# Macroeconometrics

Lecture 23 Less than 2°C warming by 2100 unlikely – partial reproduction

Topics in Climate Change Forecasting CO<sub>2</sub> Emissions for the 21st Century

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The data, the model, and prior assumptions

Matrix notation and MCMC sampler

**Estimation results** 

Probabilistic predictions

Lecture is based on

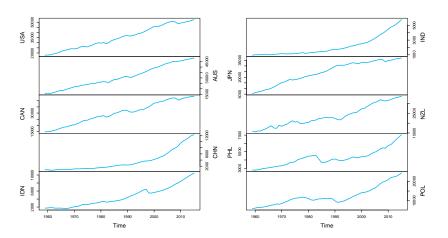
Raftery, Zimmer, Frierson, Startz, Liu (2017), Less than 2°C warming by 2100 unlikely, Nature Climate Change, Vol. 7.

Materials

A zip file L23mcxs-all.zip for the reproduction of the results

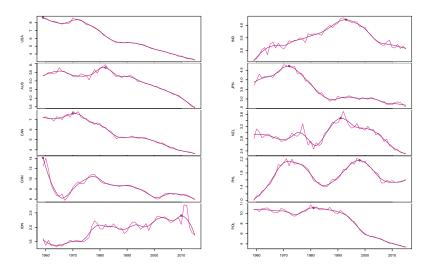


#### The data: GDP data



Annual data from 1959–2015 (T=56 of differentiated series, N=10) Model applied to logarithms of the original data Source: data files of Raftery et al. (2017)

## The data: carbon intensity



The data, loess smoothed values, and cut-off dates

### Model and prior assumptions

#### Model for the frontier economy – prior distributions.

$$F_{t} = F_{t-1} + \gamma + \gamma_{pre1973} \mathcal{I}(t \leq 1973) + \epsilon_{t}^{(f)}$$

$$\epsilon_{t}^{(f)} \sim \mathcal{N}(0, \sigma_{f}^{2})$$

$$\gamma \sim \mathcal{U}[0, 1]$$

$$\gamma_{pre1973} \sim \mathcal{U}[-0.1, 0.1]$$

$$\sigma_{f}^{2} \sim \mathcal{IG}2(\hat{s}^{2}, 3)$$

### Model and prior assumptions

Model for other economies – prior distributions.

$$(F_{t} - G_{c.t}) = \phi_{c}(F_{t-1} - G_{c.t-1}) + \epsilon_{c.t}^{(g)}$$

$$\epsilon_{c.t}^{(g)} \sim \mathcal{N}\left(0, \sigma_{g.c}^{2}\right)$$

$$\phi_{c}|\mu_{\phi}, \sigma_{\phi}^{2} \sim \mathcal{T}\mathcal{N}_{[0,1]}\left(\mu_{\phi}, \sigma_{\phi}^{2}\right)$$

$$\mu_{\phi} \sim \mathcal{U}[0, 1]$$

$$\sigma_{\phi}^{2} \sim \mathcal{U}[0, 1]$$

$$\sigma_{g.c}^{2}|\underline{s} \sim \mathcal{IG}2\left(\underline{s}, 3\right)$$

$$\underline{s} \sim \mathcal{G}(1, 1)$$

### Model and prior assumptions

#### Model for carbon intensity.

$$\begin{aligned} \tau_{c.t} &= \eta(t - \overline{t}) + \beta \tau_{c.t-1} - \delta_c + \rho \frac{\sigma_c}{\sigma_{g.c}} \epsilon_{c.t}^{(g)} + \epsilon_{c.t} \\ \epsilon_{c.t} &\sim \mathcal{N} \left( 0, \sigma_c^2 \right) \\ \eta &\sim \mathcal{N} \left( 0.1, 0.01 \right) \\ \beta &\sim \mathcal{U}[0, 1] \\ \rho &\sim \mathcal{U}[-1, 1] \\ \delta_c | \mu_\delta, \sigma_\delta^2 &\sim \mathcal{N} \left( \mu_\delta, \sigma_\delta^2 \right) \\ \mu_\delta &\sim \mathcal{N}(0, 1) \\ \sigma_\delta^2 &\sim \mathcal{IG2}(1, 1) \\ \sigma_c^2 | \underline{s}_\sigma &\sim \mathcal{IG2} \left( \underline{s}_\sigma, 3 \right) \\ \underline{s}_\sigma &\sim \mathcal{G}(1, 1) \end{aligned}$$

