Elements of Econometrics.
Lecture 17.
Modeling with Time Series Data.
Dynamic Processes.

FCS, 2022-2023

POLYNOMIAL DISTRIBUTED LAG MODEL

Polynomial Distributed lag :

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \ldots + u_t$$

Where $\beta_s = \gamma_0 + \gamma_1 \cdot s + \gamma_2 \cdot s^2 + \ldots + \gamma_m \cdot s^m; \quad s = -1,0,1,2,\ldots$

$$\beta_0 = \gamma_0 - \gamma_1 + \gamma_2 + \ldots; \quad \beta_1 = \gamma_0; \qquad \beta_2 = \gamma_0 + \gamma_1 + \gamma_2 + \ldots + \gamma_m$$

$$\beta_3 = \gamma_0 + 2\gamma_1 + 4\gamma_2 + \ldots + 2^m \gamma_m; \ldots$$

For example, for *n* = 3, *m* = 2 we get:

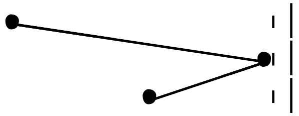
$$Y_t = \alpha + (\gamma_0 - \gamma_1 + \gamma_2) X_t + \gamma_0 X_{t-1} + (\gamma_0 + \gamma_1 + \gamma_2) X_{t-2} +$$
$$(\gamma_0 + 2\gamma_1 + 4\gamma_2) X_{t-3} + u_t = \alpha + \gamma_0 (X_t + X_{t-1} + X_{t-2} + X_{t-3}) +$$
$$\gamma_1 (-X_t + X_{t-2} + 2X_{t-3}) + \gamma_2 (X_t + X_{t-2} + 4X_{t-3}) =$$
$$\alpha + \gamma_0 Z_0 + \gamma_1 Z_1 + \gamma_2 Z_2.$$

$$Estimate \ \ \alpha \ and \ \gamma's, \quad then \ calculate \ \ \beta's.$$

POLYNOMIAL DISTRIBUTED LAG MODEL: EXAMPLE. DEPENDENT VARIABLE RUR_USD, REGRESSOR OIL_BRENT(-2). 09.01.2014-14.01.2017, 747 observations, Eviews.

ls rur_usd c pdl(oil_brent(-2),2,2)

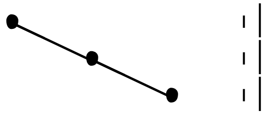
Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	89.56551	0.255745	350.2137	0.0000
PDL01	-0.008316	0.095010	-0.087526	0.9303
PDL02	0.092072	0.052901	1.740470	0.0822
PDL03	-0.244216	0.142521	-1.713537	0.0870



Lag Distribution of...	i	Coefficient	Std. Error	t-Statistic
	0	-0.34460	0.07116	-4.84242
	1	-0.00832	0.09501	-0.08753
	2	-0.16046	0.07108	-2.25761
Sum of Lags		-0.51338	0.00362	-141.876

ls rur_usd c pdl(oil_brent(-2),2,1)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	89.56151	0.256069	349.7553	0.0000
PDL01	-0.171107	0.001208	-141.6807	0.0000
PDL02	0.092063	0.052970	1.738036	0.0826



Lag Distribution of...	i	Coefficient	Std. Error	t-Statistic
	0	-0.26317	0.05304	-4.96186
	1	-0.17111	0.00121	-141.681
	2	-0.07904	0.05293	-1.49339
Sum of Lags		-0.51332	0.00362	-141.681

Estimation of Polynomial Distributed Lags in Eviews:
ls Y c pdl(X, Number of Lags, Order of Polynomial)

MODELS OF DYNAMIC PROCESSES:

