OLS with ARMA errors Time Series Analysis $G_t = \alpha + \beta \Pi_t + u_t$

↕ ARIMAX (p, d, q) $u_t = \alpha + \beta u_{t-1} + \varepsilon_t \leftarrow AR(1)$

ARMAX (p, q)

$$y_t = \beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + X_t + \underbrace{u_t + \alpha_1 u_{t-1} + \dots + \alpha_q u_{t-q}}_{\varepsilon''_t}$$

$cov(\varepsilon_t, \varepsilon_{t-1}) \neq 0 \Rightarrow$ autocorrelation of error term

① ARDL (p, q)

$$y_t = \beta_0 + \beta_1 \cdot y_{t-1} + \dots + \beta_p y_{t-p} + X_t + \alpha_1 \cdot X_{t-1} + \dots + \alpha_q X_{t-q} + \varepsilon_t$$

Where ε_t – White Noise

$E(\varepsilon_t \mid y_{t-1}, \dots, y_{t-p}, X_t, \dots, X_{t-q}) = 0$

$cov(\varepsilon_i, \varepsilon_j) = 0, \; i \neq j$

$Var(\varepsilon_t) = \sigma^2_\varepsilon$

L – lag operator

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix} \qquad Ly = \begin{pmatrix} 0 \\ y_1 \\ \\ y_{T-1} \end{pmatrix}$$

$L \cdot y_t = y_{t-1}$

$L^2 \cdot y_t = y_{t-2}$

$L^k \cdot y_t = y_{t-k}$

Weak stationarity

$E(y_t) = \mu \quad \forall t$

$Cov(y_t, y_{t-k}) = \gamma_k \quad \forall t$

$Var(y_t) = \sigma^2$

Non-stationary:

1) Difference stationary

$\Delta y_t = y_t - y_{t-1}$ - stationary

2) Trend stationary

$y_t - \alpha - \beta t$

P1

$u_t - WN$

$$y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 y_{t-1} + u_t$$

$|\beta_2| \leq 1$

$$y_t = \beta_0 + \beta_1 X_t + \beta_2 L y_t + u_t$$

$$(1 - \beta_2 L) y_t = \beta_0 + \beta_1 X_t + u_t$$

$$y_t = \frac{\beta_0}{1-\beta_2 L} + \underbrace{\overbrace{\frac{\beta_1 X_t}{1-\beta_2 L}}^{= b_0}}_{= q} + \frac{u_t}{1-\beta_2 L}$$

$$\left\{ y_t = \beta_0^* + \beta_1 X_t + \underbrace{\beta_2 L \beta_1 X_t}_{\beta_1 \beta_2 X_{t-1}} + \beta_1 \beta_2^2 X_{t-2} + \dots + u_t^* \right\}$$

X_{t-1}, u_{t-1} y_{t-2} / AR(1) $u_t = \rho u_{t-1} + \varepsilon_t$

$$y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 y_{t-1} + u_t$$

Consequences of Autocorrelation

- if y_{t-1} is RHS => endogeneity
- else the consequences the same as with heteroscedasticity => $\hat{\beta}_{OLS}$ - inefficient

$\hat{\beta}_{GLS}$ $\quad \varepsilon'\varepsilon = \sigma^2_\varepsilon \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix} \quad \varepsilon'\varepsilon = \sigma^2_\varepsilon \begin{pmatrix} X_1 & \rho & 0 & \cdots & 0 \\ \rho & \ddots & \ddots & & \vdots \\ 0 & \ddots & \ddots & \ddots & 0 \\ \vdots & & \ddots & \ddots & \rho \\ 0 & \cdots & 0 & \rho & X_n \end{pmatrix}$

$$= \Omega = V \quad \hat{\Omega}$$

TGM

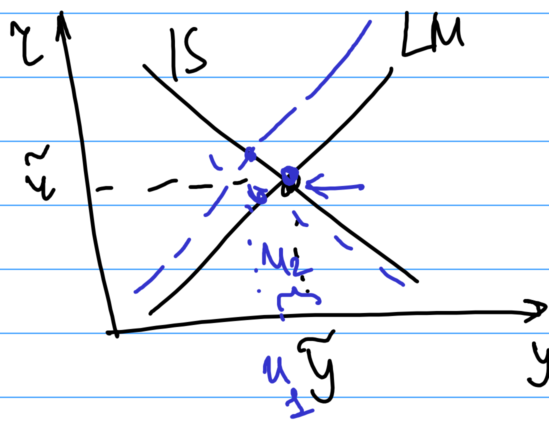
P2

ARDL(1,0)

$$y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 y_{t-1} + u_t \qquad < |1|$$

β_1 – SR effect of X on y (at t)

$\tilde{X}, \tilde{y}$ – LR equilibrium



$t \to \infty$ $\quad \tilde{y} = \beta_0 + \beta_1 \tilde{X} + \beta_2 \tilde{y}$

$$(1-\beta_2)\tilde{y} = \beta_0 + \beta_1 \cdot \tilde{X}$$

$$\tilde{y} = \frac{\beta_0}{1-\beta_2} + \underbrace{\frac{\beta_1}{1-\beta_2}} \tilde{X}$$

$$\beta_1 + \beta_1 \cdot \beta_2 + \beta_1 \cdot \beta_2^2 \ldots = \frac{\beta_1}{1-\beta_2} \quad \text{– LR effect of X on y}$$

$$\vee$$

β_1 – SR effect