Macroeconometrics

Lecture 5 Understanding unit-rooters

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Concepts

Autoregressions

Random walk processes

Inference

A helicopter tour

Useful readings:

Sims, Uhlig (1991) Understanding Unit Rooters: A Helicopter Tour, Econometrica

Materials:

An R file `05 mcxs.R`, `06 mcxs.R` for the reproduction the results
Shiny interactive apps: `05shiny-ar1.R` and `05shiny-ar2.R`

Concepts

Strict stationarity

A time series y_t is said to be strictly stationary if the joint distribution of $(y_{t_1}, \ldots, y_{t_k})$ is identical to that of $(y_{t_1+s}, \ldots, y_{t_k+s})$ for all t, where k is an arbitrary positive integer and $(t_1, \ldots, t_k)$ is a collection of k positive integers:

$$p(y_{t_1}, \ldots, y_{t_k}) = p(y_{t_1+s}, \ldots, y_{t_k+s}).$$

Covariance stationarity

A time series y_t is said to be covariance or weakly stationary if both the mean of y_t and the covariance between y_t and y_{t-s} are time invariant, where s is an arbitrary integer:

$$\begin{aligned} \mathbb{E}[y_t] &= \mu, \\ \mathbb{C}\text{ov}[y_t, y_{t-s}] &= \gamma_s, \end{aligned}$$

where:

$\gamma_0 = \mathbb{V}\text{ar}[y_t]$

$\gamma_{-s} = \gamma_s$

Unconditional moments

Expected value:

$$\mathbb{E}[y_t] = \int_{\mathbb{R}} y_t p(y_t) dy_t$$

Covariance matrix.

$$\mathbb{V}\text{ar}[y_t] = \mathbb{E}\left[(y_t - \mathbb{E}[y_t])(y_t - \mathbb{E}[y_t])'\right]$$

Autocovariance.

$$\mathbb{C}\text{ov}[y_t, y_{t-s}] = \mathbb{E}\left[(y_t - \mathbb{E}[y_t])(y_{t-s} - \mathbb{E}[y_{t-s}])'\right]$$

Sample moments

Sample mean.

$$\bar{y} = \frac{1}{T}\sum_{t=1}^{T} y_t$$

Sample covariance matrix.

$$\hat{\gamma}_0 = \frac{1}{T}\sum_{t=1}^{T} [(y_t - \bar{y})(y_t - \bar{y})']$$

Sample autocovariance.

$$\hat{\gamma}_s = \frac{1}{T-s}\sum_{t=s+1}^{T} [(y_t - \bar{y})(y_{t-s} - \bar{y})']$$

Autocorrelation

Autocorrelation function - ACF.

$$\rho_s = \mathbb{C}\text{orr}[y_{n.t}, y_{n.t-s}] = \frac{\mathbb{E}\left[\left(y_{n.t} - \mathbb{E}[y_{n.t}]\right)\left(y_{n.t-s} - \mathbb{E}[y_{n.t-s}]\right)\right]}{\sqrt{\mathbb{V}\text{ar}[y_{n.t}]\mathbb{V}\text{ar}[y_{n.t-s}]}}$$

Sample autocorrelation.

$$\hat{\rho}_s = \frac{\sum_{t=s+1}^{T}[(y_{n.t} - \bar{y}_n)(y_{n.t-s} - \bar{y}_n)]}{\sqrt{\left[\sum_{t=s+1}^{T}(y_{n.t} - \bar{y}_n)^2\right]\left[\sum_{t=1}^{T-s}(y_{n.t-s} - \bar{y}_{n.s})^2\right]}}$$

Assume covariance stationarity:

$$\rho_s = \rho_{-s} = \frac{\gamma_s}{\gamma_0}$$

White noise process

A stochastic process ϵ_t is called a white noise if $\{\epsilon_t\}$ is an i.i.d. sequence:

$$\mathbb{E}[\epsilon_t] = 0$$
$$\mathbb{V}\text{ar}[\epsilon_t] = \sigma_\epsilon^2 < \infty$$
$$\mathbb{C}\text{ov}[\epsilon_t, \epsilon_{t-s}] = 0 \qquad \forall s \neq 0$$

Denoted by

$$\epsilon_t \sim iid\left(0, \sigma_\epsilon^2\right)$$

Gaussian white noise process.

$$\epsilon_t \sim iid\mathcal{N}\left(0, \sigma_\epsilon^2\right)$$

Autoregressions

Autoregressions

AR(p) model.

$$
\begin{aligned}
&y_t = \mu_0 + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \epsilon_t \\
&\epsilon_t | y_{t-1}, \ldots, y_{t-p} \sim iid(0, \sigma_\epsilon^2)
\end{aligned}
$$