PARTIAL ADJUSTMENT MODEL

$$Y_t^* = \beta_1 + \beta_2 X_t + u_t$$

$$\boldsymbol{Y_t - Y_{t-1} = \lambda(Y_t^* - Y_{t-1})}$$
$$\boldsymbol{Y_t = \lambda Y_t^* + (1-\lambda)Y_{t-1}}$$

$$\begin{aligned} Y_t &= \lambda\left(\beta_1 + \beta_2 X_t + u_t\right) + \left(1-\lambda\right)Y_{t-1} \\ &= \beta_1\lambda + \beta_2\lambda X_t + \left(1-\lambda\right)Y_{t-1} + \lambda u_t \\ &= \alpha_1 + \alpha_2 X_t + \alpha_3 Y_{t-1} + \lambda u_t \end{aligned}$$

where $\alpha_1 = \beta_1\lambda,\ \alpha_2 = \beta_2\lambda,\ \alpha_3 = \left(1-\lambda\right).$

Let *Y be target or desired (unobserved) value of *Y.* Parameter λ is called the speed of adjustment (complete adjustment if λ=1). Substituting for Y_t^* we obtain a specification with observable variables of the ADL(1,0) form.**

TIME SERIES MODELS:

STATIC MODELS AND MODELS WITH LAGS

$$LGHOUS_t^* = \beta_1 + \beta_2 LGDPI_t + \beta_3 LGPRHOUS_t + u_t$$

$$LGHOUS_t - LGHOUS_{t-1} = \lambda(LGHOUS_t^* - LGHOUS_{t-1})$$

$$LGHOUS_t = \lambda LGHOUS_t^* + (1-\lambda) LGHOUS_{t-1}$$

$$\begin{aligned} LGHOUS_t &= \lambda\left(\beta_1 + \beta_2 LGDPI_t + \beta_3 LGPRHOUS_t + u_t\right) + \left(1-\lambda\right) LGHOUS_{t-1} \\ &= \beta_1\lambda + \beta_2\lambda LGDPI_t + \beta_3\lambda LGPRHOUS_t + \left(1-\lambda\right) LGHOUS_{t-1} + \lambda u_t \\ &= \alpha_1 + \alpha_2 LGDPI_t + \alpha_3 LGPRHOUS_t + \alpha_4 LGHOUS_{t-1} + \lambda u_t \end{aligned}$$

short-run elasticities: $\alpha_2; \quad \alpha_3$

long-run elasticities: $\beta_2 = \dfrac{\alpha_2}{1-\alpha_4} \qquad \beta_3 = \dfrac{\alpha_3}{1-\alpha_4}$

Let *LGHOUS be the desired (unobserved) demand for housing, while the actual demand LGHOUS is determined by the Partial Adjustment process. *LGDPI* is the logarithm of disposable personal income, and *LGPRHOUS* is the logarithm of the relative price index for housing services.**

PARTIAL ADJUSTMENT:DEMAND FOR HOUSING

```
============================================================
Dependent Variable: LGHOUS
Method: Least Squares
Sample(adjusted): 1960 2003
Included observations: 44 after adjusting endpoints
============================================================
     Variable      Coefficient Std.Error t-Statistic  Prob.
============================================================
        C             0.074       0.063      1.175       0.247
      LGDPI           0.283       0.047      6.031       0.000
     LGPRHOUS        -0.117       0.027     -4.271       0.000
    LGHOUS(-1)        0.707       0.044      15.93       0.000
============================================================
```

long-run income elasticity $= \dfrac{0.283}{1-0.707} = 0.97$

long-run price elasticity $= \dfrac{-0.117}{1-0.707} = -0.40$

Here is the result of logarithmic regression of expenditure on housing on *DPI* and relative price, ADL(1,0) – transformed Partial Adjustment Model.

