Macroeconometrics

Lecture 12 SVAR Tools

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SVAR Model of the Australian Economy

Impulse response functions

Forecast error variance decomposition

Compulsory readings:

Kilian & Lütkepohl (2017) Chapter 4: Structural VARs Tools, Structural Vector Autoregressive Analysis

Useful readings:

Dungey & Pagan (2009) Extending a SVAR Model of the Australian Economy, Economic Record

Materials:

R file `L12 mcxs.R` and data file `AU-SVAR-data.zip` for the reproduction of the results

Objectives.

- To present the impulse response functions as the dynamic causal effects
- To analyse shocks' contributions to business cycle and inflation
- To introduce a benchmark model of the Australian economy

Learning outcomes.

- Understanding when a structural shock is an important driver of business cycles
- Visualising of economically interpretable effects
- Interpreting IRFs and FEVDs

SVAR Model of the Australian Economy

Inspired by Dungey & Pagan (2009)

SVAR Model of the Australian Economy

$$
\begin{aligned}
y_t &= \mu_0 + A_1 y_{t-1} + \dots + A_p y_{t-p} + B u_t \\
u_t | Y_{t-1} &\sim iid(\mathbf{0}_N, I_N)
\end{aligned}
$$

SVAR for a small-open economy

- Distinguishes foreign and domestic variables
- The small-open economy assumption:
  - Australia receives foreign shocks
  - foreign sector is not affected by Australian domestic shocks
- Identification of foreign shocks
- Identification of domestic shocks

SVAR Model of the Australian Economy

$$\begin{bmatrix} y_t^f \\ y_t^d \end{bmatrix} = \begin{bmatrix} \mu_{0.1} \\ \mu_{0.2} \end{bmatrix} + \begin{bmatrix} A_{1.11} & \mathbf{0}_{6\times 6} \\ A_{1.21} & A_{1.22} \end{bmatrix} \begin{bmatrix} y_{t-1}^f \\ y_{t-1}^d \end{bmatrix} + \dots + \begin{bmatrix} B_{11} & \mathbf{0}_{6\times 6} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_t^f \\ u_t^d \end{bmatrix}$$

$$y_t^{f\prime} = \begin{bmatrix} rgdp_t & cpi_t & FFR_t & sp500_t & tot_t & rex_t \end{bmatrix}$$

$$y_t^{d\prime} = \begin{bmatrix} rgne_t & rgdp_t & cpi_t & CR_t & rtwi_t & aord_t \end{bmatrix}$$

$$u_t^{f\prime} = \begin{bmatrix} u_{1.t} & u_{2.t} & u_{3.t}^{us.mps} & u_{4.t} & u_{5.t} & u_{6.t} \end{bmatrix}$$

$$u_t^{d\prime} = \begin{bmatrix} u_{7.t} & u_{8.t} & u_{9.t} & u_{10.t}^{au.mps} & u_{11.t} & u_{12.t} \end{bmatrix}$$

Foreign block.
$rgdp_t$ – real GDP, cpi_t – CPI, FFR_t – federal funds rate, $sp500_t$ – S&P 500 index, tot_t – Australian terms of trade, rex_t – Australian real export

Australian block.
$rgne_t$ – real gross national expenditure, $rgdp_t$ – real GDP, cpi_t – CPI, CR_t – cash rate, $rtwi_t$ – real trade weighted index, $aord_t$ – All Ordinaries Index

Shocks of interest.
$u_{10.t}^{au.mps}$ – Australian monetary policy shock
$u_{3.t}^{us.mps}$ – US monetary policy shock

SVAR Model of the Australian Economy

$$\begin{bmatrix} y_t^f \\ y_t^d \end{bmatrix} = \begin{bmatrix} \mu_{0.1} \\ \mu_{0.2} \end{bmatrix} + \begin{bmatrix} A_{1.11} & \mathbf{0}_{6\times 6} \\ A_{1.21} & A_{1.22} \end{bmatrix} \begin{bmatrix} y_{t-1}^f \\ y_{t-1}^d \end{bmatrix} + \dots + \begin{bmatrix} B_{11} & \mathbf{0}_{6\times 6} \\ B_{21} & B_{22} \end{bmatrix} \begin{bmatrix} u_t^f \\ u_t^d \end{bmatrix}$$

