Macroeconometrics

Lecture 17 Bayesian estimation using simulation smoother

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A simple Unobserved Component model

Prior distributions

Derivation of Gibbs sampler

Simulation smoother

Compulsory readings:

Woźniak (2021) Bayesian estimation of simple Unobserved Component models using simulation smoother

Useful readings:

Woźniak (2021) Simulation Smoother using RcppArmadillo, Rcpp Gallery

Objectives.

- To present Bayesian estimation of state-space models
- To set the model specification through a prior distribution
- To derive a Gibbs sampler for Unobserved Component models

Learning outcomes.

- Deriving full conditional posterior distributions
- Implementing a simulation smoother
- Programming a sampler from multivariate normal distribution with special type of the mean and covariance

A simple Unobserved Component model

A simple Unobserved Component model

$$y_t = \tau_t + \epsilon_t \tag{1}$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t \tag{2}$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_p \epsilon_{t-p} + e_t \tag{3}$$

$$\begin{bmatrix} \eta_t \\ e_t \end{bmatrix} \Bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_2, \begin{bmatrix} \sigma_\eta^2 & 0 \\ 0 & \sigma_e^2 \end{bmatrix}\right)$$

$$\alpha_p(L) = 1 - \alpha_1 L - \cdots - \alpha_p L^p$$

$$\alpha_p(z) = 0 : \quad |z| > 1 \quad \forall z \in \mathbb{C}$$

$$\sigma_{\eta e} = \mathbb{C}ov[\eta_t, e_t] = 0$$

τ_0 – an estimated parameter

$$\mathbf{0}_p = (\epsilon_0, \epsilon_{-1}, \ldots, \epsilon_{-p+1})'$$

A simple Unobserved Component model

Define $T \times 1$ matrices.

$$y = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix} \quad \tau = \begin{bmatrix} \tau_1 \\ \vdots \\ \tau_T \end{bmatrix} \quad \epsilon = \epsilon_{[1:T]} = \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_T \end{bmatrix} \quad \eta = \begin{bmatrix} \eta_1 \\ \vdots \\ \eta_T \end{bmatrix} \quad e = \begin{bmatrix} e_1 \\ \vdots \\ e_T \end{bmatrix} \quad \imath_T = \begin{bmatrix} 1 \\ \vdots \\ 1 \end{bmatrix} \quad e_{1.T} = \begin{bmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Define matrices.

$$\underset{(2\times 1)}{\beta} = \begin{bmatrix} \mu & \tau_0 \end{bmatrix}' \qquad \underset{(T\times 2)}{X_\tau} = \begin{bmatrix} \imath_T & e_{1.T} \end{bmatrix}$$

$$\underset{(p\times 1)}{\alpha} = \begin{bmatrix} \alpha_1 & \dots & \alpha_p \end{bmatrix}' \qquad \underset{(T\times p)}{X_\epsilon} = \begin{bmatrix} \epsilon_{[0:(T-1)]} & \epsilon_{[-1:(T-2)]} & \dots & \epsilon_{[(-p+1):(T-p)]} \end{bmatrix}$$

Define $T \times T$ matrices.

$$H = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -1 & 1 & 0 & \dots & 0 & 0 \\ 0 & -1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -1 & 1 \end{bmatrix} \quad H_\alpha = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -\alpha_1 & 1 & 0 & \dots & 0 & 0 \\ -\alpha_2 & -\alpha_1 & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ -\alpha_p & -\alpha_{p-1} & -\alpha_{p-2} & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & 1 & 0 \\ 0 & 0 & 0 & \dots & -\alpha_1 & 1 \end{bmatrix}$$

A simple Unobserved Component model

$$y = \tau + \epsilon \tag{1}$$

$$H\tau = \mu\imath_T + \tau_0 e_{1.T} + \eta \tag{2}$$

$$H\tau = X_\tau\beta + \eta \tag{2}$$

$$\tau = H^{-1}X_\tau\beta + H^{-1}\eta \tag{2}$$

$$H_\alpha\epsilon = e \tag{3}$$

$$\epsilon = H_\alpha^{-1}e \tag{3}$$

$$= X_\epsilon\alpha + e \tag{3}$$

$$\eta \sim \mathcal{N}\left(\mathbf{0}_T, \sigma_\eta^2 I_T\right)$$

$$e \sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 I_T\right)$$

To be estimated: $\tau, \epsilon, \beta = (\mu, \tau_0), \alpha, \sigma = (\sigma_\eta^2, \sigma_e^2)$

