

**Elements of Econometrics. 2022-2023.**  
**Class 13-14. Simultaneous Equations**

**Problem 1.** Consider simple Macroeconomic Equilibrium model with Consumption function and Income Identity (time index  $t$  skipped):

$$C = \alpha + \beta Y + u \quad (1)$$

$$Y = C + I \quad (2)$$

( $C$  is aggregate consumption,  $Y$  is aggregate income (endogenous variables),  $I$  is aggregate investment (exogenous), and  $u$  is the disturbance term.

- (a) What is the difference between Structural Form model and Reduced Form Model? Derive Reduced Form model.
- (b) Explain why Gauss-Markov conditions are violated, so OLS estimates are not the best ones?
- (c) Show that OLS estimator of parameter  $\beta$  in equation (1) is inconsistent. Find its large sample bias.

**Problem 2.**

- (a) Explain what it means that an equation in the system of econometric equations is exactly identified, underidentified and overidentified?
- (b) Explain what instruments (instrumental variables) are in the system of econometric equations. What are the properties of the instruments? What conditions are imposed on them? What are predefined variables and can they be used as an instruments?
- (c) How can information about potential instruments be used to judge whether an equation is exactly identified, underidentified or overidentified (we will call this IV-rule)? Give examples.

**Problem 3.**

- (a) What is the order condition? How can it be used to judge whether an equation is exactly identified, underidentified or overidentified? Give examples.
- (b) Explain why the order condition gives the same result as the IV rule?

**Problem 4.** The following is a model of wages and prices:

$$w_t = a_0 + a_1 p_t + a_2 u_t + a_3 Z_t + e_{1t}$$

$$p_t = b_0 + b_1 w_t + b_2 u_t + b_3 Z_t + e_{2t}$$

where  $w$  is the rate of growth of money wages,  $p$  the rate of growth of prices,  $Z$  the rate of growth of productivity, and  $u$  the unemployment rate.

- (a) Derive the reduced form of the model. Is either equation identified? Explain your answer.
- (b) Examine identification under each of the following sets of restrictions.
  - (b. i)  $a_2 = a_3 = 0$
  - (b. ii)  $a_2 = b_3 = 0$
  - (b. iii)  $a_2 = a_3 = b_3 = 0$

Discuss the economic implications of these restrictions. Are any of them plausible?

- (c) What estimation methods would you use to estimate the parameters in b.i, b.ii and b.iii? Explain your reasons for your choice and give a detailed explanation of the procedures used.
- (d) How will your conclusions change in b. i) if you use a lagged variable  $w_{t-1}$  in the right part of the second equation instead of  $w_t$ ?

**Problem 5.** The following simultaneous equations model is considered:

$$Y = \beta_1 + \beta_2 X + u \quad (1)$$

$$X = \alpha_2 Y + v \quad (2)$$

where  $X$  and  $Y$  are endogenous variables, and  $u$  and  $v$  are identically and independently distributed disturbance terms with zero means. The sample consists of  $n$  observations  $(X_i, Y_i)$ .

- (a) Derive reduced form system of equations for the system above. What information gives the reduced form system on the properties of possible estimators for the coefficients of equations (1) and (2)? What can be said on the identification of these equations?
- (b) Show that OLS estimator  $\hat{\alpha}_2^{OLS}$  of  $\alpha_2$  is inconsistent and find large sample bias.
- (c) For the estimation of parameter  $\alpha_2$  of equation (2) a researcher suggests the following estimator  $\hat{\alpha}_2 = \frac{\bar{X}}{\bar{Y}}$  (where  $\bar{X}$  and  $\bar{Y}$  are the sample means of  $X$  and  $Y$ ). The researcher believes that it is consistent. Investigate this suggestion.