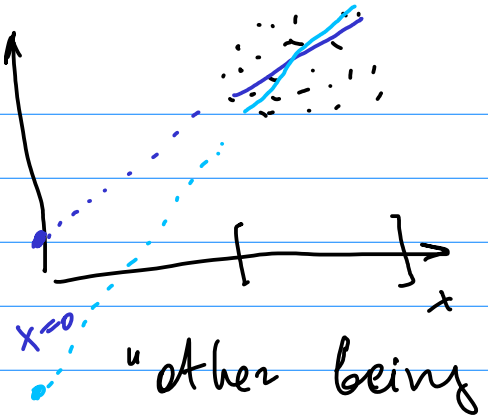
Transformation of variables

1) $y_i = \beta_0 + \beta_1 \cdot x_i + \varepsilon_i$

β_0 : $E(y_i \mid x_i = 0) = \beta_0$

β_1 : $x \uparrow 1 \Rightarrow y \uparrow \beta_1$



"other being fixed"

β_1 : "on average"

2) $y_i = e^{\beta_0} \cdot x_i^{\beta_1} \cdot e^{\varepsilon_i}$ ($\beta_0^* = e^{\beta_0}$)

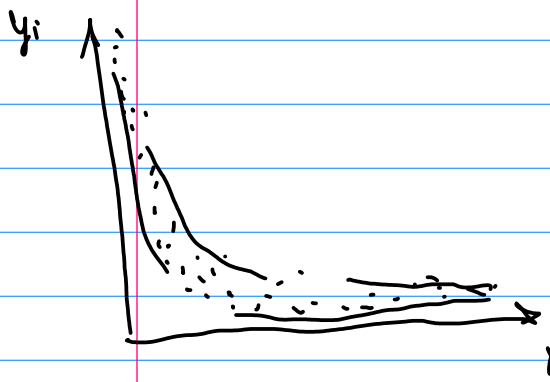
$y_i = \beta_0^* x_i^{\beta_1} \cdot u_i$

↑ multiplicative

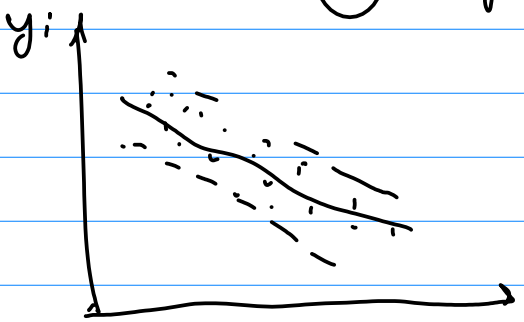
$\log y_i = \beta_0 + \beta_1 \log(x_i) + \varepsilon_i$

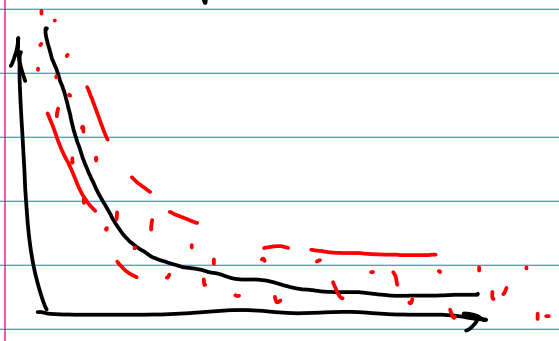
(1) $y_i = \beta_0^* \cdot x_i^{\beta_1} + u_i$

↑ additive

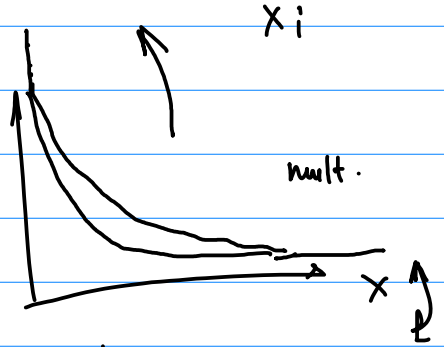


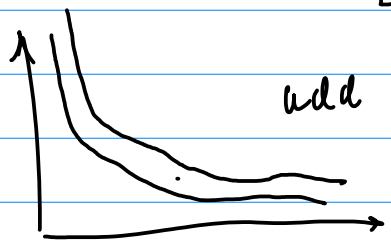
multiplicative





additive





$$\log y_i = \beta_0 + \beta_1 \log(x_i) + \varepsilon_i$$

β_1 : $x_i \uparrow 1\%$ $\quad y \uparrow \beta_1\%$

$$d\log y_i \Big/ d\log(x_i) = \frac{100 \cdot \frac{dy_i}{y_i}}{\frac{dx_i}{x_i} \cdot 100} = \beta_1$$

