

Heteroscedasticity

Part I.

$$\text{Var}(\epsilon_i) = E(\epsilon_i^2) \neq \sigma_\epsilon^2$$

WLS.

$$V_i = \alpha + \beta I_i + \epsilon_i \quad \epsilon_i \sim N(0, \sigma_\epsilon^2 I_i^2)$$

$$\text{Var}(\epsilon_i) = \sigma_\epsilon^2 \cdot h(I_i)$$

$$\text{Var}(u_i) = \text{Var}\left(\frac{\epsilon_i}{\sqrt{h(I_i)}}\right) = \sigma_\epsilon^2$$

Robust
or HC
s.e.

$$\text{Var}(\hat{\beta}) = \frac{\sigma_\epsilon^2}{\sum (x_i - \bar{x})^2}$$

unbiased
cons. $\rightarrow \hat{\beta} = (X'X)^{-1} X'y$

$$\text{Var}(\hat{\beta}) = \sigma_\epsilon^2 (X'X)^{-1}$$

(White)

$$\hat{\text{se}}(\hat{\beta}) = \sqrt{\frac{\frac{1}{n-2} \sum \hat{\epsilon}_i^2}{\sum (x_i - \bar{x})^2}}$$

$$\hat{\text{Var}}(\hat{\beta}) = \hat{\sigma}_\epsilon^2 (X'X)^{-1}$$

$$\hat{\text{se}}_{\text{hc}}(\hat{\beta}) = \sqrt{\frac{\frac{1}{n-2} \sum (x_i - \bar{x})^2 \hat{\epsilon}_i^2}{\frac{1}{h} \hat{\text{Var}}(x_i)^2}}$$

Q1(a) H_0 : homoscedastic error

H_a : heterosced. error

$$(\sigma_e^2 \sim I_i)$$

$$F = \frac{RSS_2 / 30 - 2}{RSS_1 / 20 - 2} = 7.84 \quad \begin{array}{l} \neq \text{1\% crit} (28.18) \\ = 2.98 \end{array}$$

(b) H_0 : homoscedastic errors

H_a : heterosc. errors

$$\hat{\epsilon}_i^2 \mid I, I^2, u, v, \cancel{X^2}, u^2, \cancel{v^2}, \text{cross-terms}$$

$$n \cdot R_{aux}^2 \sim \chi_q^2$$

$$q = 12 \quad (\# \text{ reg in aux model})$$

$$82 \cdot 0.57 = 46.7$$

$$\chi_{1\%, 12}^2 = 23.2$$