Macroeconometrics

Lecture 19 Modeling trend inflation

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A first look at the data... no one said it's gonna be easy!

UC models for Australian CPI inflation

UC model for Australian CPI prices

UC model for Australian Real GDP

Materials:

A zip file `L19 mcxs.zip` for the reproduction of the results

Objectives.

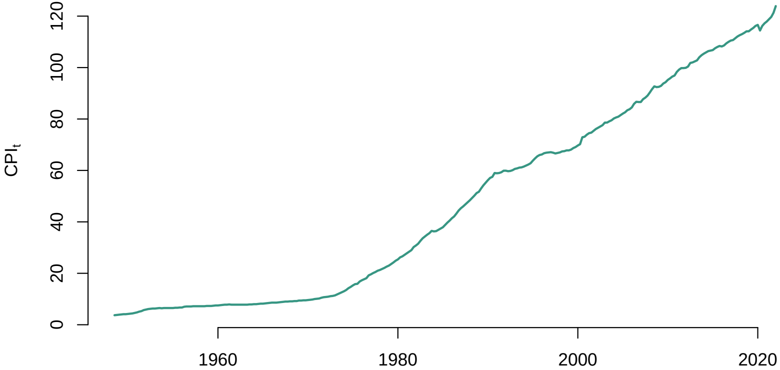
- To familiarise with the parameters, trend, and cycle estimates
- To investigate the definition of the trend via the model specification
- To document the prior dependence of the results

Learning outcomes.

- Documenting the persistence properties of the data
- Visualising the trend and cycle
- Performing prior robustness checks

A first look at the data... no one said it's gonna be easy!

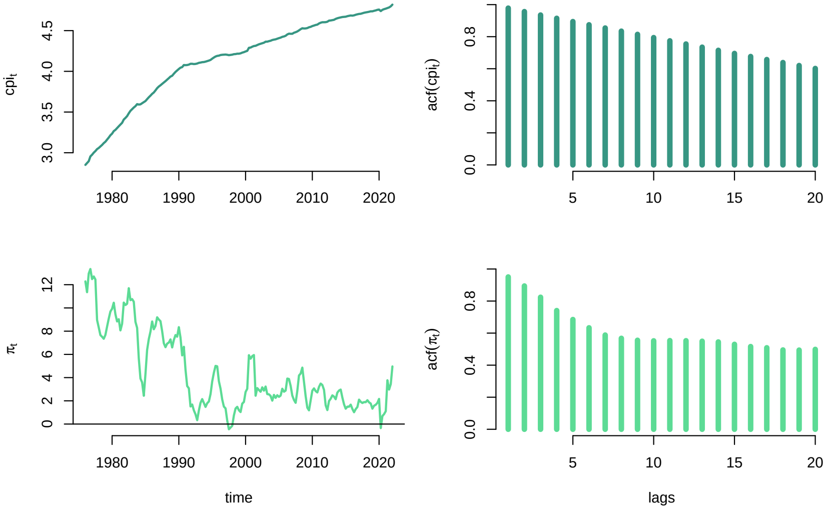
Consumer Price Index in Australia



Quarterly data from 1949Q3 to 2022Q1 ($T = 295$)
Downloaded from the ABS using

```
readabs::read_abs(series_id = "A2325846C")
```

Consumer Price Index in Australia



$cpi_t = log(CPI_t) \qquad \pi_t = 100(cpi_t - cpi_{t-4})$

Quarterly data from 1976Q1 to 2022Q1 ($T = 185$)

Consumer Price Index in Australia

Integration order verification

Quarterly data from 1976Q1 to 2022Q1

variable	deterministic terms	lag order	t_{ADF}	p-value
cpi_t	trend, constant	24	-2.776	0.252
cpi_t	constant	24	-0.922	0.714
π_t	constant	23	-2.324	0.193
π_t	none	23	-2.29	0.023
$\Delta\pi_t$	none	22	-2.868	<0.01

Computations performed using `fUnitRoots::adfTest`

UC models for Australian CPI inflation

Australian CPI inflation

UC-AR(p) model with hierarchical prior for variances.