3) $y_i = \beta_1 + \beta_2 \log x_i + \varepsilon_i$

$$dy_i \Big/ d\log x_i = dy_i \Big/ \frac{1}{100} \frac{dx_i}{x_i}\% = \frac{\beta}{100}$$

$x \uparrow 1\%$ $\quad y \uparrow \beta_2/100$

4) $\log y_i = \beta_1 + \beta_2 x_i + \varepsilon_i$

$$d(\log y_i) \Big/ dx_i = 100 \cdot \frac{dy_i}{y_i}\% \Big/ dx_i = \beta_2 \cdot 100$$

$x_i \uparrow 1 \Rightarrow y \uparrow \beta_2 \cdot 100\%$

Quadratic term

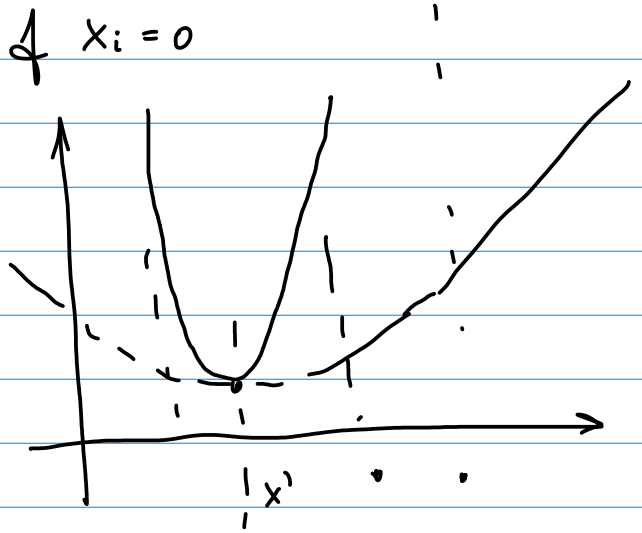
$$y_i = \beta_1 + \beta_2 X_i + \beta_3 X_i^2$$

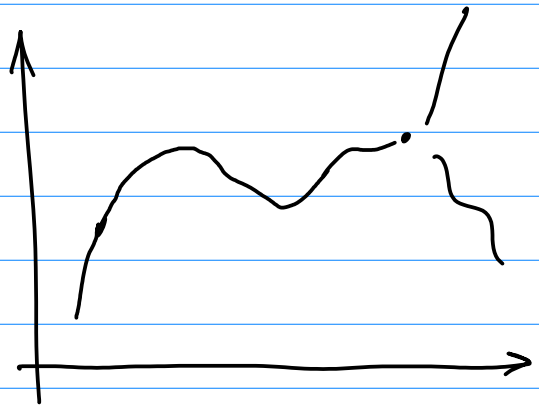
β_2 : $X \uparrow 1 \Rightarrow Y \uparrow \beta_2$ if $X_i = 0$

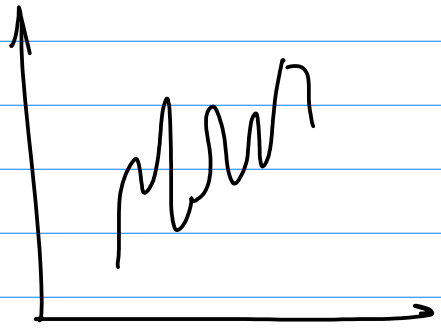
$$dy_i / dx_i = \beta_2 + \beta_3 X_i$$

β_3 : sign - direction

value - effect size







Multiplicative Term

$$y_i = \beta_1 + \beta_2 \cdot x_{1i} + \beta_3 x_{2i} + \beta_4 x_{1i} \cdot x_{2i} + e_i$$

$$dy_i / dx_{1i} = \beta_2 + \beta_4 x_{2i}$$

β_2: $x_1 \uparrow 1$ $y \uparrow \beta_2$ if $x_{2i} = 0$

$$x_1^* = x_{1i} - \bar{x}_1$$

$$y_i^* = y - \bar{y}$$

$\Rightarrow \hat{\beta}_2^*$: $x_{2i}^* = 0$ $x_2 = \bar{x}_2$

$x_1 \uparrow 1$ $y \uparrow \beta_2$ if $x_2 = \bar{x}_2$

β_4: sign - direction of effect x_2

value - effect size

RESET test (mis specification)

$$y_i = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k + u_i \Rightarrow \hat{y}_i$$

$$y_i = \beta_0 + \beta_1 X_1 + \dots + \beta_k X_k + \gamma_2 \hat{y}_i^2 + \dots + \gamma_p \cdot \hat{y}_i^p + \varepsilon_i$$

F-test for $\gamma_2 = \dots = \gamma_p = 0$

Test Box Cox

$$\frac{n}{2} \left| RSS_1 / RSS_2 \right| \sim \chi^2_1$$

$$\frac{50}{2} \left| 0,08 / 0,02 \right| = 4$$