation of SDDRs for the autoregressive parameters facilitates verification of restrictions on any conditional mean parameters of the model.

Multivariate dynamic modelling, both Bayesian and frequentist, has found some traction in R in the recent years, which resulted in many new packages available on the CRAN repository. The most relevant developments in frequentist approach include the package **MTS** by Tsay, Wood, and Lachmann (2022) covering a wide range of benchmark models for multivariate time series analysis in economics in finance. If it is about structural models, then the seminal package **vars** by Pfaff (2008) covering homoskedastic VAR and Vector Error Correction (VEC) models provides an unmatched in its reliability teaching and basic analysis tool set.

Notable Bayesian implementations include two packages focusing on specific models important from the point of view of historical developments in the field, namely, the package **BVAR** by Kuschnig and Vashold (2021) providing tools for the estimation and analysis proposed by Giannone *et al.* (2015) and package **bvarsv** by Krueger (2015) focusing on the heteroskedastic VAR proposed by Primiceri (2005). Other packages that have been archived and are no longer available on CRAN are the package **MSBVAR** by Brandt (2006) focusing on the Markov switching model by Sims and Zha (2006) and package **VARsignR** by Danne (2015) provided a treatment of Bayesian SVARs identified via sign restrictions by Uhlig (2005), Rubio-Ramirez, Waggoner, and Zha (2010), and Fry and Pagan (2011). The package **bvartools** by Mohr (2022) provides some functionality focusing on Bayesian inference of reduced form VAR and VEC models. Finally, the package **shrinkTVP** by Knaus, Bitto-Nemling, Cadonna, and Frühwirth-Schnatter (2021) implementing heteroskedastic time-varying parameters regression model with shrinkage on the state space as proposed by Bitto and Frühwirth-Schnatter (2019) and Cadonna, Frühwirth-Schnatter, and Knaus (2020) gives a possibility of estimating an SVAR model as well.

However, the most relevant package to compare **bsvars** to is the package **svars** by Lange, Dalheimer, Herwartz, and Maxand (2021) focusing on frequentist inference for SVAR models identified via exclusion restrictions, heteroskedasticity, and non-normal shocks and implementing a range of models that are feasible to estimate using the maximum likelihood method. The similarity to the functionality of package **bsvars** include the selection of models, such as the MSH and with non-normal residuals, as well as the selection of tools for structural analyses including impulse responses, historical and forecast error variance decompositions. However, the package **svars** implements maximum likelihood and bootstrap procedures for the analysis of the model parameters and offers some specification testing procedures.

In this context, the **bsvars** package implements a range of novel solutions and models for Bayesian analysis. One differentiating example is the implementation of the SVAR models with Stochastic Volatility that is not covered by the package **svars**. This model is particularly important in the context of the recent literature clearly indicating that the single extension of VARs leading to marginally largest improvements in the model fit or forecasting performance is the extension by the Stochastic Volatility as shown e.g. by Clark and Ravazzolo (2015), Chan and Eisenstat (2018), Carriero, Clark, and Marcellino (2019), Chan (2020), and Bertsche and Braun (2022). Another such example are sparse MSH and MIX models based on the hierarchical prior distribution proposed by Malsiner-Walli *et al.* (2016), which decides on the infeasibility of their frequentist implementation. Additionally, the package **bsvars** benefits from the advantage of Bayesian approach that facilitates the estimation for models with potentially many variables, autoregressive lags inflating the dimension of parameters space relative to the number of observations in macroeconomics datasets, Markov-switching regimes or normal mixture components, all of which are the factors constraining the feasibility of maximum likelihood approaches. Finally, the model specification, application of econometric advances and compiled code makes the estimation in package **bsvars** much faster than the implementations in packages **bvarsv** and **MSBVAR**.

2. Bayesian Analysis of Structural VARs

This section scrutinises the modelling framework used in the package focusing on the specification of the models, prior distributions, hypotheses verification tools, and estimation. The reader is referred to (Kilian and Lütkepohl 2017, Chapter 4: SVAR Tools) for the exposition of the standard tools for the analysis of SVAR models, such as the impulse responses, and historical and forecast error variance decompositions, as their implementation in the package closely follows this resource.

2.1. Structural VARs

All of the models in the package **bsvars** share the reduced and structural form equations, as well as the hierarchical prior distributions for these parameters following Lütkepohl *et al.* (2024). The reduced form equation is the VAR equation with p lags specified for an N-vector $\mathbf{y}_t$ collecting observations on N variables at time t:

$$\mathbf{y}_t = \mathbf{A}_1\mathbf{y}_{t-1} + \cdots + \mathbf{A}_p\mathbf{y}_{t-p} + \mathbf{A}_d\mathbf{d}_t + \boldsymbol{\varepsilon}_t, \tag{1}$$

where $\mathbf{A}_i$ are $N \times N$ matrices of autoregressive slope parameters, $\mathbf{d}_t$ is a D-vector of deterministic terms, always including a constant term, and possibly exogenous variables, $\mathbf{A}_d$ is an $N \times D$ matrix of the corresponding parameters, and $\boldsymbol{\varepsilon}_t$ collects the N reduced form error terms. Collect all the autoregressive matrices and the slope terms in an $N \times (Np + D)$ matrix $\mathbf{A} = \begin{bmatrix}\mathbf{A}_1 & \dots & \mathbf{A}_p & \mathbf{A}_d\end{bmatrix}$ and the explanatory variables in a $(Np + D)$-vector $\mathbf{x} = \begin{bmatrix}\mathbf{y}_{t-1} & \dots & \mathbf{y}_{t-p} & \mathbf{d}_t\end{bmatrix}'$. Then equation (1) can be written in the matrix form as