Exogeneity assumption in time series.

$$\mathbb{E}[\epsilon_t | y_{t-1}, \ldots, y_{t-p}] = 0$$

Autoregressions

AR(p) model: alternative notations.

$$
\begin{aligned}
y_t &= \mu_0 + \alpha_1 y_{t-1} + \cdots + \alpha_p y_{t-p} + \epsilon_t \\
y_t - \alpha_1 y_{t-1} - \cdots - \alpha_p y_{t-p} &= \mu_0 + \epsilon_t \\
\left(1 - \alpha_1 L - \cdots - \alpha_p L^p\right) y_t &= \mu_0 + \epsilon_t \\
\alpha(L) y_t &= \mu_0 + \epsilon_t
\end{aligned}
$$

L denotes the lag operator such that $L^s y_t = y_{t-s}$ and $L^s c = c$

AR(p) model: matrix notation.

$$Y = X\beta + E$$

$$
\underset{(p+1\times 1)}{\beta} = \begin{bmatrix} \mu_0 \\ \alpha_1 \\ \vdots \\ \alpha_p \end{bmatrix} \quad
\underset{(T\times 1)}{Y} = \begin{bmatrix} y_1 \\ \vdots \\ y_T \end{bmatrix} \quad
\underset{(p+1\times 1)}{x_t'} = \begin{bmatrix} 1 \\ y_{t-1} \\ \vdots \\ y_{t-p} \end{bmatrix} \quad
\underset{(T\times p+1)}{X} = \begin{bmatrix} x_1 \\ \vdots \\ x_T \end{bmatrix} \quad
\underset{(T\times 1)}{E} = \begin{bmatrix} \epsilon_1 \\ \vdots \\ \epsilon_T \end{bmatrix}
$$

Autoregressions: stationarity

An AR(p) model is called stationary if:

$$\alpha(z) = 0 \qquad \text{for complex numbers } z \text{ with } |z| > 1,$$

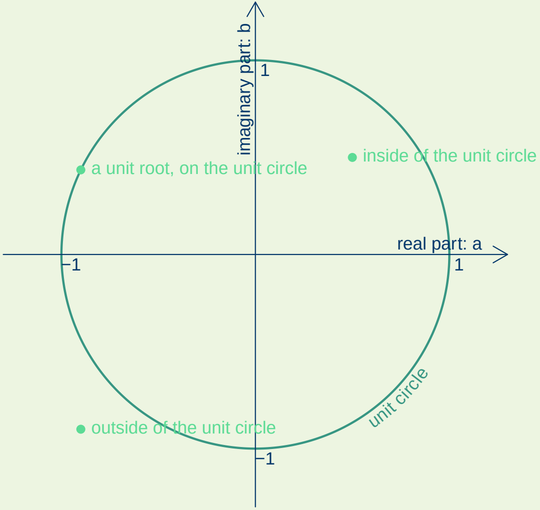
that is:

$$1 - \alpha_1 z - \cdots - \alpha_p z^p = 0 \qquad \text{for } |z| > 1 \text{ and } z \in \mathbb{C}.$$

Modulus of a complex number $z = a + ib$ is:

$$|z| = \sqrt{a^2 + b^2}$$

Unit circle



The unit circle is given by equation: $1^2 = a^2 + b^2$

Autoregressions: invertibility

A stationary AR(p) model has an MA(∞) representation:

$$\alpha(L)y_t = \mu_0 + \epsilon_t$$

$$\begin{aligned} y_t &= \alpha(L)^{-1}\mu_0 + \alpha(L)^{-1}\epsilon_t \\ &= \mu + \phi(L)\epsilon_t \\ &= \mu + \epsilon_t + \phi_1\epsilon_{t-1} + \phi_2\epsilon_{t-2} + \dots \end{aligned}$$

where:

$$\phi_j = \sum_{i=1}^{j} \phi_{j-i}\alpha_i,$$

for $j = 0, 1, 2, \dots,$ and $\phi_0 = 1$, and $\alpha_i = 0$ for $i > p$.