SVAR for a small-open economy

B_{11} – identification of foreign shocks: lower-triangular matrix

B_{22} – identification of domestic shocks: lower-triangular matrix

$B_{12} = \mathbf{0}_{6\times 6}$ – small-open economy assumption

$A_{l.12} = \mathbf{0}_{6\times 6}$ – small-open economy assumption (not imposed)

B_{21} – small-open economy assumption: foreign shocks affect domestic variables

SVAR Model of the Australian Economy

SVAR Model of the Australian Economy

Minnesota prior

All of the results are reported for $\kappa_1 \in \left\{0.02^2, 1\right\}$ and

$$\underline{A} = \begin{bmatrix} \mathbf{0}_{1\times N} \\ \kappa_3 I_N \\ \mathbf{0}_{N(p-1)\times N} \end{bmatrix}$$

with $\kappa_3 = 1$, $p = 4$, and $S = 50,000$.

The results are heavily dependent on prior hyper-parameters.

Impulse response functions

Impulse response functions

Definition.
Impulse response functions to orthogonal shocks computed for an empirically relevant SVAR model are considered the dynamic causal effects of the underlying shocks u_t on economic measurements y_t.

$$\frac{\partial y_{n.t+i}}{\partial u_{j.t}} = \theta_{nj.i}$$

$$\frac{\partial y_{t+i}}{\partial u_t} = \underset{N\times N}{\Theta_i}$$

$\theta_{nj.i}$ – response of nth variable to jth shock i periods after shock's occurrence

Θ_i – responses of all of the variables to all of the shocks i periods after shock's occurrence

for $i = 0, 1, \ldots, h$ and $n, j = 1, \ldots, N$

Impulse response functions

VAR(1) representation of RF VAR(p) model.

$$\begin{aligned} Y_t &= \mathbf{A} Y_{t-1} + E_t \\ &= E_t + \mathbf{A} E_{t-1} + \mathbf{A}^2 E_{t-2} + \dots \end{aligned}$$

$$\begin{aligned} y_t &= J Y_t \\ &= J E_t + J\mathbf{A} J' J E_{t-1} + J\mathbf{A}^2 J' J E_{t-2} + \dots \\ &= \epsilon_t + J\mathbf{A} J' \epsilon_{t-1} + J\mathbf{A}^2 J' \epsilon_{t-2} + \dots \\ &= \Phi_0 \epsilon_t + \Phi_1 \epsilon_{t-1} + \Phi_2 \epsilon_{t-2} + \dots \end{aligned}$$

$$\frac{\partial y_{t+i}}{\partial \epsilon_t} = J\mathbf{A}^i J' = \Phi_j$$

Matrices Φ_i do not identify the effects of interest because they represent responses to correlated shocks.

$$\mathbf{A} = \begin{bmatrix} A_1 & A_2 & \dots & A_p \\ & I_{N(p-1)} & & \mathbf{0}_{N(p-1)\times N} \end{bmatrix} \quad E_t' = \begin{bmatrix} \epsilon_t' & \mathbf{0}_{1\times N(p-1)} \end{bmatrix} \quad J = \begin{bmatrix} I_N & \mathbf{0}_{N\times N(p-1)} \end{bmatrix}$$

Impulse response functions

Apply the relationship between RF and SF: $\epsilon_t = Bu_t$

$$\begin{aligned} y_t &= \epsilon_t + J\mathbf{A}J'\epsilon_{t-1} + J\mathbf{A}^2J'\epsilon_{t-2} + \dots \\ &= Bu_t + J\mathbf{A}J'Bu_{t-1} + J\mathbf{A}^2J'Bu_{t-2} + \dots \\ &= \Theta_0 u_t + \Theta_1 u_{t-1} + \Theta_2 u_{t-2} + \dots \end{aligned}$$

$$\frac{\partial y_{t+i}}{\partial u_t} = \Theta_i = \Phi_i B = J\mathbf{A}^i J' B$$

Matrices Θ_i identify the IRFs, i.e. the effects of interest because they represent responses to well-isolated uncorrelated shocks.