# Matrix notation and Gibbs sampler

### Metropolis-Hastings sampler

an MCMC method for sampling from the posterior distribution of parameters  $\boldsymbol{\theta}$ 

requires only an ordinate of the kernel of the posterior density

$$k(\theta) = L(\theta, \mathbf{y})p(\theta)$$

relies on the specification of a candidate drawing density

$$\theta^* \sim q(\theta)$$

**accept** the candidate draw  $\theta^*$  with probability

$$\min \left\{ 1, \frac{k(\theta^*)q(\theta^*)}{k(\theta^{(s-1)})q(\theta^{(s-1)})} \right\}$$

Gibbs sampler is its special case with acceptance probability 1

### Metropolis-Hastings sampler

Due to a non-standard form of dependence in equation

$$\tau_{c.t} = \eta(t - \bar{t}) + \beta \tau_{c.t-1} - \delta_c + \rho \frac{\sigma_c}{\sigma_{g.c}} \epsilon_{c.t}^{(g)} + \epsilon_{c.t}$$

the full conditional posterior distributions are non-standard

**Estimation strategy:** derive the full conditional posterior densities for all of the parameters as if  $\frac{\sigma_c}{\sigma_{g,c}} \epsilon_{c,t}^{(g)}$  was a fixed regressor and use these densities as candidate drawing densities. Accept or reject the candidate draws with appropriate probabilities.

#### Uniform prior distribution

- **Uniform prior** distributions help to impose restrictions
- **Density function** of a uniform distribution  $\mathcal{U}(a,b)$  does not depend o the random variable and is equal to  $(b-a)^{-1}$
- **Full conditional posterior** distribution for a parameter  $\theta \sim \mathcal{U}(a,b)$  is a truncated density, for instance:

$$L(\theta|\mathbf{y}) = \mathcal{N}(\tilde{\theta}, \tilde{V}_{\theta})$$

$$\theta = \mathcal{U}(a, b)$$

$$\downarrow$$

$$p(\theta|\mathbf{y}, \dots) = \mathcal{N}(\tilde{\theta}, \tilde{V}_{\theta})\mathcal{I}(\theta \in [a, b])$$

## Hierarchical prior distributions: normal

$$\theta | \mu_{\theta}, \sigma_{\theta}^{2} \sim \mathcal{N}(\mu_{\theta}, \sigma_{\theta}^{2})$$

$$\mu_{\theta} \sim \mathcal{N}(\mu_{\mu}, \sigma_{\mu}^{2})$$

$$\sigma_{\theta}^{2} \sim \mathcal{IG2}(s_{\theta}, \nu_{\theta})$$

$$\downarrow$$

$$\mu_{\theta} | \theta, \sigma_{\theta}^{2}, \mu_{\mu}, \sigma_{\mu}^{2} \sim \mathcal{N}\left((\sigma_{\theta}^{-2} + \sigma_{\mu}^{-2})^{-1}(\sigma_{\theta}^{-2}\theta + \sigma_{\mu}^{-2}\mu_{\mu}), (\sigma_{\theta}^{-2} + \sigma_{\mu}^{-2})^{-1}\right)$$

$$\sigma_{\theta}^{2} | \theta, \mu_{\theta}, s_{\theta}, \nu_{\theta} \sim \mathcal{IG2}\left(s_{\theta} + (\theta - \mu_{\theta})^{2}, \nu_{\theta} + 1\right)$$

