Variable transformation

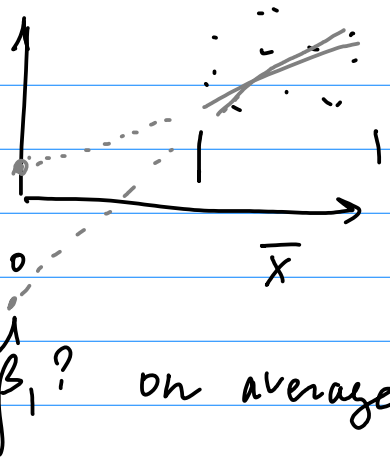
1) $y_i = \beta_0 + \beta_1 x_i + \varepsilon_i$

β_0 : $E(y_i \mid x_i = 0) = \beta_0$

β_1 : $x_i \uparrow 1$ $\quad y \uparrow \beta_1$; β_1? on average

$dy / dx = \beta_1$

and other regressors being fixed"

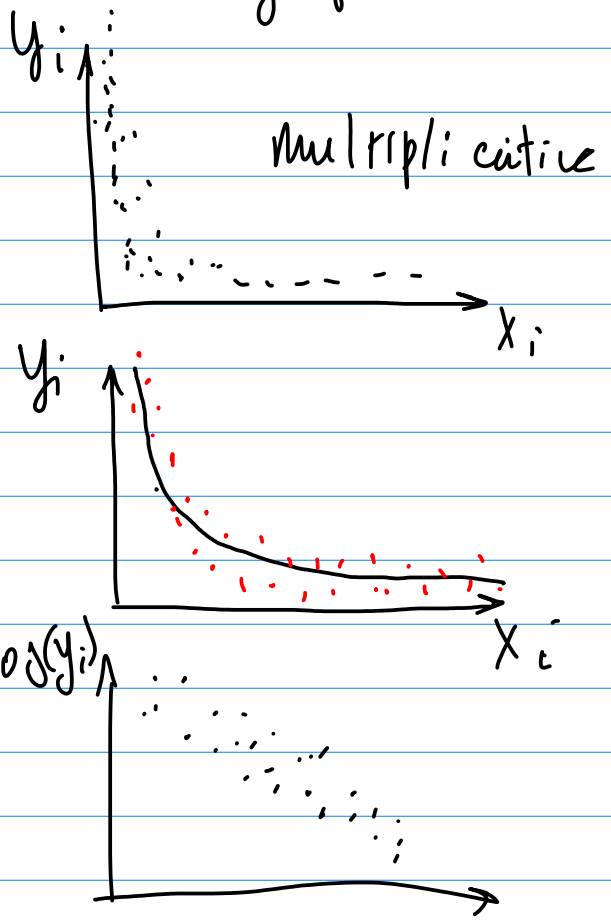


2) $y_i = \beta_0 \cdot x_i^{\beta_1} \cdot \varepsilon_i$

(?) $y_i = \beta_0 \cdot x_i^{\beta_1} + \varepsilon_i$

$\log(y_i) = \log \beta_0 + \beta_1 \log x_i + \log \varepsilon_i$

$\log(y_i) = \beta_0^* + \beta_1 \log x_i + \varepsilon_i^*$



β_1? $\quad d \log y_i / d \log x_i = \beta_2$

$100 \cdot \frac{dy_i}{y_i} \Big/ 100 \cdot \frac{dx_i}{x_i} = \beta_2 \ (\%)$

$x \uparrow 1\% \rightarrow y \uparrow \beta_2 \%$

3) $y_i = \beta_1 + \beta_2 \cdot \log(x_i) + \varepsilon_i$

$$\frac{dy_i}{d\log(x_i)} = \frac{dy_i}{100 \cdot dx_i/x_i \%} = \frac{\beta}{100}$$

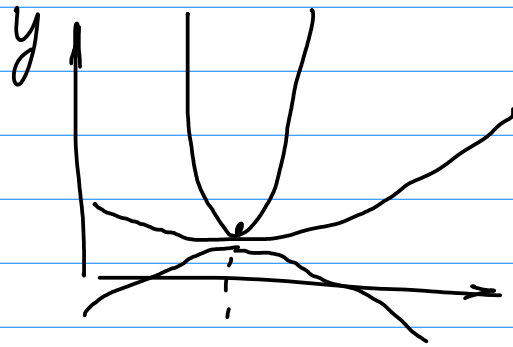
$x \uparrow 1\% \Rightarrow y \uparrow \beta/100$

4) $\log(y_i) = \beta_0 + \beta_1 x_i + \varepsilon_i$

$$\frac{d \log y_i}{dx_i} = \frac{dy_i/y_i \cdot 100}{dx_i} = 100 \cdot \beta_1$$

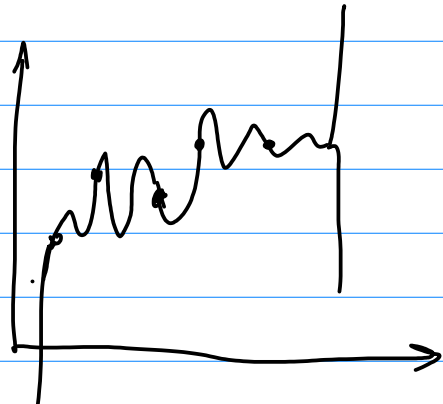
$x \uparrow 1 \quad y \uparrow 100 \cdot \beta \%$

Quadratic terms



$y_i = \beta_0 + \beta_1 x_i + \beta_2 x_i^2 + \varepsilon_i$

$dy_i/dx_i = [\beta_1 + 2\beta_2 x_i]$



β_1: if $x_i = 0$ $x_i \uparrow 1 \Rightarrow y_i \uparrow \beta_1$

β_2: (sign) direction and sharpens

Multiplicative terms

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_2 X_{2i} + \beta_3 X_{1i} X_{2i} + \varepsilon_i$$

$$dy_i / dx_{1i} = \beta_1 + \beta_3 X_{2i}$$

β_1 : $x_1 \uparrow 1$ $y \uparrow \beta_1$ if $x_2 = 0$

→ Sol: Centering $x_i^* = x_i - \bar{x}$

$y_i^* = y_i - \bar{y}$

β^* : $x_1 \uparrow 1$ $y \uparrow \beta_1$ if $x_2 = \bar{x}$

β_3 : sign → direction of effect

value → size of effect

RESET test

$$y_i = \beta_1 + \beta_2 x_{1i} + \dots + \beta_k x_{ki} \Rightarrow \hat{y}_i$$

$$y_i = \beta_1 + \beta_2 x_i + \dots + \beta_k x_{ki} + \gamma_1 \hat{y}_i^2 + \dots + \gamma_p \hat{y}_i^p$$

F-test for $H_0 : \gamma_1 = \dots = \gamma_p$

Box-Cox test:

$$F(\cdot) = \frac{y^\lambda - 1}{\lambda}$$

$\lambda = 0 \rightarrow \log y$

$\lambda = 1 \rightarrow y$

$\hat{\lambda}_1, \hat{\lambda}_2$

$$\frac{n}{2} \left| \log RSS_1 / RSS_0 \right| \sim \chi^2_1$$

$$\frac{50}{2} \left| \log 0{,}08 / 0{,}068 \right| = 4$$