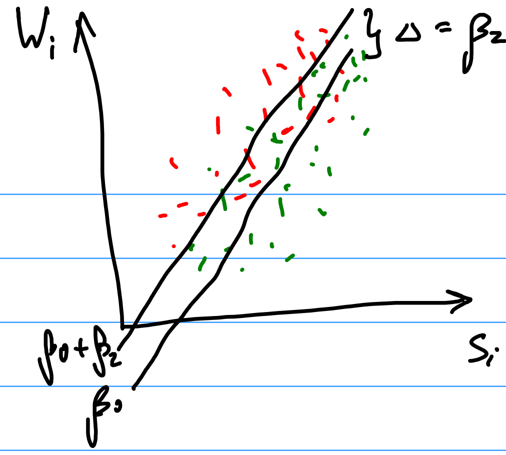
Dummy Variable



ⓐ $$D_i = \begin{cases} 1, & \text{female} \\ 0, & \text{male} \end{cases}$$

$$W_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot D_i + \varepsilon_i$$

$(D_i = 1)$, female: $W_i = (\beta_0 + \beta_2) + \beta_1 \cdot S_i + \varepsilon_i$

$(D_i = 0)$ male: $w_i = \beta_0 + \beta_1 \cdot S_i + \varepsilon_i$

$H_0: \beta_2 = 0$ (t-test)

ⓑ $$y_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot S_i \cdot D_i + \varepsilon_i$$

female: $w_i = \beta_0 + (\beta_1 + \beta_2) \cdot S_i + \varepsilon_i$

male: $W_i = \beta_0 + \beta_1 \cdot S_i + \varepsilon_i$

$H_0: \beta_2 = 0$ (t-test)

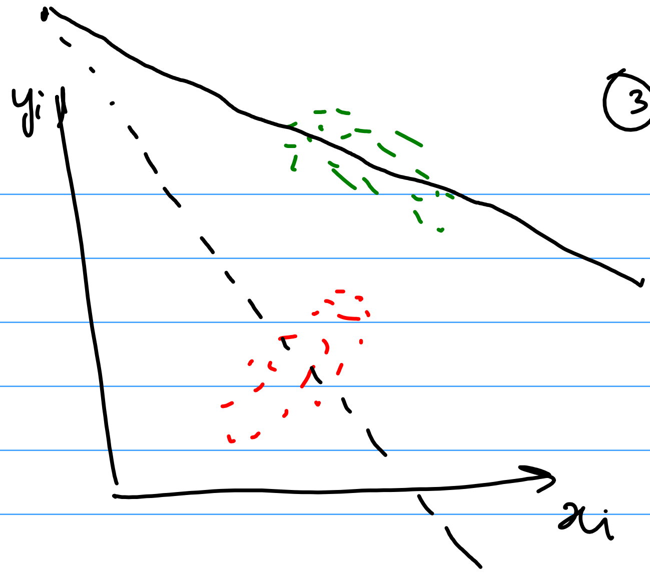
ⓒ $$W_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot D_i + \beta_3 \cdot D_i \cdot S_i + \varepsilon_i$$

female: $W_i = (\beta_0 + \beta_2) + (\beta_1 + \beta_3) S_i + \varepsilon_i$

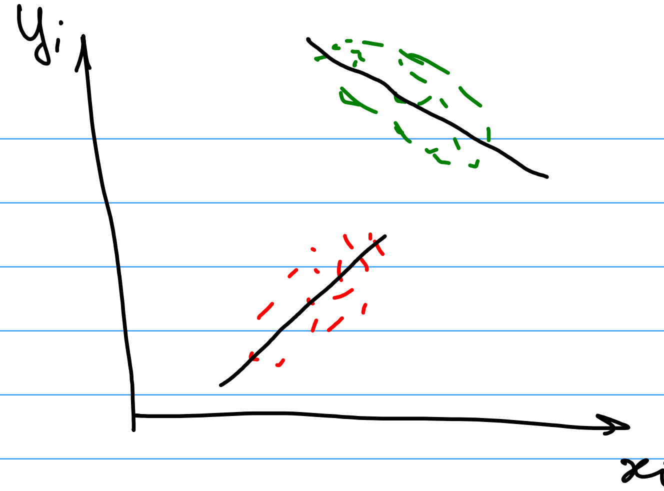
male: $W_i = \beta_0 + \beta_1 \cdot S_i + \varepsilon_i$

$H_0: \beta_2 = \beta_3 = 0$ (F-test)