# Hierarchical prior distributions: inverse gamma 2

$$\sigma^{2}|\underline{s} \sim \mathcal{IG}2(\underline{s}, \nu) \propto \underline{s}^{\frac{\nu}{2}} \left(\sigma^{2}\right)^{-\frac{\nu+2}{2}} \exp\left\{-\frac{1}{2} \frac{\underline{s}}{\sigma^{2}}\right\}$$

$$\underline{s} \sim \mathcal{G}(s, a) \propto \underline{s}^{a-1} \exp\left\{-\frac{\underline{s}}{\underline{s}}\right\}$$

$$\downarrow$$

$$\underline{s}|\sigma^{2} \sim \mathcal{G}\left(\left(s^{-1} + 0.5\sigma^{-2}\right)^{-1}, \frac{\nu}{2} + a\right)$$

# Model for the frontier economy: matrix notation.

$$F = X_F \gamma + \epsilon^{(f)}$$

$$\epsilon^{(f)} \sim \mathcal{N}_T \left( \mathbf{0}_T, \sigma_f^2 I_T \right)$$

$$\gamma \sim \mathcal{U}[0, 1]$$

$$\gamma_{pre1973} \sim \mathcal{U}[-0.1, 0.1]$$

$$\sigma_f^2 \sim \mathcal{I}\mathcal{G}2(\hat{s}^2, 3)$$

$$F_{56\times1} = \begin{bmatrix}
F_{2} - F_{1} \\
\vdots \\
F_{T} - F_{T-1}
\end{bmatrix}, \quad
X_{F}_{56\times2} = \begin{bmatrix}
I_{14} & I_{14} \\
I_{42} & \mathbf{0}_{42}
\end{bmatrix}, \quad
\epsilon_{56\times1}^{(f)} = \begin{bmatrix}
\epsilon_{1}^{(f)} \\
\vdots \\
\epsilon_{T}^{(f)}
\end{bmatrix}, \quad
\gamma_{2\times1} = \begin{bmatrix}
\gamma \\
\gamma_{pre1973}
\end{bmatrix},$$

## Model for the frontier economy: MCMC sampler.

$$\gamma|F, X_{F}, \sigma_{f}^{2} \sim \mathcal{N}_{2}(\bar{\gamma}, \bar{V}_{\gamma})\mathcal{I}\begin{pmatrix} \gamma \in [0, 1] \\ \gamma_{pre1973} \in [-.1, .1] \end{pmatrix}$$

$$\bar{V}_{\gamma} = (\sigma_{f}^{-2}X'_{F}X_{F})^{-1}$$

$$\bar{\gamma} = (X'_{F}X_{F})^{-1}X'_{F}F$$

$$\sigma_{f}^{2}|F, X_{F}, \gamma \sim \mathcal{IG}_{2}(\bar{s}_{f}, \bar{\nu}_{f})$$

$$\bar{s}_{f} = \hat{s}^{2} + (F - X_{F}\gamma)'(F - X_{F}\gamma)$$

$$\bar{\nu}_{f} = T + 3$$

### Model for other economies – matrix notation.

$$G = X_{G}\phi + \epsilon^{(g)}$$

$$\epsilon^{(g)} \sim \mathcal{N}_{(N-1)T} \left( \mathbf{0}_{(N-1)T}, \Sigma_{G} \right)$$

$$\Sigma_{G} = \operatorname{diag} \left( \sigma_{g,2}^{2}, \dots, \sigma_{g,N}^{2} \right) \otimes I_{T}$$

$$\phi | \mu_{\phi}, \sigma_{\phi}^{2} \sim \mathcal{N}_{N-1} \left( \mu_{\phi} I_{N-1}, \sigma_{\phi}^{2} I_{N-1} \right)$$

