Learning Goals for Math 151 – Calculus I for the Mathematical and Physical Sciences

Chapter 1: Functions

§1.1 Functions and Their Graphs

1.1.1 Determine a function's domain given an explicit formula for the function or a graph of the function.

1.1.2 Determine the range of a function.

1.1.3 Use inequalities, absolute values, and/or interval notation to describe a subset, e.g., the domain of a function, of the real line. Simplify such expressions and translate between different possible expressions for the same subset.

1.1.4 Determine whether a graph or an equation represents a function of the form $y = f(x)$.

1.1.5 Graph basic functions. Use the graph of a function to find its roots and the intervals on which it is increasing or decreasing.

1.1.6 Graph piecewise-defined functions. Be able to use piecewise-defined functions to solve equations.

1.1.7. Build functions that model physical phenomena such as distance, area, and volume, or are described by a graph, and be able to solve applications involving these functions.

1.1.8 Use the definitions or the function's graph to determine whether a function is even, odd, or neither.

§1.2 Combining Functions; Shifting and Scaling Graphs

1.2.1 Find the domains and ranges of the sum, difference, product, quotient, or composition of two or more functions.

1.2.2 Compute and evaluate compositions of functions. Identify a given function as a composition.

1.2.3 Find equations for graphs that have been shifted, scaled, or reflected.

1.2.4 Understand how transformations of a function affect its graph.

1.2.5 Analyze even and odd functions.

Note: In §3.6 and beyond, it will also be necessary to recognize when a function is a composition of simpler functions.

§1.3 Trigonometric Functions

1.3.1 Find the length of an arc subtended by a central angle in a circle.

1.3.2 Convert from radians to degrees and degrees to radians.

1.3.3 Calculate the value of a trigonometric function using a triangle, other trigonometric values, or at special angles (multiples of $\pi/6$ and $\pi/4$).

1.3.4 Graph trigonometric functions.

1.3.5 Use trigonometric formulas and basic identities to prove other identities, solve equations, and compute trigonometric values.

§1.4 Graphing with Software

1.4.1 Investigate the behavior of a function by graphing the function in an appropriate window.

1.4.2 Determine the most appropriate window to view a given function's graph.

§1.5 Exponential Functions

1.5.1 Graph exponential functions.

1.5.2 Apply exponential rules to simplify expressions or solve equations involving exponents.

1.5.3 Use the graph of an exponential function to estimate solutions to equations involving exponents.

1.5.4 Find the domain and range of a function involving exponentials.

1.5.5 Build models using exponentials and be able to solve applications involving these models.

§1.6 Inverse Functions and Logarithms

1.6.1 Decide if a function is one-to-one or invertible. Find a domain on which a given function is one-to-one or invertible.

1.6.2 Use information about a function to investigate the behavior of its inverse. Use the graph of a function to graph its inverse.

1.6.3 Compute the inverse of a simple function.

1.6.4 Compute values of inverse trigonometric functions.

1.6.5 Simplify compositions of trigonometric and inverse trigonometric functions by using trigonometric identities or an appropriate right triangle.

1.6.6 Apply logarithmic rules to simplify expressions or solve equations involving logarithms or exponentials.

1.6.7 Graph functions involving logarithm expressions.

Note: *Math 151 does not cover hyperbolic trigonometric functions or their inverses.*

Chapter 2: Limits and Continuity

§2.1 Rates of Change and Tangent Lines to Curves

2.1.1 Compute the average rate of change of a function over an interval.

2.1.2 Use the secant lines of a curve to find the line tangent to that curve at a point.

2.1.3 Solve applications involving rates of change.

§2.2 Limit of a Function and Limit Laws

2.2.1 Find the limits of a function and evaluate the function using a graph of the function.

2.2.2 Understand and be able to explain concepts related to the existence of limits.

2.2.3 Compute the limits of polynomial and rational functions. In particular, find the limits of difference quotients in preparation for computing derivatives.

2.2.4 Compute basic limits of trigonometric functions.

2.2.5 Find limits using the rules of limits.

2.2.6 Evaluate the limit of average rates of change.

2.2.7 Solve applications of the Sandwich Theorem.

2.2.8 Estimate limits using tables.

§2.4 One-Sided Limits

2.4.1 Use graphs of functions to find one-sided limits.

2.4.2 Compute one-sided limits algebraically.

2.4.3 Find the limits of trigonometric functions using the limit of $\sin\theta\,/\theta$.

2.4.4 Understand the relationship between one-sided and two-sided limits. Answer conceptual question about one-sided limits.

§2.5 Continuity

2.5.1 Determine where functions are continuous using the function's graph and if they can be made continuous by changing their values at certain points.
2.5.2 Determine where functions are continuous (algebraically) by applying the continuity test.
2.5.3 Find values that make a function continuous.
2.5.4 Apply the intermediate value theorem to find solutions of equations.
2.5.5 Analyze concepts involving continuous functions.

§2.6 Limits Involving Infinity; Asymptotes of Graphs

2.6.1 Find limits using a graph of the function.
2.6.2 Compute limits of functions as x approaches infinity or negative infinity using appropriate algebraic manipulation.

