ECE 6255 - Homework 6

Full name : WILD Adam
gtid : 903343514
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Problem 1 - Rabiner and Schafer 9.1

(a) We will use the following change of variable : $n' = n - m$, we have :

$$R[-m] = \sum_{n=0}^{+\infty} h[n]h[n-m] = \sum_{n'=0}^{+\infty} h[n'+m]h[n'] = R[m]$$

(b)

$$\begin{aligned}
R[-m] &= \sum_{n=0}^{+\infty} h[n]h[n-m] \\
&= \sum_{n=0}^{+\infty} \left(\left(\sum_{k=1}^{p} \alpha_k h[n-k] + G\delta[n] \right) \left(\sum_{k=1}^{p} \alpha_k h[n-k-m] + G\delta[n-m] \right) \right) \\
&= \sum_{n=0}^{+\infty} \left(\sum_{k=1}^{p} \alpha_k h[n-k]. \sum_{k=1}^{p} \alpha_k h[n-k-m] + G\delta[n] \sum_{k=1}^{p} \alpha_k h[n-k-m] + G\delta[n-m] \sum_{k=1}^{p} \alpha_k h[n-k] \right) \\
&= \sum_{n=0}^{+\infty} \left(\sum_{k=1}^{p} \alpha_k h[n-k]. \sum_{k=1}^{p} \alpha_k h[n-k-m] \right) \\
&= \sum_{k=1}^{p} \alpha_k \sum_{n=0}^{+\infty} h[n]h[n+|m-k|] \\
&= \sum_{k=1}^{p} \alpha_k R[|m-k|]
\end{aligned}$$

Problem 2 - Rabiner and Schafer 9.6

(a) By reading the table, we have :

$$A^{(4)}(z) = 1 - \sum_{k=1}^{4} \alpha_k^{(4)} z^{-k} = 1 - 0.8047z^{-1} - 0.0414z^{-2} + 0.4940z^{-3} - 0.4337z^{-4}$$

(b) We have that $\alpha_i^{(i)} = k_i$, therefore

$$\begin{aligned}
k_1 &= 0.8328 \\
k_2 &= 0.1044 \\
k_3 &= 0.1786 \\
k_4 &= -0.4337
\end{aligned}$$

(c)

$$\begin{aligned}
A^{(3)}(z) - k_4 z^{-4} A^{(3)}(z^{-1}) &= 1 - (0.7273z^{-1} - 0.0289z^{-2} + 0.1786z^{-3}) + 0.4337(0.7273z^{-3} - 0.0289z^{-2} + 0.1786z^{-4}) \\
&= 1 - 0.8047z^{-1} - 0.0414z^{-2} + 0.4940z^{-3} - 0.4337z^{-4} \\
&= A^{(4)}(z)
\end{aligned}$$

Problem 3 - Rabiner and Schafer 9.10

$$\begin{aligned}
A^{(i)}(z) &= 1 - \sum_{k=1}^{i} \alpha_k^{(i)} z^{-k} \\
&= 1 - \left(\sum_{k=1}^{i-1} \alpha_k^{(i)} z^{-k} + \alpha_i^{(i)} z^{-i} \right) \\
&= 1 - \sum_{k=1}^{i-1} \alpha_k^{(i-1)} z^{-k} + k_i z^{-i} \sum_{k=1}^{i-1} \alpha_{i-k}^{(i-1)} z^{i-k} - k_i z^{-i} \\
&= \left(1 - \sum_{k=1}^{i-1} \alpha_k^{(i-1)} z^{-k} \right) - k_i z^{-i} \left(1 - \sum_{k=1}^{i-1} \alpha_{i-k}^{(i-1)} z^{i-k} \right) \\
&= A^{(i-1)}(z) - k_i z^{-i} A^{(i-1)}(z^{-1})
\end{aligned}$$

Problem 4 - Rabiner and Schafer 9.18

(a)
The optimum value of β verifies $\frac{d\epsilon}{d\beta} = 0$

$$\begin{aligned}
\frac{d\epsilon}{d\beta} &= \sum_m \frac{d\epsilon}{d\beta} (s[m] - \beta s[m - N_p])^2 \\
&= 2\beta \sum_m (\beta s[m - N_p] - s[m]) \\
&= 0
\end{aligned}$$

That is :

$$\begin{aligned}
2\beta \sum_m (\beta s[m - N_p] - s[m]) = 0 \quad &\Longleftrightarrow \quad \beta \sum_m s[m - N_p] = \sum_m s[m] \\
&\Longleftrightarrow \quad \beta = \frac{\sum_m s[m]}{\sum_m s[m - N_p]}
\end{aligned}$$

(b) Once the β value is found, we have to minimize the error by varying only N_p which is a delay. Since the signal is finite, we can test all values of N_p and retain the value that minimize the error. The complexity of this method is linear with the size of the sample.

Problem 5

We get the best result with the autocorrelation method