$$\mu_{\phi} \sim \mathcal{U}[0, 1]$$

$$\sigma_{\phi}^{2} \sim \mathcal{U}[0, 1]$$

$$\sigma_{g,c}^{2} | \underline{s} \sim \mathcal{I}G2(\underline{s}, 3)$$

$$\underline{s} \sim \mathcal{G}(1, 1)$$

#### Model for other economies – matrix notation.

$$\frac{G}{(N-1)T\times 1} = \begin{bmatrix}
F_2 - G_{2.1} \\
\vdots \\
F_T - G_{2.T} \\
\vdots \\
F_2 - G_{N.2} \\
\vdots \\
F_T - G_{N.T}
\end{bmatrix}, \quad e^{(g)}_{(N-1)T\times 1} = \begin{bmatrix}
\epsilon^{(g)}_{2.2} \\
\vdots \\
\epsilon^{(g)}_{2.T} \\
\vdots \\
\epsilon^{(g)}_{N.2} \\
\vdots \\
\epsilon^{(g)}_{N.2}
\end{bmatrix}, \quad \phi_{N-1\times 1} = \begin{bmatrix}\phi_2 \\
\vdots \\
\phi_N\end{bmatrix}$$

$$\begin{bmatrix}
F_1 - G_{2.1} & \dots & 0 \\
\vdots \\
F_{T-1} - G_{2.T-1} & \dots & 0 \\
\vdots \\
0 & \dots & F_1 - G_{N.1} \\
\vdots \\
0 & \dots & F_{T-1} - G_{N.T-1}
\end{bmatrix}$$

## Model for other economies - MCMC sampler.

$$\begin{split} \phi | G, X_G, \sigma_g^2, \mu_{\phi}, \sigma_{\phi}^2 &\sim \mathcal{N}_{N-1} \left( \bar{\phi}, \bar{V}_{\phi} \right) \mathcal{I} \left( \phi_c \in [0, 1] \right) \\ \bar{V}_{\phi} &= \left( X_G' \Sigma_G^{-1} X_G + \sigma_{\phi}^{-2} I_{N-1} \right)^{-1} \\ \bar{\phi} &= \bar{V}_{\phi} \left( X_G' \Sigma_G^{-1} G + \sigma_{\phi}^{-2} \mu_{\phi} I_{N-1} \right) \\ \sigma_{g.c}^2 | G, X_G, \underline{s} &\sim \mathcal{I} \mathcal{G} 2 (\bar{s}_{g.c}, \bar{\nu}_{g.c}) \\ \bar{s}_{g.c} &= \underline{s} + (G - X_G \phi)' (G - X_G \phi) \\ \bar{\nu}_{g.c} &= T + 3 \end{split}$$

$$\mu_{\phi} | \phi, \sigma_{\phi}^2 &\sim \mathcal{N}_{N-1} \left( \bar{\mu}_{\phi}, \bar{V}_{\mu_{\phi}} \right) \mathcal{I} \left( \mu_{\phi} \in [0, 1] \right) \\ \bar{V}_{\mu_{\phi}} &= \sigma_{\phi}^2 / (N - 1) \\ \bar{\mu}_{\phi} &= \bar{V}_{\mu_{\phi}} \sigma_{\phi}^{-2} I_{N-1} \phi \\ \sigma_{\phi}^2 | \phi, \mu_{\phi} &\sim \mathcal{I} \mathcal{G} 2 \left( \bar{s}_{\phi}, N - 3 \right) \mathcal{I} \left( \sigma_{\phi}^2 \in [0, 1] \right) \\ \bar{s}_{\phi} &= (\phi - \mu_{\phi} I_{N-1})' (\phi - \mu_{\phi} I_{N-1}) \\ \underline{s} | \sigma_g^2 &\sim \mathcal{G} \left( \left( 1 + .5 \sum_{c} \sigma_{g.c}^{-2} \right)^{-1}, 1.5 (N - 1) + 1 \right) \end{split}$$