$$\mathbf{y}_t = \mathbf{A}\mathbf{x}_t + \boldsymbol{\varepsilon}_t. \tag{2}$$

Each of the rows of the matrix $\mathbf{A}$, denoted by $[\mathbf{A}]_{n\cdot}$ follows a multivariate conditional normal prior distribution, given the equation-specific shrinkage hyper-parameter $\gamma_{A.n}$, with the mean

vector $\underline{\mathbf{m}}_{n.A}$ and the covariance $\gamma_{A.n}\underline{\Omega}_A$, denoted by:

$$[\mathbf{A}]'_{n\cdot} \mid \gamma_{A.n} \sim \mathcal{N}_{Np+1}\left(\underline{\mathbf{m}}_{n.A}, \gamma_{A.n}\underline{\Omega}_A\right), \tag{3}$$

where $\underline{\mathbf{m}}_{n.A}$ is specified in-line with the Minnesota prior by Doan *et al.* (1984) as a vector of zeros if all of the variables are unit-root stationary, or containing value 1 in its n^{th} element if the n^{th} variable is nonstationary. By default, $\underline{\mathbf{\Omega}}_A$ is a diagonal matrix with vector $\begin{bmatrix}\mathbf{p}^{-2\prime} \otimes \boldsymbol{\imath}'_N & 100\boldsymbol{\imath}'_d\end{bmatrix}'$ on the main diagonal, where $\mathbf{p}$ is a vector containing a sequence of integers from 1 to p and $\boldsymbol{\imath}_N$ is an N-vector of ones. Both, $\underline{\mathbf{m}}_{n.A}$ and $\underline{\mathbf{\Omega}}_A$ can be modified by the user. This specification includes the shrinkage level exponentially decaying with the increasing lag order, relatively large prior variances for the deterministic term parameters, and the flexibility of the hierarchical prior that leads to the estimation of the level of shrinkage as proposed by Giannone *et al.* (2015). The latter feature is facilitated by assuming a 3-level local-global hierarchical prior on the equation-specific reduced form parameters shrinkage given by

$$\gamma_{A.n} | s_{A.n} \sim \mathcal{IG}2\left(s_{A.n}, \underline{\nu}_A\right), \tag{4}$$

$$s_{A.n} | s_A \sim \mathcal{G}\left(s_A, \underline{a}_A\right), \tag{5}$$

$$s_A \sim \mathcal{IG}2\left(\underline{s}_{s_A}, \underline{\nu}_{s_A}\right), \tag{6}$$

where $\mathcal{G}$ and $\mathcal{IG}2$ are gamma and inverted gamma 2 distributions (see Bauwens, Richard, and Lubrano 1999, Appendix A), hyper-parameters $\gamma_{A.n}$, $s_{A.n}$, and s_A are estimated, and $\underline{\nu}_A$, $\underline{a}_A$, $\underline{s}_{s_A}$, and $\underline{\nu}_{s_A}$ are all set by default to value 10 to assure appropriate level of shrinkage towards the prior mean. The values of the latter hyper-parameters can be modified by the user.

The structural form equation determines the linear relationship between the reduced-form innovations $\boldsymbol{\varepsilon}_t$ and the structural shocks $\mathbf{u}_t$ using the $N \times N$ structural matrix $\mathbf{B}_0$:

$$\mathbf{B}_0\boldsymbol{\varepsilon}_t = \mathbf{u}_t. \tag{7}$$

The structural matrix specifies the contemporaneous relationship between the variables in the system and determines the identification of the structural shocks from vector $\mathbf{u}_t$. Its appropriate construction may grant specific interpretation to one or many of the shocks. The package **bsvars** facilitates the identification of the structural matrix and the shocks via exclusion restrictions (see Kilian and Lütkepohl 2017, Chapter 8) and/or through heteroskedasticity or non-normal shocks (Kilian and Lütkepohl 2017, Chapter 14). The zero restrictions are imposed on the structural matrix row-by-row following the framework proposed by Waggoner and Zha (2003a) via the following decomposition of the n^{th} row of the structural matrix, denoted by $[\mathbf{B}_0]_{n\cdot}$:

$$[\mathbf{B}_0]_{n\cdot} = \mathbf{b}_n\mathbf{V}_n, \tag{8}$$

where $\mathbf{b}_n$ is a $1 \times r_n$ vector collecting the elements to be estimated and the $r_n \times N$ matrix $\mathbf{V}_n$ including zeros and ones placing the estimated elements in the demanded elements of $[\mathbf{B}_0]_{n\cdot}$.

The structural matrix $\mathbf{B}_0$ follows a conditional generalised-normal prior distribution by Waggoner and Zha (2003a) recently revised by Arias, Rubio-Ramírez, and Waggoner (2018) that is proportional to:

$$\mathbf{B}_0 \mid \gamma_{B.1}, \ldots, \gamma_{B.N} \sim |\det(\mathbf{B}_0)|^{\underline{\nu}_B - N} \exp\left\{-\frac{1}{2}\sum_{n=1}^{N}\gamma_{B.n}^{-1}\mathbf{b}_n\mathbf{b}'_n\right\}, \tag{9}$$

with the shape parameter $\underline{\nu}_B \geq N$ and the equation-specific structural parameter shrinkage $\gamma_{B.n}$. The default value of the shape parameter $\underline{\nu}_B$ set to N makes this prior a conditional, zero-mean r_n-variate normal prior distribution for $\mathbf{b}_n$ with the diagonal covariance and the diagonal element $\gamma_{B.n}$. The shape parameter can be modified by the user though.