$$y_t = \tau_t + \epsilon_t$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_p \epsilon_{t-p} + e_t$$

$$\begin{bmatrix} \eta_t \\ e_t \end{bmatrix} \Bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_2, \begin{bmatrix} \sigma_\eta^2 & 0 \\ & \sigma_e^2 \end{bmatrix} \right)$$

Prior hyper-parameters.

$$\underline{\alpha} = \mathbf{0}_p, \qquad \underline{V}_\alpha = \kappa_1 I_p, \qquad \kappa_1 = 1, \qquad \alpha \in A - \text{not imposed}$$
$$\underline{\beta} = \mathbf{0}_2, \qquad \underline{V}_\beta = \kappa_2 I_2, \qquad \kappa_2 = 1$$
$$\underline{\nu} = 3, \qquad s = 0.00346, \qquad a = 1$$

Estimation via Gibbs sampler as presented in Lectures 17 & 18

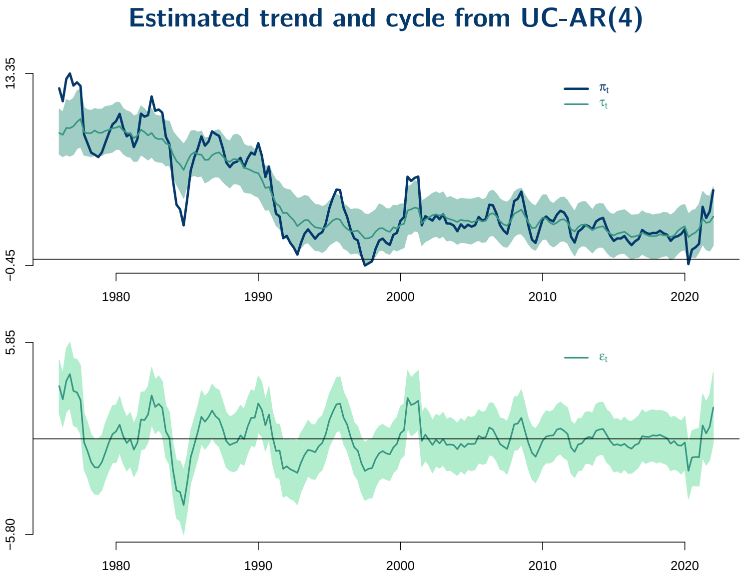
Australian CPI inflation

Estimation results from UC-AR(p) models: π_t 1976Q1–2022Q1

p	α_1	α_2	α_3	α_4	α_5	% s	μ	σ_η^2/σ_e^2
1	0.893					95.9	-0.011	2.009
	0.091						0.055	
2	1.221	-0.346				98.4	-0.015	2.288
	0.203	0.213					0.054	
3	1.092	-0.003	-0.267			99.8	-0.018	1.398
	0.131	0.181	0.111				0.048	
4	0.838	0.158	0.101	-0.395		100	-0.022	0.750
	0.108	0.130	0.130	0.115			0.039	
5	1.025	0.089	0.037	-0.624	0.330	99.2	-0.026	0.382
	0.095	0.119	0.119	0.128	0.100		0.033	

% s – fraction of posterior draws for which stationarity condition holds
σ_η^2/σ_e^2 – signal-to-noise ratio

Australian CPI inflation



Australian CPI inflation

UC-AR(p) with hierarchical prior and correlated shocks.

$$y_t = \tau_t + \epsilon_t$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_p \epsilon_{t-p} + e_t$$

$$\begin{bmatrix}\eta_t \\ e_t\end{bmatrix} \bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_2, \begin{bmatrix}\sigma_\eta^2 & \rho\sigma_\eta\sigma_e \\ & \sigma_e^2\end{bmatrix}\right)$$

Prior hyper-parameters.

$$\begin{aligned}
&\underline{\alpha} = \mathbf{0}_p, \qquad \underline{V}_\alpha = \kappa_1 I_p, \qquad \kappa_1 = 1, \qquad \alpha \in A - \text{not imposed} \\
&\underline{\beta} = \mathbf{0}_3, \qquad \underline{V}_\beta = \kappa_2 I_3, \qquad \kappa_2 = 1 \\
&\underline{\nu} = 3, \qquad s = 0.00346, \qquad a = 1
\end{aligned}$$

