

# Learning Goals for Math 152 – Calculus II for the Mathematical and Physical Sciences

## Chapter 5: Integrals

### §5.3 The Definite Integral

5.3.1 Use even/odd symmetry to integrate functions on the interval  $[-a,a]$

5.3.2 Use the Mean Value Theorem for Integrals to find the average value of a function over an interval

### §5.5 Indefinite Integrals and the Substitution Method

5.5.1 Use substitution to simplify an indefinite integral of a composite function

5.5.2 Use substitution to evaluate an indefinite/a definite integral with mixed trigonometric functions

5.5.3. Use substitution to find an indefinite/a definite integral of ratio of functions

5.5.4 Use substitution to find an indefinite/a definite integral involving composite exponential functions

5.5.5 Use substitution to evaluate an indefinite/a definite integral with  $\ln(x)$  and  $1/x$

5.5.6 Use substitution to evaluate an indefinite/a definite integral with radicals and exponents

5.5.7 Use substitution to evaluate an indefinite/a definite integral with an inverse trigonometric function

5.5.8 Use substitution to evaluate an indefinite/a definite integral with a composite function with a polynomial

### §5.6 Definite Integral Substitutions and the Area Between Curves

5.6.1 Draw the given curves/lines and indicate the specific region in a given problem

5.6.2 Find the area of a region bounded between curves/lines on a given interval

5.6.3 Find the area of a region bounded by two functions that cross twice

5.6.4 Find the area of a region bounded by two functions that cross more than twice

5.6.5 Solve area problems where on some interval  $f > g$  and on other interval,  $g > f$ .

5.6.6 Solve area problem with integration with respect to  $y$ .

## Chapter 6: Applications of Definite Integrals

### §6.1 Volumes Using Cross-Sections

6.1.1 Set and evaluate an integral for a non-revolution volume with a given base and cross sections

6.1.2 Set and evaluate an  $x$  or a  $y$  integral for a volume of revolution about the  $x$  or the  $y$  axis using the disk method

6.1.3	Set and evaluate an $x$ or a $y$ integral for a volume of revolution about the $x$ or the $y$ axis using the washer method
6.1.4	Set and evaluate an integral for a volume of revolution of a given region about the line $x = \text{nonzero \#}$ or $y = \text{nonzero \#}$ using the disk or the washer method
6.1.5	Set and evaluate a symbolic integral for a volume of revolution
<b>§6.2 Volumes Using Cylindrical Shells</b>	
6.2.1	Set and evaluate an $x$ or a $y$ integral for a volume of revolution of a region bounded by a unique function on an interval about the $x$ or the $y$ axis using the method of cylindrical shells
6.2.2	Set and evaluate an $x$ or a $y$ integral for a volume of revolution of a region bounded by functions that cross about the $x$ or the $y$ axis using the method of cylindrical shells
6.2.3	Set and evaluate an integral for a volume of revolution of a given region about the line $x = \text{nonzero \#}$ or $y = \text{nonzero \#}$ using the method of cylindrical shells
6.2.4	Set and evaluate integrals for the same volume of revolution using the disk/washer and the cylindrical shells methods
6.2.5	Set and evaluate an $x$ integral and a $y$ -integral for volume of the same solid
6.2.6	Set and evaluate the volume of a solid that is described by words and no functions are given
<b>§6.3 Arc Length</b>	
6.3.1	Set and evaluate an $x$ or a $y$ integral for the length of a given curve
6.3.2	Find a curve with a given length integral or value
<b>§6.4 Areas of Surfaces of Revolution</b>	
6.4.1	Set and evaluate an $x$ or a $y$ integral for a surface of revolution about the $x$ or the $y$ -axis