This prior specification is complemented by a 3-level local-global hierarchical prior on the equation-specific structural parameters shrinkage given by

$$\gamma_{B.n}|s_{B.n} \sim \mathcal{IG}2\left(s_{B.n}, \underline{\nu}_b\right), \tag{10}$$

$$s_{B.n}|s_B \sim \mathcal{G}\left(s_B, \underline{a}_B\right), \tag{11}$$

$$s_B \sim \mathcal{IG}2\left(\underline{s}_{s_B}, \underline{\nu}_{s_B}\right), \tag{12}$$

where hyper-parameters $\gamma_{B.n}$, $s_{B.n}$, and s_B are estimated and $\underline{\nu}_b$, $\underline{a}_B$, $\underline{s}_{s_B}$, and $\underline{\nu}_{s_B}$ are fixed to values 10, 10, 1, and 100 respectively to assure a flexible dispersed distribution *a priori* but they can be modified by the user.

Finally, all of the models share the zero-mean conditional normality of the structural shocks given the past observations with the diagonal covariance matrix with the N-vector of structural shocks' variances, $\boldsymbol{\sigma}_t^2$, on the main diagonal:

$$\mathbf{u}_t \mid \mathbf{x}_t \sim \mathcal{N}_N\left(\mathbf{0}_N, \operatorname{diag}\left(\boldsymbol{\sigma}_t^2\right)\right). \tag{13}$$

The diagonal covariance matrix, together with the joint normality, implies contemporaneous independence of the structural shocks which is the essential feature allowing for the estimation of dynamic effects to a well-isolated cause that is not influenced by other factors in the SVAR models.

The model parts described in the current section are common to all the models considered in the package **bsvars**. The alternative model specifications are distinguished by the characterisation of the conditional variances collected in vector $\boldsymbol{\sigma}_t^2$.

2.2. Models for Conditional Variances

The **bsvars** package offers five alternative specifications for the conditional variance process.

Homoskedastic model

The first such specification is a homoskedastic model for which the conditional variance of every shock is equal to

$$\sigma_{n.t}^2 = 1 \tag{14}$$

for all t. This setup results in a simple SVAR model that is quick to estimate. Note that in this model the conditional covariance of the data vector, $\mathbf{y}_t$ is equal to $(\mathbf{B}_0'\mathbf{B}_0)^{-1}$. The point at which this model is standardized as in equation (14) is the benchmark value for the all other heteroskedastic models whose conditional variances hover around value 1.

Stochastic Volatility

The heteroskedastic model with Stochastic Volatility is implemented in two versions: non-centred by Lütkepohl *et al.* (2024) and centred Chan *et al.* (2024). In these models, the

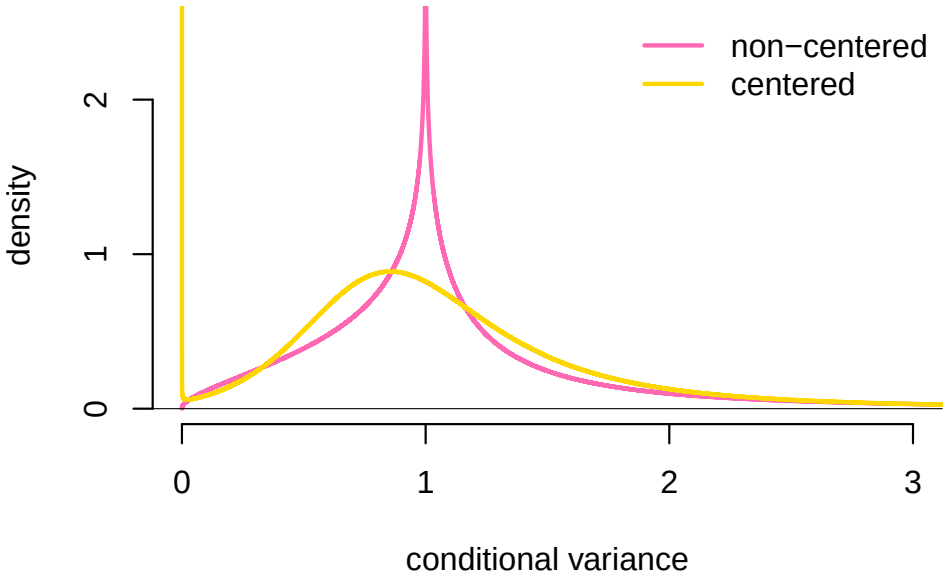

Figure 2: Marginal prior density of a structural shock conditional variance in the two SV models. Sourse: Lütkepohl et al. (2024).

conditional variances for each of the shocks are given by

$$\text{non-centred:} \qquad \sigma_{n.t}^2 = \exp\left\{\omega_n h_{n.t}\right\} \tag{15}$$

$$\text{centred:} \qquad \sigma_{n.t}^2 = \exp\left\{\tilde{h}_{n.t}\right\}, \tag{16}$$

where the parameter ω_n denotes the plus-minus square root of the conditional variance for the log-conditional variance process and will be referred to as the *volatility of the log-volatility*, whereas $h_{n.t}$ and $\tilde{h}_{n.t}$ are the log-volatility processes following an autoregressive equation

$$\text{non-centred:} \qquad h_{n.t} = \rho_n h_{n.t-1} + v_{n.t}, \qquad v_{n.t} \sim \mathcal{N}\left(0, 1\right) \tag{17}$$

$$\text{centred:} \qquad \tilde{h}_{n.t} = \rho_n \tilde{h}_{n.t-1} + \tilde{v}_{n.t}, \qquad \tilde{v}_{n.t} \sim \mathcal{N}\left(0, \sigma_v^2\right) \tag{18}$$

with the initial values $h_{n.0} = \tilde{h}_{n.0} = 0$, where ρ_n is the autoregressive parameter, $v_{n.t}$ and $\tilde{v}_{n.t}$ are the Stochastic Volatility normal innovations, and σ_v^2 is the conditional variance of $\tilde{h}_{n.t}$ in the centred parameterisation.