Australian CPI inflation

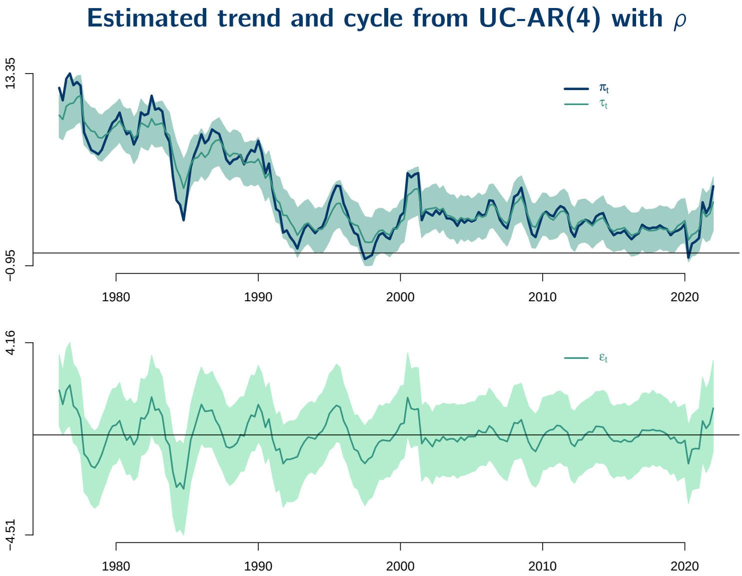
Estimation results from UC-AR(p) models: π_t 1976Q1–2022Q1

p	α_1	α_2	α_3	α_4	α_5	% s	μ	ρ	σ_η^2/σ_e^2
2	1.240	-0.374				97/7	-0.011	0.056	4.693
	0.266	0.262					0.079	0.053	
3	1.038	-0.025	-0.246			99.6	-0.014	0.079	7.583
	0.313	0.244	0.212				0.066	0.060	
4	0.828	0.139	0.105	-0.398		99.8	-0.020	0.132	2.380
	0.179	0.196	0.187	0.165			0.057	0.069	
5	1.013	0.094	0.054	-0.664	0.331	99.1	-0.023	0.102	1.184
	0.132	0.149	0.148	0.154	0.126		0.058	0.064	

% s – fraction of posterior draws for which stationarity condition holds

σ_η^2/σ_e^2 – signal-to-noise ratio

Australian CPI inflation



Australian CPI inflation

UC-AR(p) with gamma prior.

$$y_t = \tau_t + \epsilon_t$$

$$\tau_t = \mu + \tau_{t-1} + \eta_t$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_p \epsilon_{t-p} + e_t$$

$$\begin{bmatrix} \eta_t \\ e_t \end{bmatrix} \Bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_2, \begin{bmatrix} \sigma_\eta^2 & 0 \\ & \sigma_e^2 \end{bmatrix}\right)$$

$$\sigma_\eta^2 \mid \underline{s} \sim \mathcal{G}(s, a)$$

Australian CPI inflation

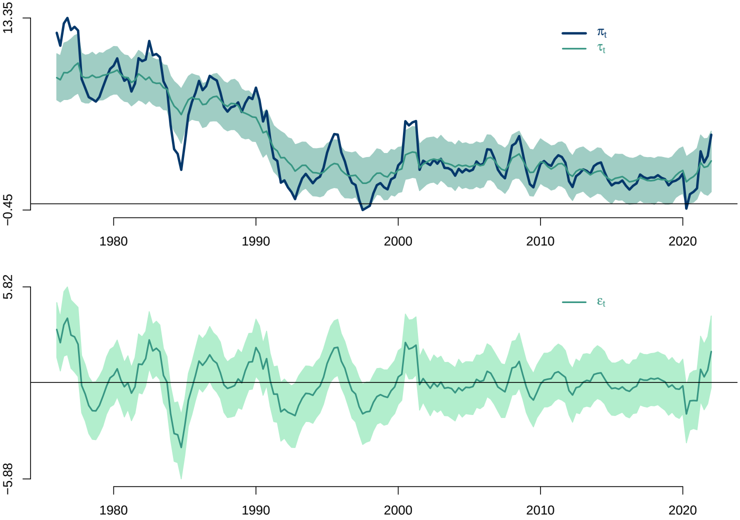
Estimation results from UC-AR(p) models: π_t 1976Q1–2022Q1