Impulse response functions

Impulse response at the infinite horizon.

$$\lim_{h \to \infty} \frac{\partial y_{t+h}}{\partial u_t} = \Theta_\infty = J \left(I_{Np} - \mathbf{A} \right)^{-1} J' B$$

Requires that $I_{Np} - \mathbf{A}$ is invertible.

Using standard SVAR parameterizations.

$$\begin{aligned} \Theta_\infty &= \left(I_N - A_1 - \cdots - A_p \right)^{-1} B \\ &= \left(B_0 - B_1 - \cdots - B_p \right)^{-1} \end{aligned}$$

Impulse response functions

Consider IRFs functions of parameters: $\Theta_i(A, B) = J\mathbf{A}^i J' B$

MLE.

Estimate the model obtaining the MLE $(\widehat{A}, \widehat{B})$

Apply the invariance property to compute

$$\Theta_i(A, B)\Big|_{\substack{A=\widehat{A}\\ B=\widehat{B}}} = J\widehat{\mathbf{A}}^i J'\widehat{B}$$

Bayesian estimation.

Obtain a sample from the posterior distribution $\left\{A^{(s)}, B^{(s)}\right\}_{s=1}^{S}$

Compute $\left\{\Theta_i\left(A^{(s)}, B^{(s)}\right)\right\}_{s=1}^{S}$ as a sample drew from the posterior distribution of Θ_i given data

Impulse response functions

Confidence intervals.

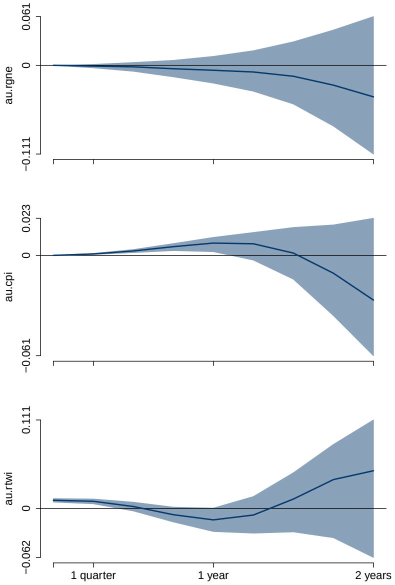
Asymptotic results based on delta rule unreliable due to a high nonlinearity of the IRFs as functions of the original parameters

Bootstrap procedures for dynamic models provide draws from the empirical distribution of the parameters that can be used to form reliable confidence intervals

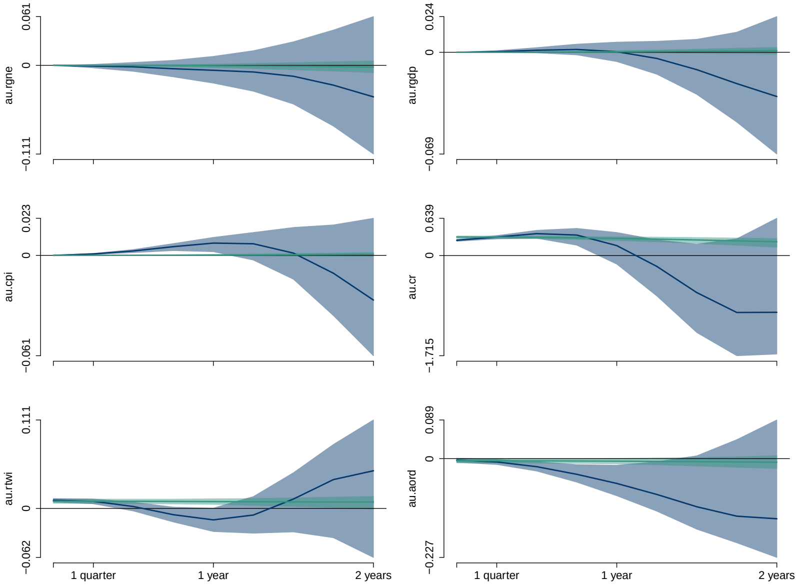
Bayesian highest posterior density intervals.

