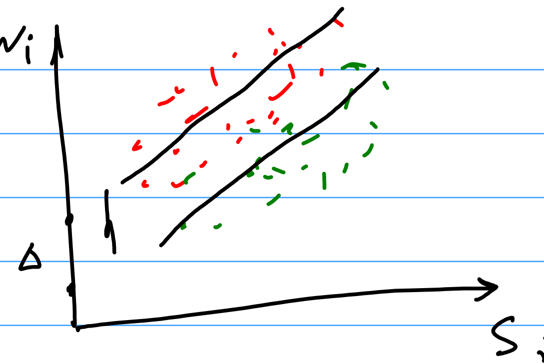
Dummy Variable

$$D_i = \begin{cases} 1, & \text{female} \\ 0, & \text{male} \end{cases}$$



1) $W_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot D_i + \varepsilon_i$

t - test for β_2 → male: $W_i = \beta_0 + \beta_1 \cdot S_i + \varepsilon_i$

refrence (all $D_i = 0$)

female: $W_i = (\beta_0 + \beta_2) + \beta_1 \cdot S_i + \varepsilon_i$

$$W_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot D_{m_i} + \beta_3 \cdot D_{f_i} + \varepsilon_i$$

$$\begin{bmatrix} 1 \\ \vdots \\ \\ \\ 1 \end{bmatrix} = \begin{bmatrix} 0 \\ \vdots \\ 0 \\ 1 \\ \vdots \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ \vdots \\ 1 \\ 0 \\ \vdots \\ 0 \end{bmatrix}$$

perfect multicollinearity

$$\text{rank}(X) = 3 < 4$$

$\hat{\beta} = (X'X)^{-1}X'y$ can't be obtained

since $X'X$ is not invertible

$W_i = \beta_1 \cdot S_i + \beta_2 \cdot D_{m_i} + \beta_3 \cdot D_{f_i} + \varepsilon_i$

const for female

const for male

2) $W_i = \beta_0 + \beta_1 \cdot S_i + \beta_2 \cdot D_i \cdot S_i + \varepsilon_i$

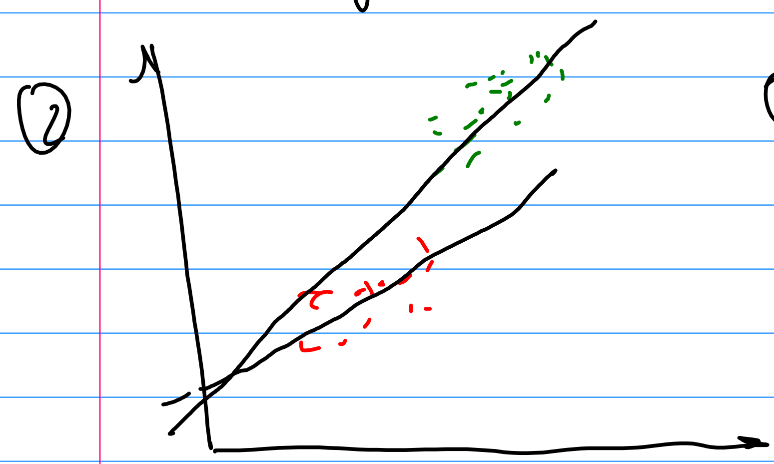
t-test for β_2

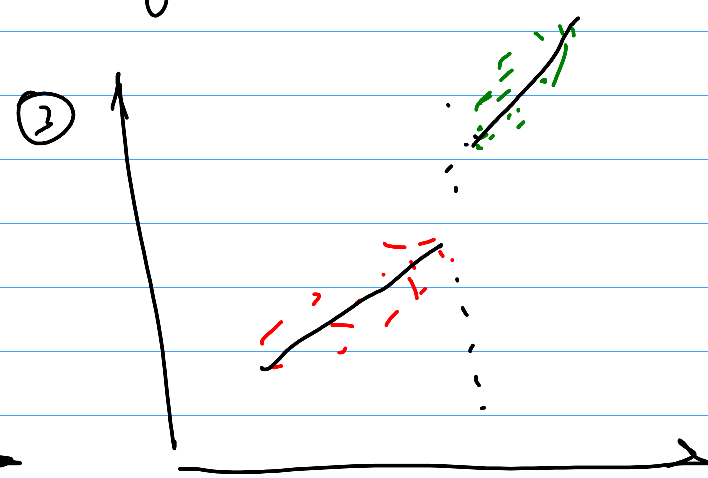
male: $w_i = \beta_0 + \beta_1 S_i$

female: $w_i = \beta_0 + (\beta_1 + \beta_2) \cdot S_i$

3) $W_i = \beta_0 + \beta_1 S_i + \beta_2 D_i + \beta_3 D_i S_i + \varepsilon_i$

F-test for $\beta_2 = 0$ and $\beta_3 = 0$





$$n = n_A + n_B$$

pooled: $y_i = \beta_0 + \beta_1 X_{1i} + \ldots + \beta_k X_{ki} + \varepsilon_i$

A: $y_i = \beta_0^A + \beta_1^A X_{1i} + \ldots + \beta_k^A X_{ki} + \varepsilon_i$

B: $y_i = \beta_0^B + \beta_1^B X_{1i} + \ldots + \beta_k^B X_{ki} + \varepsilon_i$

Chow test

H_0: $\beta_0^A = \beta_0^B, \ldots, \beta_k^A = \beta_k^B$

$$F = \frac{\left(RSS^{pooled} - (RSS^A + RSS^B)\right)/(k+1)}{(RSS^A + RSS^B)/(N - 2\cdot(k+1))} \sim F(k+1, N-2(k+1))$$

F-test:

$$y_i = \beta_0 + \beta_1 X_{1i} + \beta_k \cdot X_{ki} + \gamma_0 D_i + \gamma_1 X_{1i} \cdot D_i + \ldots + \gamma_k \cdot X_{ki} \cdot D_i + \varepsilon_i$$

H_0: $\gamma_0 = \ldots = \gamma_k$

$$F = \frac{(RSS^R - RSS^{UR})/(k+1)}{RSS^{UR}/(N - 2\cdot(k+1))} \sim F(k+1; N-2(k+1))$$

$RSS^R = RSS^{pooled}$

$RSS^{UR} = RSS^A + RSS^B$

$$\ln(y_i) = \beta_0 + \beta_1 X_{1i} + \beta_2 \cdot D_i + \varepsilon_i$$

$X \uparrow 1 \Rightarrow \beta_1 \cdot 100\ \% = d\ln(y) / dx_i$ log-linear

$D_i \uparrow 1 \Rightarrow \beta_2 \cdot 100\% = d\ln(y) / dD_i$

$$D_i = \begin{cases} 1, & \text{if } A \\ 0, & \text{else} \end{cases}$$

$\Delta D_i = 1 \uparrow \hookrightarrow 100 \cdot (e^{\beta_2} - 1)\ \% = \Delta \ln(y) / \Delta D_i$

$$y_i = e^{\beta_0 + \beta_1 X_{1i} + \beta_2 \cdot D_i + \varepsilon_i}$$

$$\Delta y = e^{\beta_0 + \beta_1 \cdot X_{1i} + \beta_2} - e^{\beta_0 + \beta_1 X_i}$$

$$= e^{\beta_0 + \beta_1 \cdot X_i}(e^{\beta_2} - 1)$$

$$100 \cdot \frac{\Delta y}{y} = 100 \cdot \frac{e^{\beta_0 + \beta_1 X_i}(e^{\beta_2} - 1)}{e^{\beta_0 + \beta_1 X_i}} = 100(e^{\beta_2} - 1)$$