Autoregressions: invertibility

Consider a stationary AR(1) model:

$$
\begin{aligned}
y_t &= \mu_0 + \alpha_1 y_{t-1} + \epsilon_t \\
y_t - \alpha_1 y_{t-1} &= \mu_0 + \epsilon_t \\
(1 - \alpha_1 L) y_t &= \mu_0 + \epsilon_t
\end{aligned}
$$

It has an MA(∞) representation:

$$
\begin{aligned}
y_t &= \frac{\mu_0}{1 - \alpha_1} + \epsilon_t + \alpha_1 \epsilon_{t-1} + \alpha_1^2 \epsilon_{t-2} + \alpha_1^3 \epsilon_{t-3} + \dots \\
&= \mu + \epsilon_t + \phi_1 \epsilon_{t-1} + \phi_2 \epsilon_{t-2} + \dots
\end{aligned}
$$

Inverting polynomial of order 1:

$$
(1 - az)^{-1} = 1 + \sum_{i=1}^{\infty} a^i z^i
$$

Autoregressions: unconditional moments

Unconditional mean.

Assume stationarity.

$$\begin{aligned}\mu = \mathbb{E}[y_t] &= \mathbb{E}[\alpha(1)^{-1}\mu_0 + \alpha(L)^{-1}\epsilon_t] \\ &= \frac{\mu_0}{\alpha(1)} + \alpha(L)^{-1}\mathbb{E}[\epsilon_t] \\ &= \frac{\mu_0}{\alpha(1)} + \alpha(L)^{-1}\mathbb{E}\left[\mathbb{E}[\epsilon_t | y_{t-1}, \ldots, y_{t-p}]\right]\end{aligned}$$

$$\mu = \frac{\mu_0}{1 - \alpha_1 - \cdots - \alpha_p}$$

given that: $\alpha_1 + \cdots + \alpha_p \neq 1$

The Law of Iterated Expectations.

$$\mathbb{E}\left[\epsilon_t\right] \overset{LIE}{=} \mathbb{E}\left[\mathbb{E}[\epsilon_t | y_{t-1}, \ldots, y_{t-p}]\right] \overset{exogeneity}{=} \mathbb{E}\left[0\right] = 0$$

Autoregressions: autocorrelations

AR(1) model.

Step 1: Use $\mu_0 = \mu(1-\alpha_1)$, to write the model as:

$$y_t - \mu = \alpha_1(y_{t-1} - \mu) + \epsilon_t$$

Step 2: Multiply by $y_{t-s} - \mu$:

$$(y_{t-s} - \mu)(y_t - \mu) = \alpha_1(y_{t-s} - \mu)(y_{t-1} - \mu) + (y_{t-s} - \mu)\epsilon_t$$

Step 3: Take the expectations:

$$\gamma_s = \alpha_1\gamma_{s-1} + \mathbb{E}\left[(y_{t-s} - \mu)\epsilon_t\right]$$

Step 4: Write equation above for $s = 0, 1, \ldots$ and solve for $\gamma_0, \gamma_1, \ldots$

Solution:

$$\begin{aligned} \gamma_0 &= \sigma_\epsilon^2/(1-\alpha_1^2) & \rho_0 &= 1 & & \\ \gamma_s &= \alpha_1\gamma_{s-1} & \rho_s &= \alpha_1\rho_{s-1} & &\text{for } s > 0 \end{aligned}$$

Autoregressions: autocorrelations

AR(2) model.

Step 1: Use $\mu_0 = \mu(1 - \alpha_1 - \alpha_2)$, to write the model as:

$$y_t - \mu = \alpha_1(y_{t-1} - \mu) + \alpha_2(y_{t-2} - \mu) + \epsilon_t$$

Step 2: Multiply by $y_{t-s} - \mu$:

$$(y_{t-s} - \mu)(y_t - \mu) =$$
$$\alpha_1(y_{t-s} - \mu)(y_{t-1} - \mu) + \alpha_2(y_{t-s} - \mu)(y_{t-2} - \mu) + (y_{t-s} - \mu)\epsilon_t$$

Step 3: Take the expectations:

$$\gamma_s = \alpha_1\gamma_{s-1} + \alpha_2\gamma_{s-2} + \mathbb{E}\left[(y_{t-s} - \mu)\epsilon_t\right]$$

Step 4: Write the equation above for $s = 0, 1, \ldots$ and solve for autocovariances.

