DLM $\qquad y \leftarrow X$

ARDL(p, q)

if ε_t - autocorr. => y_{t-1} endogen.

$$y_t = \alpha_0 + \alpha_1 y_{t-1} + \dots + \alpha_p y_{t-p} + \beta_0 \cdot X_t + \beta_1 \cdot X_{t-1} + \dots + \beta_q X_{t-q} + \varepsilon_t$$

β_0 → SR

ARDL(0, ∞)

↕

ARDL(1, 0)

DLM

Geometrically DL (Koyck's DLM)

Polynomial DL (Almon's DLM)

DLM:

$$y_t = \alpha + \sum_{j=0}^{\infty} \beta_j X_{t-j} + \varepsilon_t .$$

$\sum \beta_j < \infty$

SR: β_0

LR: $\sum \beta_j$

ARDL(0, 4)

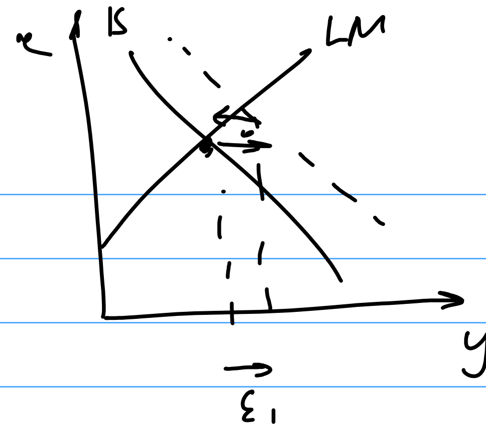
ARDL(1, 0)

→ ARDL(0, ∞)

ARDL(4, 6)

→ ARDL(0, ∞)

$$\tilde{y} = \alpha + \beta_0 \cdot \tilde{X} + \beta_1 \cdot \tilde{X} + \dots + \cancel{\varepsilon}$$



Geometrically DL (Koyck's DLM).

λ - rate of decay

$$W_j = \left[\beta_j = \underbrace{(1-\lambda)}\cdot\underbrace{\lambda^j}\right], \quad 0<\lambda<1$$

λ ≈ 0 faster decay

λ ≈ 1 slow decay

$$\sum W_j = \sum_{j=0}^{\infty} \beta_j = \frac{1-\lambda}{1-\lambda} = 1 \qquad \sum_{j=0}^{\infty} \lambda^j = \frac{b_0}{1-q} = \frac{1}{1-\lambda}$$

ARDL(0,∞): $$y_t = \alpha + \beta\cdot\underbrace{(1-\lambda)\cdot\sum_{j=0}^{\infty}\lambda^j X_{t-j}} + \varepsilon_t$$

Estimation: 1) fix $\lambda = 0{,}5$

SR: $\beta(1-\lambda)$

2) $y_t = \alpha + \beta\cdot f(X_t, \lambda) + \varepsilon_t$

LR: β

↳ RSS

3) $\min\limits_{\lambda} RSS$

Autoregressive form:

