Elements of Econometrics. Lecture 19. Autocorrelation. Part 2.

FCS, 2022-2023

AUTOCORRELATED DISTURBANCE TERM

Violated Assumption C.6 (Gauss-Markov 3 condition)
"The values of the disturbance term have independent distributions:
u_t is distributed independently of $u_{t'}$ for $t' \neq t$ "

Reasons: **disturbance term combines the influence of all factors not included in the model directly, and some of them may be autocorrelated in the case of time series data.**

Consequences: **in general, the regression coefficients remain unbiased, but OLS is inefficient. Standard errors estimated wrongly, t-tests invalid. If lagged dependent variable is a regressor, the OLS estimates are biased and inconsistent.**

Detection:
- Breusch–Godfrey LM test (autocorrelation of order p; large samples);
- Durbin-Watson d-test (AR(1) type autocorrelation, finite samples, fixed values of explanatory variables, intercept, critical values depend on X's);
- Durbin h-test (model with lagged dependent variable as a regressor; large samples);
- F-tests and t-tests (large samples only).

Remedial Measures: **to be discussed; special type of GLS.**

ELIMINATING AR(1) AUTOCORRELATION: ONE EXPLANATORY VARIABLE

$$Y_t = \beta_1 + \beta_2 X_t + u_t \qquad u_t = \rho u_{t-1} + \varepsilon_t$$

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2 X_t - \beta_2 \rho X_{t-1} + u_t - \rho u_{t-1}$$

$$Y_t = \beta_1(1-\rho) + \rho Y_{t-1} + \beta_2 X_t - \beta_2 \rho X_{t-1} + \varepsilon_t$$

Non-linear estimation in EViews:
Estimate Equation; type the formula:

Y=C(1)*(1-C(2))+C(2)*Y(-1)+C(3)*X-C(2)*C(3)*X(-1)

Doing the AR(1) transformation, we get rid of the autocorrelation in the disturbance term. Only the innovation term ε_t remains. But the revised specification involves a nonlinear restriction: the coefficient of X_{t-1} is minus the product of the coefficients of X_t and Y_{t-1}.

Hence non-linear estimation technique is needed.

ELIMINATING AR(1) AUTOCORRELATION

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t \qquad u_t = \rho u_{t-1} + \varepsilon_t$$

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{2t-1} + \beta_3 \rho X_{3t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1 (1-\rho) + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + u_t - \rho u_{t-1}$$

$$Y_t = \beta_1 (1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

$$- \quad two \;\; nonlinear \;\; restrictions.$$

EViews:
Y=C(1)*(1-C(2))+C(2)*Y(-1)+C(3)*X2-C(2)*C(3)*X2(-1)+C(4)*X3-C(2)*C(4)*X3(-1)

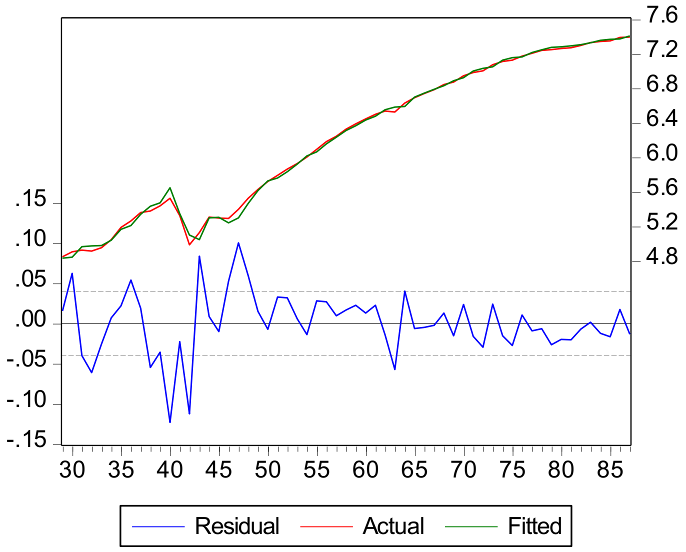
Option in the EViews: add AR(1) to the list of explanatory variables in the initial regression. If the second order autocorrelation available, add AR(1) and AR(2); higher orders dealt respectively.