# Model for carbon intensity – matrix notation.

$$\begin{split} \tau &= X_{\tau}\beta_{\tau} + \epsilon \\ \epsilon &\sim \mathcal{N}_{\left(\sum_{c=1}^{N} \tau_{c}\right)}(\mathbf{0}, \Sigma) \end{split}$$
 
$$\beta_{\tau} | \mu_{\delta}, \sigma_{\delta}^{2} \sim \mathcal{N}_{N+3} \left(\underline{\mu}_{\beta}, \underline{V}_{\beta}\right) \mathcal{I} \begin{pmatrix} \beta \in [0, 1] \\ \rho \in [-1, 1] \end{pmatrix}$$
 
$$\mu_{\delta} \sim \mathcal{N}(0, 1)$$
 
$$\sigma_{\delta}^{2} \sim \mathcal{I} \mathcal{G} 2(1, 1)$$
 
$$\sigma_{c}^{2} | \underline{s}_{\sigma} \sim \mathcal{I} \mathcal{G} 2(\underline{s}_{\sigma}, 3)$$
 
$$\underline{s}_{\sigma} \sim \mathcal{G}(1, 1)$$
 
$$\underline{\mu}_{\beta} = \begin{bmatrix} 0.1 \\ 0 \\ \mu_{\delta} I_{N} \\ 0 \end{bmatrix}, \quad \underline{V}_{\beta}^{-1} = \begin{bmatrix} 100 & 0 & \dots & 0 \\ 0 & 0 & \dots & 0 \\ \vdots & \vdots & \sigma_{\delta}^{-2} I_{N} & \vdots \\ 0 & 0 & \dots & \sigma_{N}^{2} I_{T_{N}} \end{bmatrix}, \quad \Sigma = \begin{bmatrix} \sigma_{1}^{2} I_{T_{1}} & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & \sigma_{N}^{2} I_{T_{N}} \end{bmatrix}$$

# Model for carbon intensity – matrix notation.

$$\chi_{\tau} = \begin{bmatrix} \tau_{1} \\ \vdots \\ \tau_{N} \end{bmatrix}, \quad \epsilon_{(\sum_{c=1}^{N} T_{c}) \times 1} = \begin{bmatrix} \epsilon_{1} \\ \vdots \\ \epsilon_{N} \end{bmatrix}, \quad \beta_{\tau} = \begin{bmatrix} \eta \\ \delta_{1} \\ \vdots \\ \delta_{N} \\ \rho \end{bmatrix}, \\
\chi_{\tau} = \begin{bmatrix} trend_{1} & \tau_{1.t-1} & -\iota_{T_{1}} & \mathbf{0} & \dots & \mathbf{0} & \frac{\sigma_{1}}{\sigma_{f}} \epsilon^{(f)} \\ trend_{1} & \tau_{1.t-1} & \mathbf{0} & -\iota_{T_{2}} & \dots & \mathbf{0} & \frac{\sigma_{2}}{\sigma_{g.2}} \epsilon^{(g)} \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ trend_{N} & \tau_{N.t-1} & \mathbf{0} & \mathbf{0} & \dots & -\iota_{T_{N}} & \frac{\sigma_{N}}{\sigma_{g.2}} \epsilon^{(g)}_{N} \end{bmatrix},$$