## Chapter 8: Techniques of Integration

<b>§8.2 Integration by Parts</b>	
8.2.1	Evaluate an indefinite or definite integral using integration by parts once
8.2.2	Evaluate an indefinite or definite integral by first using substitution and then integration by parts
8.2.3	Evaluate an indefinite or definite integral using twice integration by parts
8.2.4	Find reduction formula using integration by parts
8.2.5	Evaluate an indefinite or definite integral with inverse trigonometric function by integration by parts
8.2.6	Recognize when integration by parts is not needed
<b>§8.3 Trigonometric Integrals</b>	
8.3.1	Evaluate an indefinite or definite integral of product of trigonometric functions
8.3.2	Evaluate an indefinite or definite integral of a unique trigonometric function to a power

	of a constant, includes $\sec^3$
	8.3.3 Evaluate an indefinite or definite integral of quotient of trigonometric functions
	8.3.4 Find reduction formula for integrals containing trigonometric functions to a power of $n$
<b>§8.4 Trigonometric Substitutions</b>	
	8.4.1 Evaluate an indefinite and a definite integral containing radical using trigonometric substitution
	8.4.2 Evaluate an indefinite and a definite integral containing radical by first completing the square and then trigonometric substitution
	8.4.3 Evaluate an indefinite and a definite integral not containing radicals using trigonometric substitution
	8.4.4 Evaluate an indefinite integral by first substitution and then trigonometric substitution
<b>§8.8 Improper Integrals</b>	
	8.8.1 Determine whether an improper integral with at least one infinite limit is convergent or divergent by evaluating
	8.8.2 Determine whether an improper integral with unbounded integrand is convergent or divergent by evaluating
	8.8.3 Determine whether an improper integral with at least one infinite limit is convergent or divergent using the comparison test and the $p$ -theorem over $[a>0, \infty)$
	8.8.4 Determine whether an integral with unbounded integrand is convergent or divergent by using the comparison test and the $p$ -theorem over $[0, a>0]$

## Chapter 10: Infinite Sequences and Series

<b>§10.1 Sequences</b>	
	10.1.1 Find the few first terms of a sequence whose terms is defined directly as a function of $n$ or recursively
	10.1.2 Find the general term of a sequence given its first few terms
	10.1.3 Determine whether a sequence converges or diverges by directly evaluating the limit of its term
	10.1.4 Determine whether a sequence converges or diverges by identifying its term with a function and then evaluating the limit
	10.1.5 Determine whether a sequence diverges or converges and find the limit if converges using the squeeze theorem for sequences
	10.1.6 Using the Bounded Monotonic Sequences theorem to find the limit of a recursive sequence
<b>§10.2 Infinite Series</b>	
	10.2.1 Find the first terms of a series
	10.2.2 Determine whether a series converges or diverges when the term is function of a term of another series
	10.2.3 Determine whether a series converges or diverges using the sequence of partial sums

10.2.4 Evaluate the sum of a telescopic series
10.2.5 Determine if a geometric series converges and if so find its sum
10.2.6 Express repeating decimals as fractions using geometric series
10.2.7 Show that a series diverges by the Term Divergence Theorem
10.2.8 Determine the value of $x$ for which a geometric series converges (preview of power series)
<b>§10.3 The Integral Test</b>
10.3.1 Determine whether a series with all positive terms diverges or converges using the integral test
<b>§10.4 Comparison Tests</b>
10.4.1 Determine whether a series diverges or converges using either the Direct or the Limit Comparison Tests with a $p$ -series
10.4.2 Determine whether a series diverges or converges using either the Direct or the Limit Comparison Tests with a geometric series
<b>§10.5 Absolute Convergence: The Ratio and Root Tests</b>
10.5.1 Determine whether a series diverges or absolutely converges using the ratio/root tests.
<b>§10.6 Alternating Series and Conditional Convergence</b>
10.6.1 Determine whether a series diverges, conditionally converges or absolutely converges
10.6.2 Estimate the remainder of an alternating series
<b>§10.7 Power Series</b>
10.7.1 Find the interval and radius of convergence for a power series
10.7.2 Find the interval of convergence and using Theorem 20
10.7.3 Find the power series representation of a rational function using the power series representation of $1/(1+x)$
10.7.4 Find the power series representation of a new function by using the power series representation of other functions by either term by term differentiation or integration
10.7.5 Using some information on convergence at some values to find the convergence on other values.
<b>§10.8 Taylor and Maclaurin Series</b>
10.8.1 Find the $N$ th degree Taylor polynomial for a function about a given center for a finite $N$
10.8.2 Find the Maclaurin series for a function
10.8.3 Find the Taylor series for a function about a given center
10.8.4 Find the first $N$ terms of or the whole Maclaurin series for a function using either multiplication of another Maclaurin series by $x^n$ or by multiplying or dividing two other Maclaurin series
<b>§10.9 Convergence of Taylor Series</b>
10.9.1 Find the Maclaurin series for a function by substituting for $x$ and other simple algebraic manipulation of other Maclaurin series
10.9.2 Find the Maclaurin series for a function using a combination of two Maclaurin series