α_1	α_2	α_3	α_4	α_5	% s	μ	σ^2_η/σ^2_e
0.841	0.160	0.091	-0.387		100	-0.021	0.780
0.114	0.135	0.136	0.120			0.041	

% s – fraction of posterior draws for which stationarity condition holds
σ^2_η/σ^2_e – signal-to-noise ratio

Australian CPI inflation

Estimated trend and cycle from UC-AR(4) with gamma prior



Australian CPI inflation

Stochastic or deterministic trend?

Testing restriction $\sigma_\eta^2 = 0$ is difficult. However

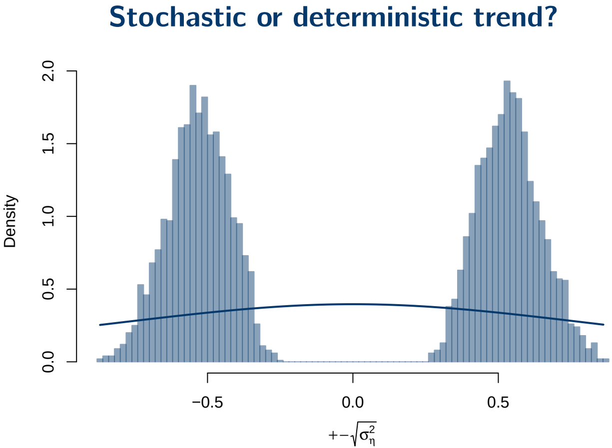
$$\sigma_\eta^2 \mid \underline{s} \sim \mathcal{G}\left(2\underline{s}, \frac{1}{2}\right) \qquad \Rightarrow \qquad \pm\sqrt{\sigma_\eta^2} \mid \underline{s} \sim \mathcal{N}\left(0, \underline{s}\right)$$

Testing restriction $\pm\sqrt{\sigma_\eta^2} = 0$ seems easier.

Savage-Dickey density ratio for $\pm\sqrt{\sigma_\eta^2} = 0$:

$$\frac{p\left(\pm\sqrt{\sigma_\eta^2} = 0 \mid data\right)}{p\left(\pm\sqrt{\sigma_\eta^2} = 0\right)} = \frac{\Pr\left[\pm\sqrt{\sigma_\eta^2} = 0 \mid data\right]}{\Pr\left[\pm\sqrt{\sigma_\eta^2} = 0\right]}$$

Australian CPI inflation



The value of the SDDR is clearly less than 1 which constitutes strong Bayesian evidence against the restriction.

UC model for Australian CPI prices

Australian CPI prices

UC-AR(4) with time-varying drift.

$$y_t = \tau_t + \epsilon_t$$

$$\tau_t = \mu_t + \tau_{t-1} + \eta_t$$

$$\mu_t = \mu_{t-1} + m_t$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_4 \epsilon_{t-4} + e_t$$

$$\begin{bmatrix} \eta_t \\ e_t \\ m_t \end{bmatrix} \Bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_3, \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ & \sigma_e^2 & 0 \\ & & \sigma_m^2 \end{bmatrix} \right)$$