# Model for carbon intensity – MCMC sampler.

$$\begin{split} \beta_{\tau}|\tau, X_{\tau}, \sigma_{c}^{2}, \mu_{\delta}, \sigma_{\delta}^{2} &\sim \mathcal{N}_{N+3} \left(\bar{\mu}_{\beta}, \bar{V}_{\beta}\right) \mathcal{I} \left(\begin{array}{c} \beta \in [0, 1] \\ \rho \in [-1, 1] \end{array}\right) \\ \bar{V}_{\beta} &= \left(X_{\tau}' \Sigma^{-1} X_{\tau} + \underline{V}_{\beta}^{-1}\right)^{-1} \\ \bar{\mu}_{\beta} &= \bar{V}_{\beta} \left(X_{\tau}' \Sigma^{-1} \tau + \underline{V}_{\beta}^{-1} \underline{\mu}_{\beta}\right) \\ \sigma_{c}^{2}|\tau, X_{\tau}, \beta_{\tau} \underline{s}_{\sigma} &\sim \mathcal{I} \mathcal{G} 2(\bar{s}_{c}, \bar{\nu}_{c}) \\ \bar{s}_{c} &= \underline{s}_{\sigma} + (\tau - X_{\tau} \beta_{\tau})_{[(\tau_{c-1}+1):\tau_{c}]}^{\prime} (\tau - X_{\tau} \beta_{\tau})_{[(\tau_{c-1}+1):\tau_{c}]}^{\prime} \\ \bar{\nu}_{c} &= T_{c} + 3 \end{split}$$

$$\mu_{\delta}|\delta, \sigma_{\delta}^{2} &\sim \mathcal{N} \left(\bar{V}_{\mu_{\delta}} \sigma_{\delta}^{-2} I_{n}^{\prime} \delta, \bar{V}_{\mu_{\delta}}\right), \quad \bar{V}_{\mu_{\delta}} &= [\sigma_{\delta}^{-2} N + 1]^{-1} \\ \sigma_{\delta}^{2}|\delta, \mu_{\delta} &\sim \mathcal{I} \mathcal{G} 2(1 + (\delta - \mu_{\delta} I_{N})^{\prime} (\delta - \mu_{\delta} I_{N}), N + 1) \\ \underline{s}_{\sigma}|\sigma^{2} &\sim \mathcal{G} \left(\left(1 + \sum_{c} \sigma_{c}^{2}\right)^{-1}, 1.5N + 1\right) \end{split}$$

## **Estimation** results

# Model for the frontier economy

$$F_t = F_{t-1} + \gamma + \gamma_{pre1973} \mathcal{I}(t \le 1973) + \epsilon_t^{(f)}, \qquad \epsilon_t^{(f)} \sim \mathcal{N}\left(0, \sigma_f^2\right)$$
$$\gamma \sim \mathcal{U}[0, 1], \quad \gamma_{pre1973} \sim \mathcal{U}[-0.1, 0.1], \quad \sigma_f^2 \sim \mathcal{IG}(\hat{s}^2, 3)$$

θ	$\gamma$	$\gamma_{pre1973}$	$\sigma_f$		
$E[\theta \mathbf{y}]$	0.016	0.012	0.019		
$sd[\theta \mathbf{y}]$	0.003	0.006	0.002		

#### Model for other economies

$$(F_{t} - G_{c,t}) = \phi_{c}(F_{t-1} - G_{c,t-1}) + \epsilon_{c,t}^{(g)}, \qquad \epsilon_{c,t}^{(g)} \sim \mathcal{N}\left(0, \sigma_{g,c}^{2}\right)$$

$$\phi_{c}|\mu_{\phi}, \sigma_{\phi} \sim \mathcal{T}\mathcal{N}_{[0,1]}\left(\mu_{\phi}, \sigma_{\phi}^{2}\right), \quad \mu_{\phi} \sim \mathcal{U}[0,1], \quad \sigma_{\phi} \sim \mathcal{U}[0,1]$$

$$\sigma_{g,c}^{2}|\underline{s} \sim \mathcal{I}\mathcal{G}2\left(\underline{s}, 3\right), \quad \underline{s} \sim \mathcal{G}\left(1, 1\right)$$