Prior distributions

Prior distributions

Prior distribution of τ is specified by equation (2)

$$
\begin{aligned}
\tau &= H^{-1}X_\tau\beta + H^{-1}\eta \\
\eta &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_\eta^2 I_T\right) \\
H^{-1}\eta &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_\eta^2 (H'H)^{-1}\right) \\
&\downarrow \\
\tau|\beta, \sigma_\eta^2 &\sim \mathcal{N}_T\left(H^{-1}X_\tau\beta, \sigma_\eta^2 (H'H)^{-1}\right) \\
&\propto \exp\left\{-\frac{1}{2}\frac{1}{\sigma_\eta^2}\left(\tau - H^{-1}X_\tau\beta\right)' H'H\left(\tau - H^{-1}X_\tau\beta\right)\right\}
\end{aligned}
$$

Prior distributions

Prior distribution of ϵ is specified by equation (3)

$$
\begin{aligned}
\epsilon &= H_\alpha^{-1} e \\
e &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 I_T\right) \\
H_\alpha^{-1} e &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 (H_\alpha' H_\alpha)^{-1}\right) \\
&\downarrow \\
\epsilon | \alpha, \sigma_e^2 &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 (H_\alpha' H_\alpha)^{-1}\right) \\
&\propto \exp\left\{-\frac{1}{2}\frac{1}{\sigma_e^2}\epsilon' H_\alpha' H_\alpha \epsilon\right\}
\end{aligned}
$$

Prior distributions

Prior distributions of $\alpha, \beta, \sigma_\eta^2, \sigma_e^2$ are assumed to be

$$\alpha \sim \mathcal{N}_p\left(\underline{\alpha}, \underline{V}_\alpha\right)\mathcal{I}(\alpha \in A) \quad \propto \exp\left\{-\frac{1}{2}(\alpha - \underline{\alpha})'\underline{V}_\alpha^{-1}(\alpha - \underline{\alpha})\right\}\mathcal{I}(\alpha \in A)$$

$$\beta \sim \mathcal{N}_2\left(\underline{\beta}, \underline{V}_\beta\right) \quad \propto \exp\left\{-\frac{1}{2}(\beta - \underline{\beta})'\underline{V}_\beta^{-1}(\beta - \underline{\beta})\right\}$$

$$\sigma_\eta^2 \sim \mathcal{IG}2\left(\underline{s}, \underline{\nu}\right) \quad \propto \left(\sigma_\eta^2\right)^{-\frac{\underline{\nu}+2}{2}} \exp\left\{-\frac{1}{2}\frac{\underline{s}}{\sigma_\eta^2}\right\}$$

$$\sigma_e^2 \sim \mathcal{IG}2\left(\underline{s}, \underline{\nu}\right) \quad \propto \left(\sigma_e^2\right)^{-\frac{\underline{\nu}+2}{2}} \exp\left\{-\frac{1}{2}\frac{\underline{s}}{\sigma_e^2}\right\}$$

$\alpha \in A$ – set of parameters α for which stationarity holds and

$$\mathcal{I}(\alpha \in A) = \begin{cases} 1 & \text{if } \alpha \in A \\ 0 & \text{otherwise} \end{cases}$$