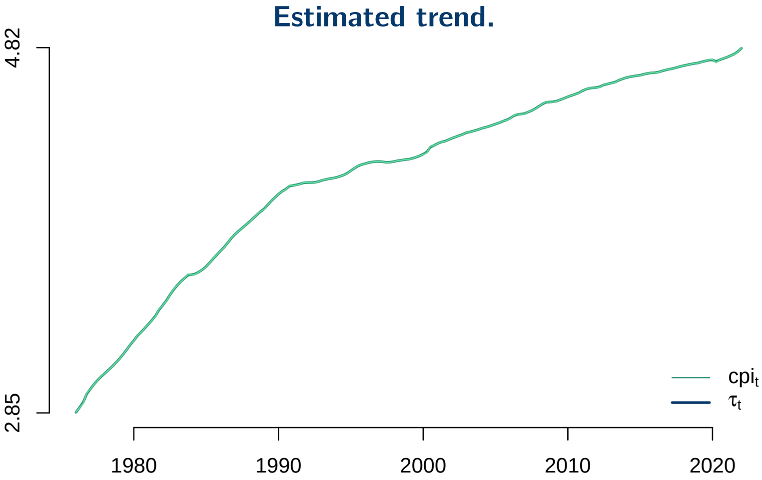
Australian CPI prices

Estimation results from UC-AR(4) *cpi_t* 1976Q1–2022Q1

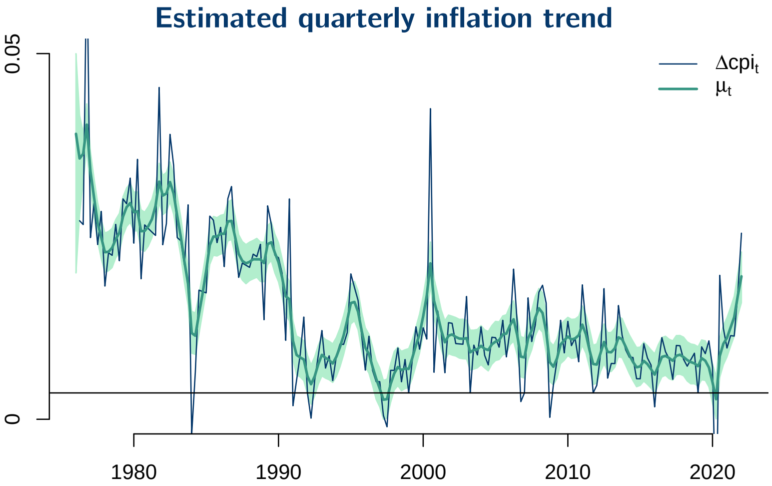
α_1	α_2	α_3	α_4	% s	σ^2_η/σ^2_e	τ_0	μ_0
0.211	-0.085	0.017	0.129	0.956	1.911	2.816	0.034
0.334	0.222	0.206	0.188			0.016	0.024

% s – fraction of posterior draws for which stationarity condition holds
σ^2_η/σ^2_e – signal-to-noise ratio