The priors for these models include a uniform distribution for the autoregressive parameter

$$\rho_n \sim \mathcal{U}(-1, 1), \tag{19}$$

which assures the stationarity of the log-volatility process and sets its unconditional expected value to $E[h_{n.t}] = E[\tilde{h}_{n.t}] = 0$. The prior distribution for the volatility of the log-volatility parameter in the non-centred and the conditional variance of the log-volatility in the centred parameterisation follow a multi-level hierarchical structures given by:

$$\text{non-centred:} \quad \omega_n \mid \sigma_{\omega.n}^2 \sim \mathcal{N}\left(0, \sigma_{\omega.n}^2\right), \qquad \sigma_{\omega.n}^2 \mid s_\sigma \sim \mathcal{G}(s_\sigma, \underline{a}_\sigma), \qquad s_\sigma \sim \mathcal{IG}2(\underline{s}, \underline{\nu}) \tag{20}$$

$$\text{centred:} \quad \sigma_v^2 \mid s_v \sim \mathcal{IG}2(s_v, \underline{a}_v), \qquad s_v \mid s_\sigma \sim \mathcal{G}(s_\sigma, \underline{a}_\sigma), \qquad s_\sigma \sim \mathcal{IG}2(\underline{s}, \underline{\nu}) \tag{21}$$

where parameters $\sigma^2_{\omega.n}$ and s_v follow gamma distribution with expected value equal to $s_\sigma \underline{a}_\sigma$. In this hierarchical structure the hyper-parameters ω_n, $\sigma^2_{\omega.n}$, σ^2_v, s_v, and s_σ are estimated, while $\underline{a}_v$, $\underline{a}_\sigma$, $\underline{\nu}$, and $\underline{s}$ are set to values 1, 1, 1, and 0.1, respectively, by default and can be modified by the user.

Both of the SV models share a number of features, such as the flexibility of hierarchical priors with estimated hyper-parameters granting better fit to data and improved forecasting performance, and they facilitate the identification of the structural matrix $\mathbf{B}_0$ and shocks through heteroskedasticity following the original ideas by Bertsche and Braun (2022) and Lewis (2021). However, they differ with respect to the features analysed by Lütkepohl *et al.* (2024) that are presented in Figure 2 plotting the marginal prior distribution of the conditional variances, $\sigma^2_{n.t}$, in the two SV models. These densities are implied by the SV model specifications presented in this section. The centred parameterisation concentrate the prior probability mass around point 1 only mildly and goes to infinity when the conditional variance goes to zero. The non-centred parameterisation, on the other hand, concentrates the prior probability mass around point 1 more strongly, goes to zero when the conditional variance goes to zero, and features fat right tail. Finally, in the non-centred parameterisation homoskedasticity can be verified by testing the restriction $\omega_n = 0$ (see Lütkepohl *et al.* 2024).

Markov-switching heteroskdasticity

In another heteroskedastic model, the MSH one, the time-variation of the conditional variances is determined by a discrete-valued Markov process s_t with M regimes:

$$\sigma^2_{n.t} = \sigma^2_{n.s_t}. \tag{22}$$

All of the variances switch their values at once according to the latent Markov process takes the values $s_t = m \in \{1, \dots, M\}$. The properties of the Markov process itself are determined by the transition matrix $\mathbf{P}$ whose $[\mathbf{P}]_{i.j}$ element denotes the transition probability from regime i to regime j over the next period. The process' initial probabilities are estimated and denoted by the M-vector $\boldsymbol{\pi}_0$. In this model proposed by Brunnermeier *et al.* (2021), the variances in the n^{th} equation sum to M, each of them has the prior expected value equal to 1, and their regimes are given equal prior probabilities of occurrence equal to M^{-1}. Therefore, the prior for the conditional variances is the M-variate Dirichlet distribution:

$$M^{-1}\left(\sigma^2_{n.1}, \dots, \sigma^2_{n.M}\right) \sim \mathcal{D}irichlet_M(\underline{e}_\sigma, \dots, \underline{e}_\sigma), \tag{23}$$

where the hyper-parameter $\underline{e}_\sigma = 1$ is fixed. Each of the rows of the transition matrix as well as the initial state probabilities follow the Dirichlet distribution as well:

$$[\mathbf{P}]_{m\cdot} \sim \mathcal{D}irichlet_M(\underline{e}, \dots, \underline{e}) \quad \text{and} \quad \boldsymbol{\pi}_0 \sim \mathcal{D}irichlet_M(\underline{e}_0, \dots, \underline{e}_0). \tag{24}$$

The package **bsvars** offers two alternative models based on MSH. The first is characterised by a stationary Markov process with no absorbing state, and with a positive minimum number of regime occurrences following Woźniak and Droumaguet (2015). In this model, the hyper-parameter $\underline{e} = \underline{e}_0$ is fixed to 1 by default and can be modified by the user. The other model represents a novel proposal of a sparse representation that fixes the number of regimes to an over-fitting value $M = 20$ (or specified by the user). In this model, many of the regimes

will have zero occurrences throughout the sample, which allows the number of regimes with non-zero occurrences to be estimated following the ideas by Malsiner-Walli *et al.* (2016). Its prior specification is complemented by a hierarchical prior for the hyper-parameter $\underline{e}$:

$$\underline{e} \sim \mathcal{IG}2\left(\underline{s}_e, \underline{\nu}_e\right). \tag{25}$$