Autoregressions: autocorrelations

Step 4: Solve the set of equations for γ_0, γ_1 and γ_2.

$$\gamma_0 = \alpha_1\gamma_1 + \alpha_2\gamma_2 + \sigma_u^2$$
$$\gamma_1 = \alpha_1\gamma_0 + \alpha_2\gamma_1$$
$$\gamma_2 = \alpha_1\gamma_1 + \alpha_2\gamma_0$$

Solution:

$$\gamma_0 = \frac{1-\alpha_2}{(1-\alpha_2^2)(1-\alpha_2) - \alpha_1^2(1+\alpha_2)}\sigma_\epsilon^2 \qquad \rho_0 = 1$$

$$\gamma_1 = \frac{\alpha_1}{(1-\alpha_2^2)(1-\alpha_2) - \alpha_1^2(1+\alpha_2)}\sigma_\epsilon^2 \qquad \rho_1 = \alpha_1/(1-\alpha_2)$$

$$\gamma_2 = \frac{\alpha_1^2 + \alpha_2(1-\alpha_2)}{(1-\alpha_2^2)(1-\alpha_2) - \alpha_1^2(1+\alpha_2)}\sigma_\epsilon^2 \qquad \rho_2 = \frac{\alpha_1^2 - \alpha_2^2 + \alpha_2}{1-\alpha_2}$$

$$\gamma_s = \alpha_1\gamma_{s-1} + \alpha_2\gamma_{s-2} \qquad \rho_s = \alpha_1\rho_{s-1} + \alpha_2\rho_{s-2}$$

for $s \geq 1$

Autoregressions: autocorrelations

Shiny interactive apps.

`shiny` is an R package to build interactive apps in R. They are html pages with the computational engine of R behind it.

Shiny apps for AR-model implied autocorrelations.

Install package `shiny`

Open in RStudio files `shiny-ar1.R` and `shiny-ar2.R`

Run the app by pressing **Run App** button.

Discover the patterns in autocorrelations that AR(1) and AR(2) models can capture.

Random walk processes

Random walk process

A time series $\{y_t\}$ is called a random walk if:

$$y_t = y_{t-1} + \epsilon_t$$
$$\epsilon_t \sim iid\left(0, \sigma_\epsilon^2\right)$$

Different representation:

$$\begin{aligned} y_t &= y_{t-1} + \epsilon_t \\ &= \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \ldots \\ &= \sum_{i=0}^{\infty} \epsilon_{t-i} \end{aligned}$$

Random walk process

Random walk process with initial value.
Let y_0 be a real number denoting the starting value of the process, then the random walk process can be written as:

$$y_t = y_0 + \sum_{i=0}^{t-1} \epsilon_{t-i}$$

Random walk process

Consider a random walk process with initial value:

$$y_t = y_0 + \sum_{i=0}^{t-1} \epsilon_{t-i}$$

Moments.

$$\mathbb{E}[y_t] = \mathbb{E}[y_0 + \sum_{i=0}^{T} \epsilon_{t-i}] = y_0$$

$$\mathbb{V}\text{ar}[y_t] = \sum_{i=0}^{t-1} \mathbb{E}[\epsilon_{t-i}^2] = t\sigma_\epsilon^2$$

$$\rho_s = \frac{\gamma_s}{\sqrt{\gamma_0}\sqrt{\mathbb{V}\text{ar}[y_{t-s}]}} = \frac{t-s}{\sqrt{t}\sqrt{t-s}} = \sqrt{\frac{t-s}{t}}$$

Random walk process

Consider a random walk process:

$$y_t = \epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \dots$$

Moments.

$$\mathbb{E}[y_t] = \mathbb{E}[\epsilon_t + \epsilon_{t-1} + \epsilon_{t-2} + \dots] = 0$$

$$\mathbb{V}\text{ar}[y_t] = \mathbb{E}[\epsilon_t^2] + \mathbb{E}[\epsilon_{t-1}^2] + \dots = \lim_{t \to \infty} t\sigma_\epsilon^2 = \infty$$

$$\rho_s = \lim_{t \to \infty} \sqrt{\frac{t-s}{t}} = 1 \qquad \forall s = 0, 1, \dots$$

Random walk process

$$y_t = y_{t-1} + \epsilon_t$$

Forecasting.

$$y_{T+h|T} = \mathbb{E}[y_{T+h}|y_T, y_{T-1}, \dots] = y_T$$

$$\begin{aligned}\sigma^2_{T+h|T} &= \mathbb{E}\left[(y_{T+h} - y_{T+h|T})^2 | \mathbf{y}_T\right] \\ &= \mathbb{E}\left[(\epsilon_{T+1} + \cdots + \epsilon_{T+h})^2 | \mathbf{y}_T\right] \\ &= h\sigma^2_\epsilon\end{aligned}$$

Random walk with drift

A random walk with drift process.

$$y_t = \mu_0 + y_{t-1} + \epsilon_t = y_0 + t\mu_0 + \sum_{i=1}^{t} \epsilon_i$$

$$\epsilon_t \sim iid\left(0, \sigma_\epsilon^2\right)$$

$$\mu_0 = \mathbb{E}[y_t - y_{t-1}] \quad \text{– a time trend}$$

Moments.