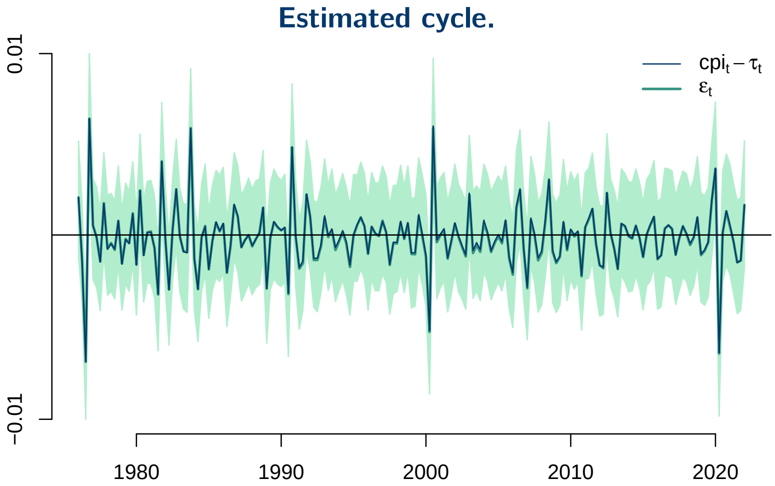
Australian CPI prices



Australian CPI prices

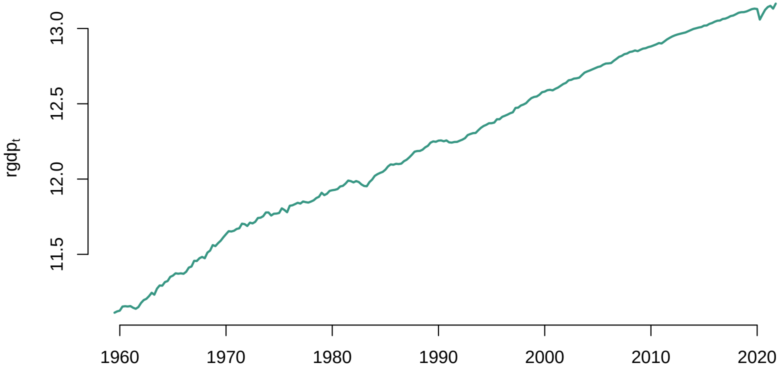


Australian CPI prices



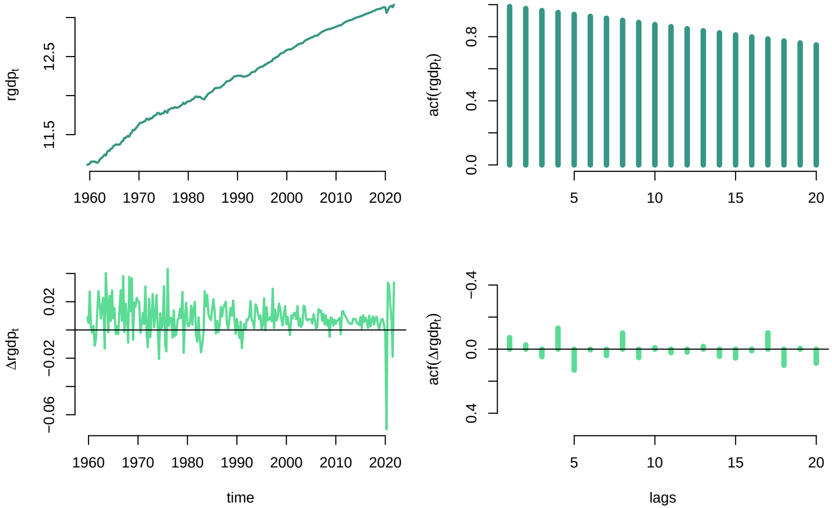
UC model for Australian Real GDP

Australian real GDP



Quarterly data from 1959Q3 to 2021Q4 ($T = 250$)
Downloaded from the ABS using
`readabs::read_abs(series_id = "A2304402X")`

Australian real GDP



$rgdp_t = log(GDP_t)$ and $\Delta rgdp_t = rgdp_t - rgdp_{t-1}$

Quarterly data from 1959Q3 to 2021Q4 ($T = 250$)

Australian real GDP

UC-AR(4) with time-varying drift.

$$y_t = \tau_t + \epsilon_t$$

$$\tau_t = \mu_t + \tau_{t-1} + \eta_t$$

$$\mu_t = \mu_{t-1} + m_t$$

$$\epsilon_t = \alpha_1 \epsilon_{t-1} + \cdots + \alpha_4 \epsilon_{t-4} + e_t$$

$$\begin{bmatrix} \eta_t \\ e_t \\ m_t \end{bmatrix} \Bigg| Y_{t-1} \sim ii\mathcal{N}\left(\mathbf{0}_3, \begin{bmatrix} \sigma_\eta^2 & 0 & 0 \\ & \sigma_e^2 & 0 \\ & & \sigma_m^2 \end{bmatrix} \right)$$