PARTIAL ADJUSTMENT: LINTNER's MODEL OF DIVIDEND ADJUSTMENT

Let D_t be actual dividend, and Π_t is profit. D_t^* is a "target" dividend dependent on profit to which the actual dividend is adjusting gradually.

$$D_t^* = \alpha + \gamma \Pi_t + u_t$$

$$D_t - D_{t-1} = \lambda (D_t^* - D_{t-1}) + v_t$$

$$D_t - D_{t-1} = \lambda \alpha + \gamma \lambda \Pi_t - \lambda D_{t-1} + \lambda u_t + v_t$$

$$D_t = \lambda \alpha + \gamma \lambda \Pi_t + (1 - \lambda) D_{t-1} + \lambda u_t + v_t$$

Here the explanatory variable D_{t-1} and the disturbance term are related, but not simultaneously correlated, therefore, the OLS estimators will be biased, but consistent (as well as in the general Partial Adjustment model).

G. Lintner has estimated the model directly on the data for the US corporate sector for 1918-1941 and has obtained the following results: $\gamma = 0.3; \;\; \lambda = 0.5.$

ADAPTIVE EXPECTATIONS

$$Y_t = \beta_1 + \beta_2 X^e_{t+1} + u_t$$

$$\boldsymbol{X^e_{t+1} - X^e_t = \lambda\left(X_t - X^e_t\right)}$$

$$\boldsymbol{X^e_{t+1} = \lambda X_t + \left(1-\lambda\right)X^e_t}$$

$$Y_t = \beta_1 + \beta_2 \lambda X_t + \beta_2\left(1-\lambda\right)X^e_t + u_t$$

$$\boldsymbol{X^e_t = \lambda X_{t-1} + \left(1-\lambda\right)X^e_{t-1}}$$

$$Y_t = \beta_1 + \beta_2 \lambda X_t + \beta_2 \lambda\left(1-\lambda\right)X_{t-1} + \beta_2\left(1-\lambda\right)^2 X^e_{t-1} + u_t$$

$$Y_t = \beta_1 + \beta_2 \lambda X_t + \beta_2 \lambda\left(1-\lambda\right)X_{t-1} + \beta_2 \lambda\left(1-\lambda\right)^2 X_{t-2} + \ldots$$
$$+ \beta_2 \lambda\left(1-\lambda\right)^{s-1} X_{t-s+1} + \beta_2\left(1-\lambda\right)^s X^e_{t-s+1} + u_t$$

X^e_{t+1} is the expected value of *X* (unobservable explanatory variable). We suppose that expectations are changing proportionally to the discrepancy between X^e_t, and the actual value X_t.

We assume $0 < \lambda \leq 1$, hence $(1 - \lambda)^s$ tends to zero as *s* grows, and the term with unobserved variable can be neglected.

ADAPTIVE EXPECTATIONS

$$X_{t+1}^e - X_t^e = \lambda\left(X_t - X_t^e\right)$$

$$Y_t = \beta_1 + \beta_2 X_{t+1}^e + u_t \qquad X_{t+1}^e = \lambda X_t + (1-\lambda)X_t^e$$

$$Y_t = \beta_1 + \beta_2 \lambda X_t + \beta_2(1-\lambda)X_t^e + u_t$$

$$Y_{t-1} = \beta_1 + \beta_2 X_t^e + u_{t-1}$$

$$\beta_2 X_t^e = Y_{t-1} - \beta_1 - u_{t-1}$$

$$\begin{aligned} Y_t &= \beta_1 + \beta_2 \lambda X_t + (1-\lambda)(Y_{t-1} - \beta_1 - u_{t-1}) + u_t \\ &= \beta_1\lambda + \beta_2 \lambda X_t + (1-\lambda)Y_{t-1} + u_t - (1-\lambda)u_{t-1} \\ &= \alpha_1 + \alpha_2 X_t + \alpha_3 Y_{t-1} + u_t - (1-\lambda)u_{t-1} \end{aligned}$$

where $\alpha_1 = \beta_1\lambda,\ \alpha_2 = \beta_2\lambda,\ \alpha_3 = (1-\lambda)$

This is again the ADL(1,0) model. The only difference with the partial adjustment model is the compound disturbance term. But the explanatory variable Y_{t-1} includes u_{t-1}, and hence C.7 assumption is violated and the OLS estimates are biased and inconsistent. The model has to be estimated as the geometrically distributed lag model.