$\varepsilon_t - \lambda\varepsilon_{t-1}$

$$y_t = \alpha_0 + \beta_0 X_t + \lambda y_{t-1} + v_t$$

SR : $\beta_0 = \beta(1-\lambda)$

LR : $\dfrac{\beta_0}{1-\lambda} = \beta$

$$\tilde{y} = \alpha_0 + \beta_0 \cdot \tilde{X} + \lambda \tilde{y}$$

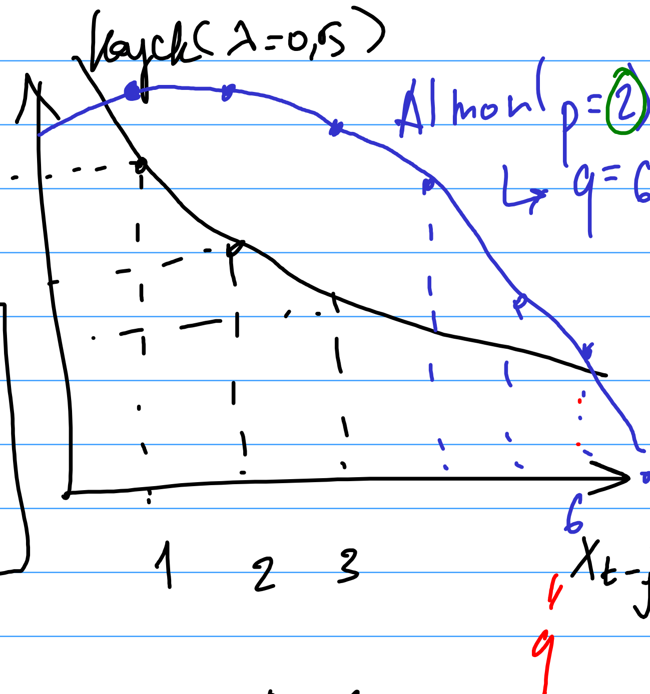
$$(1-\lambda)\tilde{y} = \alpha_0 + \beta_0 \cdot \tilde{X}$$

$$\tilde{y} = \frac{\alpha_0}{1-\lambda} + \frac{\beta_0}{1-\lambda}\tilde{X}$$

Polynomially DL (Almon's DLM)

$$y_t = \alpha + \sum_{j=0}^{q} \beta_j X_{t-j} + \varepsilon_t$$

$$\left[\beta_j = \gamma_0 + \gamma_1 \cdot j + \dots + \gamma_p \cdot j^p = \sum_{k=0}^{p} \gamma_k j^k\right]$$



$$y_t = \alpha + \sum_{k=0}^{p} \gamma_k \cdot Z_{tk} + \varepsilon_t$$

$$Z_{tk} = \sum_{j=0}^{q} j^k \cdot X_{t-k}$$

$p = 3$

$\beta_0 = \gamma_0$

$\beta_2 = \gamma_0 + \gamma_1$

$\beta_3 = \gamma_0 + 2\gamma_1 + \gamma_2$

$\cancel{\beta_4 = \gamma_0 + 3\gamma_1 + 4\gamma_2 + \gamma_3}$

SR: $\gamma_0 = \beta_0$

LR: $\sum \gamma_k \cdot \sum j^k$

Economic Models (with Koyck's DL)

1) Partial Adjustment Model $\quad y_t - y_t^* \mid K_t$

2) Adaptive Expectation Model $\quad u_t \mid \pi_t - \pi_t^e$

$$y_t^* = \alpha + \beta X_t + \varepsilon_t$$

↳ unobserved LR / equilibrium y,

s.t. Part. Adj. Hypothesis:

$0 < \lambda < 1$.

λ - speed of adjustment

$\lambda \approx 0$ fast adj

$\lambda \approx 1$ slow adj

$$y_t - y_{t-1} = (1-\lambda)(y_t^* - y_{t-1})$$

lin. comb.

$$y_t = (1-\lambda) y_t^* + \lambda y_{t-1} = \dots$$

$$= (1-\lambda) \sum \lambda^j \cdot y_{t-j}^*$$

$$y_t = \alpha + \beta \sum_{j=0}^{\infty} w_j \cdot X_{t-j} + \sum_{j=0}^{\infty} w_j \cdot \varepsilon_{t-j}$$

$$w_j = (1-\lambda) \lambda^j$$

$$y_t = \alpha_0 + \beta_0 \cdot X_t + \lambda y_{t-1} + v_t$$

SR : $\beta_0 = \beta(1-\lambda)$

LR : $\dfrac{\beta_0}{1-\lambda} = \beta$

$$\begin{cases} \alpha_0 = 1 \\ \beta_0 = 0{,}01 \\ \lambda = 0{,}9 \end{cases}$$

↳ $\dfrac{0{,}01}{0{,}1} = 0{,}1$