Prior distributions

$$p(\tau, \epsilon, \alpha, \beta, \sigma) = p\left(\tau|\beta, \sigma_\eta^2\right) p(\beta) p\left(\sigma_\eta^2\right) p\left(\epsilon|\alpha, \sigma_e^2\right) p(\alpha) p\left(\sigma_e^2\right)$$

$$\begin{aligned}
\tau|\beta, \sigma_\eta^2 &\sim \mathcal{N}\left(H^{-1} X_\tau \beta, \sigma_\eta^2 (H'H)^{-1}\right) \\
\beta &\sim \mathcal{N}_2\left(\underline{\beta}, \underline{V}_\beta\right) \\
\sigma_\eta^2 &\sim \mathcal{IG}2\left(\underline{s}, \underline{\nu}\right) \\
\epsilon|\alpha, \sigma_e^2 &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 (H_\alpha' H_\alpha)^{-1}\right) \\
\alpha &\sim \mathcal{N}_p\left(\underline{\alpha}, \underline{V}_\alpha\right) \mathcal{I}(\alpha \in A) \\
\sigma_e^2 &\sim \mathcal{IG}2\left(\underline{s}, \underline{\nu}\right)
\end{aligned}$$

Derivation of Gibbs sampler

Gibbs sampler

is an iterative algorithm at each iteration of which draws are sampled from the following full-conditional posterior distributions

$$
\begin{aligned}
\tau &\sim p\left(\tau|y,\alpha,\beta,\sigma\right)\\
\epsilon &\sim p\left(\epsilon|y,\alpha,\beta,\sigma\right)\\
\beta &\sim p\left(\beta|y,\tau,\sigma_\eta^2\right)\\
\alpha &\sim p\left(\alpha|y,\epsilon,\sigma_e^2\right)\\
\sigma_\eta^2 &\sim p\left(\sigma_\eta^2|y,\tau,\beta\right)\\
\sigma_e^2 &\sim p\left(\sigma_e^2|y,\epsilon,\alpha\right)
\end{aligned}
$$

Full-conditional posterior distribution of $\tau|y,\alpha,\beta,\sigma$

Conditional likelihood is based on equation (1)

$$\tau = y - \epsilon$$

$$\epsilon \sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 (H_\alpha' H_\alpha)^{-1}\right)$$

$$L\left(\tau|y,\alpha,\sigma_e^2\right) \propto \exp\left\{-\frac{1}{2}\sigma_e^{-2}(\tau - y)' H_\alpha' H_\alpha (\tau - y)\right\}$$

Prior distribution is given by

$$p\left(\tau|\beta,\sigma_\eta^2\right) \propto \exp\left\{-\frac{1}{2}\frac{1}{\sigma_\eta^2}\left(\tau - H^{-1}X_\tau\beta\right)' H'H\left(\tau - H^{-1}X_\tau\beta\right)\right\}$$

Full-conditional posterior distribution.

$$\begin{aligned}
p\left(\tau|y,\alpha,\beta,\sigma\right) &\propto L\left(\tau|y,\alpha,\sigma_e^2\right) p\left(\tau|\beta,\sigma_\eta^2\right) \\
&= \mathcal{N}_T\left(\overline{\tau}, \overline{V}_\tau\right) \\
\overline{V}_\tau &= \left[\sigma_e^{-2} H_\alpha' H_\alpha + \sigma_\eta^{-2} H'H\right]^{-1} \\
\overline{\tau} &= \overline{V}_\tau \left[\sigma_e^{-2} H_\alpha' H_\alpha y + \sigma_\eta^{-2} H' X_\tau \beta\right]
\end{aligned}$$

Full-conditional posterior distribution of $\epsilon|y,\alpha,\beta,\sigma$

Conditional likelihood is based on equation (1)

$$\epsilon = y - \tau$$

$$y - \tau \sim \mathcal{N}\left(y - H^{-1}X_\tau\beta, \sigma_\eta^2(H'H)^{-1}\right)$$

$$L\left(\epsilon|y,\beta,\sigma_\eta^2\right) \propto \exp\left\{-\frac{1}{2}\sigma_\eta^{-2}\left(\epsilon - (y - H^{-1}X_\tau\beta)\right)' H'H\left(\epsilon - (y - H^{-1}X_\tau\beta)\right)\right\}$$

Prior distribution is given by

$$p\left(\epsilon|\alpha,\sigma_e^2\right) \propto \exp\left\{-\frac{1}{2}\frac{1}{\sigma_e^2}\epsilon' H_\alpha' H_\alpha \epsilon\right\}$$