ADAPTIVE EXPECTATIONS: FRIEDMAN'S PERMANENT INCOME HYPOTHESIS

$$C_t^P = \beta_2 Y_t^P$$

$$C_t = C_t^P + C_t^T$$

$$Y_t = Y_t^P + Y_t^T$$

$$Y_t^P - Y_{t-1}^P = \lambda(Y_t - Y_{t-1}^P)$$

$$Y_t^P = \lambda Y_t + (1-\lambda)Y_{t-1}^P$$

$$Y_{t-1}^P = \lambda Y_{t-1} + (1-\lambda)Y_{t-2}^P$$

$$C_t - C_t^T = \beta_2\left(\lambda Y_t + (1-\lambda)Y_{t-1}^P\right)$$

$$C_t = \beta_2 \lambda Y_t + \beta_2(1-\lambda)Y_{t-1}^P + C_t^T$$

$$C_t = \beta_2 \lambda Y_t + \beta_2 \lambda(1-\lambda)Y_{t-1} + \beta_2(1-\lambda)^2 Y_{t-2}^P + C_t^T$$

$$C_t = \beta_2 \lambda Y_t + \beta_2 \lambda(1-\lambda)Y_{t-1} + \beta_2 \lambda(1-\lambda)^2 Y_{t-2} + \ldots$$
$$+ \beta_2 \lambda(1-\lambda)^{s-1} Y_{t-s+1} + \beta_2(1-\lambda)^s Y_{t-s+1}^P + C_t^T$$

The specification is nonlinear in parameters and Friedman fitted it using nonlinear iterative estimation method.

ADAPTIVE EXPECTATIONS: FRIEDMAN'S PERMANENT INCOME HYPOTHESIS

$$C_t^P = \beta_2 Y_t^P$$

$$C_t = C_t^P + C_t^T$$

$$Y_t = Y_t^P + Y_t^T$$

$$Y_t^P - Y_{t-1}^P = \lambda(Y_t - Y_{t-1}^P)$$

$$Y_t^P = \lambda Y_t + (1-\lambda) Y_{t-1}^P$$

$$Y_{t-1}^P = \lambda Y_{t-1} + (1-\lambda) Y_{t-2}^P$$

$$C_t - C_t^T = \beta_2 \left(\lambda Y_t + (1-\lambda) Y_{t-1}^P \right)$$

$$C_t = \lambda \beta_2 Y_t + (1-\lambda) \beta_2 Y_{t-1}^P + C_t^T$$

$$\beta_2 Y_{t-1}^P = C_{t-1}^P = C_{t-1} - C_{t-1}^T$$

$$\begin{aligned} C_t &= \lambda \beta_2 Y_t + (1-\lambda)\left(C_{t-1} - C_{t-1}^T\right) + C_t^T \\ &= \lambda \beta_2 Y_t + (1-\lambda) C_{t-1} + C_t^T - (1-\lambda) C_{t-1}^T \end{aligned}$$

The short-run marginal propensity to consume is given by the coefficient of Y_t, $\lambda\beta_2$ and the long-run propensity is β_2. If to estimate in this form, C.7 assumption is violated, and the estimates are biased and inconsistent.