In the sparse MSH model the regimes feature label switching, which excludes regime-specific interpretation of parameters. Instead, the estimated sequence of conditional variances, $\sigma^2_{n.t}$, enjoys standard interpretations. Furthermore, both MSH models provide identification through heteroskedasticity following the ideas by Lanne, Lütkepohl, and Maciejowska (2010) and Lütkepohl and Woźniak (2020). The latter paper provides framework for verifying the identification, which in the **bsvars** package is implemented by verifying the homoskedasticity hypothesis represented by the restriction

$$\sigma^2_{n.1} = \cdots = \sigma^2_{n.M} = 1. \tag{26}$$

2.3. Models with Non-Normal Shocks

Identification via non-normality of the structural shocks is implemented in the package using the mixture of normal component model (MIX) following the proposal by Woźniak and Droumaguet (2015). In this model, the structural shocks follow a conditional N-variate normal distribution given the state variable $s_t = m \in \{1, \ldots, M\}$;

$$u_t \mid s_t = m \sim \mathcal{N}_N\left(\mathbf{0}_n, \boldsymbol{\sigma}^2_m\right), \tag{27}$$

and where the states are predicted to occur in the next period with probability π which is an M-vector. Therefore, unconditionally, the shocks follow a mixture of zero-mean normal components with variance $\boldsymbol{\sigma}^2_m$ for $m \in \{1, \ldots, M\}$. The prior distribution for the conditional variances is the same as for the MSH model and is given by equation (23). The predictive state probabilities follow a Dirichlet distribution:

$$\boldsymbol{\pi} \sim \mathcal{D}irichlet_M(\underline{e}, \ldots, \underline{e}). \tag{28}$$

Despite the predictive state probabilities being constant the classification of the observations into the states is performed via the estimated filtered probabilities $\Pr[s_t = m \mid y_t]$ (see Song and Woźniak 2021, for a recent review of the method).

The MIX model comes in two versions as well. The first is the finite mixture model (see e.g. Frühwirth-Schnatter 2006) in which the number of states, M, is fixed and the state probabilities are positive. The latter condition requires non-zero regime occurrences over the sample, a condition that is imposed in the package implementation. An alternative specification is referred to as the sparse mixture model and is based on the proposal by Malsiner-Walli *et al.* (2016). In this model, the number of the finite mixture model's components is set to be larger than the real number of the components. The number of components with non-zero probability of occurrence is, therefore, estimated. This sparse structure of normal components many of which are allowed to have zero probability is implemented thanks to the prior specified for the hyper-parameter $\underline{e}$ of Dirichlet distribution as in equation (25).

The MIX models facilitate the identification through non-normality as proposed by Lanne and Lütkepohl (2010). The hypothesis of normality contradicting the identification is verified by checking whether restriction as in equation (26) holds.

2.4. Model diagnostics using SDDRs

Verdinelli and Wasserman (1995)

Verifying heteroskedasticity

Verifying autoregressive specification

2.5. Posterior Samplers and Computational Details

In this section we explain the the **bsvars** package implementation of fast and efficient estimation algorithms obtained thanks to the application of appropriate model specification and frontier econometric techniques best described in Lütkepohl *et al.* (2024) and Woźniak and Droumaguet (2015), and compiled code written in `cpp`.

The objective for choosing the model equations and the prior distributions was to make the estimation using Gibbs sampler technique (see e.g. Casella and George 1992) and well-specified easy-to-sample-from full conditional posterior distributions. Therefore, the package relies on the reduced form equation (2) for the VAR model. This choice is fairly uncommon in the SVAR literature but it simplifies the estimation of the autoregressive parameters $\mathbf{A}$ directly in the form as they are used for the computations of impulse responses or forecast error variance decomposition. This choice combined with the specification of the structural form equation (7) facilitates the application of the row-by-row sampler by Chan *et al.* (2024). As shown by Carriero *et al.* (2019), relative to the joint estimation of the matrix $\mathbf{A}$ in one step, a usual practice in reduced form VARs, the row-by-row estimation in SVARs can reduce computational complexity of Bayesian estimation from $\mathcal{O}(N^6)$ to $\mathcal{O}(N^4)$.

The estimation of the structural form equation (7) is implemented following the quickly converging, efficient, and providing excellent mixing sampling algorithm by Waggoner and Zha (2003a). It offers a flexible framework for setting exclusion restrictions and was also adapted to the SVARs identified through heteroskedasticity and non-normality by Woźniak and Droumaguet (2015). The unique formulation of this equation is particularly convenient for complex heteroskedastic models facilitating the row-by-row estimation of matrix $\mathbf{A}$ and Gibbs sampler for the heteroskedastic process. None of the existing `R` packages implements any of these algorithms.