$$\begin{aligned}
\mathbb{E}[y_t] &= \mathbb{E}\left[y_0 + t\mu_0 + \epsilon_t + \epsilon_{t-1} + \cdots + \epsilon_1\right] \\
&= y_0 + t\mu_0 \\
\mathbb{V}\text{ar}[y_t] &= \mathbb{E}\left[(\epsilon_t + \epsilon_{t-1} + \cdots + \epsilon_1)^2\right] = t\sigma_\epsilon^2
\end{aligned}$$

Random walk with drift

$$y_t = \mu_0 + y_{t-1} + \epsilon_t = y_0 + t\mu_0 + \sum_{i=1}^{t} \epsilon_i,$$

Forecasting.

$$\begin{aligned} y_{T+h|T} = \mathbb{E}[y_{T+h}|\mathbf{y}_T] &= \mathbb{E}[\epsilon_{T+h} + \epsilon_{T+h-1} + \cdots + \epsilon_{T+1} + h\mu_0 + y_T|\mathbf{y}_T] \\ &= h\mu_0 + y_T \\ \sigma^2_{T+h|T} &= \mathbb{E}\left[(y_{T+h} - y_{T+h|T})^2|\mathbf{y}_T\right] \\ &= h\sigma^2_\epsilon \end{aligned}$$

Inference

Random walk process

Data generating process: $y_t = y_{t-1} + \epsilon_t$

Estimated model.

$$y_t = \alpha_1 y_{t-1} + \epsilon_t$$

$$\hat{\alpha}_1 = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}$$

Asymptotic distribution.

$$T\left(\hat{\alpha}_1 - 1\right) \xrightarrow{d} \frac{B(1)^2 - 1}{2\int_0^1 B(s)^2 ds}$$

T – convergence rate implying superconsistency

$B(s)$ – Brownian motion

$B(1)^2 - 1 = \chi_1^2$ – a χ^2 distribution

$2\int_0^1 B(s)^2 ds$ – non-normal distribution

Random walk process

Data generating process: $y_t = y_{t-1} + \epsilon_t$

Estimated model.

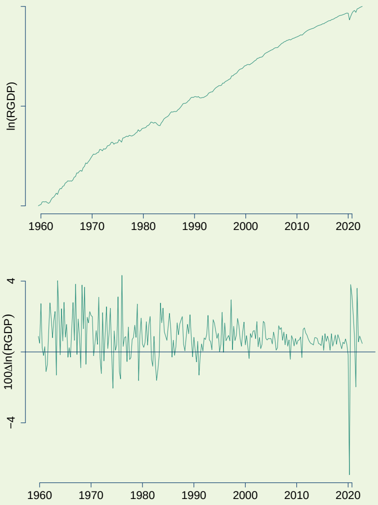
$$y_t = \alpha_1 y_{t-1} + \epsilon_t$$
$$\epsilon_t | y_{t-1} \sim \mathcal{N}\left(0, \sigma_\epsilon^2\right)$$
$$\hat{\alpha}_1 = \frac{\sum_{t=2}^{T} y_t y_{t-1}}{\sum_{t=2}^{T} y_{t-1}^2}$$

Likelihood function.

$$\alpha_1 \sim \mathcal{N}\left(\hat{\alpha}_1, \sigma_\epsilon^2 (X'X)^{-1}\right)$$

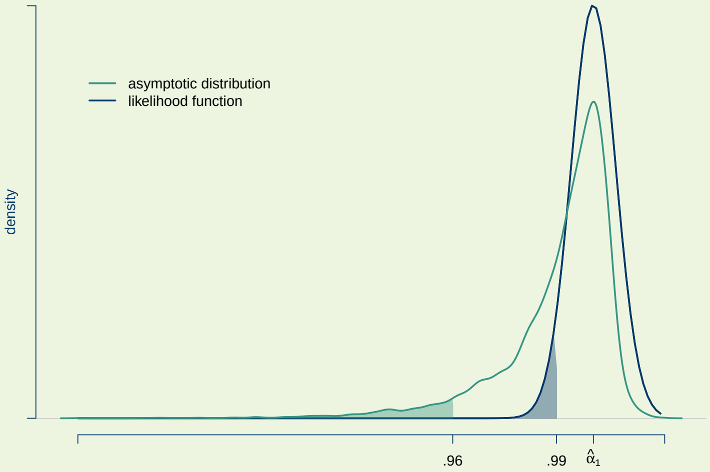
Illustration for Australian real GDP

Australian real GDP



Based on real GDP data for period Q3 1959 – Q4 2022
Data source: Australian Macro Database www.ausmacrodata.org