2.6.3 Find infinite limits.
2.6.4 Graph rational functions and identify any asymptotes of the function.
2.6.5 Use limits to find domains, ranges, and asymptotes of functions.
2.6.6 Find and graph functions with given conditions.
2.6.7 Compute the limits of differences of functions at infinity.
2.6.8 Find and graph oblique asymptotes of rational functions.
2.6.9 Answer conceptual questions involving limits at infinity, infinite limits, and asymptotes.

Chapter 3: Derivatives

§3.1 Tangent Lines and the Derivative at a Point

3.1.1 Estimate a derivative by visual inspection of a graph.
3.1.2 Compute a derivative as a limit of a difference quotient. Recognize a function as non-differentiable at a point when this limit does not exist.
3.1.3 Interpret the derivative as the slope of a graph or of a tangent line. Find the equation of a line tangent to a graph using the derivative at a point.

3.1.4 Interpret the derivative as an instantaneous rate of change.

§3.2 The Derivative as a Function

3.2.1 Understand Leibniz notation for derivatives.
3.2.2 Compute the derivative of a function using limits.
3.2.3 Understand how the derivative of a function f relates to the graph of f. Recognize a function as non-differentiable at a point based on the behavior of its graph.

3.2.4 Compute the one-sided derivative of a function at a point using one-sided limits. Use one-sided limits and continuity properties to determine whether a function is differentiable at a certain point.
3.2.5 Answer conceptual questions involving differentiation.

§3.3 Differentiation Rules

3.3.1 Compute derivatives of simple functions by using the rules of differentiation. Decide which rules to use and in what order to use them.
3.3.2 Compute higher-order derivatives.

3.3.3 Use the derivative to answer conceptual questions about derivatives, tangent lines, or instantaneous rates of change.

§3.4 The Derivative as a Rate of Change

3.4.1 Interpret the derivative as an instantaneous rate of change. Solve applications involving rate of change.

3.4.2 Solve linear motion problems using derivatives. Find an object's displacement, velocity, speed, and acceleration.

3.4.3 Analyze the motion of an object given a graph of its position, velocity, or acceleration.

§3.5 Derivatives of Trigonometric Functions

3.5.1 Compute derivatives of trigonometric functions using the formulas for the six basic trigonometric functions.

3.5.2 Evaluate limits involving trigonometric functions. When appropriate, recognize a limit as a derivative of a trigonometric function and use this information to evaluate.

3.5.3 Solve applications involving trigonometric functions and their derivatives.

3.5.4 Answer conceptual questions involving trigonometric functions.

§3.6 The Chain Rule

3.6.1 Identify “inside” and “outside” functions in a composition and apply the chain rule, along with any other appropriate rules of differentiation.

3.6.2 Answer conceptual questions involving the chain rule.

§3.7 Implicit Differentiation

3.7.1 Compute derivatives for implicitly defined functions by applying the Chain Rule.

3.7.2 Compute the slope of the line tangent or normal to the graph of an implicitly defined curve at a given point. Find points at which the curve has a given slope.

3.7.3 Find higher order derivatives using implicit differentiation.

3.7.4 Answer conceptual questions using implicit differentiation.

§3.8 Derivatives of Inverse Functions and Logarithms

3.8.1 Understand how the derivatives of a function and its inverse behave graphically. Use Theorem 3 to compute the derivative of an inverse function, or to compute the derivative of the inverse function at a given point $x = f(a)$.

3.8.2 Know the formulas for the derivatives of logarithmic and exponential functions of any base. Use these formulas to compute derivatives of related functions.

3.8.3 Use logarithmic differentiation to compute derivatives. Recognize when this technique is helpful, and when it is necessary.

3.8.4 Answer conceptual questions involving inverse functions and logarithms.

Note: The discussion of *e* as a limit will be continued in §4.4. See Exercises 3.8.102 and 4.4.84-85.

§3.9 Inverse Trigonometric Functions

3.9.1 Compute angles in a right triangle using inverse trigonometric functions.

3.9.2 Use special values or information about the graphs of the six basic trigonometric functions to compute special values or limits of their inverses.

3.9.3 Use trigonometric identities and the methods of §3.8 to find formulas for the derivatives of the six basic trigonometric functions.

3.9.4 Know the derivatives of the six basic trigonometric functions and use them to compute related derivatives.

3.9.5 Answer conceptual questions involving the inverse trigonometric functions and their derivatives.

Note: *Math 151 does not cover hyperbolic trigonometric functions or their inverses.*

§3.10 Related Rates

3.10.1 Solve related rates problems in which variables are related by a given equation.

3.10.2 Solve related rates problems using familiar geometric or trigonometric identities.

§3.11 Linearization and Differentials

3.11.1 Estimate the value of a function using an appropriate linearization.

3.11.2 Compute differentials. Use them to estimate the error that propagates through a computation.

Chapter 4: Applications of Derivatives

§4.1 Extreme Values of Functions on Closed Intervals

4.1.1 Find extreme values and where they occur using the graph of the function.
4.1.2 Find absolute extrema and where they occur on finite closed intervals.
4.1.3 Find critical points, local and absolute extrema, and domain endpoints for functions.
4.1.4 Understand concepts related to extreme values of functions.
4.1.5 Solve applications involving extreme values.