	10.9.3 Estimate error using either the Remainder estimation Theorem or the alternating Series Estimation Theorem and find the number of term to attain a given error.
<b>§10.10 Applications of Taylor Series</b>	
	10.10.1 Find a limit of functions using their Maclaurin series
	10.10.2 Use series to estimate values of integrals and transcendental numbers

## Chapter 11: Parametric Equations and Polar Coordinates

<b>§11.1 Parameterizations of Plane Curves</b>	
	11.1.1 Express/match to a curve as $y=f(x)$ (with direction and initial and terminal points) by eliminating the parameter
	11.1.2 Draw a parametric curve
	11.1.3 Parameterize a curve
<b>§11.2 Calculus with Parametric Curves</b>	
	11.2.1 Find the tangent line to a parametric curve at a given parameter
	11.2.2 Find the first and the second derivative of a parametric equation
	11.2.3 Find the slope of an implicitly defined parametric curves at a given parameter.
	11.2.4 Find an area enclosed by parametric curves
	11.2.5 Find the arc length of a parametric curve
	11.2.6 Find the surface area of a surface that is generated by revolving a parametric curve about either the x or the y axis
<b>§11.3 Polar Coordinates</b>	
	11.3.1 Find different labels to the same point.
	11.3.2 Convert polar coordinates to rectangular coordinates and vice versa
	11.3.3 Sketch a polar curve
	11.3.4 Convert polar curves to Cartesian curves and vice versa
<b>§11.4 Graphing Polar Coordinate Equations</b>	
	11.4.1 Identify symmetries and sketch a polar curve in the xy-plane.
	11.4.2 Find a slope and a tangent line to a polar curve at a given theta
<b>§11.5 Areas and Lengths in Polar Coordinates</b>	
	11.5.1 Find the area of a region bounded by either a polar curve or part of it
	11.5.2 Find the arc length of a polar curve

## Complex Number Packet

### Chapter 1: Arithmetic of Complex Numbers

<b>§Cplx1.1 Definition</b>	
	Cplx1.1.1 Find the real and the imaginary part of a complex number
	Cplx1.1.2 Express a complex number in rectangular form
<b>§Cplx1.2 Addition, Subtraction, and Scalar Multiplication</b>	
	Cplx1.2.1 Evaluate sum, difference and scalar multiplication of complex numbers

<b>§Cplx1.3 Multiplication of Complex Numbers</b>	
	Cplx1.3.1 Multiply two complex numbers
<b>§Cplx1.4 Complex Conjugate and Division</b>	
	Cplx1.4.1 Find the conjugate of a complex number and use the properties of the Complex Conjugate to evaluate expressions containing conjugates
	Cplx1.4.2 Divide two complex numbers
<b>Chapter 2: Geometry of Complex Numbers</b>	
<b>§Cplx2.1 Rectangular Coordinates and Points in the Plane</b>	
	Cplx2.1.1 Express a complex number in rectangular form and polar form and vice versa
<b>§Cplx2.2 Complex Numbers in Polar Coordinates</b>	
	Cplx2.2.1 Express a complex number in rectangular form and polar form and vice versa
<b>§Cplx2.3 The Exponential Notation - Euler's Formula</b>	
	Cplx2.3.1 Express a complex number in rectangular form and polar form and vice versa
	Cplx2.3.2 Use Euler's Formula to express a complex number in exponential form
<b>§Cplx2.4 The Product and the Quotient of Complex Numbers in Polar Coordinates - De Moivre's Formulas</b>	
	Cplx2.4.1 Find product and quotient of complex numbers
<b>§C2.5 Roots of Complex Numbers</b>	
	Cplx2.5.1 Find power and root of a complex number
	C2.5.2 Find the n complex (and real) roots of a polynomial of degree n
	Cplx2.5.3 Find a polynomial with a given degree and a given conjugate zeros.