$$y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$$

(a) $H_0: \beta_2 = 0$

$H_a: \beta_2 \neq 0$

$$t = \frac{\hat{\beta}_2 - 0}{se(\hat{\beta}_2)} \sim t_{n-k}$$

obs

est. coef.

(b) $H_0: \beta_2 = 1$

$$t = \frac{\hat{\beta}_2 - 1}{se(\hat{\beta}_2)} \sim t_{n-k}$$

(c) $H_0: \beta_2 = \beta_3 = 0$

$H_a: \beta_2 \neq 0 \lor \beta_3 \neq 0$

$$F = \frac{R^2/2}{1-R^2/n-3} \sim F(2, n-3)$$

F-test for linear restrictions

$H_0: \beta_4 = \beta_7 = \beta_8 = 0 \quad \Rightarrow RSS_R$

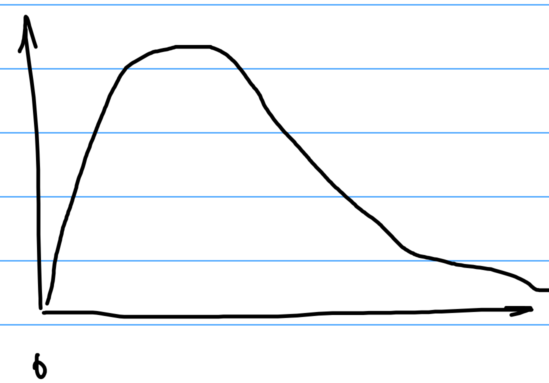
H_a: one of restrictions isn't satysfied $\quad \Rightarrow RSS_{UR}$

# of linear restrictions

$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n-k)} \sim F(q, n-k)$$

est. coef. in UR model

$RSS_R > RSS_{UR}$



F-test for goodness of fit

$H_0: \beta_2 = \dots \beta_k = 0 \qquad y_i \overset{R}{=} \beta_1 + \varepsilon_i$

$H_a: \exists i \ \beta_i \neq 0$

$$F = \frac{(TSS - RSS)/k-1}{RSS/n-k} = \frac{ESS/k-1}{RSS/n-k} = \frac{R^2/k-1}{(1-R^2)/n-k} \sim F(k-1, n-k)$$

d) $H_0: \beta_2 = \beta_3$ $\qquad \theta = \beta_2 - \beta_3 = 0$

$\beta_3 = \beta_2 - \theta$

UR: $y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$

R: $y_i = \beta_1 + \beta_2 (X_{2i} + X_{3i}) + \varepsilon_i$

t: $y_i = \beta_1 + \beta_2 (X_{2i} + X_{3i}) - \theta X_{3i} + \varepsilon_i$

$$F = \frac{(RSS_R - RSS_{UR})/q}{RSS_{UR}/(n-k)} \sim F(q, n-k)$$

$$= \frac{\left(RSS_R/TSS - RSS_{UR}/TSS\right)/q}{RSS_{UR}/TSS \,/\, n-k} =$$

$$= \frac{(1 - R^2_R - 1 + R^2_{UR})/q}{(1 - R^2_{UR})/n-k} = \frac{R^2_{UR} - R^2_R / q}{1 - R^2_{UR} / n-k}$$

(e) $H_0: \beta_2 + \beta_3 = 1$

UR: $y_i = \beta_1 + \beta_2 X_{2i} + \beta_3 X_{3i} + \varepsilon_i$

R: $y_i = \beta_1 + \beta_2 X_{2i} + (1-\beta_2) X_{3i} + \varepsilon_i$

$y_i - X_{3i} = \beta_1 + \beta_2 (X_{2i} - X_{3i}) + \varepsilon_i$

$\Rightarrow$ F-test using RSS

(not R^2!)