AR(1) in the Cobb-Douglas Production Function (USSR, 1928-1987)

Dependent Variable: Log(GNP) Method: Least Squares
Sample (adjusted): 1929 1987 Included observations: 59 after adjustments

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	-5.402	1.757	-3.074	0.0033
Log(K)	0.739	0.107	6.90	0.0000
Log(L)	0.974	0.109	8.922	0.0000
AR(1)	0.981	0.015	65.17	0.0000
R-squared		0.998	Mean dependent var	6.210
S.D. dependent var		0.860	S.E. of regression	0.040
Sum squared resid		0.087	F-statistic	9023.25
Durbin-Watson stat		1.580	Prob(F-statistic)	0.0000

For n=60; k=4; 5% significance:
d_L=1.48; d_U=1.69, hence d=1.58 lies in the uncertainty zone. But d-test can not be applied since AR(1) regression includes Y(-1) as a regresor. Apply Breusch-Godfrey LM test.
The residual plot (right) shows that probably the problem was different,- incorrect specification.



AR(1) in the Cobb-Douglas Production Function (USSR, 1928-1987): LM Test

Breusch-Godfrey Serial Correlation LM Test:

F-statistic	2.083800	Prob. F(2,53)	0.134535
Obs*R-squared	4.301185	Prob. Chi-Square(2)	0.116415

Test Equation: Dependent Variable: RESID Method: Least Squares
Sample: 1929 1987 Included observations: 59
Presample missing value lagged residuals set to zero.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	1.650	1.904	0.866	0.39
Log(K)	-0.103	0.117	-0.887	0.38
Log(L)	-0.0086	0.107	-0.080	0.94
AR(1)	-0.0079	0.015	-0.515	0.609
RESID(-1)	0.233	0.146	1.594	0.117
RESID(-2)	0.152	0.139	1.092	0.280

R-squared	0.073	S.E. of regression	0.039
Sum squared resid	0.081	Durbin-Watson stat	1.952
F-statistic	0.834	Prob(F-statistic)	0.532

H_o hypothesis of no serial correlation is not rejected even at 10% level.

COCHRANE–ORCUTT ITERATIVE PROCESS

$$Y_t = \beta_1 + \beta_2 X_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

$$\rho Y_{t-1} = \beta_1 \rho + \beta_2 \rho X_{t-1} + \rho u_{t-1}$$

$$Y_t - \rho Y_{t-1} = \beta_1(1-\rho) + \beta_2 X_t - \beta_2 \rho X_{t-1} + u_t - \rho u_{t-1}$$

$$\tilde{Y}_t = \beta_1' + \beta_2 \tilde{X}_t + \varepsilon_t$$

$$\tilde{Y}_t = Y_t - \rho Y_{t-1}$$
$$\tilde{X}_t = X_t - \rho X_{t-1}$$
$$\beta_1' = \beta_1(1-\rho)$$

1. Regress Y_t on X_t using OLS
2. Calculate $\hat{u}_t = Y_t - \hat{\beta}_1 - \hat{\beta}_2 X_t$ and regress $\hat{u}_t$ on $\hat{u}_{t-1}$ to obtain an estimate of ρ.
3. Calculate $\tilde{Y}_t$ and $\tilde{X}_t$ and regress $\tilde{Y}_t$ on $\tilde{X}_t$ to obtain revised estimates $\hat{\beta}_1$ and $\hat{\beta}_2$. Return to (2) and continue until convergence.

CO procedure allows to apply linear OLS for iterative estimation of non-linear model. After steps 1-3, we keep alternating between Step 2 and Step 3 until convergence is obtained.

Heteroscedasticity and Autocorrelation Consistent Covariances and Variances (Newey-West)

In the initial model with autocorrelated disturbamce term, standard errors are calculated incorrectly, and t-tests invalid.

The White covariance matrix (heteroscedasticity) assumes that the residuals are serially uncorrelated. Newey and West (1987) have proposed a more general covariance estimator that is consistent in the presence of both heteroscedasticity and autocorrelation of unknown form.

To use the Newey-West method in the EViews, select the Options tab in the Equation Estimation. Check the box labeled Heteroscedasticity Consistent Covariance and press the Newey-West button.

If Feasible GLS applied, then the autocorrelation is eliminated, and no need to use Newey-West method.