Full-conditional posterior distribution.

$$\begin{aligned}
p\left(\epsilon|y,\alpha,\beta,\sigma\right) &\propto L\left(\epsilon|y,\beta,\sigma_\eta^2\right) p\left(\epsilon|\alpha,\sigma_e^2\right)\\
&= \mathcal{N}_T\left(\overline{\epsilon}, \overline{V}_\epsilon\right)\\
\overline{V}_\epsilon &= \left[\sigma_e^{-2}H_\alpha' H_\alpha + \sigma_\eta^{-2}H'H\right]^{-1}\\
\overline{\epsilon} &= \overline{V}_\epsilon \sigma_\eta^{-2} H'H\left(y - H^{-1}X_\tau\beta\right)
\end{aligned}$$

Full-conditional posterior distribution of $\beta|y,\tau,\sigma^2_\eta$

Conditional likelihood is based on equation (2)

$$
\begin{gathered}
H\tau - X_\tau\beta = \eta \\
\eta \sim \mathcal{N}\left(\mathbf{0}_T, \sigma^2_\eta I_T\right) \\
L\left(\beta|y,\tau,\sigma^2_\eta\right) \propto \exp\left\{-\frac{1}{2}\sigma_\eta^{-2}\left(X_\tau\beta - H\tau\right)'\left(X_\tau\beta - H\tau\right)\right\}
\end{gathered}
$$

Prior distribution is given by

$$
p(\beta) \propto \exp\left\{-\frac{1}{2}(\beta-\underline{\beta})'\underline{V}_\beta^{-1}(\beta-\underline{\beta})\right\}
$$

Full-conditional posterior distribution.

$$
\begin{aligned}
p\left(\beta|y,\tau,\sigma^2_\eta\right) &\propto L\left(\beta|y,\tau,\sigma^2_\eta\right)p(\beta) \\
&= \mathcal{N}_2\left(\overline{\beta},\overline{V}_\beta\right) \\
\overline{V}_\beta &= \left[\sigma_\eta^{-2}X_\tau'X_\tau + \underline{V}_\beta^{-1}\right]^{-1} \\
\overline{\beta} &= \overline{V}_\beta\left[\sigma_\eta^{-2}X_\tau'H\tau + \underline{V}_\beta^{-1}\underline{\beta}\right]
\end{aligned}
$$

Full-conditional posterior distribution of $\alpha|y, \epsilon, \sigma_e^2$

Conditional likelihood is based on equation (3)

$$\epsilon - X_\epsilon \alpha = e$$

$$e \sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 I_T\right)$$

$$L\left(\alpha|y, \epsilon, \sigma_e^2\right) \propto \exp\left\{-\frac{1}{2}\sigma_e^{-2}\left(X_\epsilon\alpha - \epsilon\right)'\left(X_\epsilon\alpha - \epsilon\right)\right\}$$

Prior distribution is given by

$$p(\alpha) \propto \exp\left\{-\frac{1}{2}(\alpha - \underline{\alpha})'\underline{V}_\alpha^{-1}(\alpha - \underline{\alpha})\right\}\mathcal{I}(\alpha \in A)$$

Full-conditional posterior distribution.

$$p\left(\alpha|y, \epsilon, \sigma_e^2\right) \propto L\left(\alpha|y, \epsilon, \sigma_e^2\right)p(\alpha)\mathcal{I}(\alpha \in A)$$
$$= \mathcal{N}_p\left(\overline{\alpha}, \overline{V}_\alpha\right)\mathcal{I}(\alpha \in A)$$
$$\overline{V}_\alpha = \left[\sigma_e^{-2}X_\epsilon' X_\epsilon + \underline{V}_\alpha^{-1}\right]^{-1}$$
$$\overline{\alpha} = \overline{V}_\alpha\left[\sigma_e^{-2}X_\epsilon'\epsilon + \underline{V}_\alpha^{-1}\underline{\alpha}\right]$$