§4.2 The Mean Value Theorem

4.2.1 Find the values that satisfy the conclusion of the Mean Value Theorem.

4.2.2 Identify functions that satisfy the hypotheses of the Mean Value Theorem.

4.2.3 Use Rolle's theorem to investigate the number of zeroes of a function on given intervals.
4.2.4 Find functions and values of functions given the derivatives.

4.2.5 Find the position of an object given its velocity or acceleration with appropriate initial value(s).
4.2.6 Solve applications involving the Mean Value Theorem.

4.2.7 Answer conceptual questions involving the Mean Value Theorem.

§4.3 Monotonic Functions and the First Derivative Test

4.3.1 Find intervals where the function is increasing or decreasing, and extrema given the function, its derivative, or the graph of either.

4.3.2 Find local and absolute extrema in given domains.

4.3.3 Discuss extreme-value behavior of functions by analyzing their first derivatives.

4.3.4 Create a function that has given extreme values.

4.3.5 Show that given functions have inverses over their domains.

§4.4 Concavity and Curve Sketching

4.4.1 Identify inflection points, local extrema, intervals of increasing/decreasing, and concavity from graphs.

4.4.2 Graph functions and find any extrema and inflection points.

4.4.3 Graph $f(x)$ given information about the first and second derivatives, such as functions, graphs, or sign behavior.
4.4.4 Solve applications involving graphs of functions and their derivatives.
4.4.5 Answer conceptual questions involving curve sketching using information about the function or its derivatives.

§4.5 Indeterminate Forms and L'Hôpital's Rule

4.5.1 Decide whether L'Hôpital's Rule can be used to evaluate a limit. Apply the rule when appropriate and decide how many times to apply it.
4.5.2 Identify and classify indeterminate forms. Use limit laws and algebraic manipulations, including logarithms, to make an indeterminate limit suitable for evaluation by L'Hôpital's Rule.
4.5.3 Answer conceptual questions involving L'Hôpital's Rule.

§4.6 Applied Optimization

4.6.1 Given a word problem about optimization, determine appropriate variables, a function to optimize, and the interval on which it should be optimized. Use these to solve the word problem.
4.6.2 Decide whether a function has a max or min on an open, unbounded interval. If so, find it. Justify your conclusions.

§4.8 Antiderivatives

4.8.1 Compute general antiderivatives by inverting derivative rules. Verify that an antiderivative is correct by differentiating.

4.8.2 Use antiderivatives to solve initial value problems.

4.8.3 Construct an initial value problem to model given information.

4.8.4 Answer conceptual questions involving antiderivatives.

Chapter 5: Integrals

§5.1 Area and Estimating with Finite Sums

5.1.1 Approximate areas, distances, and averages by using upper sums, lower sums, and midpoint approximations.

§5.2 Sigma Notation and Limits of Finite Sums

5.2.1 Understand how to use the sigma notation to describe and evaluate sums.

5.2.2 Understand the notation for and be able to compute Riemann sums.

5.2.3 Sketch rectangles associated with Riemann sums.

5.2.4 Compute the limit of a Riemann sum.

5.2.5 Compute the area under the graph of a nonnegative function using a Riemann sum with a regular partition and a given set of sample points.

§5.3 The Definite Integral

5.3.1 Express limits of Riemann sums as definite integrals.

5.3.2 Evaluate definite integrals using either a limit of a Riemann sum, properties of definite integrals, or known area formulas.

5.3.3 Find average values of functions.

5.3.4 Use the properties of the definite integral to analyze integrals and prove relationships relating them.

§5.4 The Fundamental Theorem of Calculus

5.4.1 Compute definite integrals using the Fundamental Theorem of Calculus, Part 2 (FTC2).
5.4.2 Find derivatives of area functions (i.e., accumulation functions) using the Fundamental Theorem of Calculus, Part 1 (FTC1)
5.4.3 Find the area of a region between two graphs of functions.
5.4.4 Use the FTC to solve an initial value problem.
5.4.5 Use the FTC to solve for unknowns and analyze functions.

§5.5 Indefinite Integrals and the Substitution Method

5.5.1 Compute indefinite integrals using substitution.
5.5.2 Solve initial value problems.
5.5.3 Solve applications involving indefinite integrals and the substitution method.
5.5.4 Answer conceptual questions involving the substitution method.

§5.6 Definite Integral Substitutions and the Area Between Curves

5.6.1 Compute definite integrals using substitution.
5.6.2 Find the area of a region in the plane using integration with respect to x or y.
5.6.3 Apply properties of integrals to even or odd functions and function shifts.
5.6.4 Use substitutions to verify equations.
5.6.5 Answer conceptual questions involving the substitution method for definite integrals or areas.
5.6.6 Solve applications involving areas.