Compute the 68% highest posterior density intervals for $\left\{\Theta_i\left(A^{(s)}, B^{(s)}\right)\right\}_{s=1}^{S}$ for $i = 0, 1, \ldots, h$

IRFs of domestic sector to $u_{10.t}^{au.mps}$ for $\kappa_1 = 1$



IRFs to $u_{10.t}^{au.mps}$ for $\kappa_1 = 1$ and $\kappa_1 = 0.02^2$



Forecast error variance **decomposition**

Forecast error variance decomposition

Definition.

Forecast error variance decomposition provides information regarding the fraction of variability of the h-period ahead forecast of a particular variable attributed by each individual structural shock occurring at time t.

Forecast error variance decomposition

Forecast error variance.

$$\begin{aligned}
\mathbb{V}\text{ar}[\mathbf{e}_{t+h|t}] &= \mathbb{E}[(y_{t+h} - y_{t+h|t})(y_{t+h} - y_{t+h|t})'] \\
&= \mathbb{E}[\mathbf{e}_{t+h|t}\mathbf{e}'_{t+h|t}] \\
&= \mathbb{E}[(\Phi_0\epsilon_{t+h} + \cdots + \Phi_{h-1}\epsilon_{t+1})(\Phi_0\epsilon_{t+h} + \cdots + \Phi_{h-1}\epsilon_{t+1})'] \\
&= \mathbb{E}[(\Theta_0 u_{t+h} + \cdots + \Theta_{h-1} u_{t+1})(\Theta_0 u_{t+h} + \cdots + \Theta_{h-1} u_{t+1})'] \\
&= \Theta_0\mathbb{E}\left[\mathbb{E}_{t+1}[u_{t+h}u'_{t+h}]\right]\Theta'_0 + \cdots + \Theta_{h-1}\mathbb{E}\left[\mathbb{E}_t[u_{t+1}u'_{t+1}]\right]\Theta'_0 \\
&= \Theta_0\Theta'_0 + \cdots + \Theta_{h-1}\Theta'_{h-1}
\end{aligned}$$

Focus on the diagonal elements of the matrix above.

Forecast error variance decomposition

Individual shock contribution.

The contribution of the ith shock to the mean square forecast error (MSFE) of the nth variable $y_{n.t+h}$ h periods ahead is

