Time Series

Stationarity (weak sence):

$E(y_t) = \mu \quad \forall t$

$Var(y_t) = \sigma^2 \quad \forall t$

$cov(y_t, y_{t-k}) = \gamma_k \quad \forall t$

ARIMA(p, d, q)

ARMA: $y_t = \alpha + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + u_t + \gamma_1 \cdot u_{t-1} + \dots + \gamma_q u_{t-q}$

Difference stationary

Δy_t - stationary

Trend stationary

$y - \alpha - \beta t$

L - lag operator

$L y_t = y_{t-1} \qquad L^k y_t = y_{t-k}$

Autocorrelation:

$$Cov(\varepsilon_i, \varepsilon_j) \neq 0$$

$$y_t = \beta_0 + \beta_1 X_t + \underbrace{u_t}_{}$$

$u_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1}$

$$Cov(u_t, u_{t-1}) = \gamma_1$$

$$y_t = \beta_0 + \beta_1 X_t + \beta_2 y_{t-1} + u_t$$

$y_{t-1} = f(X_{t-1}, \varepsilon_{t-1}, \varepsilon_{t-2})$

$u_t = \varepsilon_t + \alpha_1 \varepsilon_{t-1}$

Consequences of Autocorr.

(*) if y_{t-1} is in RHS

=> endogeneity

(*) else consequences are the same as with Heterosced.

=> $\hat{\beta}_{OLS}$ - inefficient

$\hat{\beta}_{GLS}$ - efficient

TGM

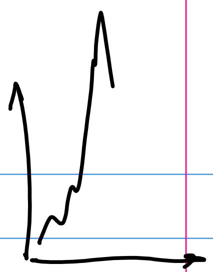
$$\sigma_\varepsilon^2 \begin{pmatrix} 1 & & 0 \\ & \ddots & \\ 0 & & 1 \end{pmatrix}$$

Heterosc.

$$\sigma_\varepsilon^2 \begin{pmatrix} x_1 & & 0 \\ & \ddots & \\ 0 & & x_n \end{pmatrix}$$

Autocor. (of order 1)

$$\sigma_\varepsilon^2 \begin{pmatrix} 1 & \gamma_1 & \cdots & 0 \\ \gamma_1 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & \gamma_1 \\ 0 & \cdots & \gamma_1 & 1 \end{pmatrix}$$



$$\left[y_t = \beta_1 + \beta_2 \cdot X_t + \beta_3 \big(\overset{y_{t-1}}{\overset{\shortparallel}{L y_t}}\big) + u_t \right]$$

$|\beta_3| < 1$ $\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ $\uparrow$ w.n.

if

$\beta_3 > 1$

$$(1 - \beta_3 L)\, y_t = \beta_1 + \beta_2 X_t + u_t$$

$$y_t = \frac{\beta_1}{1-\beta_3 L} + \frac{\overset{b_0}{\overset{\shortparallel}{\beta_2 X_t}}}{1-\underset{q}{\underset{\shortparallel}{\beta_3 L}}} + \frac{u_t}{1-\beta_3 L}$$

$$y_t = \beta_1^* + \beta_2 X_t + L \cdot \underset{\beta_2 \cdot \beta_3 \cdot X_{t-1}}{\underset{\shortparallel}{(\beta_2 \cdot X_t \cdot \beta_3)}} + \beta_2 \cdot \beta_3^2 \cdot X_{t-2} + \dots + u_t^*$$

$$b_0 + b_0 \cdot q + b_0 \cdot q^2 + \dots = \frac{b_0}{1-q}$$

$$y = \begin{pmatrix} y_1 \\ \vdots \\ y_t \end{pmatrix} \qquad L y = \begin{pmatrix} 0 \\ y_1 \\ \vdots \\ y_{t-1} \end{pmatrix}$$

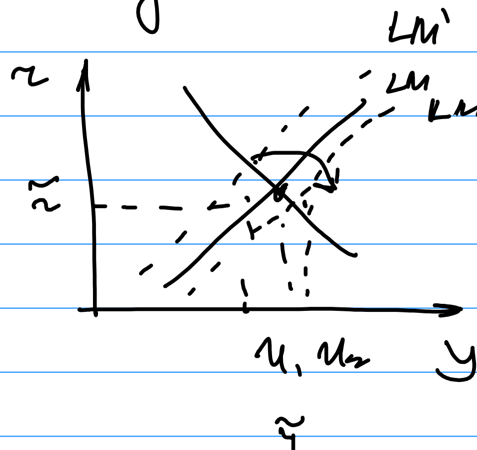
$(0 \ 1 \ \dots \ 1)$

≤ 1

$$y_t = \beta_0 + \beta_1 X_t + \beta_2 y_{t-1} + u_t$$

β_1 – SR effect (shows change at t)

$\tilde{y}, \tilde{X}$ – LR equilibrium



$$t \to \infty : \tilde{y} = \beta_0 + \beta_1 \tilde{X} + \beta_2 \cdot \tilde{y}$$

$$(1-\beta_2)\tilde{y} = \beta_0 + \beta_1 \tilde{X}$$

$$\tilde{y} = \frac{\beta_0}{1-\beta_2} + \frac{\beta_1}{1-\beta_2}\tilde{X}$$

$$\beta_1 + \beta_1 \cdot \beta_2 + \beta_1 \cdot \beta_2^2 + \dots = \frac{\beta_1}{1-\beta_2}$$ – LR effect X on y

$\vee$

β_1 – SR effect

ARDL(p, q) ; ADL(p, q).

$$y_t = \beta_0 + \beta_1 X_t + \dots + \beta_q \cdot X_{t-q} + \alpha_1 \cdot y_{t-1} + \dots + \alpha_p y_{t-p} + u_t$$