Australian real GDP



A helicopter tour

A helicopter tour: simulation setup

Data generating process.

$$y_t = \rho y_{t-1} + \epsilon_t \quad \text{and} \quad \epsilon_t \sim \mathcal{N}\left(0, \sigma_\epsilon^2 = 1\right)$$

$$\rho \in \{.8, .81, .82, \dots, 1.09, 1.1\} \quad \text{– a grid of 31 values for } \rho$$

$$y_0 = 0$$

Matrix notation.

$$\mathbf{R}y = E \quad \text{and} \quad E \sim \mathcal{N}\left(\mathbf{0}_T, I_T\right)$$

$$y = \mathbf{R}^{-1}E \quad \text{– generate data from AR(1)} \textit{DGP}$$

$$\underset{(T\times T)}{\mathbf{R}} = \begin{bmatrix} 1 & 0 & 0 & \dots & 0 & 0 \\ -\rho & 1 & 0 & \dots & 0 & 0 \\ 0 & -\rho & 1 & \dots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \dots & -\rho & 1 \end{bmatrix} \qquad y = \begin{bmatrix} y_1 \\ y_2 \\ \vdots \\ y_{T-1} \\ y_T \end{bmatrix}$$

A helicopter tour: simulation setup

Estimated parameter.

$$\hat{\rho}_{ML} = (X'X)^{-1}X'Y$$
$$Y' = \begin{bmatrix} y_2 & y_3 & \dots & y_T \end{bmatrix}$$
$$X' = \begin{bmatrix} y_1 & y_2 & \dots & y_{T-1} \end{bmatrix}$$

Generate the graph.

Generate $S = 50,000$ times $T = 100$ random numbers from $\mathcal{N}(0, 1)$

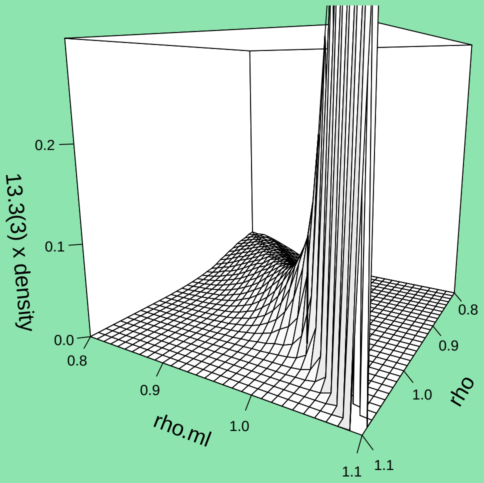
Create For $50,000$ $T \times 1$ normal vectors and for 31 values of ρ from the grid compute y according to the DGP

Estimate $\hat{\rho}_{ML}$ for each of the generated y vector

Compute histograms of $\hat{\rho}_{ML}$ for each of grid values ρ using bins: $(-\infty, .795], (.795, .805], (.805, .815], \dots, (1.095, 1.105], , (1.105, \infty]$

Plot the joint distribution of $\hat{\rho}_{ML}$ and ρ

A helicopter tour



A helicopter tour: conditional distributions

$$p(\hat{\rho}_{ML}|\rho = 1)$$

Is a simulated distribution of MLE of ρ given that the data is generated using a fixed value of $\rho = 1$

Is equivalent to the asymptotic distribution of $\hat{\rho}_{ML}$ with $T \to \infty$

Is used to compute the critical values for unit root tests

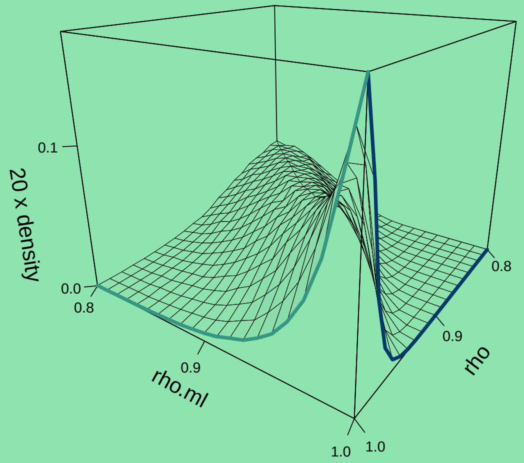
$$p(\rho|\hat{\rho}_{ML} = 1)$$

Is equivalent to the posterior distribution of ρ given that data y is such that $\hat{\rho}_{ML} = 1$ and σ_ϵ^2 is fixed to 1

Prior distribution in this case is set to an improper distribution $p(\rho) \propto 1$, e.g., $p(\rho) = 1$