The estimation of the Stochastic Volatility models is particularly requiring due to the N independent T-valued latent volatility processes estimation that it involves. The implementation of crucial techniques is particularly important here. The sampling algorithms use the 10-component auxiliary mixture technique by Omori, Chib, Shephard, and Nakajima (2007) that facilitates the estimation of the log-volatility using the simulation smoother by McCausland, Miller, and Pelletier (2011) for conditionally Gaussian state-space models greatly speeding up the computations (see Woźniak 2021, for the computational times comparison for various estimation algorithms). Additionally, our specification facilitates the algorithms to estimate heteroskedastic process if the signal from the data is

strong, but it also allows them to heavily shrink the posterior towards homoskedasticity, as in equation (14), otherwise. The package implements the adaptation of the ancillarity-sufficiency interweaving strategy that is shown by Kastner and Frühwirth-Schnatter (2014) to improve the efficiency of the sampler when the heteroskedasticity is uncertain. Our implementation of the sampling algorithm closely follows the algorithms from package **stochvol** with necessary adaptations for the SVAR modelling.

The estimation of the Markov switching and mixture models benefits mainly from the implementation of the forward-filtering backward-sampling estimation algorithm for the Markov process s_t by Chib (1996) in cpp. However, an additional step of choosing the parameterisation of the conditional variances as in equation (23), requiring sampling from a new distribution defined by Woźniak and Droumaguet (2015), assures excellent mixing and sampling efficiency improvements relative to alternative ways of standardising these parameters.

All of the estimation routines for the Markov chain Monte Carlo estimation of the models and those for low level processing of the rich estimation output are implemented based using compiled code in **cpp**. This task is facilitated by the **Rcpp** package by Eddelbuettel, François, Allaire, Ushey, Kou, Russel, Chambers, and Bates (2011) and Eddelbuettel (2013). The **bsvars** package relies heavily on linear algebra and pseudo-random number generators (RNGs). The former is implemented using the package **RcppArmadillo** by Eddelbuettel and Sanderson (2014) that is a collection of headers linking to the cpp library **armadillo** by Sanderson and Curtin (2016), as well as on several utility functions for operations on tri-diagonal matrices from package **stochvol** by Hosszejni and Kastner (2021). Another essential element are the pseudo-random number generators. The latter refers to the RNGs from the standard normal distribution using package **RcppArmadillo**, truncated normal distribution **RcppTN** by Olmsted (2017) implementing the efficient sampler by Robert (1995), and generalised inverse Gaussian distribution using package **GIGrvg** by Leydold and Hörmann (2017) implementing the sampler by Hörmann and Leydold (2014). All of these developments make the algorithms computationally fast. Still, Bayesian estimation of multivariate dynamic structural models is a requiring task that might take from a minute to even several hours depending on the model specification. To give a better idea of the remaining time the current package displays a progress bar implemented using the package **RcppProgress** by Forner (2020). Finally, the rich structure of the model specification including the prior distributions, identification pattern, and starting values, as well as the rich outputs from the estimation algorithms are organised using dedicated classes within the **R6** package by Chang (2021) functionality.

3. Workflows for SVAR analysis

4. An Example for US Fiscal Policy Model

5. Conclusion

The **bsvars** package offers fast and efficient algorithms for Bayesian estimation of a range of homo- and heteroskedastic Structural VARs. This is a distinguishing feature amongst existing R packages that either focus on a specific model, do not consider heteroskedastic shocks, or lack the implementation using compiled code. Additionally, thanks to the application of the frontier econometric techniques the package makes the estimation of multivariate dynamic structural models feasible even for a larger number of variables, complex identification strategies, or many Markov-switching regimes.

References

Arias JE, Rubio-Ramírez JF, Waggoner DF (2018). "Inference Based on Structural Vector Autoregressions Identified with Sign and Zero Restrictions: Theory and Applications." *Econometrica*, **86**(2), 685–720.

Bańbura M, Giannone D, Reichlin L (2010). "Large Bayesian Vector Auto Regressions." *Journal of Applied Econometrics*, **92**, 71– 92. doi:10.1002/jae.1137.

Bauwens L, Richard J, Lubrano M (1999). *Bayesian inference in dynamic econometric models*. Oxford University Press, USA.

Bertsche D, Braun R (2022). "Identification of structural vector autoregressions by stochastic volatility." *Journal of Business & Economic Statistics*, **40**(1), 328–341. doi:10.1080/07350015.2020.1813588.

Bitto A, Frühwirth-Schnatter S (2019). "Achieving shrinkage in a time-varying parameter model framework." *Journal of Econometrics*, **210**(1), 75–97. doi:https://doi.org/10.1016/j.jeconom.2018.11.006.

Brandt PT (2006). ***MSBVAR**: Bayesian Vector Autoregression Models, Impulse Responses and Forecasting*. R package version 0.1.1, URL https://CRAN.R-project.org/package=MSBVAR.

Brunnermeier M, Palia D, Sastry KA, Sims CA (2021). "Feedbacks: Financial Markets and Economic Activity." *American Economic Review*, **111**(6), 1845–1879. ISSN 0002-8282. doi:10.1257/aer.20180733.

Cadonna A, Frühwirth-Schnatter S, Knaus P (2020). "Triple the Gamma—A Unifying Shrinkage Prior for Variance and Variable Selection in Sparse State Space and TVP Models." *Econometrics*, **8**(2). doi:10.3390/econometrics8020020.

Carriero A, Clark TE, Marcellino M (2019). "Large Bayesian vector autoregressions with stochastic volatility and non-conjugate priors." *Journal of Econometrics*, **212**(1), 137–154. doi:10.1016/j.jeconom.2019.04.024.

Casella G, George EI (1992). "Explaining the Gibbs Sampler." *The American Statistician*, **46**(3), 167–174. doi:10.1080/00031305.1992.10475878.

Chan JC (2020). "Large Bayesian VARs: A flexible Kronecker error covariance structure." *Journal of Business & Economic Statistics*, **38**(1), 68–79. doi:10.1080/07350015.2018.1451336.

