ARDL (p,q)

$$y_t = \gamma + \alpha_1 \cdot y_{t-1} + \dots + \alpha_p y_{t-p} +$$

$$\beta_0 X_t + \beta_1 X_{t-1} + \dots + \beta_q \cdot X_{t-q} + \varepsilon_t$$

$\uparrow$ SR

$\varepsilon_t \sim N(0, \sigma^2)$ iid

Distributed Lag Models

Geometrically (Koyck's) Dist. lags

Polynomial (Almond's) Dist. Lags

$$Y_t = \alpha + \sum_{j=0}^{\infty} \beta_j \cdot X_{t-j} + \varepsilon_t$$

$$\tilde{Y} = \alpha + \beta_0 \tilde{X} + \beta_1 \cdot \tilde{X} + \dots + \beta_\infty \tilde{X} + \cancel{\varepsilon}$$

SR : β_0

$$\tilde{Y} = \alpha + \sum \beta_j \cdot \tilde{X}$$

LR : $\sum \beta_j < \infty$

Geom. (Koyck's) Lag Model :

$$W_j = (1 - \lambda) \cdot \lambda^j \quad , : \quad 0 < \lambda < 1$$

$$\sum_{j=0}^{\infty} W_j = \frac{1-\lambda}{1-\lambda} = 1 \qquad \sum \lambda^j = \frac{1}{1-\lambda}$$

$$y_t = \alpha + \beta(1-\lambda) \sum \lambda^j X_{t-j} + \varepsilon_t$$

Autoregressive form: $\lambda \approx 1$ slow decay

$\lambda \approx 0$ fast decay

$$y_t = \alpha(1-\lambda) + \beta(1-\lambda) X_t + \lambda y_{t-1} + \varepsilon_t - \lambda \varepsilon_{t-1} =$$

$$= \alpha_0 + \beta_0 X_t + \lambda y_{t-1} + u_t$$

SR: $\beta_0 = \beta(1-\lambda)$ $\qquad \tilde{y} = \alpha_0 + \beta_0 \tilde{X} + \lambda \tilde{y}$

LR: $\dfrac{\beta_0}{1-\lambda} = \beta$ $\qquad \tilde{y} = \dfrac{\alpha_0}{1-\lambda} + \dfrac{\beta_0}{1-\lambda} \tilde{X}$

Polynomial (Almondis) Lag Model

$$y_t = \alpha_0 + \sum_{j=0}^{q} \beta_j X_{t-j} + \varepsilon_t$$

$p \leq q$, p - order polynomial

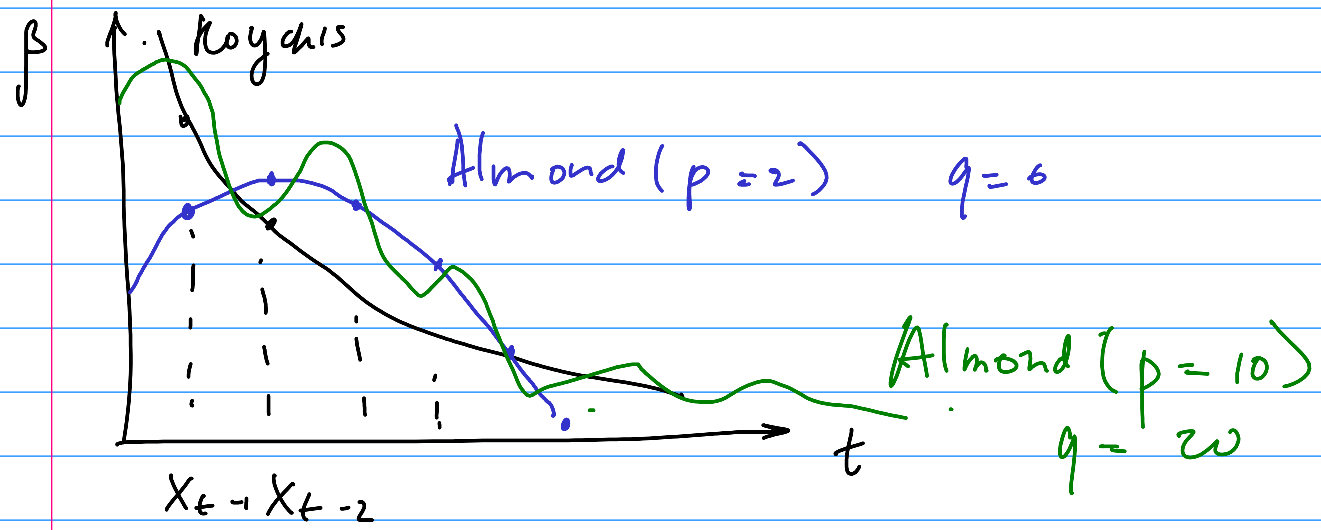
$$\beta_j = \gamma_0 + \gamma_{1j} j^1 + \dots + \gamma_{pj} \cdot j^p = \sum_{k=0}^{p} \gamma_k j^k$$

$$y_t = \alpha_0 + \sum_{k=0}^{p} \gamma_k \cdot Z_{ik} + \varepsilon_t$$

$$Z_{ik} = \sum_{j=0}^{q} j^k \cdot X_{i-j}$$

SR: $\beta_0 = \gamma_0$

LR: $\beta = \sum \beta_j = \sum \gamma_k \sum j^k$



Economic Models (with Koyck's Lags)

1) Partial Adjustment $\quad y_t - y_t^* \mid K_t$

2) Adaptive Expectations $\quad u_t \mid \pi_t - \pi_t^e$

① PAM

$$\left[y_t^* = \alpha + \beta X_t + \varepsilon_t \right]$$

↳ unobserved long-run/equilibrium y_t,

λ - PA coef. s.t. <u>PA hypothesis</u>:

$\lambda \approx 0$ fast adj $\quad y_t - y_{t-1} = (1-\lambda)(y_t^* - y_{t-1})$

$\lambda \approx 1$ slow adj

lin. comb ← $\{ y_t = (1-\lambda) y_t^* + \lambda y_{t-1} \}$

$$= (1-\lambda) y_t^* + \lambda(1-\lambda) y_{t-1}^* +$$

$$+ \lambda^2 (1-\lambda) y_{t-2}^* + \dots =$$

$$= (1-\lambda) \Sigma \lambda^j y_{t-j}^*$$

$$y_t = (1-\lambda)(\alpha + \beta X_t + \varepsilon_t) +$$

$$\lambda(1-\lambda) \cdot \sum_{j=1}^{\infty} \lambda^j \cdot (\alpha + \beta X_{t-1} - \varepsilon_{t-1})$$

$$y_t = \alpha + \beta \cdot \sum_{j=0}^{\infty} w_j \cdot X_{t-j} + \sum_{j=0}^{\infty} w_j \varepsilon_{t-j}$$

$$w_j = (1-\lambda)\lambda^j$$

Autoregressive form:

$$y_t = \alpha_0 + \beta_0 X_t + \lambda y_{t-1} + v_t$$

SR: $\beta_0 = \beta(1-\lambda)$

LR: $\beta_0 / 1-\lambda = \beta$

$$\alpha_0 = \alpha \cdot (1-\lambda)$$

$$\beta_0 = \beta \cdot (1-\lambda)$$

$$v_t = \Sigma w_j^* \varepsilon_{t-j}$$

$$= (1-\lambda) \cdot \varepsilon_t$$

E.g. $\lambda = 0{,}9 \leftarrow$ P.A. coef.

$$\beta_0 = 0{,}01$$

$$\beta = 0{,}01 / 0{,}1 = 0{,}1$$