②



③



$\alpha \neq 0$ (1) $y = \alpha + \beta x$

$\alpha = 0$ (2) $y = \beta x$

$$p_t = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot D_{W_t} + \beta_3 \cdot D_{sp_t} + \beta_4 \cdot D_{A_t} + \varepsilon_t$$

Winter: $p_t = (\beta_0 + \beta_2) + \beta_1 t + \varepsilon_t$

(7) $$p_t = \beta_0 + \beta_1 \cdot t + \beta_2 \cdot D_{W_t} + \beta_3 \cdot D_{sp_t} + \beta_4 \cdot D_{A_t} + \beta_5 \cdot D_{sum_t} + \varepsilon_t$$

$$\begin{bmatrix} 1 \\ \vdots \\ \vdots \\ \vdots \\ 1 \end{bmatrix} = \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix} + \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

Perfect multicol. (Dummy-variable)

$$\text{rank}(X) = 5 < 6$$

$$\hat{\beta} = (X'X)^{-1} \cdot X'y$$

$\det(X'X) = 0 \Rightarrow X'X$ not invertible $\Rightarrow \hat{\beta}$

$$p_t = \cancel{\beta_0} + \beta_1 \cdot t + \beta_2 \cdot D_{W_t} + \beta_3 \cdot D_{sp_t} + \beta_4 \cdot D_{A_t} + \beta_5 \cdot D_{sum_t} + \varepsilon_t$$

Winter: $p_t = \beta_2 + \beta_1 t + \varepsilon_t$

Summer: $p_t = \beta_5 + \beta_1 t + \varepsilon_t$

Chow Test : n pool: $y_i^R = \beta_0 + \beta_1 \cdot X_{1i} + \dots + \beta_k \cdot X_{ki} + \varepsilon_i$

$$\begin{cases} n_A & A: y_i = \beta_0^A + \beta_1^A \cdot X_{1i} + \dots + \beta_k^A \cdot X_{ki} + \varepsilon_i \\ + & \\ n_B & B: y_i = \beta_0^B + \beta_1^B \cdot X_{1i} + \dots + \beta_k^B \cdot X_{ki} + \varepsilon_i \end{cases}$$

$H_0: \beta_0^A = \beta_0^B, \dots, \beta_k^A = \beta_k^B \qquad \partial = \beta^B - \beta^A$

$$F = \frac{(RSS_{pool} - (RSS_A + RSS_B)) / (k+1)}{(RSS_A + RSS_B) / (N - 2(k+1))} \sim F(k+1, N-2(k+1))$$

$$y_i^{UR} = \beta_0 + \beta_1 \cdot X_{1i} + \dots + \beta_k \cdot X_{ki} + \partial_0 D_i + \partial_1 \cdot D_i \cdot X_{1i} + \dots + \partial_k D_i X_{ki} + \varepsilon_i$$

$H_0: \partial_0 = \dots = \partial_k = 0$

$$F = \frac{(\underbrace{RSS^R}_{= RSS_{pool}} - \underbrace{RSS^{UR}}_{= RSS_A + RSS_B}) / (k+1)}{RSS^{UR} / (N - 2 \cdot (k+1))}$$

$$\log(y_i) = \beta_0 + \beta_1 \cdot x_i + \beta_2 \cdot D_i + \varepsilon_i$$

β_1 : $x_i \uparrow 1$ $\quad y \uparrow \beta_1 \cdot 100\%$ $\quad \partial \ln y / \partial x_i$

β_2 : $D_i \uparrow 1$ $\quad y \uparrow \beta_2 \cdot 100\%$? $\quad \partial \ln y / \partial D_i$

$\Delta \ln y / \Delta D_i$

$\Delta D = 1$ $\quad \Delta y = 100 \cdot \dfrac{\Delta y}{y_0} \%$

$$\Delta y = y(D_i = 1) - y(D_i = 0) = e^{\beta_0 + \beta_1 \cdot X_i + \beta_2} - e^{\beta_0 + \beta_1 \cdot X_i} =$$

$$e^{\beta_0 + \beta_1 \cdot X_i}(e^{\beta_2} - 1)$$

$$100 \cdot \frac{e^{\beta_0 + \beta_1 \cdot X_i}(e^{\beta_2} - 1)}{e^{\beta_0 + \beta_1 \cdot x_i}} = 100 \cdot (e^{\beta_2} - 1) \%$$

$\Rightarrow$ $\quad D_i \uparrow 1$ $\quad y \uparrow 100 \cdot (e^{\beta_2} - 1)\%$