$\phi_c$	AUS	CAN	CHN	IDN	IND	JPN	NZL	PHL	POL
$E[\phi_c \mathbf{y}]$ $sd[\phi_c \mathbf{y}]$	0.98	0.99	0.99	0.99	0.99	0.95	0.99	0.99	0.99
$\sigma_{g.c}$	AUS	CAN	CHN	IDN	IND	JPN	NZL	PHL	POL
$E[\sigma_{g.c} \mathbf{y}]$ $sd[\sigma_{g.c} \mathbf{y}]$	0.02	0.01	0.06	0.05	0.04	0.03	0.03	0.04	0.04
θ	$\mu_{\phi}$	$\sigma_{\phi}$	<u>s</u>						
$E[\theta \mathbf{y}]$ $sd[\theta \mathbf{y}]$	0.37 .28	0.56 .25	0.04						25-/

<del>25 /</del> 38

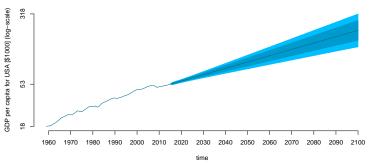
## Model for carbon intensity

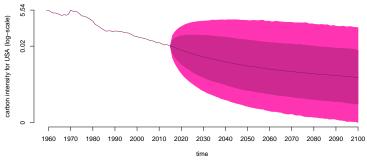
$$\begin{split} \tau_{c.t} &= \eta(t - \overline{t}) + \beta \tau_{c.t-1} - \delta_c + \rho \frac{\sigma_c}{\sigma_{g.c}} \epsilon_{c.t}^{(g)} + \epsilon_{c.t}, \quad \epsilon_{c.t} \sim \mathcal{N} \left( 0, \sigma_c^2 \right) \\ & \eta \sim \mathcal{N} \left( 0.1, 0.01 \right), \quad \beta \sim \mathcal{U}[0, 1], \quad \rho \sim \mathcal{U}[-1, 1] \\ & \delta_c | \mu_\delta, \sigma_\delta^2 \sim \mathcal{N} \left( \mu_\delta, \sigma_\delta^2 \right), \quad \mu_\delta \sim \mathcal{N}(0, 1), \quad \sigma_\delta^2 \sim \mathcal{IG}2(1, 1) \\ & \sigma_c^2 | \underline{s}_\sigma \sim \mathcal{IG}2 \left( \underline{s}_\sigma, 3 \right), \quad \underline{s}_\sigma \sim \mathcal{G}(1, 1) \end{split}$$

θ	η	β	ρ				$\mu_\delta$	$\sigma_{\delta}$	<u>s</u> <sub>σ</sub>	
$E[\theta \mathbf{y}]$ $sd[\theta \mathbf{y}]$	-0.001 .0003	0.96 .02	-0.13 .05				0.12 .57	0.59 .35	0.05 .007	
$\delta_c$	USA	AUS	CAN	CHN	IDN	IND	JPN	NZL	PHL	POL
$E[\delta_c \mathbf{y}]$ $sd[\delta_c \mathbf{y}]$	.042 .03	.047 .03	.045 .03	.061 .04	.043 .10	.045 .03	.031 .02	.033 .02	.018 .02	.042
$\sigma_c$	USA	AUS	CAN	CHN	IDN	IND	JPN	NZL	PHL	POL
$E[\sigma_c \mathbf{y}]$ $sd[\sigma_c \mathbf{y}]$	.022 .002	.016 .002	.023 .002	.077 .007	.185 .060	.031 .005	.033 .004	.036 .005	.043 .008	.039
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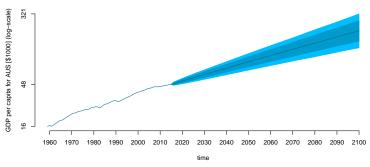
## **Probabilistic predictions**