Chan JC, Eisenstat E (2018). "Bayesian model comparison for time-varying parameter VARs with stochastic volatility." *Journal of Applied Econometrics*, **33**(4), 509–532. doi:10.1002/jae.2617.

Chan JCC, Koop G, Yu X (2024). "Large Order-Invariant Bayesian VARs with Stochastic Volatility." *Journal of Business & Economic Statistics*, **42**(2), 825–837. doi:10.1080/07350015.2023.2252039.

Chang W (2021). ***R6**: Encapsulated Classes with Reference Semantics*. R package version 2.5.1, URL https://CRAN.R-project.org/package=R6.

Chib S (1996). "Calculating posterior distributions and modal estimates in Markov mixture models." *Journal of Econometrics*, **75**(1), 79–97. doi:10.1016/0304-4076(95)01770-4.

Clark TE, Ravazzolo F (2015). "Macroeconomic Forecasting Performance Under Alternative Specification of Time-Varying Volatility." *Journal of Applied Econometrics*, **30**, 551–575. doi:10.1002/jae.2379.

Danne C (2015). "**VARsignR**: Estimating VARs using sign restrictions in R." *MPRA Paper*, **69429**, 1–16. URL https://mpra.ub.uni-muenchen.de/68429/.

Doan T, Litterman RB, Sims CA (1984). "Forecasting and Conditional Projection Using Realistic Prior Distributions." *Econometric Reviews*, **3**(June 2013), 37–41. doi:10.1080/07474938408800053.

Eddelbuettel D (2013). *Seamless R and C++ Integration with **Rcpp***. Springer, New York, NY.

Eddelbuettel D, François R, Allaire J, Ushey K, Kou Q, Russel N, Chambers J, Bates D (2011). "**Rcpp**: Seamless R and C++ integration." *Journal of statistical software*, **40**(8), 1–18.

Eddelbuettel D, Sanderson C (2014). "**RcppArmadillo**: Accelerating R with high-performance C++ linear algebra." *Computational Statistics & Data Analysis*, **71**, 1054–1063.

Forner K (2020). ***RcppProgress**: An Interruptible Progress Bar with OpenMP Support for C++ in R Packages*. R package version 0.4.2, URL https://CRAN.R-project.org/package=RcppProgress.

Frühwirth-Schnatter S (2006). *Finite Mixture and Markov Switching Models*. Springer. doi:10.1007/978-0-387-35768-3.

Fry R, Pagan A (2011). "Sign restrictions in structural vector autoregressions: A critical review." *Journal of Economic Literature*, **49**(4), 938–60. doi:10.1257/jel.49.4.938.

Giannone D, Lenza M, Primiceri GE (2015). "Prior selection for Vector Autoregressions." *Review of Economics and Statistics*, **97**(2), 436–451. doi:10.1162/REST_a_00483.

Hörmann W, Leydold J (2014). "Generating generalized inverse Gaussian random variates." *Statistics and Computing*, **24**(4), 547–557. doi:10.1007/s11222-013-9387-3.

Hosszejni D, Kastner G (2021). "Modeling Univariate and Multivariate Stochastic Volatility in R with **stochvol** and **factorstochvol**." *Journal of Statistical Software*, **100**(12).

Kastner G, Frühwirth-Schnatter S (2014). "Ancillarity-sufficiency interweaving strategy (ASIS) for boosting MCMC estimation of stochastic volatility models." *Computational Statistics & Data Analysis*, **76**, 408 – 423. doi:https://doi.org/10.1016/j.csda.2013.01.002.

Kilian L, Lütkepohl H (2017). *Structural Vector Autoregressive Analysis*. Cambridge University Press, Cambridge.

Knaus P, Bitto-Nemling A, Cadonna A, Frühwirth-Schnatter S (2021). "Shrinkage in the Time-Varying Parameter Model Framework Using the R Package **shrinkTVP**." *Journal of Statistical Software*, **100**(13), 1–32. doi:10.18637/jss.v100.i13.

Krueger F (2015). ***bvarsv**: Bayesian Analysis of a Vector Autoregressive Model with Stochastic Volatility and Time-Varying Parameters*. R package version 1.1, URL https://CRAN.R-project.org/package=bvarsv.

Kuschnig N, Vashold L (2021). "**BVAR**: Bayesian Vector Autoregressions with Hierarchical Prior Selection in R." *Journal of Statistical Software*, **100**(14), 1–27. doi:10.18637/jss.v100.i14.

Lange A, Dalheimer B, Herwartz H, Maxand S (2021). "**svars**: An R Package for Data-Driven Identification in Multivariate Time Series Analysis." *Journal of Statistical Software*, **97**(5), 1–34. doi:10.18637/jss.v097.i05.

Lanne M, Lütkepohl H (2010). "Structural Vector Autoregressions With Nonnormal Residuals." *Journal of Business & Economic Statistics*, **28**(1), 159–168. doi:10.1198/jbes.2009.06003.

Lanne M, Lütkepohl H, Maciejowska K (2010). "Structural vector autoregressions with Markov switching." *Journal of Economic Dynamics and Control*, **34**, 121–131. doi:10.1016/j.jedc.2009.08.002.

Lewis DJ (2021). "Identifying Shocks via Time-Varying Volatility." *The Review of Economic Studies*, **88**(6), 3086–3124. doi:10.1093/restud/rdab009.

Leydold J, Hörmann W (2017). ***GIGrvg**: Random Variate Generator for the GIG Distribution*. R package version 0.5, URL https://CRAN.R-project.org/package=GIGrvg.