$TSS_R \neq TSS_{UR}$

PE?

$$y_i = \beta_1 + \beta_2 x_i + \varepsilon_i \qquad i = \overline{1, n}$$

$$\hat{y}_{n+1} = \hat{\beta}_1 + \hat{\beta}_2 x_{n+1}$$

$$E(\hat{y}_{n+1}) = E(\hat{\beta}_1 + \hat{\beta}_2 x_{n+1}) = \beta_1 + \beta_2 x_{n+1}$$

$$E(y_{n+1}) = E(\beta_1 + \beta_2 x_{n+1} + \varepsilon_i) = \beta_1 + \beta_2 x_{n+1} + 0$$

$$\Rightarrow \quad E(y_{n+1}) = E(\hat{y}_{n+1})$$

$$Var(\hat{y}_{n+1} - y_{n+1}) = E(\hat{y}_{n-1} - y_{n+1})^2 =$$

$$= E(\hat{\beta}_1 + \hat{\beta}_2 \cdot x_{n+1} - \beta_1 - \beta_2 x_{n+1} - \varepsilon_{n+1})^2 =$$

$$= E((\hat{\beta}_1 - \beta_1) + (\hat{\beta}_2 - \beta_2) x_{n+1} - \varepsilon_{n+1})^2 =$$

$$= E(\hat{\beta}_1 - \beta_1)^2 + x_{n+1} \cdot E(\hat{\beta}_2 - \beta_2)^2 + E(\varepsilon_{n+1})^2 +$$

$$2 x_{n+1} E((\hat{\beta}_1 - \beta_1)(\hat{\beta}_2 - \beta_2)) - 2 E((\hat{\beta}_1 - \beta_1) \cdot \varepsilon_{n+1}) -$$

$$-2 x_{n+1} E((\hat{\beta}_2 - \beta_2) \varepsilon_{n+1}) =$$

$$= var(\hat{\beta}_1) + x_{n+1} \cdot var(\hat{\beta}_2) + \sigma^2 +$$

$$2 x_{n+1} \operatorname{cov}(\hat{\beta}_1, \hat{\beta}_2) - 0 - 0 =$$

$$= \frac{\sigma^2 \Sigma x_i^2}{n \Sigma (x_i - \bar{x})^2} + x_{n+1} \frac{\sigma^2}{\Sigma (x_i - \bar{x})^2} + \sigma^2 + 2 x_{n+1} \frac{-\bar{x} \sigma^2}{\Sigma (x_i - \bar{x})^2} =$$

$$= \sigma^2 \left(1 + \frac{1}{n} + \frac{(X_{n+1} - \bar{X})^2}{\sum (X_i - \bar{X})^2} \right)$$

$$\hat{\Updownarrow}$$

$$s^2$$

$$\sigma^2(\beta) = \sigma^2 (X'X)^{-1} = \begin{bmatrix} \frac{\sigma^2 \sum X_i^2}{n \sum (X_i - \bar{X})^2} & \frac{-\bar{X}\sigma^2}{\sum (X_i - \bar{X})^2} \\ \frac{-\bar{X}\sigma^2}{\sum (X_i - \bar{X})^2} & \frac{\sigma^2}{\sum (X_i - \bar{X})^2} \end{bmatrix}$$

$$X'X = \begin{bmatrix} n & \sum X_i \\ \sum X_i & \sum X_i^2 \end{bmatrix}$$

Multicollinearity

- perfect multicollinearity
  ↳ one regressor is a lin. com. of others

$$\hat{\beta} = (X'X)^{-1}X'y$$

X - not full rank

$X'X$ - cannot be inversed

$$\begin{pmatrix} 1 & 1 & 0 & x_1 \\ \vdots & \vdots & \vdots & \vdots \\ \vdots & 1 & 0 & \vdots \\ \vdots & 0 & 1 & \vdots \\ \vdots & \vdots & \vdots & \vdots \\ 1 & 0 & 1 & x_n \end{pmatrix}$$

$\bar{1} = m_i + f_i$

dummy variable trap (example of perfect MC) multicollinearity:

Consequences:

- $se(\hat{\beta}_i)$ are inflated
- t-stat. are decreased
- instability of estimates (change in sign)