Full-conditional posterior distribution of $\sigma^2_\eta | y, \tau, \beta$

Conditional likelihood is based on equation (2)

$$H\tau - X_\tau \beta = \eta$$

$$\eta \sim \mathcal{N}\left(\mathbf{0}_T, \sigma^2_\eta I_T\right)$$

$$L\left(\sigma^2_\eta | y, \tau, \beta\right) \propto \left(\sigma^2_\eta\right)^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\sigma_\eta^{-2}\left(X_\tau\beta - H\tau\right)'\left(X_\tau\beta - H\tau\right)\right\}$$

Prior distribution is given by

$$p\left(\sigma^2_\eta\right) \propto \left(\sigma^2_\eta\right)^{-\frac{\underline{\nu}+2}{2}} \exp\left\{-\frac{1}{2}\frac{\underline{s}}{\sigma^2_\eta}\right\}$$

Full-conditional posterior distribution.

$$\begin{aligned} p\left(\sigma^2_\eta | y, \tau, \beta\right) &\propto L\left(\sigma^2_\eta | y, \tau, \beta\right) p\left(\sigma^2_\eta\right) \\ &= \mathcal{IG}2\left(\overline{s}_\eta, \overline{\nu}_\eta\right) \\ \overline{s}_\eta &= \underline{s} + (H\tau - X_\tau\beta)'(H\tau - X_\tau\beta) \\ \overline{\nu}_\eta &= \underline{\nu} + T \end{aligned}$$

Full-conditional posterior distribution of $\sigma_e^2 | y, \epsilon, \alpha$

Conditional likelihood is based on equation (3)

$$
\begin{aligned}
\epsilon - X_\epsilon \alpha &= e \\
e &\sim \mathcal{N}\left(\mathbf{0}_T, \sigma_e^2 I_T\right) \\
L\left(\sigma_e^2 | y, \epsilon, \alpha\right) &\propto \left(\sigma_e^2\right)^{-\frac{T}{2}} \exp\left\{-\frac{1}{2}\sigma_e^{-2} \left(X_\epsilon \alpha - \epsilon\right)' \left(X_\epsilon \alpha - \epsilon\right)\right\}
\end{aligned}
$$

Prior distribution is given by

$$
p\left(\sigma_e^2\right) \propto \left(\sigma_e^2\right)^{-\frac{\underline{\nu}+2}{2}} \exp\left\{-\frac{1}{2}\frac{\underline{s}}{\sigma_\eta^2}\right\}
$$

Full-conditional posterior distribution.

$$
\begin{aligned}
p\left(\sigma_e^2 | y, \epsilon, \alpha\right) &\propto L\left(\sigma_e^2 | y, \epsilon, \alpha\right) p\left(\sigma_e^2\right) \\
&= \mathcal{IG}2\left(\overline{s}_e, \overline{\nu}_e\right) \\
\overline{s}_e &= \underline{s} + (\epsilon - X_\epsilon \alpha)'(\epsilon - X_\epsilon \alpha) \\
\overline{\nu}_e &= \underline{\nu} + T
\end{aligned}
$$

Simulation smoother

Simulation smoother

Sampling from a multivariate normal distribution using a precision matrix

$$\mathcal{N}\left(D^{-1}b, D^{-1}\right)$$

D^{-1} – an $N \times N$ covariance matrix

D – an $N \times N$ precision matrix that is a tridiagonal

$O\left(N^3\right)$ operations needed to invert D with usual computing routines

$O\left(N\right)$ operations are required to invert D using dedicated routines for special types of matrices

Simulation smoother

$$\mathcal{N}\left(D^{-1}b, D^{-1}\right)$$

Let $L = chol(D)$ be a lower-triangular matrix such that $D = LL'$

Suppose that L^{-1} can be computed efficiently

Compute the mean of the distribution by:

$$L^{-1\prime}L^{-1}b = (LL')^{-1}b = D^{-1}b$$

Let x denote an $N \times 1$ vector with elements drawn independently from a standard normal distribution

Sample a draw from the target normal distribution

$$L^{-1\prime}\left(L^{-1}b + x\right)$$

The method in the next slide further simplifies the algorithm and bypasses inverting L

Simulation smoother

Let $L \setminus b$ denote the unique solution to the triangular system $Lx = b$ obtained by forward (backward) substitution, that is, $L \setminus b = L^{-1}b$.

