Heteroscedasticity

Part II

WLS. $y_i = \alpha + \beta x_i + \varepsilon_i \quad | : x_i^{3/2} \quad | : \sqrt{h(x_i)}$

$Var(\varepsilon_i) = \sigma_\varepsilon^2 \cdot x_i^3 \qquad Var(u_i) = Var\left(\frac{\varepsilon_i}{\sqrt{h(x_i)}}\right) = \sigma_\varepsilon^2$

$Var(\varepsilon_i) = \sigma_\varepsilon^2 \cdot h(x_i)$

$\hat{\beta} = (X'X)^{-1} X'y$

Robust or HC s.e. (White)

$Var(\hat{\beta}) = \frac{\sigma_\varepsilon^2}{\sum (X_i - \bar{X})^2}$

$Var(\hat{\beta}) = \sigma_{\varepsilon_1}^2 \cdot (X'X)^{-1}$

$\widehat{Var}(\hat{\beta}) = \hat{\sigma}_\varepsilon^2 (X'X)^{-1}$

↑ biased and inconsistent

$\rightarrow \hat{se}(\hat{\beta}) = \sqrt{\frac{\frac{1}{n-2} \cdot \sum \hat{\varepsilon}_i^2}{\sum (X_i - \bar{X})^2}}$ ← SER

$\rightarrow \hat{se}_{HC}(\hat{\beta}) = \sqrt{\frac{\frac{1}{n-2} \sum (X_i - \bar{X})^2 \hat{\varepsilon}_i^2}{\frac{1}{n} \left(\sum (X_i - \bar{X})^2\right)^2}}$

Problem 1.

(a) H_0: homosced. errors

H_a: heteroscedasticity $\quad (\sigma_{\varepsilon_i}^2 \sim I_i)$

$F = \frac{RSS_2 / 30-2}{RSS_1 / 20-2} = 7,84 \qquad F_{crit}^{1\%}(28, 18) = 2,98$

(b) $n \cdot R^2_{aux} = 82 \cdot 0,57 \sim \chi^2_q$ q - # reg. in aux model.

$= 46,7$

$\hat{\varepsilon}_i^2 \mid I, I^2, u, R, I^4, u^2, \cancel{R^2}, C_2^4$

$\| \quad 6$

$q = 12 \Rightarrow \chi^2_{1\%,12} = 23,2$

H_0 is rejected