Econometrics-2022-2023.
Home assignment 7. Dynamic Models
To be submitted by **January, 30, 23:59**

1. [30 marks] In a certain bond market the demand for bonds, B_t, in period t is negatively related to the expected interest rate, i_{t+1}^e, in period $t+1$:

$$B_t = \beta_1 + \beta_2 i_{t+1}^e + u_t \qquad (1)$$

where u_t is a disturbance term not subject to autocorrelation. The expected interest rate is determined by an adaptive expectations process:

$$i_{t+1}^e - i_t^e = \lambda(i_t - i_t^e) \qquad (2)$$

where i_t is the actual rate of interest in period *t*. A researcher uses the following model to fit the relationship:

$$B_t = \gamma_1 + \gamma_2 i_t + \gamma_3 B_{t-1} + v_t \qquad (3)$$

where v_t is a disturbance term.

1.1 [10 marks] Show how the model (3) may be derived from the demand function and the adaptive expectations process. Use finite (one-step) Koyck transformation.

1.2 [10 marks] Explain why inconsistent estimates of the parameters will be obtained if equation (3) is fitted using ordinary least squares (OLS).
(A mathematical proof is not required. Do not attempt to derive an expression for the bias. But all formulas and expressions used in your explanation should be clearly and explicitely indicated)

1.3 [10 marks] Describe a method for fitting the model that would yield consistent estimates.

2. [20 marks] The regression output gives the result of regressing *LGELEC*, the logarithm of consumer expenditure on electricity, on *LGDPI*, the logarithm of disposable personal income, *LGPRELEC*, the logarithm of a relative price index for electricity, and *LGELEC* lagged one year, using annual aggregate data for the United States for the period 1960 - 1994. Potential problems of nonstationarity may be ignored.

$$\underset{\quad}{LGELEC} = \underset{(0.836)}{-0.0367} + \underset{(0.135)}{0.0753}LGDPI - \underset{(0.052)}{0.0447}LGPRELEC + \underset{(0.100)}{0.9161}LGELEC(-1) \qquad R^2 = 0.998$$

2.1. [10 marks] □ Show how this regression specification could be derived from a partial adjustment model.

□ Explain the short-run and long-run dynamics inherent in such a model.

2.2. [10 marks] □ Give an economic interpretation of the regression results, paying attention to both short-run and long-run dynamics. Comment on the plausibility of the estimated short-run and long-run effects.

□ At a seminar, a commentator points out that the regression specification could also have been derived from an adaptive expectations model. State whether the adaptive expectations model might be a more suitable framework for this model. (*Note*: *a mathematical demonstration that the regression specification could have been derived from an adaptive expectations model is not required and no credit will be given for it*).

□ Based on which economic and/or econometric considerations can one choose between two interpretations of the constructed model (partial adjustment scheme or adaptive expectation scheme)?

In answering the practical questions, no general theoretical/mathematical explanations are acceptable, nor any information on the topic that is not directly related to the question. The use of extended mathematical comments instead of meaningful discussion of the questions asked will result in a lower grade.

3. [50 marks] Use data file **ha07_data_NN.** Full description of variables can be found in the file HA07 Data Description.pdf). The purpose of this exercise is to estimate and compare in various ways the long-run and short-run income DPI_t effects on personal business $BUSI_t$ in the United States on the basis of annual time series, using logarithmic models, including the lagged variables. The control variable in all models is the current relative prices of personal business services. For the simplicity, in all tasks we will refer to $Y_t = LGBUSI_t$, $X_t = LGDPI_t$, $Z_t = LGPRBUSI_t$. In your analysis, ignore the problems associated with autocorrelation and non-stationarity of the time series.

3.1. [10 marks] □ As a starting point, estimate a multiple regression model

$$Y_t = \beta_1 + \beta_2 X_t + \gamma Z_t + u_t \qquad \textbf{(1)}$$

Discuss the economic meaning of its coefficients, their plausibility and the economic and econometric problems associated with their estimation.

□ Why is there a need to estimate models with a more complex structure? What is the purpose of including a control variable in the model Z_t?

□ Step by step, include additional lag variables in the model

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \gamma Z_t + u_t \qquad (2)$$

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + \gamma Z_t + u_t \qquad (3)$$

$$Y_t = \beta_1 + \beta_2 X_t + \beta_3 X_{t-1} + \beta_4 X_{t-2} + \beta_5 X_{t-3} + \gamma Z_t + u_t \qquad (4)$$

Comment on the estimated coefficients of the models. What are the comparative advantages and disadvantages of these models? Would it be appropriate to include also lagged variables related to the variable Z_t?

3.2. [10 marks] □ Using an assumption about the geometric distribution of the coefficients under the lag variables (Koyck distribution) obtain and estimate an appropriate model.

Perform model estimation in two ways:

□ **1)** by a stepwise variation of the Koyck distribution parameter (use a step of 0.2 or 0.1), and

□ **2)** by a non-linear least squares method. Give and comment on the Eviews commands used in the process.

It is sufficient to limit the lag value to 3, which corresponds to model (4) with constraints.

3.3. [10 marks] □ Perform the Koyck transformation in the context of the model in question. Write out the model resulting from the Koyck transformation, comment its structure.

□ Evaluate it using available data. What are the properties of the resulting estimates? Compare them with the properties of the estimates obtained for the Koyck distribution model in the previous paragraph 3.3.

□ Interpret the obtained results in the context of the partial adjustment model and in the context of the adaptive expectation model (no mathematics is expected here)

3.4. [10 marks] □ Using the estimated model show graphically that the line representing long term effect of income has steeper slope than short term line (represent on the graph Y_t as a function of X_t and draw the lines corresponding to the different years starting in 1960 at 5-year intervals (1960, 1965, ….2000) – to evaluate parameters of lines and points on them substitute values of X_t, Y_{t-1}, and Z_t to the estimated model, mark the obtained points and connect them (*see details in the textbook by Ch.Dougherty 4th ed. p.402, 5th ed. p.417*). Present and explain your work.

3.5. [10 marks] □ Construct the model using polynomial Almon lags, extend number of lags up to 3,

$$Y_t = \alpha + \beta_0 X_t + \beta_1 X_{t-1} + \beta_2 X_{t-2} + \beta_3 X_{t-3} + \beta_4 Z_t + u_t$$

and set polynomial distribution being quadratic $\beta_s = \gamma_0 + \gamma_1 \cdot s + \gamma_2 \cdot s^2; \quad s = 0, 1, 2, 3.$

Use **1)** direct method based on the Eviews function pdl(*LGDPI*,3,2)

2) manually write down all coefficients of estimated equation and then use non-linear least squares for estimation of obtained equation. Comment the results.