Time Series Analysis

ARIMA(p, d, q)

→ ARMAX(p, q) : $y_t = \beta_0 + \beta_1 \cdot y_{t-1} + \dots + \beta_p y_{t-p} + \gamma \cdot X_t + \underbrace{\varepsilon_t + \alpha_1 \varepsilon_{t-1} + \dots + \alpha_q \varepsilon_{t-q}}_{u_t}$

→ ARDL(p, q) : $y_t = \beta_0 + \beta_1 y_{t-1} + \dots + \beta_p y_{t-p} + \alpha_0 X_t + \alpha_1 X_{t-1} + \dots + \alpha_q X_{t-q} + \varepsilon_t$

$$\begin{cases} E(\varepsilon_t) = 0 & \forall t \\ Var(\varepsilon_t) = \sigma_\varepsilon^2 & \forall t \\ Cov(\varepsilon_t, \varepsilon_{t-k}) = \gamma_k = 0 & \forall t, \ \forall k \geq 1 \end{cases}$$

ε_t — WN

Difference stationary

$\Delta y_t = y_t - y_{t-1}$ — stat.

Trend stationary

$y_t - \alpha - \beta t$ — stat

L — lag operator

$L^k \cdot y = y_{t-k}$

$y = \begin{pmatrix} y_1 \\ \vdots \\ y_T \end{pmatrix} \qquad L \cdot y = \begin{pmatrix} 0 \\ y_1 \\ \vdots \\ y_{T-1} \end{pmatrix} \qquad Ly = y_{t-1}$

P1

ARDL(1, 0)

$$y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 \cdot y_{t-1} + \overset{WN}{u_t}$$

$$y_t = \beta_0 + \beta_1 X_t + \beta_2 L y_t + u_t \qquad |\beta_2| \le 1$$

$\beta_2 > 1$

$$(1 - \beta_2 L)\, y_t = \beta_0 + \beta_1 \cdot X_t + u_t$$

$$\frac{b_0}{1-q}$$

$$y_t = \frac{\beta_0}{1-\beta_2 \cdot L} + \frac{\overset{= b_0}{\beta_1 X_t}}{\underset{= q}{1-\beta_2 L}} + \frac{u_t}{1-\beta_2 L}$$

$$y_t = \beta_0^* + \beta_1 X_t + \overset{= \beta_1 \beta_2 X_{t-1}}{\beta_2 L \beta_1 X_t} +$$

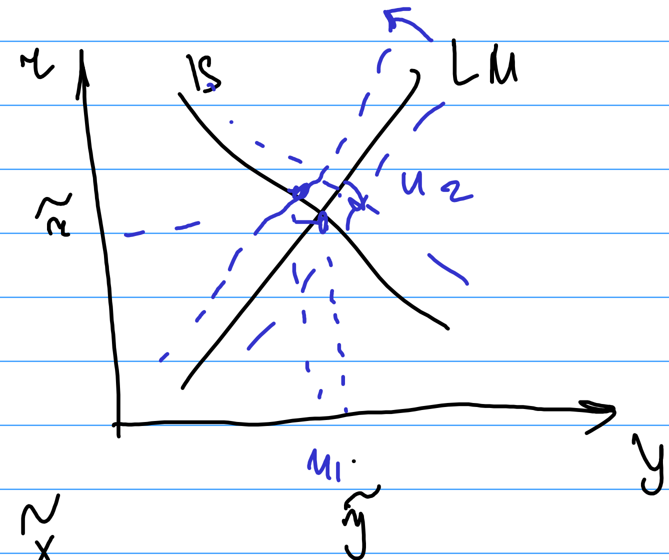
$$+ \beta_2^2 \beta_1 \cdot X_{t-2} + \dots + u_t^*$$

ARDL(1, 0) $\leftrightarrow$ ARDL(0, ∞)

(P2) $y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 \cdot y_{t-1} + u_t$

β_1 – SR effect of X on y

$\tilde{y}, \tilde{X}$ – equilibrium X and



$t \to \infty$ $\quad \tilde{y} = \beta_0 + \beta_1 \tilde{X} + \beta_2 \tilde{y}$

$$(1-\beta_2)\tilde{y} = \beta_0 + \beta_1 \tilde{X}$$

$$\tilde{y} = \frac{\beta_0}{1-\beta_2} + \frac{\beta_1}{1-\beta_2} \cdot \tilde{X}$$

$|\beta_2| < 1$

$$\beta_1 + \beta_1 \cdot \beta_2 + \dots = \frac{\beta_1}{1-\beta_2} \text{ – LR effect of } X \text{ on } y$$

$$\vee$$

β_1 – SR effect of X on y

Q3 $y_t = \beta_0 + \beta_1 \cdot X_t + \beta_2 \cdot y_{t-1} + u_t$

$\hookrightarrow$ AR(1)

$X_{t-1}, y_{t-2}, u_{t-1}$ $\longleftrightarrow$ $u_t = \rho u_{t-1} + \varepsilon_t$

Consequences of Autocorrelation

- if y_{t-1} in RHS $\Rightarrow$ endogeneity
- else consequences the same as heteroscedasticity

$\hat\beta_{OLS}$ – inefficient $\quad se(\hat\beta_{OLS})$

$\shortparallel \qquad\qquad\qquad \vee$

$\hat\beta_{GLS}$ – efficient $\quad se(\hat\beta_{GLS})$

$$\varepsilon'\varepsilon = \sigma_\varepsilon^2 \begin{pmatrix} 1 & 0 & \cdots & 0 \\ 0 & \ddots & \ddots & \vdots \\ \vdots & \ddots & \ddots & 0 \\ 0 & \cdots & 0 & 1 \end{pmatrix}$$

TGM

$(X'X)^{-1}X'y$

$$\Omega = \varepsilon'\varepsilon = \sigma^2 \begin{pmatrix} X_1 & \rho & 0 & \cdots & 0 \\ \rho & \ddots & \ddots & \ddots & 0 \\ 0 & \ddots & \ddots & \ddots & \rho \\ 0 & \cdots & 0 & \rho & X_n \end{pmatrix}$$

$\hat\Omega$