Simulation smoother.

Compute $L = chol(D)$ such that $D = LL'$

Sample $x \sim \mathcal{N}\left(0_{N\times 1}, I_N\right)$

Compute a draw from the distribution via the affine transformation:

$$L' \setminus (L \setminus b + x)$$

Simulation smoother in R for tridiagonal D matrix

```
library(mgcv)

N              = dim(D)[1]
lead.diag      = diag(D)
sub.diag       = sdiag(D, -1)

D.chol         = trichol(ld = lead.diag, sd=sub.diag)
D.L            = diag(D.chol$ld)
sdiag(D.L,-1) = D.chol$sd

x              = matrix(rnorm(n*N), ncol=n)
a              = forwardsolve(D.L, b)
draw           = backsolve(t(D.L), a + x)
```

Simulation smoother in R comparison

```
rmvnorm.tridiag.precision = function(n, D, b){
  N           = dim(D)[1]
  lead.diag   = diag(D)
  sub.diag    = sdiag(D, -1)

  D.chol      = trichol(ld = lead.diag, sd=sub.diag)
  D.L         = diag(D.chol$ld)
  sdiag(D.L,-1) = D.chol$sd

  x           = matrix(rnorm(n*N), ncol=n)
  a           = forwardsolve(D.L, b)
  draw        = backsolve(t(D.L),
                    matrix(rep(a,n), ncol=n) + x)

  return(draw)
}
```

Simulation smoother in R comparison

```
rmvnorm.usual = function(n, D, b){
  N             = dim(D)[1]
  D.chol        = t(chol(D))
  variance.chol = solve(D.chol)

  x             = matrix(rnorm(n*N), ncol=n)
  draw          = t(variance.chol) %*%
             (matrix(rep(variance.chol%*%b,n), ncol=n) + x)

  return(draw)
}
```

Simulation smoother in R comparison

```
library(mgcv); library(microbenchmark)

set.seed(12345)
T     = 240
md    = rgamma(T, shape=10, scale=10)
od    = rgamma(T-1, shape=10, scale=1)
D     = 2*diag(md)
sdiag(D, 1)  = -od
sdiag(D, -1) = -od
b     = as.matrix(rnorm(T))

microbenchmark(
  trid  = rmvnorm.tridiag.precision(n=100, D=D, b=b),
  usual = rmvnorm.nothing.special(n=100, D=D, b=b),
  check = "equal", setup=set.seed(123456)
)

Unit: milliseconds
 expr       min        lq      mean    median        uq       max neval
 trid  3.489267  4.539225  6.655018  4.666931  4.963046 154.70131   100
usual 13.769023 15.355094 16.746697 15.694483 17.626655  28.79406   100
```

Simulation smoother in R comparison

```
set.seed(12345)
T     = 720
md    = rgamma(T, shape=10, scale=10)
od    = rgamma(T-1, shape=10, scale=1)
D     = 2*diag(md)
sdiag(D, 1) = -od
sdiag(D, -1) = -od
b     = as.matrix(rnorm(T))

microbenchmark(
  trid  = rmvnorm.tridiag.precision(n=100, D=D, b=b),
  usual = rmvnorm.nothing.special(n=100, D=D, b=b),
  check = "equal", setup=set.seed(123456)
)

Unit: milliseconds
 expr       min        lq      mean    median        uq      max neval
 trid  22.46832  26.55893  45.00241  29.93325  35.21184 164.7311   100
usual 249.23538 263.19570 286.56741 270.97134 289.85116 583.5321   100
```

Bayesian estimation of Unobserved Component models

proceeds via Gibbs sampling with well-specified full conditional posterior distributions

bypasses the application of Kalman filter that requires sequential computer calculations

uses the simulation smoother instead in order to sample draws of latent processes

applies dedicated algorithms for special types of matrices for fast computations