HETEROSCEDASTICITY AND AUTOCORRELATED DISTURBANCE TERM: PRECISION OF $\hat{\boldsymbol{\beta}}_2$ ESTIMATOR, SLR

$$\sigma^2_{\hat{\beta}_2} = \mathrm{E}\left\{\left(\hat{\beta}_2 - \mathrm{E}(\hat{\beta}_2)\right)^2\right\} = \mathrm{E}\left\{\left(\hat{\beta}_2 - \beta_2\right)^2\right\} = \mathrm{E}\left\{\left(\sum_{i=1}^{n} a_i u_i\right)^2\right\} =$$

$$= \mathrm{E}\left\{\sum_{i=1}^{n} a_i^2 u_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j u_i u_j\right\} = \sum_{i=1}^{n} a_i^2 \mathrm{E}\left(u_i^2\right) + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j \mathrm{E}\left(u_i u_j\right) =$$

$$= \sum_{i=1}^{n} a_i^2 \sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} a_i a_j \sigma_{ij} = \frac{\sum_{i=1}^{n} {x_i}^2\, \sigma_i^2 + \sum_{i=1}^{n}\sum_{j\neq i} x_i x_j\, \sigma_{ij}}{\left(\sum_{j=1}^{n} {x_j}^2\right)^2}$$

This formula is used for the estimation of Heteroscedasticity and Autocorrelation Consistent standard errors (Newey-West)

Dependent Variable: LOG(HOUS)
Method: Least Squares
Date: 10/13/16 Time: 20:31
Sample: 1959 2003
Included observations: 45

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005625	0.167903	0.033501	0.9734
LOG(DPI)	1.031918	0.006649	155.1976	0.0000
LOG(PRHOUS)	-0.483421	0.041780	-11.57056	0.0000

R-squared	0.998583
Adjusted R-squared	0.998515
S.E. of regression	0.016859
Sum squared resid	0.011937
Log likelihood	121.4304
Durbin-Watson stat	0.633113

The standard errors and test statistics estimated using standard formula and Newey-West method.

Durbin-Watson test:

For n=45; k=3:

$d_{L,5\%}$=1.43; $d_{U,5\%}$=1.62:

AR(1) autocorrelation.

Dependent Variable: LOG(HOUS)
Method: Least Squares
Date: 01/31/18 Time: 17:12
Sample: 1959 2003
Included observations: 45
HAC standard errors & covariance (Bartlett kernel, Newey-West fixed bandwidth = 4.0000)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.005625	0.217090	0.025911	0.9795
LOG(DPI)	1.031918	0.010821	95.36011	0.0000
LOG(PRHOUS)	-0.483421	0.053260	-9.076663	0.0000

R-squared	0.998583	Mean dependent var	6.359334
Adjusted R-squared	0.998515	S.D. dependent var	0.437527
S.E. of regression	0.016859	Akaike info criterion	-5.263574
Sum squared resid	0.011937	Schwarz criterion	-5.143130
Log likelihood	121.4304	Hannan-Quinn criter.	-5.218673
F-statistic	14797.05	Durbin-Watson stat	0.633113
Prob(F-statistic)	0.000000	Wald F-statistic	5091.963
Prob(Wald F-statistic)	0.000000		

AR(1) as a Special Case of ADL(1,1) Model

$$Y_t = \beta_1 + \beta_2 X_{2t} + \beta_3 X_{3t} + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Restricted model (transformed AR(1)):

$$Y_t = \beta_1(1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

Unrestricted ADL(1,1) model

$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

Restrictions embodied in the AR(1) process

$$\lambda_3 = -\lambda_1 \lambda_2 \qquad \lambda_5 = -\lambda_1 \lambda_4$$

The AR(1) model can be considered as a special (restricted) case of more general (unrestricted) ADL(1,1) model.

COMMON FACTOR TEST

Test statistic: $n \log \dfrac{RSS_R}{RSS_U}$

Restricted model RSS_R

$$Y_t = \beta_1(1-\rho) + \rho Y_{t-1} + \beta_2 X_{2t} - \beta_2 \rho X_{2t-1} + \beta_3 X_{3t} - \beta_3 \rho X_{3t-1} + \varepsilon_t$$

Unrestricted model RSS_U

$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

Restrictions embodied in the AR(1) process

$$\lambda_3 = -\lambda_1 \lambda_2 \qquad \lambda_5 = -\lambda_1 \lambda_4$$

Under the null hypothesis that the restrictions are valid, the test statistic has a χ^2 (chi-squared) distribution with degrees of freedom equal to the number of restrictions. It is a large-sample test.