THE ERROR CORRECTION MODEL: TYPE 1

$$Y_t^* = \beta_1 + \beta_2 X_t + u_t$$

$$\begin{aligned}\Delta Y_t &= \lambda\left(Y_t^* - Y_{t-1}\right) + \delta \Delta X_t \\ &= \lambda\left(\beta_1 + \beta_2 X_t - Y_{t-1}\right) + \delta\left(X_t - X_{t-1}\right) + \lambda u_t \\ &= \lambda\beta_1 + (\lambda\beta_2 + \delta) X_t - \delta X_{t-1} - \lambda Y_{t-1} + \lambda u_t\end{aligned}$$

$$Y_t = \alpha_1 + \alpha_2 X_t + \alpha_3 Y_{t-1} + \alpha_4 X_{t-1} + \lambda u_t$$

$$\alpha_1 = \lambda\beta_1 \qquad \alpha_2 = \lambda\beta_2 + \delta$$

$$\alpha_3 = 1 - \lambda \qquad \alpha_4 = -\delta$$

Y* is a desirable (appropriate) unobserved value of Y. In the short run, $\Delta Y_t = Y_t - Y_{t-1}$, is determined by two components: closing the discrepancy between its "appropriate" and previous actual values, $Y^*_t - Y_{t-1}$, and a straightforward response to ΔX_t. This is ADL(1,1) model. The ADL(1,0) model is a special case with the testable restriction $\alpha_4 = -\delta = 0$.

THE ERROR CORRECTION MODEL: TYPE 2

$$Y_t^* = \beta_1 + \beta_2 X_t + u_t$$

$$\begin{aligned}
\Delta Y_t &= \lambda\left(Y_{t-1}^* - Y_{t-1}\right) + \delta \Delta X_t \\
&= \lambda\left(\beta_1 + \beta_2 X_{t-1} - Y_{t-1}\right) + \delta\left(X_t - X_{t-1}\right) + \lambda u_{t-1} \\
&= \lambda \beta_1 + \delta X_t + \left(\lambda \beta_2 - \delta\right) X_{t-1} - \lambda Y_{t-1} + \lambda u_{t-1}
\end{aligned}$$

$$Y_t = \alpha_1 + \alpha_2 X_t + \alpha_3 Y_{t-1} + \alpha_4 X_{t-1} + \lambda u_{t-1}$$

$$\alpha_1 = \lambda \beta_1 \qquad \alpha_2 = \delta$$

$$\alpha_3 = 1 - \lambda \qquad \alpha_4 = \lambda \beta_2 - \delta$$

Y* is a desirable (appropriate) unobserved value of Y. In the short run, $\Delta Y_t = Y_t - Y_{t-1}$, is determined by two components: closing the discrepancy between its previous "appropriate" and actual values, $Y^*_{t-1} - Y_{t-1}$, and a straightforward response to ΔX_t.

ERROR CORRECTION MODEL (TYPE 1): EXAMPLE

$$Y_t^* = \beta_1 + \beta_2 X_t + u_t$$

$$\Delta Y_t = \lambda\left(Y_t^* - Y_{t-1}\right) + \delta \Delta X_t + \lambda u_t = \lambda\beta_1 + (\lambda\beta_2 + \delta) X_t - \delta X_{t-1} - \lambda Y_{t-1} + \lambda u_t$$

$$RUR\_USD_t = \alpha_1 + \alpha_2 OIL\_BRENT_{t-2} +$$
$$+ \alpha_3 RUR\_USD_{t-1} + \alpha_4 OIL\_BRENT_{t-3} + u_t$$

Dependent Variable: RUR_USD; 14.01.2014-14.01.2017, 744 obs.

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.069175	1.183033	5.130184	0.0000
OIL_BRENT(-2)	-0.257502	0.025500	-10.09801	0.0000
OIL_BRENT(-3)	0.223103	0.025962	8.593349	0.0000
RUR_USD(-1)	0.932065	0.013166	70.79289	0.0000

$$\hat{\alpha}_1 = \hat{\lambda}\hat{\beta}_1 = 6.07; \quad \hat{\alpha}_2 = \hat{\lambda}\hat{\beta}_2 + \hat{\delta} = -0.2575; \hat{\alpha}_3 = 1 - \hat{\lambda} = 0.932; \quad \hat{\alpha}_4 = -\hat{\delta} = 0.223;$$

$$Hence \quad \hat{\beta}_2 = -0.51; \quad \hat{\beta}_1 = 89.3.$$