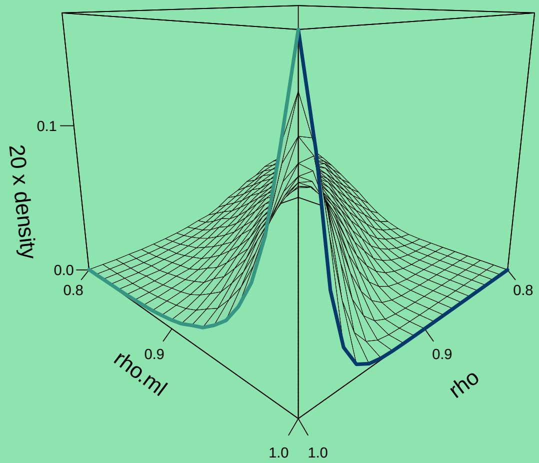
Posterior distribution is then proportional to $\exp\left\{-\frac{1}{2}\left(\rho - \hat{\rho}_{ML}\right)' X'X \left(\rho - \hat{\rho}_{ML}\right)\right\}$

A helicopter tour: joint distribution



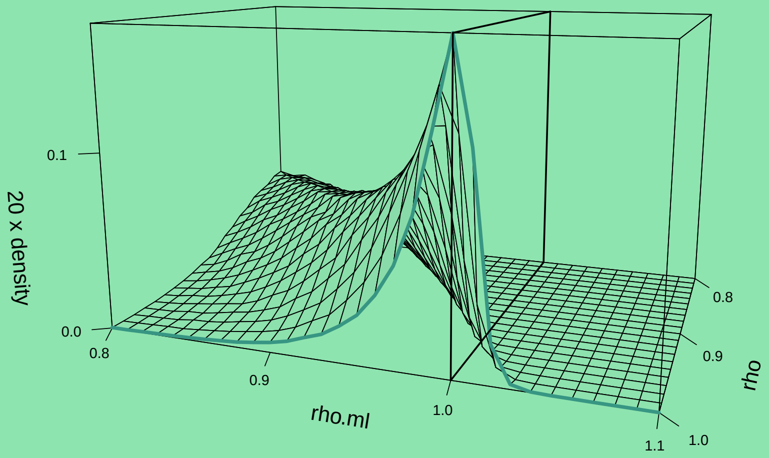
$p(\hat{\rho}_{ML}|\rho = 1)$ $\quad$ $p(\rho|\hat{\rho}_{ML} = 1)$

A helicopter tour: asymmetry



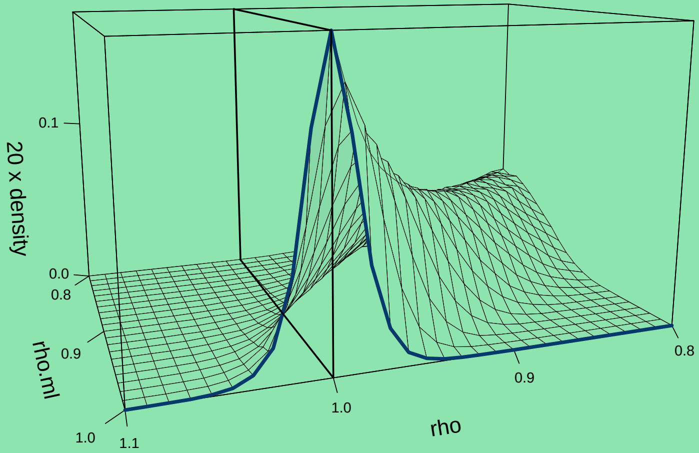
$p(\hat{\rho}_{ML}|\rho = 1)$ $p(\rho|\hat{\rho}_{ML} = 1)$