COMMON FACTOR TEST: EXAMPLE

$$LGHOUS_t = \beta_1 + \beta_2 LGDPI_t + \beta_3 LGPRHOUS_t + u_t$$

$$u_t = \rho u_{t-1} + \varepsilon_t$$

Restricted model

$$\begin{aligned} LGHOUS_t = {} & \beta_1(1-\rho) + \rho LGHOUS_{t-1} \\ & + \beta_2 LGDPI_t - \beta_2 \rho LGDPI_{t-1} \\ & + \beta_3 LGPRHOUS_t - \beta_3 \rho LGPRHOUS_{t-1} + \varepsilon_t \end{aligned}$$

Unrestricted model

$$\begin{aligned} LGHOUS_t = {} & \lambda_0 + \lambda_1 LGHOUS_{t-1} \\ & + \lambda_2 LGDPI_t + \lambda_3 LGDPI_{t-1} \\ & + \lambda_4 LGPRHOUS_t + \lambda_5 LGPRHOUS_{t-1} + \varepsilon_t \end{aligned}$$

We compare the initial regression with AR(1) term (RSS_R=0.006084) with the estimated with OLS the ADL(1,1) model with no restrictions on the parameters (RSS_U=0.001456).

COMMON FACTOR TEST: OUTPUT FOR UNRESTRICTED MODEL

Dependent Variable: LGHOUS   Method: Least Squares
Sample (adjusted): 1960 2003
Included observations: 44 after adjusting endpoints

Variable	Coefficient	Std.Error	t-Statistic	Prob.
C	0.041	0.065	0.636	0.53
LGDPI	0.276	0.068	4.057	0.0002
LGPRHOUS	-0.229	0.075	-3.034	0.0043
LGHOUS(-1)	0.726	0.0585	12.41	0.0000
LGDPI(-1)	-0.011	0.087	-0.123	0.903
LGPRHOUS(-1)	0.126	0.084	1.498	0.142

R-squared	0.9998	Mean dependent var	6.379
S.D. dependent var	0.422	S.E. of regression	0.0062
Sum squared resid	0.001456	F-statistic	39944.40
Durbin-Watson stat	1.764		

Test for serial correlation (h-test, LM test): no serial correlation:

$$h = (1 - 0.5 \times 1.764) \times \sqrt{\frac{44}{1 - 44 \times 0.0585^2}} = 0.86$$

COMMON FACTOR TEST

The unrestricted model is not subject to autocorrelation (tested with the Breusch–Godfrey test and Durbin's *h*-test.

$$n\log\left(\frac{RSS_R}{RSS_U}\right) = 44\log\left(\frac{0.006084}{0.001456}\right) = 62.9$$

$$\chi^2_{\text{crit}} = 13.8 \quad (2,\ 0.1\%)$$

We reject the restrictions. We should choose more general model instead of assuming that the disturbance term is subject to an AR(1) process.

Then we test if the lagged regressors *LGDPI* and *LGPRHOUS* are needed in the ADL(1,1) model, using *F*-test:

$$F(2,38) = \frac{(0.001566 - 0.001456)/2}{0.001456/38} = 1.44 \qquad F(2,35)_{\text{crit},5\%} = 3.27$$

The H_0 is not rejected, and we finally come to the ADL(1,0) model:

$$LGHOUS_t = \lambda_0 + \lambda_1 LGHOUS_{t-1} + \lambda_2 LGDPI_t + \lambda_3 LGPRHOUS_t + \varepsilon_t$$

COMMON FACTOR TEST: AR(1,0) FINAL SPECIFICATION

Dependent Variable: LGHOUS
Method: Least Squares Sample (adjusted): 1960 2003
Included observations: 44 after adjusting endpoints

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	0.074	0.063	1.1755	0.247
LGDPI	0.283	0.047	6.0312	0.000
LGPRHOUS	-0.117	0.027	-4.271	0.0001
LGHOUS(-1)	0.707	0.044	15.928	0.000

