Distrubuted Lag Models

ARDL(p, q)

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} +$$

$$\beta_0 \cdot X_t + \beta_1 \cdot X_{t-1} + \dots + \beta_q \cdot X_{t-q} + \varepsilon_t$$

DLM

- Geometrically (Koyck's) DLM
- Polynomial (Almond's) DLM

$$y_t = \alpha + \sum_{j=0}^{\infty} \beta_j X_{t-j} + \varepsilon_t$$

$$\tilde{y} = \alpha + \beta_0 \tilde{X} + \beta_1 \tilde{X} + \dots$$

SR: β_0

LR: $\sum_{j=0}^{\infty} \beta_j < \infty$

Geometrical (Koyck's) DLM:

$$W_j = (1-\lambda) \cdot \lambda^j \qquad 0 < \lambda < 1$$

$$\sum W_j = \frac{1-\lambda}{1-\lambda} = 1 \qquad \sum_{j=0}^{\infty} \lambda^j = \frac{b_0}{1-q} = \frac{1}{1-\lambda}$$

$$\left\{ \; y_t = \alpha + \beta(1-\lambda) \cdot \sum_{j=0}^{\infty} \lambda^j X_{t-j} + \varepsilon_t \right.$$

ARDL(0, ∞)

λ — rate of decay

$\lambda \approx 1$ slow decay

$\lambda \approx 0$ fast decay

Est. 1) fix $\lambda = 0{,}1$

2) $y_t = \alpha + \beta \cdot \underbrace{\left[0{,}9 \cdot \sum_{j=0}^{9} 0{,}1^j x_{t-j}\right]}_{= X'} + \varepsilon$

3) RSS $\to$ min

$$y_t = \alpha + \beta(1-\lambda) \cdot \sum_{j=0}^{\infty} \lambda^j x_{t-j} + \varepsilon_t$$

SR: $\beta \cdot (1-\lambda)$

$= \frac{1}{1-\lambda}$

LR: $\beta(1-\lambda) \cdot \sum_{j=0}^{\infty} \lambda^j = \beta$

$\tilde{y} = \alpha + \beta(1-\lambda)\tilde{x} + \beta(1-\lambda)\lambda\tilde{x} + \dots$

Autoregressive form: $\underset{\smile}{A}R\underset{\smile}{DL}(1,0)$

$$y_t = \alpha_0 + \beta_0 \cdot x_t + \underbrace{\lambda y_{t-1}}_{AR} + v_t$$

SR: $\beta_0 = \beta(1-\lambda)$

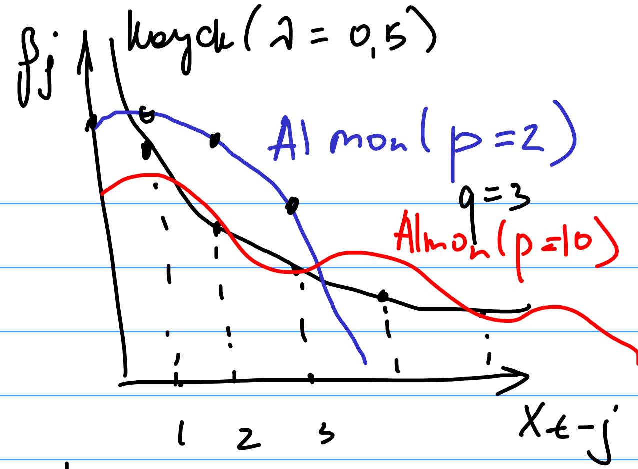
$\{ \tilde{y} = \alpha_0 + \beta_0 \tilde{x} + \lambda \tilde{y} \}$

LR: $\frac{\beta_0}{1-\lambda} = \beta$

$(1-\lambda)\tilde{y} = \alpha_0 + \beta_0 \tilde{x}$

$\tilde{y} = \frac{\alpha_0}{1-\lambda} + \frac{\beta_0}{1-\lambda}\tilde{x}$

Polynomial (Almon's) Lag Model :



$$y_i = \alpha + \sum_{j=0}^{q} \beta_j X_{t-j} + \varepsilon_t$$

$$\beta_j = \gamma_0 + \gamma_1 j + \dots + \gamma_p j^p = \sum_{k=0}^{p} \gamma_k j^k$$

$$p \le q$$

$$y_t = \alpha + \sum_{k=0}^{p} \gamma_k \cdot Z_{tk} + \varepsilon_t$$

$$Z_{tk} = \sum_{j=0}^{q} j^k X_{t-j}$$

Economic Model (with Koyck's DLM):

1) Partial Adjustment Model $\quad y_t - y_t^* \mid K_t$
2) Adaptive Expectations Models $\quad u_t \mid \pi_t - \pi_t^e$

$$y_t^* = \alpha + X_t + \varepsilon_t$$

↳ unobserved LR or equilibrium value Y_t,

s.t. Partial Adjustment (PA) hypothesis

$$y_t - y_{t-1} = (1-\lambda)(y_t^* - y_{t-1})$$

lin. comb. $\underbrace{y_t = (1-\lambda) y_t^* + \lambda y_{t-1}}$ $= \dots$

λ - speed of adjustment

$$= (1-\lambda) \sum \lambda^j y^*_{t-j}$$

DL form

$\lambda \approx 0$ - fast adj.

$\lambda \approx 1$ - slow adj

$$y_t = \dots = \alpha + \beta \cdot \sum w_j X_{t-j} +$$

$$+ \sum w_j \cdot \varepsilon_{t-j}, \quad w_j = (1-\lambda)\lambda^j$$

$$y_t = \alpha_0 + \beta_0 \cdot X_t + \lambda y_{t-1} + v_t$$

AR form

SR: $\beta(1-\lambda) = \beta_0$

LR: $\beta = \dfrac{\beta_0}{1-\lambda}$

E.g. SR: $\beta_0 = 0{,}01$

$\lambda = 0{,}9$ – speed of adjustment

LR: $\beta = \dfrac{0{,}01}{0{,}1} = 0{,}1$