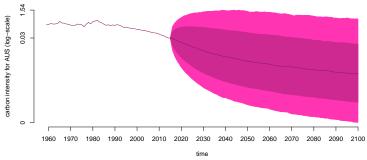
#### Predictions: USA



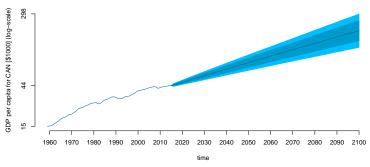


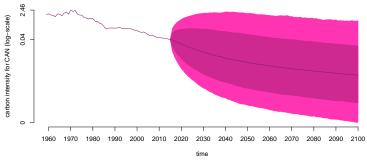
#### Predictions: Australia



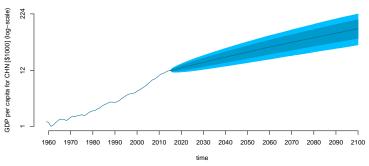


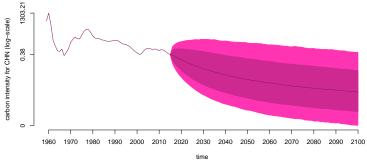
#### Predictions: Canada



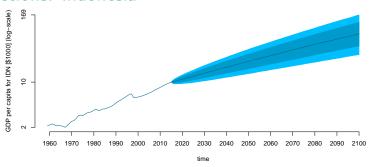


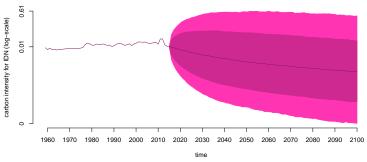
### Predictions: China



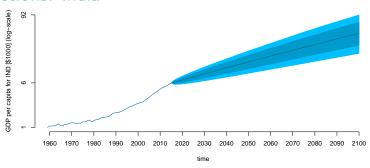


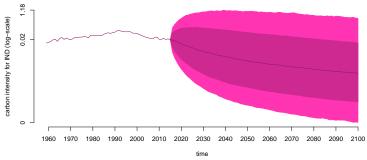
#### Predictions: Indonesia



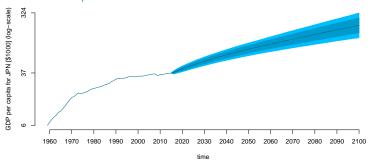


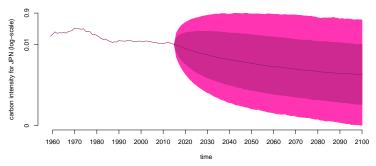
#### Predictions: India



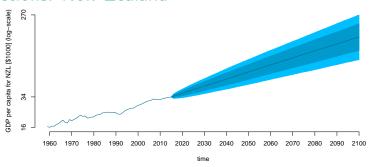


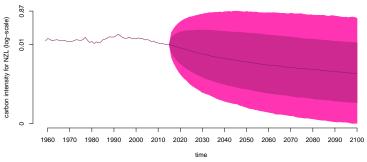
## Predictions: Japan



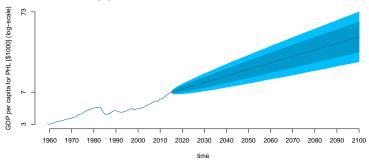


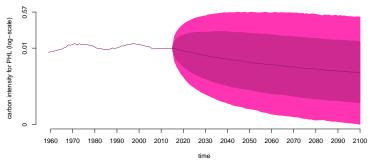
#### Predictions: New Zealand



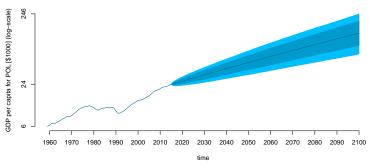


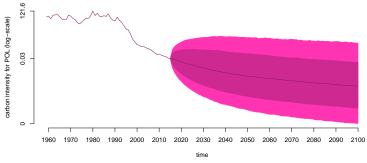
## Predictions: Philippines





#### Predictions: Poland





## Less than 2°C warming by 2100 unlikely

- Long-run forecasting of quantities that are essential for decision-makers faces multiple challenges
- Probabilistic forecasting is crucial for realistic assessment of future tendencies
- Hierarchical Bayesian modeling provides additional tools to calibrate the model to the objective of the research
- Much stricter policies lowering the carbon intensity of economies are required to keep the increase in global temperatures below the level triggering multiple climate change tipping points