Macroeconometrics: ECOM90007

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Notes from the whiteboard.

These are the whiteboard notes from lecture 3.

Definitions.

The transformations below can be understood as definitions, decompositions, and the way to construct the distributions. Consider random variables X and Y.

$$\begin{aligned}
p(X) &- \text{denotes a marginal distribution of } X\\
p(X,Y) &- \text{denotes a joint distribution of } X \text{ and } Y\\
p(X \mid Y) &- \text{denotes a conditional distribution of } X \text{ given } Y
\end{aligned}$$

The joint distribution can be decomposed as or be constructed from a conditional and marginal distribution

$$\begin{aligned}
p(X,Y) &= p(X \mid Y)p(Y)\\
&= p(Y \mid X)p(X)
\end{aligned}$$

If X and Y are independent then their joint distribution can be constructed as a product of marginal distributions:

$$p(X,Y) = p(X)p(Y)$$

A conditional distribution is constructed as:

$$\begin{aligned}
p(X \mid Y) &= \frac{p(X,Y)}{p(Y)}\\
p(Y \mid X) &= \frac{p(X,Y)}{p(X)}
\end{aligned}$$

Marginal distributions can be constructed as:

$$\begin{aligned}
p(X) &= \frac{p(X,Y)}{p(Y \mid X)}\\
&= \int p(X,Y)dY\\
p(Y) &= \frac{p(X,Y)}{p(X \mid Y)}\\
&= \int p(X,Y)dX
\end{aligned}$$

Expected values.

$$\begin{aligned}
E(X) &= \int Xp(X)dX\\
E(X \mid Y) &= \int Xp(X \mid Y)dX\\
E\left(X^2\right) &= \int X^2 p(X)dX\\
E(f(X)) &= \int f(X)p(X)dX
\end{aligned}$$

Task. Using the formulae above write down how you would compute/construct the variance of X, $Var(X)$.