SEM

1) omitted variable

2) measurement error

3) simultaneity

lgdp | prot

1) relevance $cov(X_i, Z_i) \neq 0$

2) exogeneity $cov(\varepsilon_i, X_i) = 0$

$$se(\hat{\beta}_{IV}) = se(\hat{\beta}_{OLS}) \cdot \sqrt{\frac{1}{\hat{\rho}^2_{X,Z}}}$$

SEM

(P1)

$$\begin{cases} y_i = \beta_1 + \beta_2 X_i + \varepsilon_i, & \beta_2 < 0 \\ X_i = \alpha_1 + \alpha_2 y_i + u_i, & \alpha_2 > 0 \end{cases} \quad \leftarrow \hat{\beta}_2^{OLS}$$

$cov(u_i, \varepsilon_i) = 0$

structural equation form

$$\hat{\beta} \xrightarrow{p} \beta + \frac{cov(X_i, \varepsilon_i)}{Var(X_i)}$$

$$X_i = \alpha_1 + \alpha_2 (\beta_1 + \beta_2 X_i + \varepsilon_i) + u_i$$

$$\begin{cases} X_i = \dfrac{\alpha_1 + \alpha_2 \beta_1 + \alpha_2 \cdot \varepsilon_i + u_i}{1 - \alpha_2 \beta_2} \\[2ex] y_i = \dfrac{\beta_1 + \alpha_1 \beta_2 + \varepsilon_i + \beta_2 u_i}{1 - \alpha_2 \beta_2} \end{cases}$$

reduced form

$$\operatorname{cov}(X_i, \varepsilon_i) = \operatorname{cov}\left(\frac{\cancel{\alpha_1} + \cancel{\alpha_2} \beta_1 + \alpha_2 \cdot \varepsilon_i + u_i}{1 - \alpha_2 \beta_2}, \varepsilon_i \right) =$$

$$= \sigma_\varepsilon^2 \cdot \frac{\alpha_2 \quad > 0}{1 - \alpha_2 \underset{\overset{\wedge}{0}}{\beta_2} \quad > 0}$$

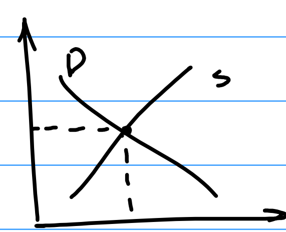
P2

$$\begin{aligned} D: \quad & \underset{\parallel}{\ln Q_i} = \beta_1 + \beta_2 \ln P_i + \varepsilon_i, \quad \beta_2 < 0 \\ S: \quad & \ln Q_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \cdot \ln T_i + u_i, \quad \gamma_2 > 0 \end{aligned}$$

$\operatorname{cov}(\ln T_i, \ln P_i) \neq 0$

$\operatorname{cov}(\ln T_i, \varepsilon_i) = 0$

$$\hat{\beta}_2 \xrightarrow{p} \beta_2 + \frac{\operatorname{cov}(\ln P_i, \varepsilon_i)}{\operatorname{var}(\ln P_i)}$$



$$\beta_1 + \beta_2 \ln P_i + \varepsilon_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \ln T_i + u_i$$

$$\operatorname{cov}\left(\frac{\cancel{\beta_1} + \varepsilon_i - \cancel{\gamma_1} - \gamma_3 \cdot \ln T_i - u_i}{\gamma_2 - \beta_2}, \varepsilon_i\right) = \begin{cases} \operatorname{cov}(\ln T_i, \varepsilon_i) = 0 \\ \operatorname{cov}(\varepsilon_i, u_i) = 0 \end{cases}$$

$$= \underbrace{\frac{\sigma_\varepsilon^2}{\gamma_2 - \beta_2}}_{\neq 0} \neq 0 \quad \Rightarrow \hat{\beta}_{OLS} \text{ inconsistent}$$

$$\begin{cases} W_i = \alpha_0 + \alpha_1 p_i + \alpha_2 \cdot u_i + \alpha_3 z_i + \varepsilon_i \\ p_i = \beta_0 + \beta_1 W_i + \beta_2 u_i + \beta_3 z_i + v_i \end{cases}$$

a) $\alpha_2 = \alpha_3 = 0$

$$\begin{cases} W_i = \alpha_0 + \alpha_1 p_i + \varepsilon_i & \text{over} \\ p_i = \beta_0 + \beta_1 W_i + \beta_2 u_i + \beta_3 z_i + v_i & \text{under} \end{cases}$$

b) $\alpha_2 = \beta_3 = 0$

$$\begin{cases} W_i = \alpha_0 + \alpha_1 p_i + \alpha_3 z_i + \varepsilon_i & \text{exact} \\ p_i = \beta_0 + \beta_1 W_i + \beta_2 u_i + v_i & \text{exact} \end{cases}$$

c) $\alpha_2 = \alpha_3 = \beta_3 = 0$

$$\begin{cases} W_i = \alpha_0 + \alpha_1 p_i + \varepsilon_i & \text{exact} \\ p_i = \beta_0 + \beta_1 W_i + \beta_2 u_i + v_i & \text{under} \end{cases}$$