Lütkepohl H, Shang F, Uzeda L, Woźniak T (2024). "Partial Identification of Heteroskedastic Structural VARs: Theory and Bayesian Inference." *Working paper*, University of Melbourne. doi:10.48550/arXiv.2404.11057.

Lütkepohl H, Woźniak T (2020). "Bayesian inference for structural vector autoregressions identified by Markov-switching heteroskedasticity." *Journal of Economic Dynamics and Control*, **113**, 103862. doi:10.1016/j.jedc.2020.103862.

Malsiner-Walli G, Frühwirth-Schnatter S, Grün B (2016). "Model-based clustering based on sparse finite Gaussian mixtures." *Statistics and computing*, **26**(1), 303–324. doi:10.1007/s11222-014-9500-2.

McCausland WJ, Miller S, Pelletier D (2011). "Simulation smoothing for state–space models: A computational efficiency analysis." *Computational Statistics & Data Analysis*, **55**(1), 199–212. doi:10.1016/j.csda.2010.07.009.

Mohr FX (2022). ***bvartools**: Bayesian Inference of Vector Autoregressive and Error Correction Models*. R package version 0.2.1, URL https://CRAN.R-project.org/package=bvartools.

Olmsted J (2017). ***RcppTN**: Rcpp-Based Truncated Normal Distribution RNG and Family*. R package version 0.2-2, URL https://CRAN.R-project.org/package=RcppTN.

Omori Y, Chib S, Shephard N, Nakajima J (2007). "Stochastic Volatility with Leverage: Fast and Efficient Likelihood Inference." *Journal of Econometrics*, **140**(2), 425–449. doi:10.1016/j.jeconom.2006.07.008.

Pfaff B (2008). "VAR, SVAR and SVEC Models: Implementation Within R Package **vars**." *Journal of Statistical Software*, **27**(4), 1–32. doi:10.18637/jss.v027.i04.

Primiceri GE (2005). "Time Varying Structural Vector Autoregressions and Monetary Policy." *The Review of Economic Studies*, **72**(3), 821–852. doi:10.1111/j.1467-937X.2005.00353.x.

R Core Team (2021). *R: A Language and Environment for Statistical Computing*. R Foundation for Statistical Computing, Vienna, Austria. URL https://www.R-project.org/.

Rigobon R (2003). "Identification through heteroskedasticity." *Review of Economics and Statistics*, **85**, 777–792. doi:https://doi.org/10.1162/003465303772815727.

Robert CP (1995). "Simulation of truncated normal variables." *Statistics and Computing*, **5**(2), 121–125. doi:10.1007/BF00143942.

Rubio-Ramirez JF, Waggoner DF, Zha T (2010). "Structural vector autoregressions: Theory of identification and algorithms for inference." *The Review of Economic Studies*, **77**(2), 665–696. doi:10.1111/j.1467-937X.2009.00578.x.

Sanderson C, Curtin R (2016). "**Armadillo**: a template-based C++ library for linear algebra." *Journal of Open Source Software*, **1**(2), 26.

Sims CA (1980). "Macroeconomics and reality." *Econometrica*, **48**(1), 1–48. doi:10.2307/1912017.

Sims CA, Zha T (2006). "Were There Regime Switches in U.S. Monetary Policy?" *American Economic Review*, **96**(1), 54–81. doi:10.1257/000282806776157678.

Song Y, Woźniak T (2021). "Markov Switching." In *Oxford Research Encyclopedia of Economics and Finance*. doi:10.1093/acrefore/9780190625979.013.174.

Tsay RS, Wood D, Lachmann J (2022). ***MTS**: All-Purpose Toolkit for Analyzing Multivariate Time Series (MTS) and Estimating Multivariate Volatility Models*. R package version 1.2.1, URL http://CRAN.R-project.org/package=MTS.

Uhlig H (2005). "What are the effects of monetary policy on output? Results from an agnostic identification procedure." *Journal of Monetary Economics*, **52**(2), 381–419. doi:10.1016/j.jmoneco.2004.05.007.

Verdinelli I, Wasserman L (1995). "Computing Bayes Factors Using a Generalization of the Savage-Dickey Density Ratio." *Journal of the American Statistical Association*, **90**(430), 614–618. doi:10.1080/01621459.1995.10476554.

Waggoner DF, Zha T (2003a). "A Gibbs sampler for structural vector autoregressions." *Journal of Economic Dynamics and Control*, **28**(2), 349–366. doi:10.1016/S0165-1889(02)00168-9.

Waggoner DF, Zha T (2003b). "Likelihood preserving normalization in multiple equation models." *Journal of Econometrics*, **114**(2), 329–347. doi:https://doi.org/10.1016/S0304-4076(03)00087-3.

Woźniak T (2016). "Bayesian Vector Autoregressions." *Australian Economic Review*, **49**(3), 365–380. doi:10.1111/1467-8462.12179.

Woźniak T (2021). "Simulation Smoother using **RcppArmadillo**." *RcppGallery*, URL: https://gallery.rcpp.org/articles/simulation-smoother-using-rcpparmadillo/, (Accessed on 5 September 2021).

Woźniak T (2024). ***bsvars**: Bayesian Estimation of Structural Vector Autoregressive Models*. R package version 3.0.0, URL http://CRAN.R-project.org/package=bsvars.

Woźniak T, Droumaguet M (2015). "Assessing Monetary Policy Models: Bayesian Inference for Heteroskedastic Structural VARs." *University of Melbourne Working Papers Series*, **2017**. doi:10.13140/RG.2.2.19492.55687.

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