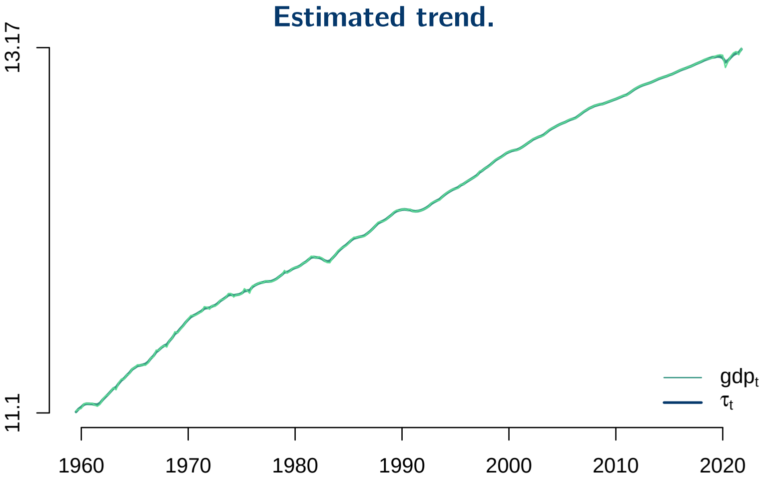
Australian real GDP

Estimation results from UC-AR(4) gdp_t 1959Q3–2022Q1

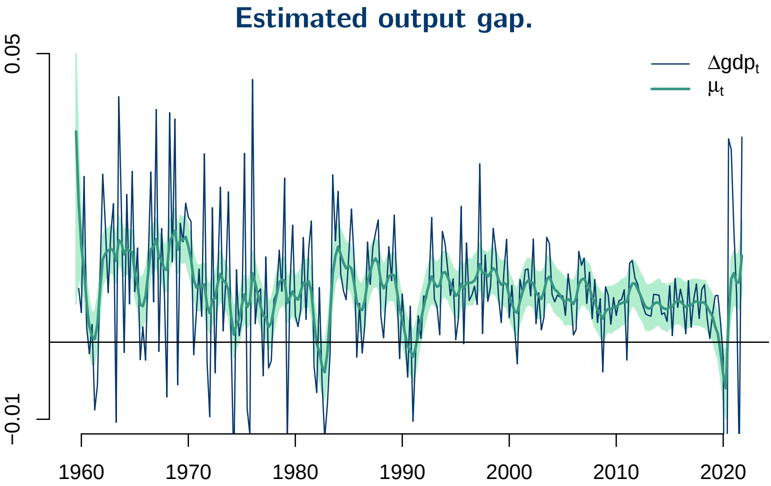
α_1	α_2	α_3	α_4	% s	σ_η^2/σ_e^2	τ_0	μ_0
0.007	-0.262	-0.149	-0.297	0.992	1.311	11.072	0.046
0.281	0.208	0.191	0.141			0.023	0.031

% s – fraction of posterior draws for which stationarity condition holds
σ_η^2/σ_e^2 – signal-to-noise ratio

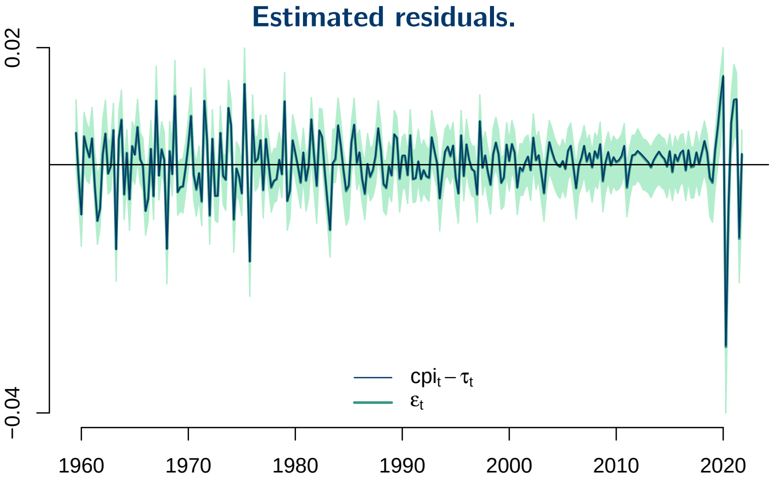
Australian real GDP



Australian real GDP



Australian real GDP



Australian CPI prices and inflation

Unobserved Component models correctly capture the dynamics in Australian CPI prices and inflation.

Trend-cycle decomposition indicates increasing long-run inflation trend.

Hierarchical priors are the essential extension.