$$\frac{MSFE_i^n(h)}{\sum_{j=1}^{N} MSFE_j^n(h)}$$

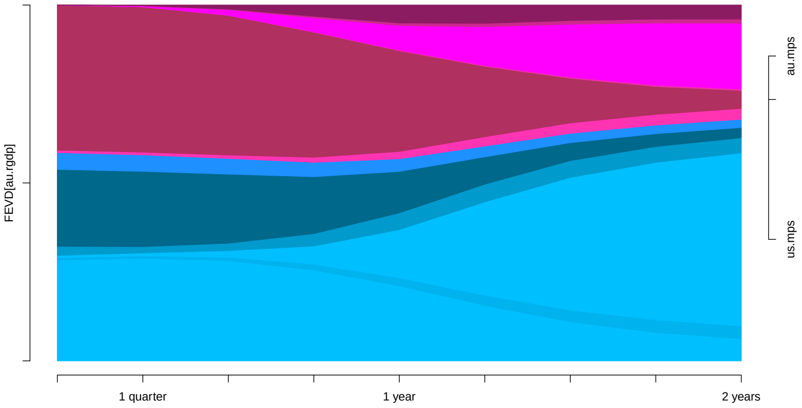
where

$$MSFE_i^n(h) = \theta_{ni.0}^2 + \cdots + \theta_{ni.h-1}^2$$

such that

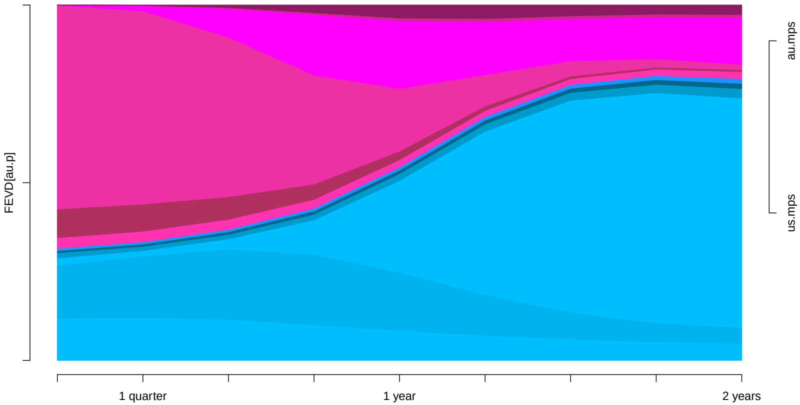
$$\frac{MSFE_1^n(h)}{\sum_{j=1}^{N} MSFE_j^n(h)} + \cdots + \frac{MSFE_N^n(h)}{\sum_{j=1}^{N} MSFE_j^n(h)} = 1$$

Forecast error variance decomposition of $au.rgdp_{t+h|t}$



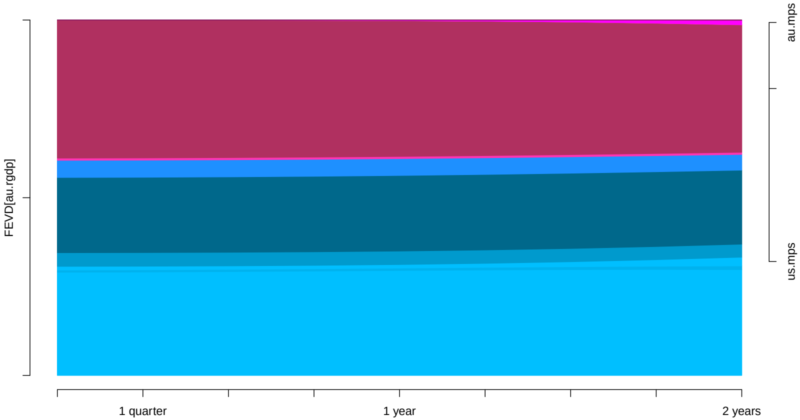
Results for $\kappa_1 = 1$

Forecast error variance decomposition of $au.cpi_{t+h|t}$



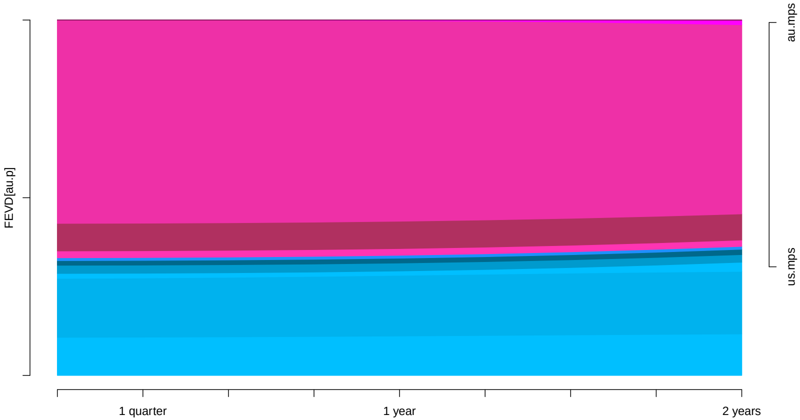
Results for $\kappa_1 = 1$

Forecast error variance decomposition of $au.rgdp_{t+h|t}$



Results for $\kappa_1 = 0.02^2$

Forecast error variance decomposition of $au.cpi_{t+h|t}$



Results for $\kappa_1 = 0.02^2$

SVAR Tools

Estimation output for structural models can be used to compute a wide range of economically interpretable values.

Structural shocks that have statistically significant IRFs over some period and contribute to a large extent to the FEVDs are considered the main drivers for economic measurements

The monetary policy shock can be considered a non-negligible determinant of the business cycle in Australia