SEM

1. omitted variables
2. measurement errors
3. simultaneity
   $y \leftarrow x$ & $y \rightarrow x$

lgdp | prot

P1.

$$\begin{cases} y_i = \beta_1 + \beta_2 X_i + \varepsilon_i, & \beta_2 < 0 \\ X_i = \alpha_1 + \alpha_2 y_i + u_i, & \alpha_2 > 0 \end{cases}$$

structural form $\qquad cov(\varepsilon_i, u_i) = 0$

$$X_i = \alpha_1 + \alpha_2 \cdot (\beta_1 + \beta_2 X_i + \varepsilon_i) + u_i$$

$$\begin{cases} X_i = \dfrac{\alpha_1 + \alpha_2 \beta_1 + \alpha_2 \cdot \varepsilon_i + u_i}{1 - \alpha_2 \beta_2} \\ y_i = \dfrac{\beta_1 + \alpha_1 \cdot \beta_2 + \varepsilon_i + \beta_2 u_i}{1 - \alpha_2 \beta_2} \end{cases}$$

reduced form

$$\operatorname{cov}(X_i, \varepsilon_i) = \operatorname{cov}\left(\frac{\cancel{\alpha_1 + \alpha_2 \beta_1} + \alpha_2 \cdot \varepsilon_i + u_1}{1 - \alpha_2 \beta_2}, \varepsilon_i\right) =$$

$$= \frac{\alpha_2 \sigma_\varepsilon^2}{1 - \alpha_2 \beta_2} > 0 \quad \Leftarrow \begin{cases} \alpha_2 > 0 \\ \beta_2 < 0 \end{cases}$$

$$\left[\ \hat{\beta}_2 \xrightarrow{p} \beta + \underbrace{\frac{\operatorname{cov}(X_i, \varepsilon_i)}{\operatorname{Var}(X_1)}}_{\text{bias}}\ \right]$$

$\operatorname{cov}(\ln P_i, \ln T_i) \neq 0$

$\operatorname{cov}(\ln T_i, \varepsilon_i) = 0$

P2

$$\begin{cases} D_i: & \ln Q_i = \beta_1 + \beta_2 \cdot \ln P_i + \varepsilon_i, \quad \beta_2 < 0 \leftarrow \\ S: & \ln Q_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \cdot \ln T_i + u_i, \end{cases}$$

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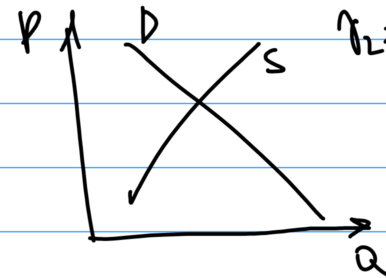
$$\hat{\beta}_2 \xrightarrow{p} \beta_2 + \frac{\operatorname{cov}(\ln P_i, \varepsilon_i)}{\operatorname{Var}(\ln P_i)}$$



$\gamma_2 > 0$

$$\beta_1 + \beta_2 \cdot \ln P_i + \varepsilon_i = \gamma_1 + \gamma_2 \cdot \ln P_i + \gamma_3 \cdot \ln T_i + u_i$$

$$\operatorname{cov}(\ln P_i, \varepsilon_i) = \operatorname{cov}\left(\frac{\beta_1 - \gamma_1 - \gamma_3 \cdot \ln T_i - u_i + \varepsilon_i}{\gamma_2 - \beta_2}, \varepsilon_i\right) =$$

$$= \left\{ \begin{matrix} \operatorname{cov}(T_i, \varepsilon_i) = 0 \\ \operatorname{cov}(\varepsilon_i, u_i) = 0 \end{matrix} \right\} = \frac{\sigma_\varepsilon^2 > 0}{\gamma_2 - \beta_2 > 0} > 0$$

$\ln Q_i$ β_2

$\ln P_i$ $\frac{1}{\gamma_2}$