A helicopter tour: asymptotic distribution



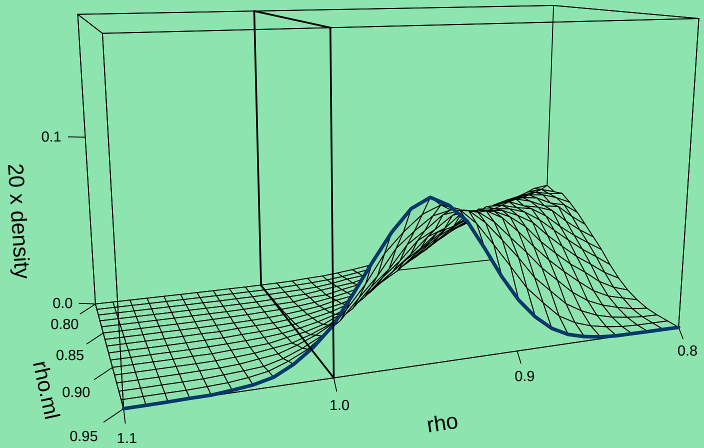
$$p(\hat{\rho}_{ML}|\rho = 1)$$

A helicopter tour: posterior distribution



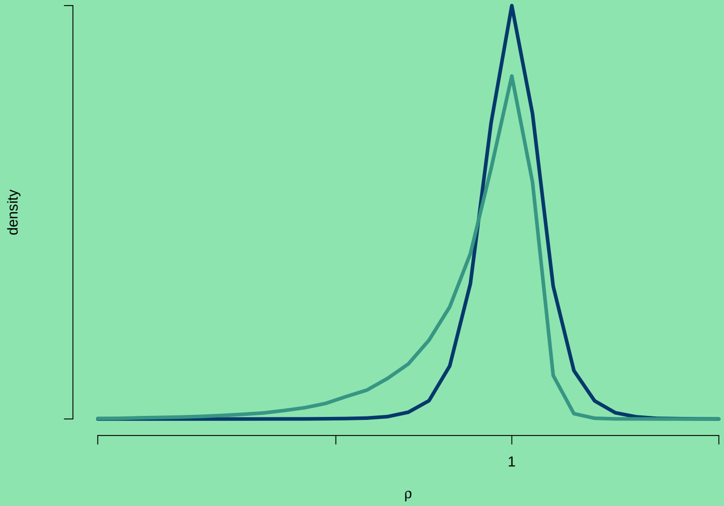
$p(\rho|\hat{\rho}_{ML} = 1)$

A helicopter tour: posterior distribution



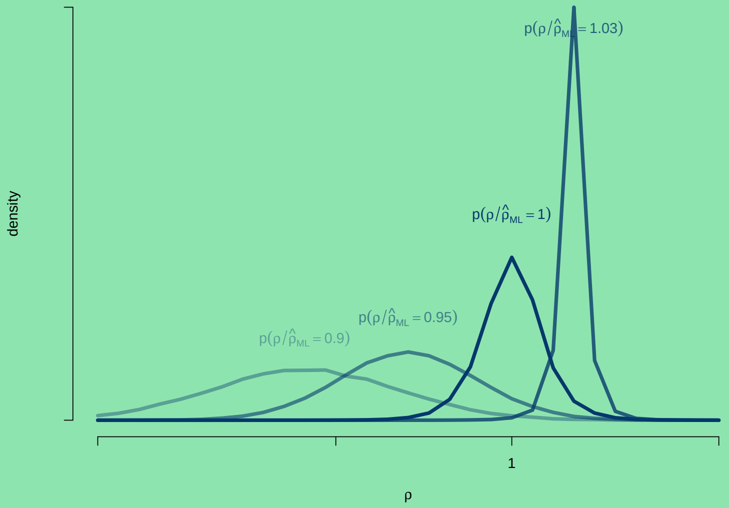
$p(\rho|\hat{\rho}_{ML} = .95)$

A helicopter tour

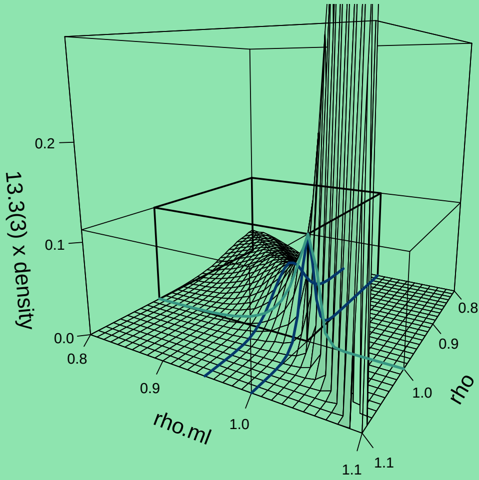


$p(\hat{\rho}_{ML}|\rho = 1)$ $p(\rho|\hat{\rho}_{ML} = 1)$

A helicopter tour



A helicopter tour: behind the scenes



$p(\hat{\rho}_{ML}|\rho = 1)$ $\quad$ $p(\rho|\hat{\rho}_{ML} = 1)$ $\quad$ $p(\rho|\hat{\rho}_{ML} = .95)$

Random walk process

Autoregressions provide convenient parameterization for modeling decaying patterns in autocorrelations of time series

Random walk process imply long-memory and unit autocorrelations at all lags

Note: seemingly conflicting evidence from the asymptotic distribution and the likelihood function

The posterior distribution is normal

Unit-root inference seems to be the only case when Bayesian and frequentist approaches do not converge asymptotically