R-squared	0.9999	Mean dependent var	6.379
S.D. dependent var	0.422	S.E. of regression	0.0063
Sum squared resid	0.001566	F-statistic	65141.75
Durbin-Watson stat	1.811		

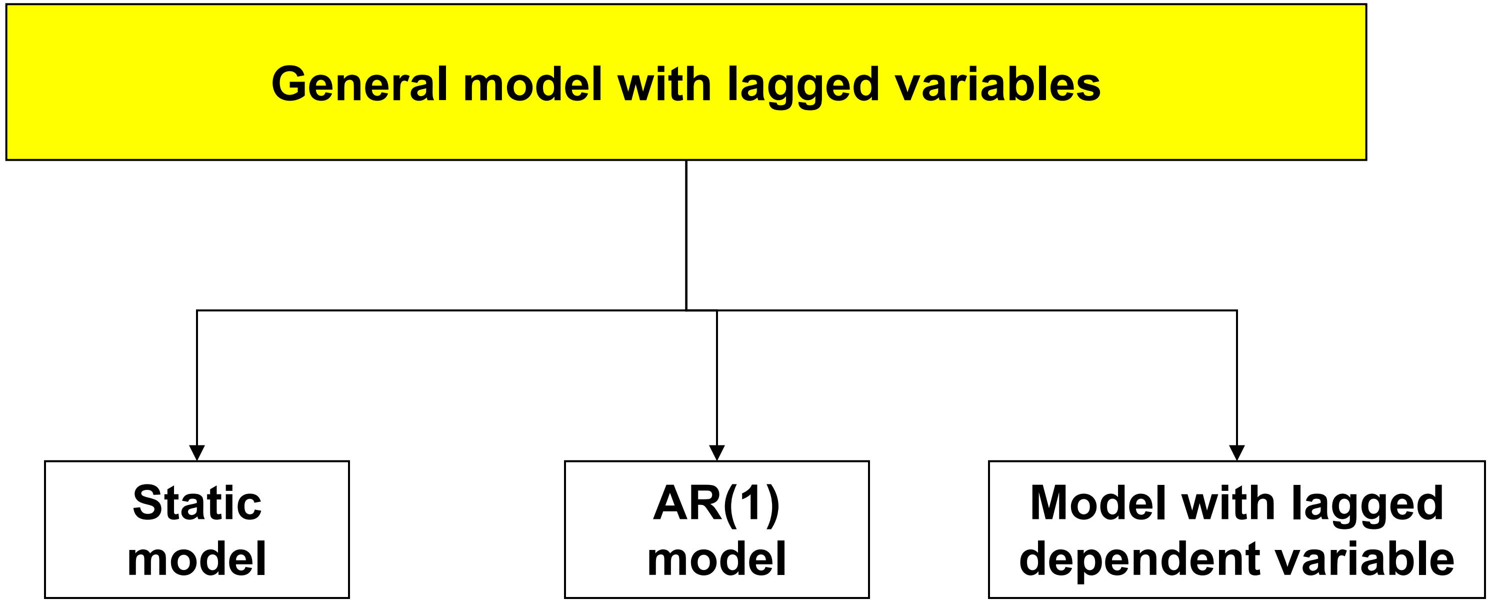
Test for autocorrelation (Breusch-Godfrey LM Test, lag=1):

F-statistic	0.162	Prob. F(1,39)	0.689
Obs*R-squared	0.182	Prob. Chi-Square(1)	0.669

No serial correlation.

Omission of the lagged dependent variable was responsible for the apparent autocorrelation in the original OLS regression.

DYNAMIC MODEL SPECIFICATION



$$\lambda_1 = \lambda_3 = \lambda_5 = 0$$

$$\lambda_3 = -\lambda_1 \lambda_2$$
$$\lambda_5 = -\lambda_1 \lambda_4$$

$$\lambda_3 = \lambda_5 = 0$$

$$Y_t = \lambda_0 + \lambda_1 Y_{t-1} + \lambda_2 X_{2t} + \lambda_3 X_{2t-1} + \lambda_4 X_{3t} + \lambda_5 X_{3t-1} + \varepsilon_t$$

General-to-specific approach should be used. We start with a model sufficiently general, and then simplify it if possible.