Elements of Econometrics. 2022-2023.
Seminar 1. Introduction.
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PLAN

1. **Expectation, variance, covariance, correlation**
2. **Properties of estimators: unbiasedness, efficiency, consistency**
3. **Convergence in distribution, convergence in probability**

EXPECTATION, VARIANCE, COVARIANCE, CORRELATION

Problem 1. Swedish scientists have found that people who ride a bicycle are significantly less likely to have heart attacks. Correlation coefficient between hours of riding bicycle and number of cases of heart attacks was -0.67 (it is significant). How can you interpret this result?

Problem 2. *"Correlation does not imply causation".* You have data from ten Russian cities. You know overall city damage caused by fire and number of fireman teams. You calculate the correlation coefficient between damage and number of teams and get a value of 0.85 (it is significant). How can you interpret this result?

Problem 3: If X and Y are independent then $\rho_{X,Y} = 0$. (*Independence Imply zero correlation).*

Problem 4: If $\rho_{XY} = 0$, then X and Y are not necessarily independent.
(*Zero Covariance Does Not Necessarily Imply Independence*).

Problem 5. Let $E(X) = \mu$ and $\operatorname{var}(X) = \sigma^2$. Define $Z = X - \dfrac{X-\mu}{\sigma}$. Find $E(Z)$ and $\operatorname{var}(Z)$.

Problem 6. Let $E(X) = \mu$ and $b \neq \mu$. Show that $E(X-\mu)^2 < E(X-b)^2$.

PROPERTIES OF ESTIMATORS

Problem 7. A random variable X has unknown population mean μ_X and population variance σ_X^2. A sample of n observations $\{X_1, X_2, ..., X_n\}$ is generated. The average of the odd-numbered observations is used to estimate μ_X.
a) Determine whether this estimator is unbiased.
b) Determine whether this estimator is efficient.
c) Determine whether this estimator is consistent.

Problem 8. A random variable X has unknown population mean μ_X and population variance σ_X^2. A sample of n observations $\{X_1, ..., X_n\}$ is generated. Show that

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + ... + \frac{1}{2^{n-1}}X_{n-1} + \frac{1}{2^{n-1}}X_n$$

is an unbiased estimator of μ_X. Show that the variance of Z does not tend to zero as n tends to infinity and that therefore Z is an inconsistent estimator, despite being unbiased.
Comment: Some students during the seminar made a remark that the formula for the estimator in this question is probably mistaken

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + ... + \frac{1}{2^{n-1}}X_{n-1} + \frac{1}{2^{n-1}}X_n$$

and should be corrected in the following way

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + \ldots + \frac{1}{2^{n-1}}X_{n-1} + \frac{1}{2^n}X_n$$

The teacher replied that there is no mistake in the question, but their correction is sensible as a version of the question for the future examinations.

Questions to discuss

1. What is the methodology of econometrics?
2. What are main types of data in econometrics?
3. What is consistency?
4. Why sample size is important in econometrics?

Additional problems.

Problem 9. A random variable X has unknown population mean μ_X and population variance σ_X^2. A sample of n observations $\{X_1, ..., X_n\}$ is generated. Investigate whether estimator

$$Z = \frac{1}{2}X_1 + \frac{1}{4}X_2 + \frac{1}{8}X_3 + ... + \frac{1}{2^{n-1}}X_{n-1} + \frac{1}{2^n}X_n$$

is consistent estimator of μ_X.

Problem 10. A random variable X has population mean μ and variance σ^2. Given a sample of n independent observations X_i, $i = 1, ..., n$, determine whether the following estimator of μ

$$\frac{n+2}{n^2+3n+1}\sum_{i=1}^{n} X_i$$

is consistent (you may assume that $\overline{X}$ is a consistent estimator).

Problem 11. A random variable X has population mean μ and variance σ^2. Given a sample of n independent observations X_i, $i = 1, ..., n$, determine whether the following estimator of μ

$$\frac{\sum_{i=1}^{n} X_i^2}{\sum_{i=1}^{n} X_i}$$

is consistent (you may assume that $\overline{X}$ is a consistent estimator of μ_X).

CONVERGENCE IN PROBABILITY AND CONVERGENCE IN DISTRIBUTION

Problem 12. Let X be a discrete random variable with probability density function

$$p_X(x) = \begin{cases} \frac{1}{3} \; if \; x = 1 \\ \frac{2}{3} \; if \; x = 0 \\ 0 \; othewise \end{cases}$$

Consider a sequence of random variables $\{X_n\}$ whose generic term is $X_n = (1 + \frac{1}{n})X$. Prove that $\{X_n\}$ converges in probability to X.

Problem 13: Let $\{X_n\}$ be a sequence of random variables having distribution functions

$$F_n(x) = \begin{cases} 0 & if \; x < 0 \\ 1-(1-x)^n & if \; 0 \le x \le 1. \\ 1 & if \; x > 1 \end{cases}$$

Find the limit in distribution (if it exists) of the sequence $\{X_n\}$.