Elements of Econometrics. 2022-2023.
Class 11. Stochastic Regressors

Problem 1. (Revision of theory) What are different types of violation of the assumptions relating disturbance term in the regression model with stochastic regressors? What are consequences of this violation.

Problem 2. **(UoL Exam).** Explain what is correct, mistaken, confused or incomplete in the following statement:
"When an explanatory variable in a regression model has a random component, it is described as a stochastic regressor. When a stochastic regressor is used in a regression model, the Gauss-Markov condition that the explanatory variables should be independent of the disturbance term is violated. Consequently OLS regression estimates will be biased. However, they will be consistent because the bias will disappear in large samples."

Problem 3. **(UoL Exam)** A variable Y is determined by the model

$$Y = \beta_1 + \beta_2 Z + v,$$

where Z is a stochastic variable and v is a disturbance term that satisfies the Gauss–Markov conditions. The explanatory variable is subject to measurement error and is measured as X where $X = Z + w$ and w is the measurement error, distributed independently of v.
Describe analytically the consequences of using OLS to fit this model. It is assumed that expected value of w is 0, and w is distributed independently of Z.

Problem 4. **(UoL Exam).** Suppose that the true model is $Y = \beta_2 Z + u$, but Z is measured with measurement error w, the observed variable being $X = Z + w$. It may be assumed that w has zero mean and constant variance, and that it is distributed independently of Z. Derive an expression for the limiting value of the usual estimator of the slope coefficient, $\sum_{i=1}^{n} X_i Y_i \Big/ \sum_{i=1}^{n} X_i^2$.

Problem 5. (Short multiple choice test) Look at the demo on the screen and choose the correct answers on the following questions. Explain your answer.

Question 7. **(UoL Exam).** In the model

$$y_t = \beta x_t + u_t ; t = 1, 2, ..., T$$

x_t is measured with error. Data is only available on x_t^*, where

$$x_t^* = x_t + v_t ; t = 1, 2, ..., T$$

and $Eu_t = Ev_t = 0$, $E(u_t v_t) = E(x_t u_t) = E(x_t v_t) = 0$. y_t, x_t, and x_t^* have zero means.

(a) If $\hat{\beta}$ is the ordinary least squares (OLS) estimator of β from regressing y_t, on x_t^*, show that $\hat{\beta}$ is inconsistent.

(b) In the above given model, suppose x_t was measured without error, y_t was measured with error and data was only available on y_t^* where

$$y_t^* = y_t + w_t \text{ and } E(w_t) = 0;\ E(u_t w_t) = E(x_t w_t) = 0 \text{ and } E(v_t w_t) = 0.$$

Let $\hat{\beta}$ be the OLS estimator of β from regressing y_t^* on x_t. Is $\hat{\beta}$ consistent? Explain in detail.

(c) Suppose in the above given model, both y_t and x_t are measured with errors and data is available only on y_t^* and x_t^* where x_t^* and y_t^* are defined above, respectively. Discuss (without derivation) whether the OLS estimator of β, from regressing y_t^* on x_t^* will be consistent or inconsistent.